Mechanical Design Engineering Handbook

Peter R.N. Childs



ELSEVIER

AMSTERDAM • BOSTON • HEIDELBERG • LONDON • NEW YORK • OXFORD
PARIS • SAN DIEGO • SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO

Butterworth-Heinemann is an imprint of Elsevier



Butterworth-Heinemann is an imprint of Elsevier
The Boulevard, Langford Lane, Kidlington, Oxford, OX5 1GB, UK
225 Wyman Street, Waltham, MA 02451, USA

First published 2014

Copyright © 2014 Elsevier Ltd. All rights reserved.

No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or any information storage and retrieval system, without permission in writing from the publisher. Details on how to seek permission, further information about the Publisher's permissions policies and our arrangement with organizations such as the Copyright Clearance Center and the Copyright Licensing Agency, can be found at our website: www.elsevier.com/permissions

This book and the individual contributions contained in it are protected under copyright by the Publisher (other than as may be noted herein).

Notice
Knowledge and best practice in this field are constantly changing. As new research and experience broaden our understanding, changes in research methods, professional practices, or medical treatment may become necessary.

Practitioners and researchers must always rely on their own experience and knowledge in evaluating and using any information, methods, compounds, or experiments described herein. In using such information or methods they should be mindful of their own safety and the safety of others, including parties for whom they have a professional responsibility.

To the fullest extent of the law, neither the Publisher nor the authors, contributors, or editors, assume any liability for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions, or ideas contained in the material herein.

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication Data
A catalog record for this book is available from the Library of Congress

ISBN: 978-0-08-097759-1

For information on all Butterworth-Heinemann publications visit our website at www.store.elsevier.com

Printed and bound in the United Kingdom

14 15 16 17 10 9 8 7 6 5 4 3 2 1



Preface

The prior knowledge base in design and engineering, as with nearly all domains, is extensive. We have been designing and producing a wide range of sophisticated machines and systems for centuries. The detailed design of many machine elements and types of machinery has been codified and significant expertise is readily available. Just knowing that someone else has successfully produced a product or system can act as inspiration or provide direction for a similar product or the design of a subsystem that uses associated technology, or needs to have comparable functions.

Many attempts have been made at defining design. The word can be used as a verb or noun, describing the process and the outcome of design, respectively. Referring to design as a process, it can be considered to include all the activities of market assessment and user requirements, specification, concept generation and idea development, embodiment of details, risk mitigation, consideration of manufacture and production, and implementation. Referring to design as a noun, the term is commonly used to describe an artifact such as a vehicle, item of fashion or other product, with the associated features and merits. This may include description or commentary on aesthetic, ergonomic, and technical features. In this book, an inclusive approach to design is used with consideration of a range of functionalities ranging from technical, aesthetic, social, economic, and latent. Design is considered to be the process of conceiving, developing, and realizing products, artifacts, processes, systems, services, and experiences with the aim of fulfilling identified or perceived needs or desires typically working within defined or negotiated constraints.

Engineering involves significant overlap with design and, indeed, it is often difficult to make a clear distinction. A common distinction is the use of quantitative analysis in engineering to aid and inform the development, simulation, testing, and refinement of a system or product. Engineering can thus be considered to be the application of scientific and mathematic principles in combination with professional and domain knowledge, in order to design, develop, and deliver artifacts, products, and systems to realize a societal, commercial, or organization requirement or opportunity. Mechanical engineering refers to the use of engineering processes to applications of a mechanical nature, typically involving moving components or energy processes. Mechanical engineering heavily relies on the engineering sciences of thermodynamics and mechanics, often integrated through design, in the

development and refinement of a solution for a specific requirement. The title for this book comes from the blend of mechanical and design engineering, the latter with its heritage in the on-going collaborations between Imperial College London and the Royal College of Art through the Innovation Design Engineering double masters, of which I have the honor of being joint course director.

The detailed design of a wide range of machine elements that are commonly used in mechanisms and machines has been the focus of attention of many companies and research groups around the world. Examples of such machine elements include bearings, shafts, gears, belts, chains, clutches, brakes, seals, and springs. Many of these are available as stock items or standardized components that are produced by a range of companies, enabling the engineer or designer to focus attention on selection of appropriate stock items and the integration of these within the application of interest, rather than having to repeat the development of previous knowledge. For other machine elements such as shafts, some clutches and brakes, and seals, their design can take advantage of prior knowledge and design guidelines.

Industry is typically concerned with the development of return for investors, or owners, within their specific product or service sector. A wide range of approaches is employed to manage the development and delivery process for new products and systems, or the manufacture and production, and delivery, of artifacts and services. Many models and approaches for the design process are employed in industry and area often bespoke to the particular organization concerned. Studies across a range of industries have identified some commonality in approach and a number of models have been proposed as representative of a standardized approach to design, and the management of design. A selection of these is presented in Chapter 1, providing an overview of typical approaches to the design process and its management.

The objective or quantitative definition of the attributes and requirements for a product, artifact, or system, represents a common activity in design and engineering. This enables an assessment of what the design is required to deliver and is often used to monitor the success of a design proposal and in contractual agreements. Two common approaches to specification are introduced in Chapter 2, including the use of product design specification pro-forma tables and quality functional deployment.

Whether the engineering or design activity involves integration of existing machine elements within a mechanism, or the design of original features and components, in the design of a brand new product or system, ideas are important. The development of ideas and the use of a range of creativity tools are introduced in Chapter 3.

An overview of the variety of machine elements is introduced in Chapter 4. The design and selection of plain surface bearings and rolling element bearings are considered in

Chapters 5 and 6. Chapter 5 includes an introduction to the wide range of bearing types and presents a selection method for boundary lubricated bearings, as well as a modeling approach for full-film hydrodynamic bearings that can be used to assess the suitability of an initial design proposal. The selection of rolling element bearings based on the classical bearing life equation is introduced in Chapter 6 along with a brief consideration of bearing arrangements. Shafts are considered in Chapter 7 with a particular focus on sizing a shaft to limit deflection to adequate limits and assessing dynamic characteristics.

Gears, critical for transforming speed and torque in transmission systems, are introduced in Chapters 8–11. Chapter 8 provides an overview of a wide range of types of gears and also presents an initial selection procedure for spur gears based on the Lewis formula for bending. The introduction of more comprehensive analysis methods using the American Gear Manufacturer Association (AGMA) equations for bending and contact stress for spur and helical gears, bevel, and worm and wheel gears, is presented in Chapters 9, 10, and 11, respectively. The related transmission elements of belts and chains are introduced in Chapter 12, with a particular focus on the selection of belts and chains from stock suppliers using standard selection procedures. Clutches and brakes are introduced in Chapter 13. This chapter includes simple design procedures for single and multiple disk clutches, long- and short-shoe drum brakes, and calliper disk brakes.

Seals are used in a wide range of machinery to prevent or limit the flow of fluids between regions in a machine and to exclude dirt or contaminants. Chapter 14 provides an overview of the wide range of types of seals and also gives guidance for the sizing of O rings for static and dynamic applications using data from British Standards, as well as modeling methods for assessing the leakage of labyrinth and bush seals.

Springs are manufactured by a wide range of world-wide manufacturers and available as both stock items or bespoke designs. Chapter 15 introduces the design of a wide range of types of springs, including helical compression, helical extension, helical torsion, leaf, and Belleville springs.

A wide range of fastening technologies is available for permanent and semipermanent joining of components. A range of these including threaded fasteners and rivets is introduced in Chapter 16, along with a brief consideration of adhesives, welding, and snap fasteners. The selection of wire rope is briefly introduced in Chapter 17. Pneumatic and hydraulic components, the majority available as stock items, are introduced in Chapter 18.

Although a component, or process, may have a designated nominal dimension or performance measure, the actual dimension or performance will likely vary within limits of this value. Chapter 19 introduces the subject of tolerance with particular consideration of component tolerances for the assembly of cylindrical components, a brief consideration of geometric tolerances, and an introduction to statistical tolerance approaches.

The majority of material presented in this book represents a classic approach to the design and selection of machine elements. A cook-book recipe approach has been adopted for many of these, enabling the rapid development of a proposal for the machine element concerned and its evaluation. It is important to acknowledge the limitations of such an approach as the configuration, or selection, may be far from optimal. A typical approach to mitigate against nonoptimal selections is to explore additional selections with variation of a parameter, say a size above and below, or a slightly stronger and weaker material, to see if the outcome is favorable and then follow this lead. In addition, a keen eye on fundamental physics and engineering principles is necessary in the development of any solution or design proposal.

A considerable quantity of the material in this book has arisen from two editions of my earlier book, *Mechanical Design*. This new handbook incorporates extensions to the treatment of former chapters on bearings, gears, belts and chains, clutches and brakes, seals, springs, fasteners, and tolerance. Specifically for this handbook, new chapters on design, specification, ideation, bevel gears, worm and wheel gears, wire rope, and pneumatics and hydraulics have been developed.

The aims of this book are thus to present an overview of the design process and to introduce the technology and selection of a number of specific machine elements that are fundamental to a wide range of mechanical engineering design applications. I hope it is useful to you.

Peter R.N. Childs

– July 2013

Acknowledgments

Many colleagues, associates in industry and engineering institutions, and students have helped me throughout my career and in the forming of the material presented in this book. I am particularly indebted to my former colleagues at the University of Sussex, and my current colleagues at Imperial College London and the Royal College of Art, who have consistently commented on and enabled refinement of notes, designs, and methodologies. I have significantly benefited over the years from membership of both the Institution of Mechanical Engineers (IMechE) and the American Society of Mechanical Engineers, in particular through the IMechE's Automobile Southern Centre and the Southern Region, and also the International Gas Turbine Institute.

I would also like to acknowledge my indebtedness to the wide number of companies and organizations who have sponsored research or development contracts with me over the years, enabling the development of invaluable experience, including Rolls-Royce plc, Alstom, Snecma, DaimlerChrysler, MTU, Volvo, Johnson Matthey, Siemens, Industriales Turbinas Propulsores, Fiat Avio, Airbus, Ricardo Consulting Engineers, Ford, Rio Tinto, the EPSRC, Technology Strategy Board and the EU, as well and many small and medium enterprises. I have also been fortunate to be able to form a number of companies over the years, with a wide range of accompanying experiences and interactions that all help to provide a richer basis from which to develop the material presented here.

Students, and novice engineers and designers, have played a huge role in my professional career. I am indebted to them all for the diverse experiences, and challenges to material, examples, and notes, as well as endorsements.

Thank you to the various authors and publishers who have granted permission to reproduce or adapt images and material. In particular, I would like to acknowledge Audi, the Design Council, Chapman and Hall, J.C. Bamford, Wiley, Pearson, Fiskars, John Crane, SKF, Rolls-Royce plc, Cross and Morse, NASA, McGraw Hill, Grainger, the American Gear Manufacturers Association, British Standards, Fenner Drives, Renold Chain, IHS-ESDU, Associated Spring, Noble & Son Ltd, AISI, Wire Rope Technical Board, and Ford.

I would also like to thank Sol Lee and Ashley Hall for their parts in the first image in the book. Thank you to Dr Marco Aurisicchio for his contributions toward Chapter 2. The staff at Arnold, Butterworth Heinemann, and Elsevier have been professional and encouraging throughout the production of the two editions of *Mechanical Design* and this new handbook. Finally, thank you to my family, and in particular my wife Jo, for accommodating the hours of writing, drawing, and distraction associated with producing any book.

About the Author

Peter R.N. Childs is the Professorial Lead in Engineering Design at Imperial College London. His general interests include: fluid flow and heat transfer, particularly rotating flow and flow system design for gas turbine engine applications; creativity tools and innovation; mechanical design, machine elements, and detailed design; sustainable energy component, concept, and system design. Prior to his current post at Imperial, he was director of the Rolls-Royce supported University Technology Centre for Aero-Thermal Systems, director of InQbate, and professor at the University of Sussex.

He has contributed to over 140 refereed journal and conference papers since completion of his doctorate in 1991, and several books including monographs on rotating flow (Elsevier and ESDU (Engineering Sciences Data Unit)), labyrinth seals (ESDU) and temperature measurement (Elsevier and ESDU), and a text book on mechanical design (Butterworth Heinemann). He has been involved in research and development contracts for Rolls-Royce plc, Alstom, Snecma, DaimlerChrysler, MTU, Volvo, Johnson Matthey, Siemens, Industriales Turbinas Propulsores, Fiat Avio, Airbus, Ricardo Consulting Engineers, Ford, Rio Tinto, the EPSRC, TSB and the EU, as well as a number of SMEs (small and medium enterprises).

He has been the Chairman of the South Eastern Region of the Institution of Mechanical Engineers. He has won the American Society of Mechanical Engineers (ASME) – International Gas Turbine Institute John P. Davis award for exceptional contribution to the literature of gas turbine technology and Institution of Mechanical Engineers and American Society of Mechanical Engineers best paper awards. He is a Fellow of the American Society of Mechanical Engineers, a Fellow of the Institution of Mechanical Engineering, and a Fellow of the Royal Society for the Arts and Manufactures. He is a member of the European Turbomachinery Network, an active member of the ASME K14 Heat Transfer Committee, and has been a Vanguard Chair for the ASME Turbo Expo, as well as a regular session chair. His role at Imperial includes being the joint course director for the Innovation Design Engineering degree with the Royal College of Art, leading the Design Engineering Group, CTO of QBot Ltd, Design lead for the Manufacturing Futures Lab, and a member of the educational executive for the Climate Knowledge Innovation Centre. He is also the Creative Director for Iceni Labs.

CHAPTER 1

Design

Chapter Outline

1.1 Introduction 1
1.2 The Design Process 4
1.2.1 Case Study 8
1.3 Total Design 10
1.3.1 Market 10
1.3.2 Specification 10
1.3.3 Conceptual Design 12
1.3.4 Detailed Design 12
1.3.5 Manufacturing 12
1.3.6 Marketing/Sales 13
1.3.7 Total Design Information Flows and Activities 13
1.4 Systematic Design 15
1.5 Double Diamond 17
1.6 Conceive, Design, Implement, Operate (CDIO) 17
1.7 Design for Six Sigma 17
1.8 Design Optimization 18
1.9 Stage-Gate Process 19
1.10 The Technology Base 20
1.11 Conclusions 22
References 22
Further Reading 23
Nomenclature 24

Abstract

The aims of this book are to present an overview of the design process and to introduce the technology and selection of a number of specific machine elements that are fundamental to a wide range of mechanical engineering design applications. This chapter introduces the design process from an inventor's perspective, and double diamond, to more formal models such as "total design" and stage- or phase-gate reviews.

1.1 Introduction

The term "design" is popularly used to refer to an object's a esthetic appearance, with specific reference to its form or outward appearance as well as its function. For example, we

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00001-0

Copyright © 2014 Elsevier Ltd. All rights reserved.

often refer to designer clothes, design icons, and beautiful cars, examples of which are given in Figures 1.1 and 1.2. In these examples, the products fulfill a range of requirements with regard to visual impact, i.e. something that appeals to our visual perception, and technical function, both of which are important in defining so-called "good design."

The word "design" is used as both a noun and a verb, and it carries a wide range of context-sensitive meanings and associations. As stated by George Cox in the *Cox Review* (Cox, 2005), "Design is what links creativity and innovation. It shapes ideas to become practical and attractive propositions for users or customers. Design may be described as creativity deployed to a specific end." In essence design can be considered to be the process of conceiving, developing, and realizing products, artifacts, processes, systems, services, and experiences with the aim of fulfilling identified or perceived needs or desires typically working within defined or negotiated constraints. This process may draw upon and synthesize principles,



Figure 1.1
Piaggio's Vespa, which was launched in 1946. The Vespa was an early example of monocoque construction where the skin and frame are combined as a single construction to provide appropriate rigidity and mounting for the vehicle's components and riders.



Figure 1.2
The Audi TT, which was originally launched in 1998 and is a contender for the most attractive sports car of the twentieth century. *Figure courtesy of Audi.*

knowledge, methods skills, and tools from a broad spectrum of disciplines depending on the nature of the design initiative and activity.

The word design comes from the Latin "designare," which means to designate or mark out. Design can be taken to mean all the processes of conception, invention, visualization, calculation, refinement, and specification of details that determine the form of a product. Design generally begins with either a need or a requirement or, alternatively, an idea. It ends with a set of drawings or computer representations and other information that enables a product to be manufactured and utilized.

Design can be regarded as "the total activity necessary to provide a product or process to meet a market need." This definition comes from the SEED (Sharing Experience in Engineering Design) organization, which is now the DESIG (Design Education Special Interest Group of the Design Society) (see Pugh, 1990).

According to a Royal Academy of Engineering pamphlet, engineering can be defined as,

> *The discipline, art and profession of acquiring and applying scientific, mathematical, economic, social and practical knowledge to design and build structures, machines, devices, systems, materials and processes that safely realize solutions to the needs of society.*

This definition is not attributed to a single individual, and the Accreditation Board for Engineering and Technology (ABET, 2011), the Institution of Mechanical Engineers, and the National Academy of Engineering (2004) all have similar definitions for engineering where scientific and mathematic principles are applied to design, develop, and deliver artifacts, products, and systems to realize a societal, commercial, or organization requirement or opportunity.

The terms "engineering design" and "design engineering" are often used interchangeably. The inclusion of the word engineering in both suggests that they involve the application of scientific and mathematical knowledge and principles. It may be useful to think of "engineering design" in connection with "engineering science" as the strand of engineering

that is concerned with application, designing, manufacturing, and building. Design engineering suggests a process in which engineering (scientific and mathematical) approaches are applied in the realization of activities that began with a design concept or proposal. However, such distinctions remain subtle and subject to context.

1.2 The Design Process

Design processes abound and have been widely documented, with many design schools, design consultancies, and engineering corporations developing their own brand of approaches (e.g. see Clarkson and Eckert, 2005). Commonly cited methods include the educational approach CDIO (conceive, develop, implement, operate), total design, double diamond, six sigma, MDO (multiobjective design optimization), and gated reviews. Design processes can be broadly categorized as activity-based, involving generation, analysis, and evaluation, and stage-based, involving distinct phases of, for example, task clarification and conceptual design. It is also widely recognized that experienced practitioners approach design in a different manner to novice designers (e.g. see Ahmed et al., 2003).

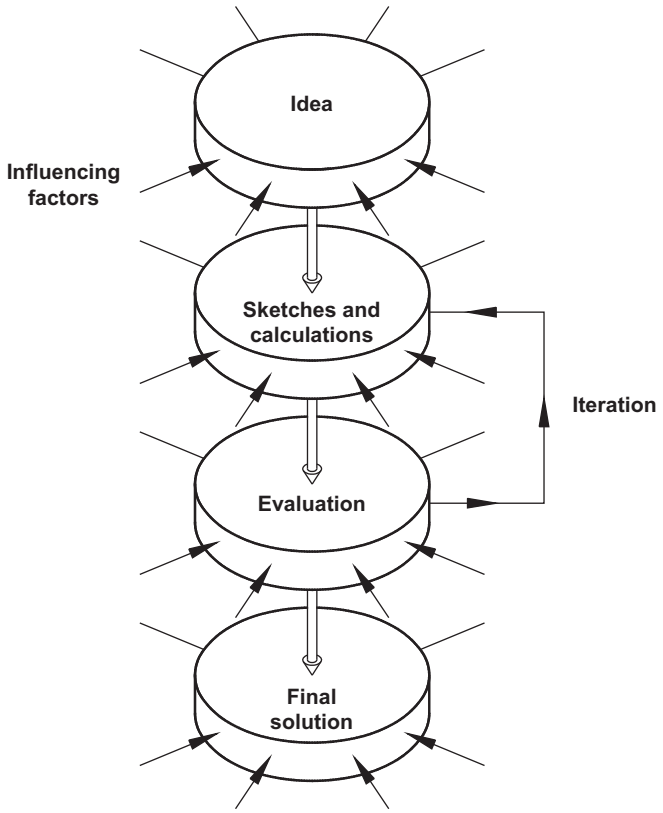


Figure 1.3
The traditional and familiar "inventor's" approach to design.

From your own experience, you probably know that design can consist of examining a design need and working on the problem by means of sketches, models, brainstorming, calculations as necessary, development of styling as appropriate, making sure the product fits together and can be manufactured, and calculation of the costs. The process of design can be represented schematically to levels of increasing formality and complexity. Figure 1.3 represents the traditional approach associated with lone inventors. This model comprises the generation of the "bright idea," drawings and calculations giving form or shape to the idea, judgment of the design, and reevaluation if necessary, resulting in the generation of the end product. The process of evaluation and reworking an idea is common in design and is represented in the model by the iteration arrow taking the design activity back a step so that the design can be improved. Figure 1.4 illustrates the possible results from this process for a helmet providing peripheral and reverse vision.

Figure 1.5 shows a more formal description of the design process that might be associated with engineers operating within a formal company management structure. The various terms used in Figure 1.5 are described in the following text.

Recognition of need	Often design begins when an individual or company recognizes a need, or identifies a potential market, for a product, device, or process. Alternatively "need" can be defined as the situation in which a company decides to reengineer one of its existing products (for example, producing a new car model). The statement of need is sometimes referred to as the brief or the market brief.
Definition of problem	This involves all the specifications of the product or process to be designed. For example, this could include inputs and outputs, characteristics, dimensions, and limitations on quantities.
Synthesis	This is the process of combining the ideas developed into a form or concept, which offers a potential solution to the design requirement. The term "synthesis" may be familiar from its use in chemistry, where it is used to describe the process of producing a compound by a series of reactions of other substances.
Analysis	This involves the application of engineering science, i.e. subjects explored extensively in traditional engineering courses such as statics and dynamics, mechanics of materials, fluid flow, and heat transfer. These engineering "tools" and techniques can be used to examine the design to give quantitative information such as whether it is strong enough or will operate at an acceptable temperature. Analysis and synthesis invariably go together. Synthesis means putting something together and analysis means resolving something into its constituent parts or taking it to pieces. Designers have to synthesize something before it can be analyzed. This is the famous chicken and the egg scenario! When a product is analyzed, some kind of deficiency or inadequacy may be identified that requires the synthesis of a new solution prior to reanalysis and repetition of the process until an adequate solution is obtained.
Optimization	This is the process of repetitively refining a set of often-conflicting criteria to achieve the best compromise.
Evaluation	This is the process of identifying whether the design satisfies the original requirements. It may involve assessment of the analysis, prototype testing, and market research.

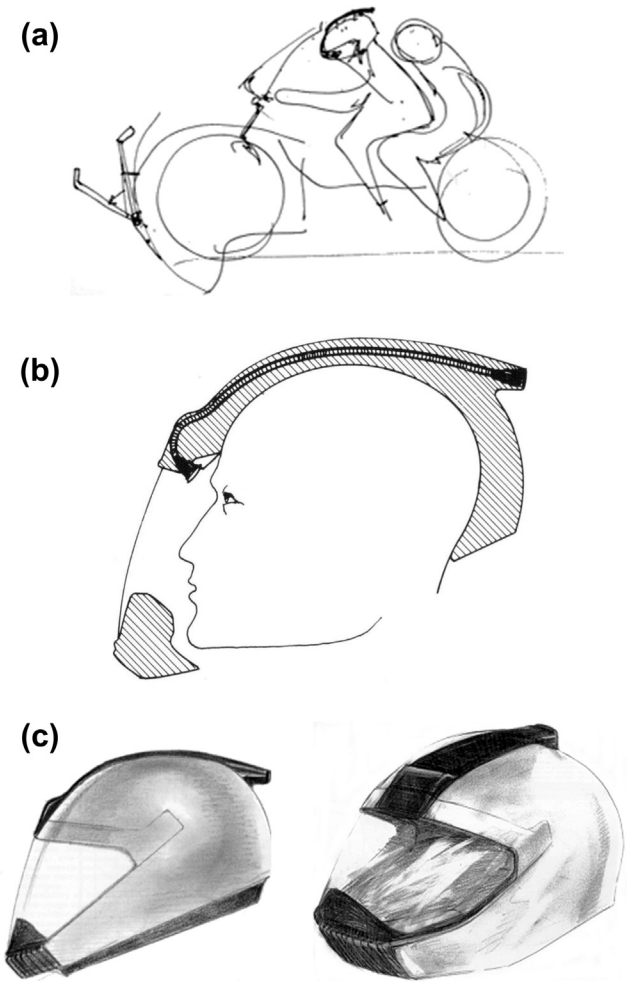


Figure 1.4
Panoramic helmet by Alberto Meda and Denis Santachiara. (a) The need: to be able to view behind you. (b) The idea: An optical link using fiber optics and lenses. (c) Practical sketches showing the concept. *Source: Manzini, 1989.*

Although Figures 1.3 and 1.5 at first glance show design occurring in a sequential fashion, with one task following another, the design process may actually occur in a step forward, step back fashion. For instance, you may propose a solution to the design need and then perform some calculations or judgments, which indicate that the proposal is inappropriate. A new solution will need to be put forward and further assessments made. This is known as the iterative process of design and forms an essential part of refining and improving the product proposal. The nonlinear nature of design is considered by Hall and Childs (2009).

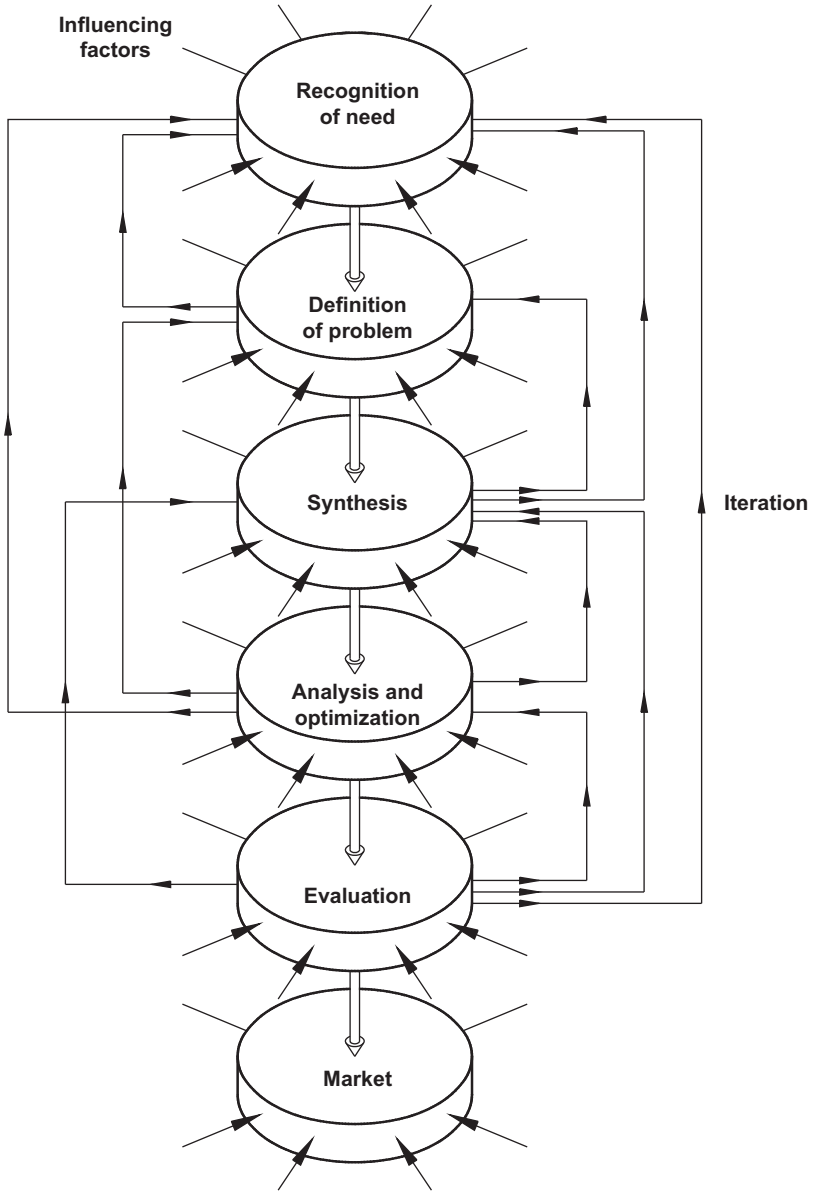


Figure 1.5
The design process illustrating some of the iterative steps associated with the process.

Note that the flow charts shown in Figures 1.3 and 1.5 do not represent a method of design but rather a description of what actually occurs within the process of design. The method of design used is often unique to the engineer or design team. Design methodology is not an exact science and there are indeed no guaranteed methods of design. Some designers work in a progressive fashion, while others work on several aspects simultaneously.

1.2.1 Case Study

The process identified in Figure 1.6 can be illustrated by example. Following some initial market assessments, the Board of a plant machinery company has decided to proceed with the design of a new product for transporting pallets around factories. The Board has in mind a forklift truck, but it does not wish to constrain the design team to this concept alone. The process of the design can be viewed in terms of the labels used in Figure 1.5.

Recognition of Need (or Market Brief)

The company has identified a potential market for a new pallet-moving device.

Definition of Problem

A full specification of the product desired by the company should be written. This allows the design team to identify whether their design proposals meet the original request. Here a long list of information needs to be developed and clarified before design can proceed. For example, for the pallet-moving device being explored here this would likely include aspects for consideration such as:

1. What sizes of pallet are to be moved?
2. What is the maximum mass on the pallet?
3. What is the maximum size of the load on the pallet?
4. What range of materials is to be moved, and are the materials packaged?
5. To what maximum height must the pallet be lifted?
6. On what terrain must the pallet-moving device operate?

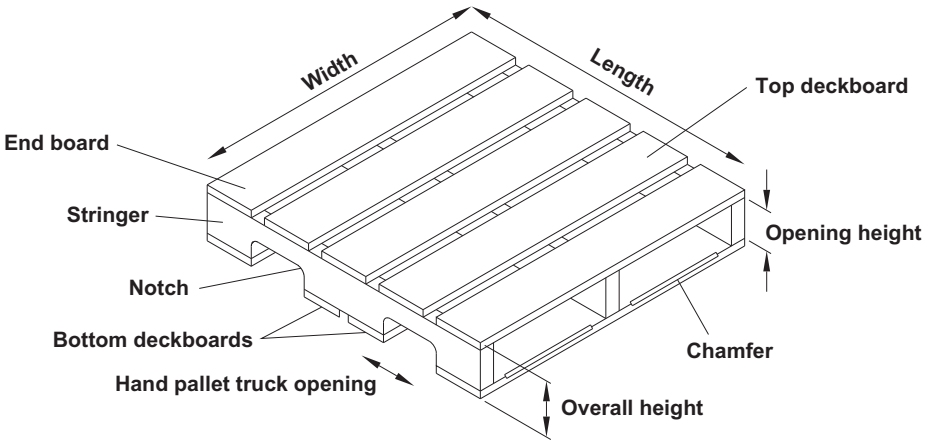


Figure 1.6
Pallet dimensions and terminology. *Source: See BS ISO 509, 99/712,554 DC, and 99/712,555 DC.*

7. What range is required for the pallet-moving device?
8. Is a particular energy source/fuel to be used? What lifetime is required?
9. Are there manufacturing constraints to be considered?
10. What is the target sales price?
11. How many units can the market sustain?
12. Is the device to be automatic or manned?
13. What legal constraints need to be considered?

This list is not exhaustive and would require further consideration. The next step is to quantify each of the criteria. For instance, the specification may yield that standard sized pallets, see Figure 1.6, are involved; the maximum load to be moved is 1000 kg; the maximum volume of the load is 2 m^3; the reach must be up to 3 m; use is on the factory floor and asphalt surfaces; the pallet-moving device must be capable of moving a single pallet 100 m and must be able to repeat this task at least 300 times before refueling if necessary; electricity, gas, or diesel fuel; seven-year lifetime; production in an East European country; target selling price 9000 Euros; 12,000 units per year; manned use; design to ISO (International Organization for Standardization) and target country national standards (e.g. see BS ISO 509, BS ISO 6780, BS EN ISO 445, BS EN 1726-1, BS EN 13,545, 99/705,213 DC, ISO, 18,334, 99/712,554 DC, BS 3726, BS 5639-1, and BS ISO 2330).

Synthesis

This is often identified as the formative and creative stage of design. Some initial ideas must be proposed or generated in order for them to be assessed and improved. Concepts can be generated by imagination, experience, or by the use of design techniques such as morphological charts. Some evaluation should be done at this stage to reduce the number of concepts requiring further work. Various techniques are available for this, including merit and adequacy assessments.

Analysis

Once a concept has been proposed, it can then be analyzed to determine whether constituent components can meet the demands placed on them in terms of performance, manufacture, cost, and any other specified criteria. Alternatively, analysis techniques can be used to determine what size the components need to be to meet the required functions.

Optimization

Inevitably there are conflicts between requirements. In the case of the forklift truck, size, maneuverability, cost, esthetic appeal, ease of use, stability, and speed are not necessarily all in accordance with each other. Cost minimization may call for compromises on material

usage and manufacturing methods. These considerations form part of the optimization of the product producing the best or most acceptable compromise between the desired criteria. Optimization is considered further in Section 1.8.

Evaluation

Once a concept has been proposed and selected and the details of component sizes, materials, manufacture, costs, and performance worked out, it is then necessary to evaluate it. Does the proposed design fulfill the specification? If it appears to, then further evaluation by potential customers and use of prototype demonstrators may be appropriate to confirm the functionality of the design, judge customer reaction, and provide information on whether any aspects of the design need to be reworked or refined.

1.3 Total Design

The process of design has been the focus of research for many years, and a number of design models and methodologies are available. Design methodology is a framework within which the designer can practice with thoroughness. One such approach, called "total design," has been proposed by the SEED programme (1985) and Pugh (1990) and is illustrated schematically in Figure 1.7. This shows the core activities of design: marketing, specification, conceptual design, detailed design, and marketing/selling. As in Figures 1.3 and 1.5, the iterative nature of design is accounted for where work on a design results in the need to go back and redo previous work in order to produce a better overall design to meet the requirements. Indeed, it is sometimes necessary to go back several levels. An example might be the discovery at manufacture stage that an item cannot be made as envisaged and a new concept is required. Ideally such a discovery should not occur, as every other level of the design process illustrated in Figure 1.7 should be considered at each stage. Each of the design activities illustrated in Figure 1.7 is described in more detail in the following text. As it is the same process being described, these descriptions are similar to those described for Figure 1.5.

1.3.1 Market

The market phase refers to the assessment of sales opportunities or perceived need to update an existing product resulting in a statement sometimes called the market brief, design brief, brief, or statement of need.

1.3.2 Specification

Specification involves the formal statement of the required functions, features, and performance of the product or process to be designed. Recommended practice from the

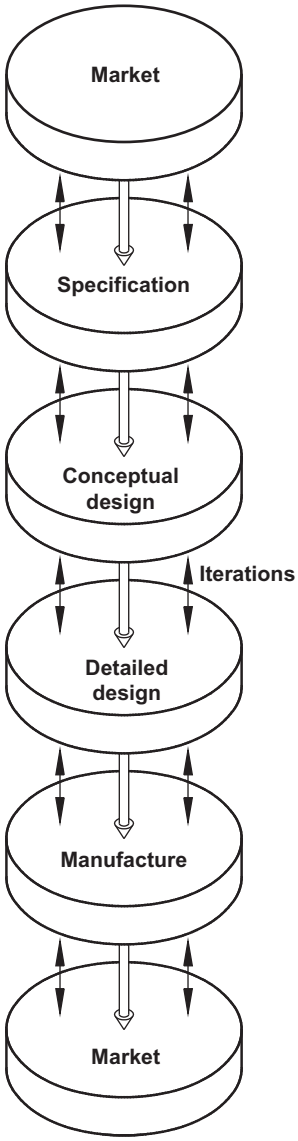


Figure 1.7
The total design core. *After Pugh (1990).*

outset of design work is to produce a product design specification that should be formulated from the statement of need. The product design specification is the formal specification of the product to be designed. It acts as the control for the total design activity because it sets the boundaries for the subsequent design. Further details of the product design specification are described in Chapter 2.

1.3.3 Conceptual Design

The early stages of design when the major decisions are to be made is sometimes called conceptual design. During this phase, a rough idea is developed as to how a product will function and what it will look like. The process of conceptual design can also be described as the definition of the product's morphology, how it is made up, and its layout. Conceptual design is the generation of solutions to meet specified requirements. Conceptual design can represent the sum of all subsystems and component parts that go on to make up the whole system. Ion and Smith (1996) describe conceptual design as an iterative process comprised of a series of generative and evaluative stages that converge to the preferred solution. At each stage of iteration, the concepts are defined in greater detail allowing more thorough evaluation. It is important to generate as many concepts and ideas as possible or as are economically expedient. There is a temptation to accept the first promising concept and proceed toward detailed design and the final product. This should be resisted, as such results can invariably be bettered. It is worth noting that sooner or later your design will have to compete against those from other manufacturers, so the generation of developed concepts is prudent. Some methods such as brainstorming, SCAMPER, and morphological analysis used to aid the generation of concepts are described in Chapter 3.

1.3.4 Detailed Design

The detailed design phase consists of the determination of the specific shape and size of individual components, what materials should be used, how they fit together, and the method of manufacture. Detailed design makes use of the many skills acquired and developed by engineers in the areas of analysis. It is the detailed design phase that can take up the bulk of the time spent on a design. However, as implied earlier, it is wise to only spend time on details once a sensible concept has been selected.

1.3.5 Manufacturing

The manufacture phase, although identified as distinct within the structure, is typical of other phases in that it influences all the others. The design of any item must be such that it is feasible to manufacture it! The materials selected must be compatible with the manufacturing facilities and skills available and at acceptable costs to match marketing requirements. Manufacturing is so important that design strategies to reinforce its importance have been developed, such as design for assembly (DFA) and design for manufacture (DFM) (see, for instance, Boothroyd, 1997). More recently, the concept of concurrent engineering has become popular. It is a systematic approach that encourages the developer from the outset to consider all the elements of a product lifecycle or process from concept through disposal including quality control, scheduling, and user requirements.

1.3.6 Marketing/Sales

The last phase, selling, is of course essential and should match the expectations of the initial market phase. This phase also has an impact on other phases within the design core. Information such as customer reaction to the product, any component failures or wear, damage during packaging, and transportation should be fed back to influence the design of product revisions and future designs.

1.3.7 Total Design Information Flows and Activities

The double arrows shown in Figure 1.7 represent the flow of information and control from one activity to another as well as the iterative nature of the process. For instance, detailed design activity may indicate that an aspect of the conceptual design is not feasible and must be reconsidered. Alternatively, conceptual work may yield features that have the potential for additional marketing opportunities. In other words, the activity on one level can and does interact dynamically with activities on other levels. Figure 1.8 illustrates the possible flow of this process during the development of a product.

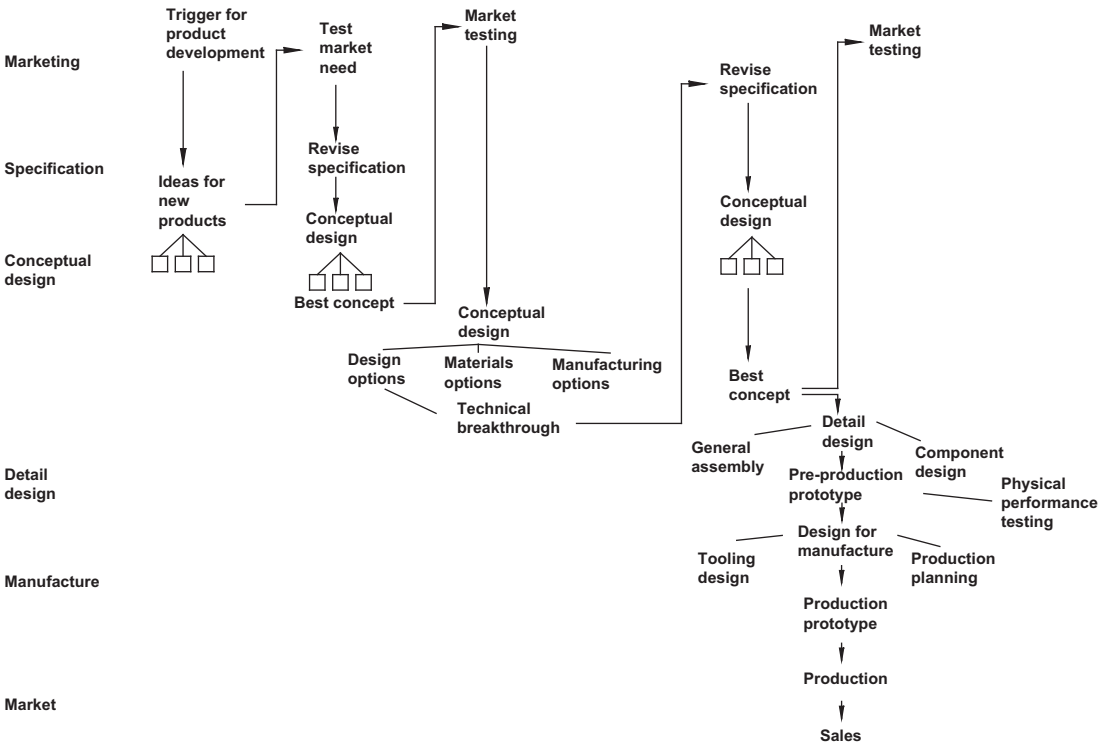


Figure 1.8
Design activities at different stages in product development. *Adapted from Baxter (1995).*

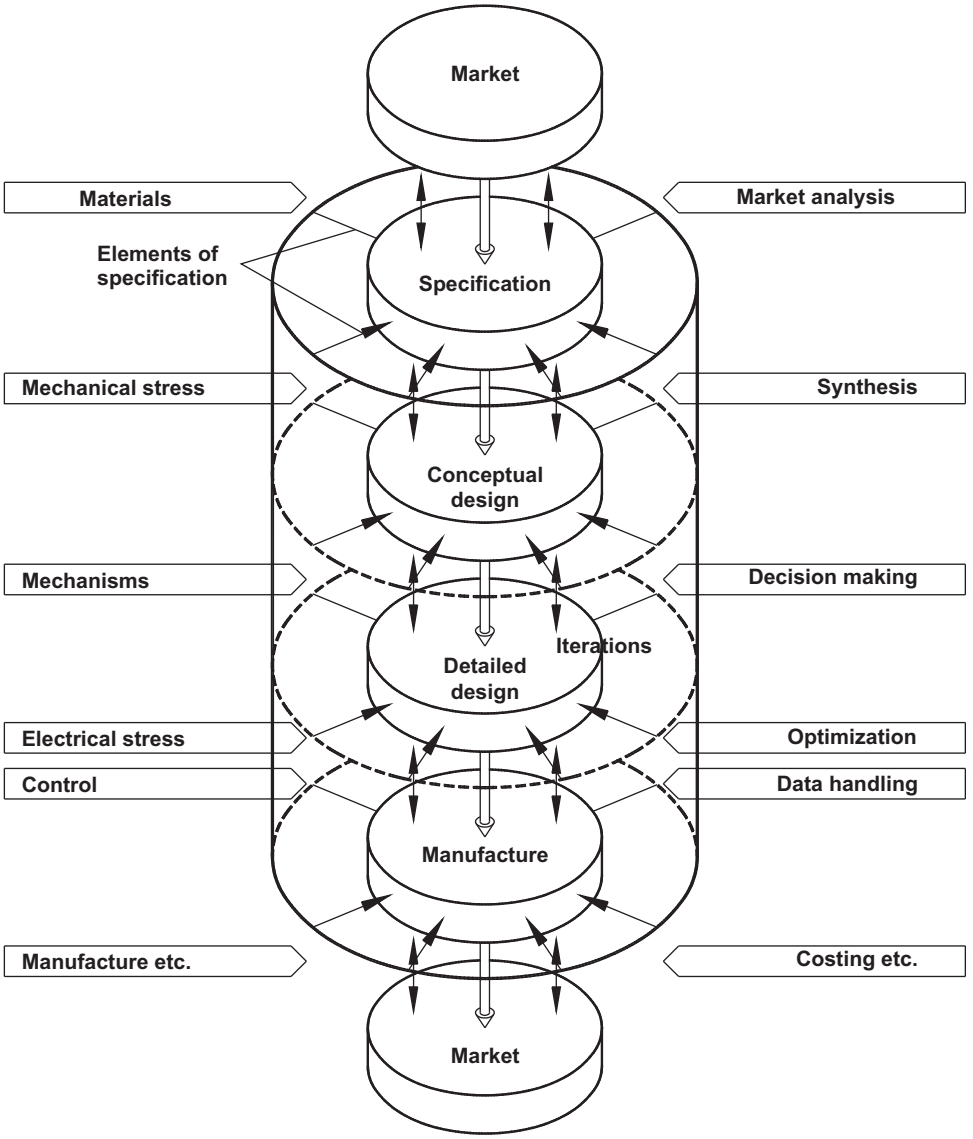


Figure 1.9
The total design process. *After Pugh (1990).*

Almost any product, such as a vacuum cleaner, kettle, automobile, or cordless hand-tool, requires input from people of many disciplines, including engineering, legal, and marketing, and this requires considerable coordination. In industrial terms, the integration comes about as a result of the partial design inputs from each discipline. In Figure 1.9, additional activities, such as market analysis, stressing, and optimization, have been added to the design core as inputs. The effective and efficient design of any product invariably requires

the use of different techniques and skills. The disciplines indicated are the designer's tool-kit and indicate the multidisciplinary nature of design. The forklift truck example mentioned in Section 1.2.1 will require engine management and control systems as well as the design of mechanical components. Although this text concentrates on mechanical design, this is just one, albeit important, interesting, and necessary, aspect of the holistic or total design activity.

A number of circumferential inputs have been shown as arrows in Figures 1.3, 1.5, 1.7, and 1.9. These represent elements of the specification listed in order of importance for each phase of design. The priority order of these specifications may alter for different phases of the design activity. The exact number will depend on the actual case under consideration.

Industry is usually concerned with total design. Total design is the systematic activity necessary from the identification of a market need to the commercialization of the product to satisfy the market need. Total design can be regarded as having a central core of activities consisting of the market potential, product specification, conceptual design, detailed design, manufacture, and marketing.

1.4 Systematic Design

A systematic approach to design has been developed and proposed by Pahl and Beitz (1996), who divide their model, see Figure 1.10, into four phases:

1. Product planning and clarifying the task;
2. Conceptual design;
3. Embodiment design;
4. Detail design.

The approach acknowledges that because of the complex nature of modern technology, it is now rarely possible for a single person to undertake the design and development of a major project on his or her own. Instead, a large team is involved and this introduces the problems of organization and communication within a larger network. The aim is to provide a comprehensive, consistent, and clear approach to systematic design.

Design models and methodologies encourage us to undertake careful marketing and specification. Because of their sequential presentation, "design starts with a need" or "design starts with an idea," they inherently encourage us to undertake tasks sequentially. This is not necessarily the intention of the models, and indeed this approach is countered within the descriptions and instructions given by the proponents of the model, who instead encourage an iterative feedback working methodology.

A criticism of the design models of Pahl and Beitz and Pugh is that they tend to be encyclopedic, with consideration of every possible scenario. As such, though, their use can be

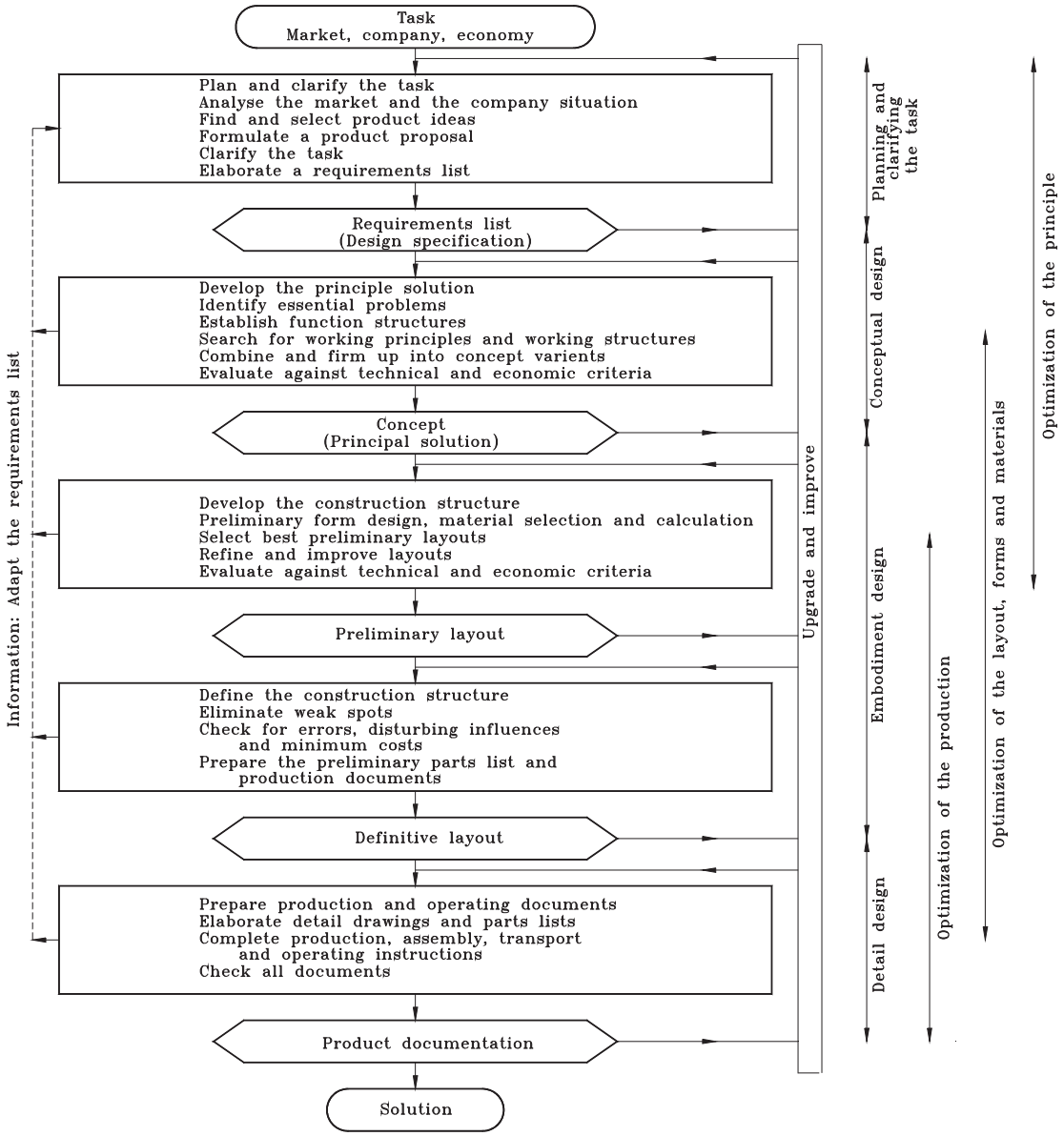


Figure 1.10

The design process proposed by Pahl and Beitz. *Figure adapted from Pahl and Beitz (1996).*

viewed as a checklist against which a personal model can be verified. A further criticism of design models is that they are too serialistic as opposed to holistic, and that because of the serious manner in which the models are portrayed and documented, they have the tendency to put the intuitive and impulsive designer off!

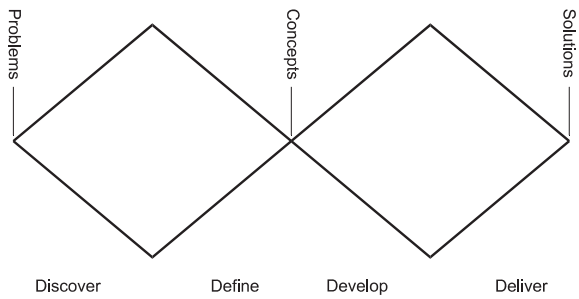


Figure 1.11
Schematic describing the design process. *Design Council (2007).*

1.5 Double Diamond

The Design Council (2007) reported a study of the design process in 11 leading companies and identified a four-step design process called the "double diamond" design process model. This model involves: discover, define, develop, and deliver. In Figure 1.11, the divergent and convergent stages of the design process are indicated, showing the different modes of thinking that designers use.

1.6 Conceive, Design, Implement, Operate (CDIO)

The CDIO framework is widely used in design and engineering education. The framework explicitly recognizes the importance of holistic considerations for effective design outcomes with application of both engineering practice skills such as design, manufacture, personal, professional, interpersonal, and business in combination with disciplinary knowledge from the sciences and mathematics as well as the humanities (Crawley, 2001).

1.7 Design for Six Sigma

Design for six sigma (DFSS) is an approach for designing a new product or service with a measurable high performance. This requires the development of an understanding of customer needs prior to launch rather than afterward.

There are a number of methodologies applying six sigma principles, including DMADV (design, measure, analyze, design, verify), IDOV (identify, design, optimize, verify) as well as DFSS. IDOV tends to focus on the final stages of engineering optimization and may not address the selection of product features and attributes that actually address customer requirements (Tennant, 2002).

DFSS comprises a number of defined activities as outlined in Table 1.1.

Table 1.1: Design for six sigma.

Phase	Characteristic activities
New product introduction	Selection of the concept or service to fulfill a perceived need. Benchmarking. Customer surveys. R&D, sales and marketing input. Risk analysis.
Define	Benchmarking, customer surveys and analysis. Development of a team charter to provide a solid foundation for the project.
Customer (measure)	Identification of the full characteristics and needs of the customer. Use of QFD (quality function deployment (see Chapter 2)) to identify the set of critical to quality (CTQ) metrics. This arises from the set of customer needs along with a list of potential parameters that can be measured and quantified defining the targets for each CTQ metric.
Concept (analyze—conceptual design)	A design is explored and developed for the new product and service. This requires a further round of QFD in order to identify the best features that have the potential to deliver the critical to quality metrics. During this phase, there is a move from critical to quality to critical to process (CTP) metrics. At the end of this stage, a concept or concepts together with a set of critical to process metrics that constrain the formal and technical design will have been produced.
Design (technical design)	The team hands over the design brief for the design team to complete using the critical to process metrics. Typical tools applied in this phase include Design of Experiments (DoE) and statistical optimization.
Implement	Use of failure mode effect analysis (FMEA) and other prototyping and product or system testing approaches to enable fine-tuning of the design.
Handover	Full-scale commercial roll-out.

1.8 Design Optimization

Inevitably there are conflicts between the diversity of requirements driven by the stakeholders. Optimization can be viewed as the process of repetitively refining a set of often-conflicting criteria to achieve the best compromise. In the case of a transportation system, size, maneuverability, cost, esthetic appeal, ease of use, stability, safety, and speed are not necessarily all in accordance with each other; for example, see Hall et al., (2013). Priorities can change within the lifetime of a product and vary within markets and cultures. Cost minimization may call for compromises on material usage and manufacturing methods. These considerations form part of the optimization of the product producing the best or most acceptable compromise between the desired criteria.

A traditional engineering design process comprises a series of often sequential steps, beginning with defined requirements or an opportunity and proceeding through ideation, synthesis, analysis, and optimization, to production. This process can be controlled by a series of gate reviews in coordination with the stakeholders and the process can be iterative with

phases being revisited when rework is recognized as necessary. This type of process can lead to bottlenecks in activity and a tendency to stick to a particular suboptimal solution as so much time and effort have already been allocated to it.

The range of optimization tools used in design is reviewed by Roy et al. (2008). Multidisciplinary design optimization (MDO), for example, combines tools and approaches from a number of disciplines in order to tackle the refinement of a set of parameters for a given problem area in order to deliver the best compromise between those parameters, and it has been widely applied in aerospace applications. A key characteristic of multidisciplinary design optimization is that the solution is better than that obtained by optimizing each of the parameters sequentially. The approach is resource-intensive in terms of computational power, however the Moore's law enhancement of processing means that this is not a hindrance to application of the approach. Optimization approaches and numerical strategies typically employed have included decomposition, approximation, evolutionary and mimetic algorithms, response surface methodologies, reliability, and multiobjective. The METUS methodology (METUS, 2012), for example, has been used in Airbus development programs to help provide a holistic approach to product development, covering the phases of conception and optimization of product architecture, visualization, and integration of partners in the supply chain.

MDO can be considered to be a methodology for the design of an engineering system that exploits the synergies between interacting parameters. The principle of MDO is that it provides the collection of tools and methods that enables and permits the tradeoff between different disciplines inherent in design. Proponents of MDO suggest that this provides the justification for its application at an early stage in a product development program (e.g. see Sobieszczanski-Sobieski et al., 1984).

Ideally an MDO environment should permit the definition of the brief and specification constraints for all the various stakeholders (e.g. see Kroll et al., 2009). This is typically achieved using a single parametric model for the whole system facilitating effective communication between the different stakeholders. MDO offers the potential for the interactions between subsystems and systems to be explored from an early stage in the design process by a number of stakeholders. The purpose is to find the minima for the cost functions and reach an optimal solution for the holistic system.

1.9 Stage-Gate Process

The stage-gate product innovation process was originally developed for new product development, taking a team or organization from an initial idea to launch. The process was originally developed by Cooper (1986) and has since found many diverse

applications. The process has been emulated by processes such as the phase-gate process and gated reviews. In a typical stage-gate model, the key stages and gates are outlined in Table 1.2.

1.10 The Technology Base

The range of existing and well-tried technology is extensive. Many items are available as standard components and can be purchased from specialist suppliers. In addition, standard

Table 1.2: Typical stage-gate process activities.

Phase	Activity
Discovery	Prework to reveal new opportunities and produce ideas.
Gate 1. Idea screen	Decision to commit tentative or initial resources to a particular project.
Stage 1. Scoping	A preliminary investigation of the project to enable the field of the project to be focused upon.
Gate 2. Second screen	A more rigorous screen where the project is evaluated again in light of the information found in stage 1. If the project is deemed go at this stage, more significant resources are provided for stage 2 to go ahead.
Stage 2. Build the business case	A more detailed investigation involving primary research to produce a business case.
Gate 3. Go to development	This is the last point at which a project can be halted before committing to very significant resources for the organization concerned. The gate 3 evaluation will involve review and detailed scrutiny of the activities and material produced in stage 2.
Stage 3. Development	Detailed design and development of the new product or system along with some product testing, preparation of production, and launch plans.
Gate 4. Go to testing	Review of development work to check that the outcome is attractive and consistent with the agreed upon requirements. Financial analysis undertaken based on more accurate data.
Stage 4. Testing and validation	In-house and marketplace tests and trials to validate and verify the new product and determine customer acceptance and the economics of the enterprise. Stage 4 can result in requirements for rework of some aspects and the need to go back to stage 3.
Gate 5. Go to launch	Decision to go to full commercialization and market launch, production, and operations setup.
Stage 5. Launch	Commercialization, full production, or full operation of the enterprise. Execution of the production, operations, distribution, quality assurance, and post-launch monitoring. Sales and revenue streams commence.
Post-launch reviews	Typically at 3–5 months after launch, with initial data, and then again at 12–19 months, once the project is stable.

Table 1.3: An overview of the scope of machine elements.

Energy conversion	Energy transmission	Energy storage	Locating	Friction reduction	Switching	Sealing	Sensors	Miscellaneous mechanisms
Turbomachinery	Gears	Flywheel	Threaded fasteners	Rolling element bearings	Clutches	Dynamic seals	Motion	Hinges, pivots
Gas turbine engines	*Spur*	Springs	*Bolts*	*Deep groove*	*Square jaw*	*Mechanical face*	Dimensional	Linkages
Rotodynamic pumps and compressors	*Helical*	*Helical*	*Nuts & lock nuts*	*Cylindrical roller*	*Multiple serration*	*Lip ring*	Mass	Levers
Fans	*Bevel*	*Leaf*	*Grub screws*	*Needle roller*	*Sprag*	*Bush*	Force	Tools
Propellers	*Worm*	*Belleville*	*Studs*	*Taper roller*	*Roller*	*Labyrinth*	Torque	*Cutters*
Turbines	*Conformal*	*Rubber*	*Screws*	*Angular contact*	*Disk*	*Brush*	Power	*Shears*
Internal combustion engines	Belts	*Spiral*	Washers	*Self-aligning*	*Drum*	*Ferrofluidic*	Pressure	*Drills*
Rotary	*Flat*	*Garter*	Nails	*Thrust*	*Cone*	*Rim seals*	*Pitot tubes*	*Formers*
Reciprocating	*Vee*	Fluid accumulator	Pins	*Recirculating ball*	*Magnetic*	*O rings*	*Static tappings*	Grips
Boilers and combustors	*Wedge*	Gas spring	*Cylindrical*	Sliding bearings	*Synchromesh*	*Packings*	*Manometers*	Guides
Electric motors	*Round*	Reservoir	*Taper*	*Plain rubbing*	Ratchet and pawl	*Piston rings*	*Piezoelectric*	Followers
Alternators and generators	*Synchronous*	Pressure vessel	*Spring*	*Hydrodynamic*	Geneva Mechanisms	Static seals	Sound	Housings
Solenoids	Chains	Solid mass	Rivets	*Hydrostatic*	Valves	*Gaskets*	Flow	*Frames*
Pneumatic and hydraulic actuators	*Roller*	Chemical Charge	Tolerance rings	*Hydrodynamic slideway*	Latches	*O rings*	*Laser Doppler*	*Casings*
Brakes	*Leaf*	Torsion bar	Expanding bolts	Wheels and rollers	Triggers	*Gaskets*	*Hot wire*	*Enclosures*
Disk	*Conveyor*		Keys	Brushes	Bimetallic strips	*Sealants*	*Ultrasonic*	Sprayers
Drum & band	*Silent*		*Flat, round*				Level	Shutters
Pumps	Cables and ropes		*Profiled*				Humidity	Hooks
Rockets	Couplings		*Gib head*				Temperature	Pulleys
Heat exchangers	*Rigid*		*Woodruff*				*Thermocouples*	Handles
Guns	*Flexible*		Splines				*Resistance thermometers*	Rollers and drums
Dampers and shock absorbers	*Universal*		Circlips				*Pyrometers*	Centrifuges
Fuel cells	Cranks		Snap rings				Heat flux	Filters
	Cams		Clamps				*Thermopile*	
	Power screws and threads		Retaining rings				*Gardon gauges*	
	Levers and linkages		Shoulders				Strain and stress	
	Pipes, hoses and ducts		Spacers				Time	
	Ball screws		Grooves				Chemical composition	
			Fits					
			Clearance					
			Transition					
			Interference					
			Adhesives					
			Welds					

practice and design methodology have been specified for many others. The designer is able to exploit available technology as it is or in combination and adaptation with other technology. The scope of mechanical machine elements available to the designer is outlined in Table 1.3. This list is not exhaustive but does give an idea of some of the "building blocks" available. A significant proportion of the machine elements defined in the table are considered within this book.

Developing an overview of the available technology base is useful to the designer as it avoids time spent repeating the design process on a technology that has already been developed. In addition, knowledge of science and engineering enables a designer to judge what is technically and scientifically feasible so that "far-fetched" ideas can be objectively ruled out.

1.11 Conclusions

Design can be considered to be the process of conceiving, developing, and realizing products, artifacts, processes, systems, services, and experiences with the aim of fulfilling identified or perceived needs or desires typically working within defined or negotiated constraints. Engineering involves the application of scientific and mathematic principles in combination with professional and domain knowledge, to design, develop, and deliver artifacts, products, and systems to realize a societal, commercial, or organization requirement or opportunity. This chapter has briefly reviewed a range of design processes relevant to both engineering and design domains. The models presented provide an indication of process, although a given individual or organization is likely to develop their own bespoke approach relevant to their specific requirements or context.

References

Books and Papers

ABET, 2011. Criteria for Accrediting Engineering Programs. Effective Evaluations during the 2010–2011 Accreditation Cycle. www.abet.org (Last accessed 24.11.12.)

Ahmed, S., Wallace, K.M., Blessing, L.S., 2003. Understanding the differences between how novice and experienced designers approach design tasks. Research in Engineering Design 14 (1), 1–11.

Baxter, M., 1995. Product Design. Chapman and Hall.

Boothroyd, G., 1997. Design for Manufacture and Assembly, pp. 676–686 in ASM Handbook. In: Material Selection and Design, vol. 20. ASM International.

Clarkson, J., Eckert, C., 2005. Design Process Improvement. A Review of Current Practice. Springer.

Cooper, R.G., 1986. Winning at New Products. Wadsworth.

Cox, G., 2005. Cox Review of Creativity in Business: Building on the UK's Strengths. HM Treasury, UK.

Crawley, E.F., January, 2001. MIT CDIO Report #1. The CDIO Syllabus. A Statement of Goals for Undergraduate Engineering Education. Department of Aeronautics and Astronautics, Massachusetts Institute of Technology.

Design Council. 2007. The 'double diamond' design process model. www.designcouncil.org.uk/en/About-Design/managingdesign/The-Study-of-the-Design-Process/ (accessed December 2007).
Hall, A., Childs, P.R.N., 2009. Innovation design engineering: non-linear progressive education for diverse intakes. In: Clarke, A., Ion, B., McMahon, C., Hogarth, P. (Eds.), Proceedings of the 11th EPDE International Conference, pp. 312–317.
Hall, A., Wuggetzer, I., Mayer, T., Childs, P.R.N., 2013. Future aircraft cabins and design thinking: optimisation vs. win-win scenarios. Journal of Propulsion and Power Research 2(2), 85–95.
Ion, B., Smith, D., 1996. The conceptual design phase. SEED.
Kroll, N., Schwamborn, D., Becker, K., Rieger, H., Thiele, F. (Eds.), 2009. MEGADESIGN and MegaOpt—German Initiatives for Aerodynamic Simulation and Optimization in Aircraft Design: Results of the Closing Symposium of the MEGADESIGN and MegaOpt Projects. Springer.
Manzini, E., 1989. The Material of Invention. Design Council.
McMahon, C., 1997. Private Communication. University of Bristol.
METUS. http://www.id-consult.com/en/metus-software/ (Last accessed 23.06.12.)
National Academy of Engineering, 2004. The Engineer of 2020: Visions of Engineering in the New Century. National Academies Press.
Pahl, G., Beitz, W., 1996. Engineering Design a Systematic Approach, second ed. Springer.
Pugh, S., 1990. Total Design. Addison Wesley.
Roy, R., Hinduja, S., Teti, R., 2008. Recent advances in engineering design optimisation: challenges and future trends. CIRP Annals—Manufacturing Technology 57, 697–715.
Sharing Experience in Engineering Design, 1985. Curriculum for Design. Engineering Undergraduate Courses. SEED.
Sobieszczanski-Sobieski, J., Barthelemy, J.-F.M., Giles, G.L., 1984. Aerospace engineering design by systematic decomposition and multilevel optimization. In: 14th Congress of the International Council of the Aeronautical Sciences (ICAS).
Tennant, G., 2002. Design for Six Sigma. Gower.

Further Reading

March, L.J., 1976. The Architecture of Form. Cambridge University Press.

Standards

99/705213 DC. ISO 18334. Timber pallets for materials handling. Quality of new wood pallet assembly.
99/712554 DC. PrEN 13698-1. Pallet product specification. Part 1. Construction specification for 800 mm × 1200 mm wooden pallet.
99/712555 DC. PrEN 13698-2. Pallet product specification. Part 2. Construction specification for 1000 mm × 1200 mm wooden pallet.
British Standards Institution, 1978. BS 3726:1978, ISO 1074-1975. Specification for counterbalanced lift trucks—stability—basic tests.
British Standards Institution, 1978. BS 5639-1:1978, ISO 2331-1974. Fork arms for fork lift trucks. Vocabulary for hook-on type form arms.
British Standards Institution, 1988. BS ISO 6780:1988. General purpose flat pallets for through transit of goods. Principal dimensions and tolerances.
British Standards Institution, 1996. BS ISO 509:1996. Pallet trucks. Principal dimensions.
British Standards Institution, 1999. BS EN 1726-1:1999. Safety of industrial trucks. Self-propelled trucks up to and including 10000 kg capacity and industrial tractors with a drawbar pull up to and including 20000 N. General requirements.
British Standards Institution, 1999. BS EN ISO 445:1999. Pallets for materials handling. Vocabulary.

British Standards Institution, 2002. BS EN 13545:2002. Pallet superstructures. Pallet collars. Test methods and performance requirements.
British Standards Institution, 2002. BS ISO 2330:2002. Fork-lift trucks. Fork arms. Technical characteristics and testing.

Web Sites

At the time of going to press the world-wide-web contained useful information relating to this chapter at the following sites.

www.bsi.org.uk
www.iso.org

Nomenclature

The following symbols have been used in this chapter:

BS	British Standard
CDIO	conceive, design, implement, operate
CTP	critical to process
CTQ	critical to quality
DFA	design for assembly
DFM	design for manufacture
DFSS	design for six sigma
DMADV	design, measure, analyze, design, verify
DoE	design of experiments
FMEA	failure mode and effects analysis
IDOV	identify, design, optimize, verify
ISO	International Organization for Standardization
MDO	multidisciplinary design optimization
PDS	product design specification
QFD	quality function deployment

CHAPTER 2

Specification

Chapter Outline

2.1 Introduction 25
2.2 Product Design Specification 27
2.3 Quality Function Deployment 36
2.3.1 QFD1 and QFD2 Matrices 37
2.4 Conclusions 47
References 48
Further Reading 49
Nomenclature 49

Abstract

Specification represents an important part of the design process to ensure that a design activity is addressing the task required. Specification may be undertaken prior to design, or as part of a gated process where some design work is necessary in order to define the scope of the product, artifact, or system being considered. This chapter provides an overview of the principles of specification and describes a standard product design specification pro forma, and also quality function deployment—a technique widely used in industry that has been developed to ensure that the voice of the customer is addressed.

2.1 Introduction

Design usually starts with a need or requirement which, when satisfied, will allow a product or process to be offered to an existing market or new market. The market need may be to meet competition, a response to social changes, opportunities to reduce hazards, inconvenience or costs, or exploitation of a new market opportunity. This is the "front end" of design. People with front-end skills, such as market researchers and information scientists, are fundamental to the design process. The starting point for the majority of design activities in industry is the establishment of the market need situation. Common practice is to produce a document or statement of need, called "the brief" or "market brief," at this initial stage. The brief varies from a simple statement of the requirements to a comprehensive document that describes the true needs of the user.

Specification is usually critical to effective design. Thorough specification provides limits defining the attributes and functions that are requisites of a design. Common practice is to develop a product design specification (PDS) where each attribute and function required of

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00002-2
Copyright © 2014 Elsevier Ltd. All rights reserved.

the product or system is described and quantified. An alternative is the use of quality function deployment (QFD), where matrices are used to capture the attributes, functions, and features associated with a product or system with a particular focus on ensuring that the voice of the customer (VOC) is captured in the specification—geared toward ensuring that the product development cycle results in an outcome that matches the customer requirement.

There are many variants and forms of specification. Specification can represent a significant undertaking. The specification documentation associated with a new turbofan engine for an aviation application, for example, may occupy over 500 pages. The precise form used will likely depend on precedence within a particular organization. The automotive industry, for example, has tended to use QFD over the last few decades, and such approaches may demand more resources than available and simpler pro formas may enable a more timely and accessible approach to specification.

Specification is usually incorporated as a fundamental part of new product development project management. It is not always possible to produce a complete specification at the outset of a project. It may be necessary to take an iterative approach to the specification with elaboration of details as a result of initial phases of design activity and concept ideas with these providing insights to what attributes and functions for a product or system are possible and desirable. In essence, the early concepts and designs serve to inform the specification. Such an approach fits well with stage gated project management processes, perhaps with phases of system design review, performance design review, and technical design review, with each referring back to an original specification and enabling the development of an agreed technically feasible specification that provides a design that is fit for purpose. A common occurrence in industry is the use of an agreed specification to provide the basis for contractual negotiation, enabling the associated parties to be able to define the completion of tasks. Any variation on the specification should be addressed by a change management process, where changes to the specification need to be agreed upon by all associated parties. When agreement cannot readily be obtained, these changes must be managed by an arbitration process.

On completion of the specification, conceptual design can proceed. This tends to be the most creative part of the design process, where solutions to the specification are given form. Having generated the conceptual solutions, the next step is to express these solutions so that they are communicable to all involved in the design process, perhaps following the total design process illustrated in Figure 1.7. In practice, this may take the form of a drawn scheme or three-dimensional model, which may be physical or computer generated. By the time the general scheme has been completed and calculations undertaken to determine the solution's compatibility with the PDS, the basis will have been established for the detailed design phase to commence.

Specification using a pro forma table is considered in Section 2.2. QFD is described in Section 2.3.

2.2 Product Design Specification

Specification involves the formal statement of the required functions, features, and performance of the product or process to be designed. One approach from the outset of design work is to produce a PDS that should be formulated from the statement of need. The PDS is the formal specification of the product to be designed. It acts as the control for the total design activity because it sets the boundaries for the subsequent design.

Specification has different meanings depending on the context and the individuals concerned. Here, specification refers to an activity that assists in providing a framework within which the design can take place. Work on a design may commence with an identification of a need, which is sometimes summarized in the form of a statement commonly known as the "design brief" or "design intent." A design brief may provide some direction to initial design activity but rarely sets sufficient bounds on it to allow a design team to know when their efforts are adequate, or not, to fulfill the requirements. In order to overcome this shortcoming and compel the individuals concerned to explore what is actually required of the product or process concerned, a PDS should be developed.

In developing a PDS, bounds should be set on a wide range of relevant parameters for the product or process concerned. The spider chart shown in Figure 2.1 provides a helpful checklist of aspects for consideration. A useful rule associated with specification, which will be stated frequently, is the mantra: "quantify wherever possible."

Having identified a general need or requirement and documenting this in the form of a brief, the specification of the product to be developed should be generated. This is called the PDS, which acts as the controlling mechanism, mantle, or envelope for the total design activity. Whatever you are concerned with, the PDS gives you the terms of reference. The starting point for design activity is market research, analysis of the competition, literature, and patent searching. Once this has been undertaken, the PDS can be developed. Figure 2.2

Table 2.1: Example format for the documentation of the product design specification. All relevant aspects identified in Figure 2.1 should be considered.

Date:.....Product:.....Issue:.....

Aspect	Objective	Criteria	Test Conditions
Aspect 1			
Aspect 2			

Source: After Pugh (1990).

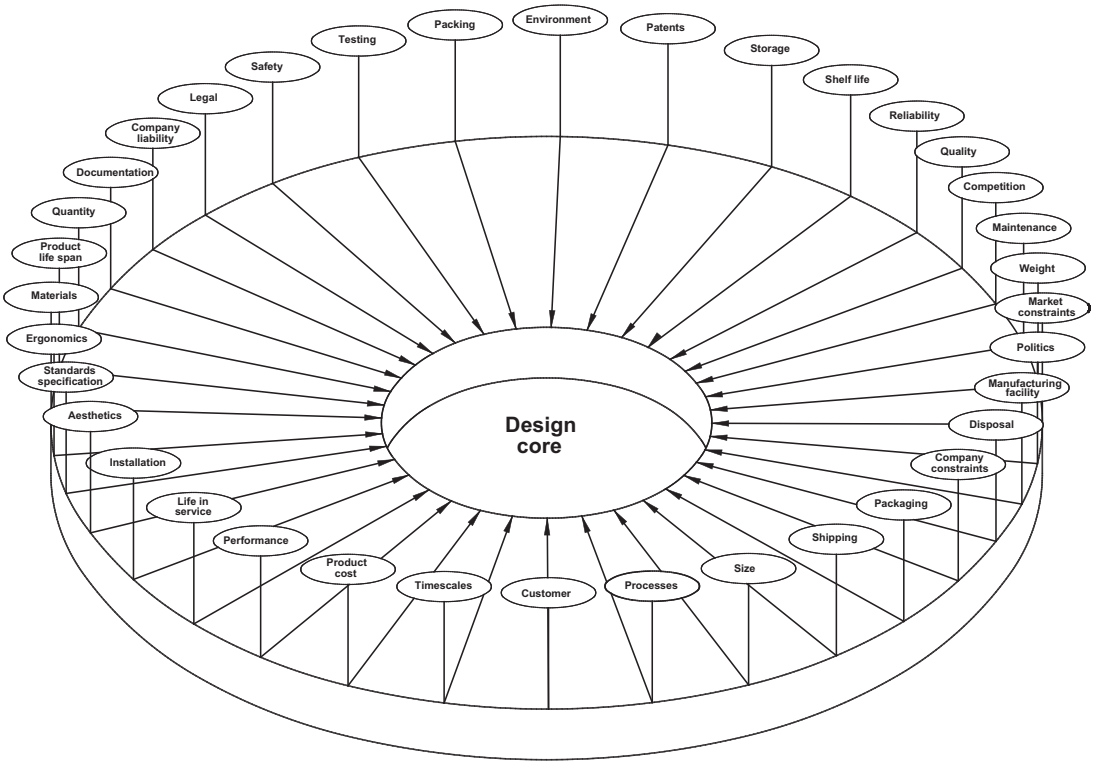


Figure 2.1
Information content of a typical product design specification. *After Pugh (1990).*

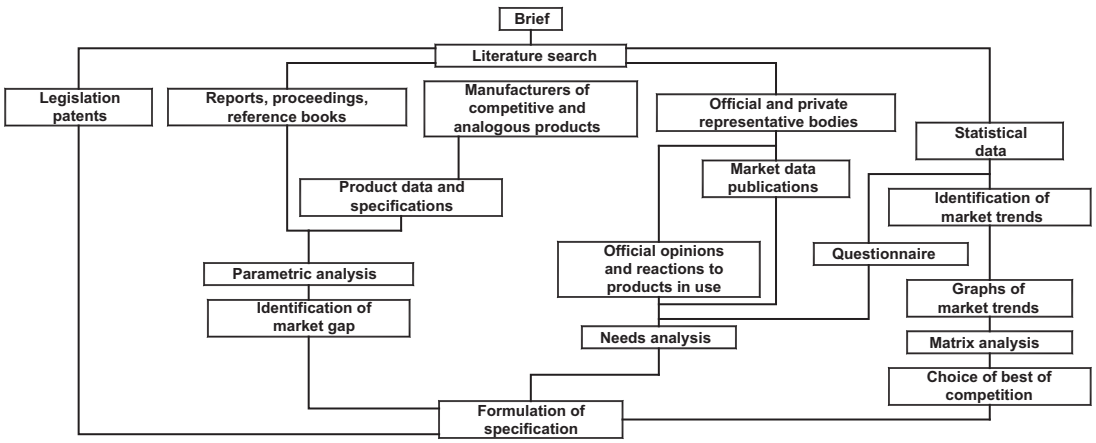


Figure 2.2
Information required for the production of the product design specification. *After Pugh (1990).*

Table 2.2: Example format for the documentation of the product design specification.

Date:.....Product:.....Issue:.....

	Parameters			
	Competition Best	Current Model (Ours)	This Design (Intent)	World Class (Target)
Performance description				
Safety description				

Source: After Pugh (1990).

shows the areas of information and research required to produce the PDS, and Tables 2.1 and 2.2 show the information content of a typical PDS.

In a new design, it is preferable to consider each aspect identified in Figure 2.1. However for the reengineering of a current product, engineering expediency dictates that some aspects will be passed over. The PDS thus consists of a document covering a wide range of considerations. The PDS is dynamic. If, during the design of a product there is a good reason for changing the basic PDS, this can be done. It should be noted, however, that the PDS may actually form part of contractual obligations and legal implications must be addressed.

Undertaking a PDS will typically be an iterative process where initial parameters are identified and limits set, but more information may be required in order to confirm them. The process of developing a PDS will require input from people with a variety of skills and knowledge. It will very rarely be simply one day's work and may extend many person months. Understand that conceptual design, let alone detailed design, cannot sensibly proceed toward any firm status until the specification has been completed.

Example 2.1

After a demonstration by competitive companies at a national ski exposition, the managing director of a recreational equipment company has outlined a requirement for a snow sports activity vehicle. The director has marched into your office and asked you to develop the PDS for this vehicle. Although you know that doing this job will take orders of magnitude more time and resources than are available, you need to outline the overall requirement in a short paragraph using a standard pro forma for the PDS and develop the PDS. Example information for the resulting pro-forma is given in Table 2.3.

Table 2.3: Product design specification for the snow sports activity vehicle.

ASPECT	OBJECTIVE	CRITERIA	TEST CONDITIONS
Performance	Operation at low temperatures	Assume arctic conditions. Operation at temperatures between −40°C and 25°C.	Test components at extreme temperatures.
	High speed	60 mph maximum.	Rolling road test or dynamometer test.
	High acceleration	Limit to the ability of an average person to hold on to and control vehicle.	Test existing products and identify maximum acceleration suitable. Test prototype on rolling road or dynamometer.
	High maneuverability	Less than 5 m turning circle.	Test model.
	Easy steering	Steering must be responsive to driver intent and provide adequate feedback of driving conditions.	Use a focus group to evaluate steering on a prototype model.
	Easy starting	Push button start.	Expose CAD model to a focus group for assessment. Expose a prototype model of starting to a focus group.
	High range	2 hours operation or 120 miles.	Dynamometer or rolling road test.
	Good braking	Must be capable of braking from 30 kph in 30 m on fresh snow.	Rolling road test.
Accommodation	2 people	Driver plus one passenger.	Test accommodation of two people in the product using digital manikins in the CAD model. Use a focus group to evaluate the layout and accommodation using a prototype mock-up of the design.
Cost	Acceptable cost for market sector	The works cost price must not exceed £3000.	Obtain quotations for constituent components from OEMs and estimates for in-house manufactured components and assembly costs.
Size	Small	Must be able to transport device on a small trailer.	Determine size of concept from CAD model. Test concept on a CAD trailer model.
Noise	Low	Noise levels must not exceed national standard limits.	Test to national standards.

Table 2.3: Product design specification for the snow sports activity vehicle.—cont'd

ASPECT	OBJECTIVE	CRITERIA	TEST CONDITIONS
Materials	Appropriate to design	Compatible with low temperatures and fuel (fire retardant, see standards).	Test to national standards.
Weight	Low	One person must be able to move the device. Two people must be able to lift the device. Mass not to exceed 200 kg.	Determine weight during design from CAD model. Weigh prototype components.
Safety	High	Power plant must cut out when driver loses control.	Bench test.
		Materials must be fire retardant to national standards.	Bench test.
Fuel	Readily available	Any readily available fuel such as petrol, diesel, kerosene or electricity.	
Propulsion	Available technology	Any available technology power plant	
Aesthetics	Attractive styling.	Current trend/modern sports style.	Expose concept design ideas to focus groups for appraisal. Expose prototypes to focus groups for appraisal.
Life	Maximise	The warrantee life of the machine should exceed 500 hours use.	Test design under accelerated stress conditions.
Manufacture	Ensure ready supply of components	Use stock machine elements where possible.	Identify components in stock supply inventories.
		Design using manufacturing techniques that are readily available from a number of local plants.	Use established suppliers.
Design time	Minimise	Detailed design must take less than a maximum of 6 elapsed months	Use standard project management methods to track progress and rectify excursions from the plan.

Example 2.2

A small and medium enterprise (SME) company has investigated in the potential market for a small electric motor driven air compressor. The market analysis has confirmed the requirement for an 80 m^3/h, 7 bar air supply driven by a 5 kW electric motor. Preliminary design has already resulted in the selection of an electric motor, with a synchronous speed of 2850 rpm, and the piston, cylinder, and associated components for a reciprocating compressor. The compressor design speed is 1500 rpm. A general arrangement for the compressor concept is given in Figure 2.3. The choice of transmission and its detailed design is yet to be undertaken. The task here is to explore the specification, conceptual design, and detailed design of the transmission for this compressor.

The brief given outlines the general requirement for a transmission of a reciprocating compressor. Not all the information necessary to develop the design is given in the brief and must be developed by means of liaison with the company and application of design skills and experience.

A specification for the transmission is developed as follows:

1. The transmission must transmit at least 5 kW.

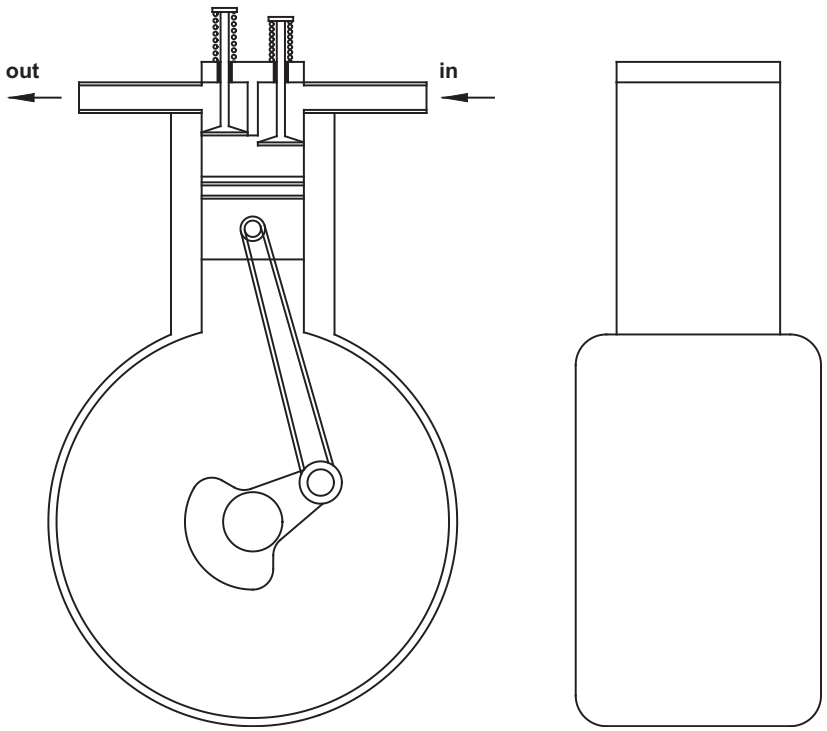


Figure 2.3
Schematic arrangement for a 7 bar, 80 m^3/h reciprocating compressor. *Childs (2004).*

2. The input is from an electric motor with a synchronous speed of approximately 2850 rpm. Details of the electric motor are not given in the brief. Contact with the company has identified the critical interface motor dimensions.
3. The output from the transmission is to a reciprocating compressor running at 1500 rpm. The load is reciprocating and, therefore, constitutes medium to light shock.
4. Following contact with the preliminary design team, dimensions for the compressor shaft have been provided.
5. The preliminary design team has confirmed that the electric motor torque speed characteristic is sufficient to start the compressor, provided the transmission gives the required ratio with reasonable efficiency.
6. A mechanical efficiency for the transmission has not been specified. However, a mechanical efficiency of at least 95% should be achievable and is proposed for the specification.
7. The means of connecting the motor and compressor to the transmission have not been detailed in the brief. Contact with the SME has confirmed that the connection of the transmission to the compressor and electric motors forms part of the brief. Flexible couplings permit some degree of misalignment and isolation of vibration and are specified for connecting the input and output shafts of the transmission to the motor and compressor.
8. Following discussion with the company and examination of the market analysis, compact size for the compressor and the transmission is desirable. Limits have not been identified in the market analysis.
9. The life of the compressor is not defined in the brief. Following discussion with the SME business team, the following information has been identified. The compressor is expected to operate for a maximum of 2 h a day on 200 days of the year. An 18 month trouble-free warranty life is desirable.
10. The compressor will be enclosed to prevent casual contact with hot or moving components. However, the transmission should include means to support its functional components and retain all lubricants, if present.
11. The transmission will operate in an internal environment at temperatures between -10 and 50 °C. The design should be tolerant to dust and high humidity.
12. Up to 1000 units a year will be manufactured.
13. The cost of the transmission should be minimized. A maximum limit for the works' cost price has been set as 600 Euros.
14. The weight is not critical but very high weights are deemed undesirable. A maximum mass for the transmission has been set by the SME at 30 kg.
15. Ideally, the transmission should use readily available components.
16. Unique transmission components should be manufactured locally to the SME in order to ensure reliability of parts supply.
17. The design should take a maximum of 6 person months and be undertaken over a 2 month period.

18. Although aesthetics are not a primary factor in the design, the design should augment the overall aesthetics of the overall product.
19. The transmission should comply with all relevant safety criteria and legislation.
20. The design should not violate any existing patents in order to protect against royalty payments.
21. The design should provide recommendations for future recycling of components in order to comply with expected legislation on sustainability.
22. The transmission must be robust enough for transportation on standard pallets via road.

The aspects of the specification listed do not yet provide sufficient bounds to enable conceptual or detailed design to proceed sensibly at this stage. Use of a pro forma such as those illustrated in Section 2.2 might be helpful to further develop definite limits on the design parameters to ensure that the work effort is being sensibly directed toward the product need defined by the marketing analysis referred to in the brief. The PDS is developed in Table 2.4.

As part of the specification, it is also important to identify the evaluation criteria before the design process proceeds. These may be altered as the design proceeds, if aspects of the evaluation are deemed inappropriate. Evaluation criteria are necessary because it is possible

Table 2.4: Product design specification for the compressor transmission.

Aspect	Objective	Criteria	Test conditions
Performance	Transmit torque	Transmission must transmit 5 kW at 2850 rpm.	Test transmission on a dynamometer
	Speed ratio	The transmission ratio between driving and driven shafts should be 2850/1500 = 1.9.	If positive drive is employed then assume specified ratio or if a positive drive is not used measure speed of output shaft at design conditions.
	Efficiency	At least 95%.	Dynamometer test.
	Drive	Must connect to specified 1.5″ driving shaft and 1.5″ driven shaft using flexible couplings.	Use standard coupling methods that are rated for the maximum torque and speed with a factor of safety suitable for the application.
	Temperature	Operation in an internal environment at temperatures between −10 and 50 °C.	Bench test prototype.
	Humidity	Operate in an environment with a humidity of 100% at 20 °C.	Bench test prototype.

Table 2.4: Product design specification for the compressor transmission.—cont'd

Aspect	Objective	Criteria	Test conditions
Size	Minimize	Must be smaller or similar to competition. Competition best transmission fits within an envelope space of 120 mm × 250 mm × 250 mm.	Measure.
Cost	Minimize cost	The cost for the transmission per unit should be less than 600 Euros assuming 1000 units per year.	Calculate work-cost price.
Life	Maximize	The warranty life of the machine should exceed $2850 \times 60 \times 2 \times 200 \times 1.5 = 1.02 \times 10^8$ cycles.	Test design under accelerated stress conditions.
Weight	Minimize	Must be less than 30 kg.	Weigh it.
Manufacture	Ensure ready supply of components	Use stock machine elements where possible. Design using manufacturing techniques that are readily available from a number of local plants.	Identify components in stock supply inventories. Use established suppliers.
Design time	Minimize	Detailed design of the transmission must take less than a maximum of 6 person months and occur over a 2 month period.	Use standard project management methods to track progress and rectify excursions from the plan.
Patents	Avoid	Do not infringe existing patents.	Patent search.
Transport	Robust design	The transmission must be robust enough for transportation on standard pallets by road.	Vibration test.
Aesthetics	Attractive design	Design should augment the overall aesthetics of the overall product.	Use focus group on concept graphics model.
Safety	High	Must comply with existing and envisaged legislation.	Check national and international standards and relevant legislation.
	Noise	Must be less than legal limits.	Measure.
Recycling	Compliant	The design should provide recommendations for future recycling of components to comply with expected legislation on sustainability.	Check national and international standards and legislation.
Maintenance	Easy	Provide access to components that might fail to enable their replacement.	Use graphics model of design to verify maintenance access.

that a number of competing designs may be identified, all of which meet the specification. The evaluation criteria can be used to determine which of the design solutions should be adopted. The evaluation criteria that seem appropriate at this stage are as follows:

- Cost,
- Safety,
- Size,
- Reliability,
- Maintenance, and
- Noise and vibration.

2.3 Quality Function Deployment

QFD, also sometimes known as the VOC, is an approach widely used in industry to help a company achieve the goal of high quality products and/or services. QFD can be described as an overall concept that provides a means of translating customer requirements into the appropriate technical requirements for each stage of the product development process.

Quality in this context is as follows:

- Defined by the customer;
- Problem avoidance; and
- Aspects that are value for money to the customer.

Quality in this context is not:

- Inspection alone (i.e. problem detection);
- Delivered by the manufacturing stage alone; and
- Achieving high standards of manufacture without direct link to a proven customer requirement.

QFD is a method to transform customer requirements into design quality and to deploy the functions forming quality and the methods for achieving the design quality into subsystems and component parts. Ultimately, QFD specifies elements of the manufacturing process.

The four phases associated with QFD are as follows:

- Product Planning (QFD1 or Phase I);
- Design Deployment (QFD2 or Phase II);
- Process Planning (QFD3 or Phase III); and
- Production Planning (QFD4 or Phase IV).

The relationship between the phases is illustrated schematically in Figure 2.4.

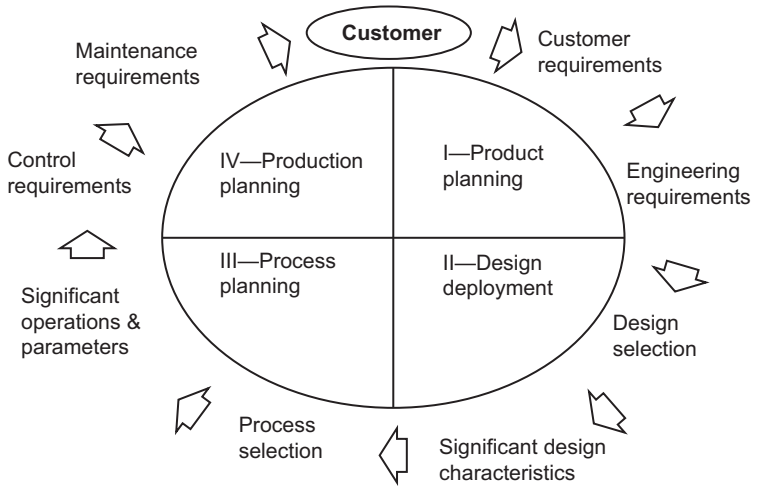


Figure 2.4
QFD phases of product delivery process.

Each of the phases of QFD relates to ensuring both the development of the specification and its subsequent implementation with a product development cycle. The characteristic activities with each of the phases are outlined in Table 2.5.

The QFD cycle of phases begins and ends at the customer. It is the customer who defines the quality standard in terms of their requirements, and it is the customer who ultimately determines if the product meets these requirements.

Note that the four phases of QFD are sequential in completion, but can be conducted in parallel. Process planning (QFD3) and design deployment (QFD2) must normally be conducted in parallel. Product planning (QFD1) should, in principle, be completed before design deployment (QFD2) commences, see Figure 2.5.

QFD uses matrix diagrams to assess the relationship between two sets of elements. E.g. two sets of requirements or a set of requirements and a set of solution concepts. The method is implemented by filling in four matrices as shown in Figure 2.6. In forming each of the matrices, the engineering design team needs to consider and define key requirements and the characteristics associated with these requirements. In this note, the focus is on QFD1 and QFD2 matrices, product planning and product design, respectively.

2.3.1 QFD1 and QFD2 Matrices

A QFD1 matrix helps capture and structure requirements during engineering projects. It is a systematic approach to translate customer requirements (or product attributes) into engineering requirements. The QFD1 method is especially useful when the opportunity for

Table 2.5: Characteristic activities associated with each of the QFD phases.

QFD phase	Activity
QFD1/Phase I Product planning	Listen to and capture the *voice of the customer*. Interpret the customer's requirements. Translate into engineering requirements. Document in a product design specification using a QFD template.
QFD2/Phase II Design deployment	Develop and select designs. Identify significant design characteristics (the specific assemblies, subassemblies, parts, dimensions, properties, and attributes of the designs considered that directly relate to achievement of customer requirements). Demonstrate that the design fulfills the requirements of the PDS (by applying the appropriate verification plan, appropriate calculations, simulations, tests, etc.).
QFD3/Phase III Process planning	Develop and select manufacturing processes. Identify significant process operations and parameters (the specific operations, e.g. critical assembly steps, and parameters, e.g. feed rates, and material temperatures that directly relate to achievement of significant design characteristics).
QFD4/Phase IV Production planning	Implement the manufacturing processes of phase III in the production environment (i.e. design, build, install, and prove out the required tooling, machinery, fixtures, jigs, and inspection equipment for the selected manufacturing processes). Develop the control requirements (i.e. the factors that should be inspected and monitored, in order to adequately control the significant process operations and parameters). Maintenance requirements (i.e. prepare the maintenance regime for tooling, machinery, etc. to deliver appropriate availability and function).

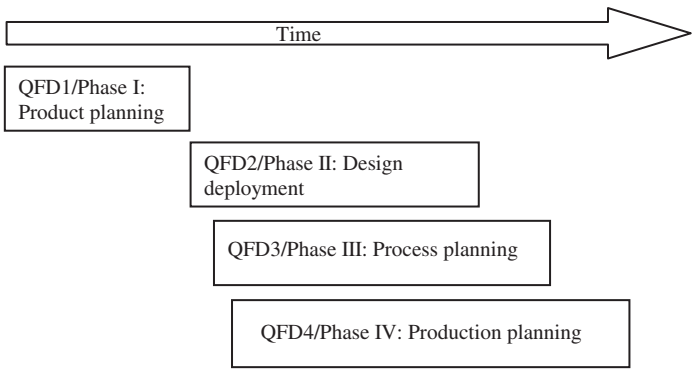


Figure 2.5
QFD phase timeline.

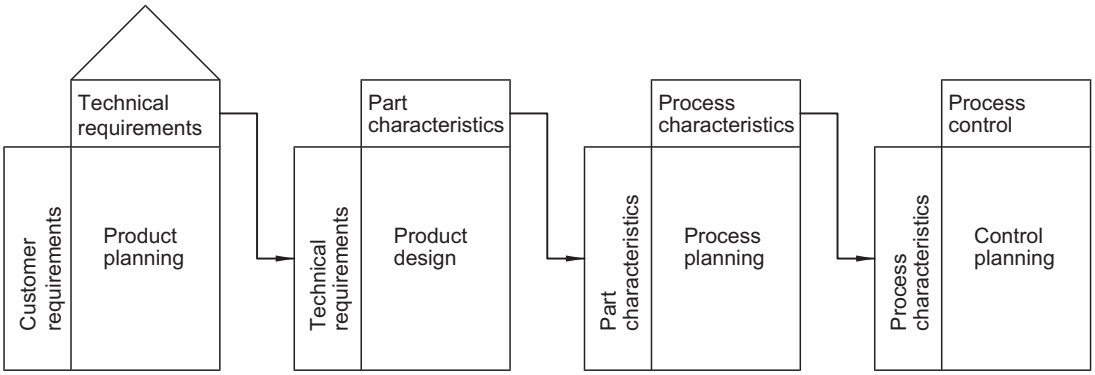


Figure 2.6
Generic QFD matrices.

commercial success is significant, and there is a need for diligence in mapping the project requirements. QFD1, compared to use of a standard PDS pro forma table, represents a more structured approach to requirement management. The QFD1 matrix allows an individual or a team to identify customer and engineering requirements and sets the relationships between these two groups of requirements. A generic QFD1 matrix is illustrated in Figure 2.7. This provides an understanding of the correlation between customer requirements and functions.

The QFD matrices follow a prescribed format and process:

1. Fill in the rows;
2. Fill in the columns.
   - Customer and engineering requirements are captured in the rows and columns of the matrix respectively.

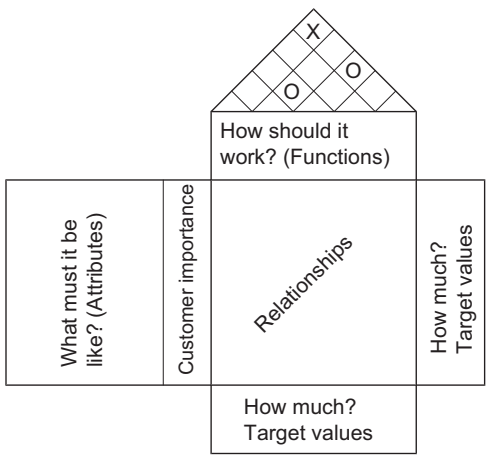


Figure 2.7
Generic example of a QFD1 matrix. o = synergy; x = conflict.

		Engineering requirements				
Attributes	**Customer importance**	Loosening earth	Removing slug of earth	Holding bulb	Inserting bulb	**Target values**
Sufficient torque for producing hole	40%	9	1			>4 N m
Price associated with a quality product	20%	4		4	4	<60 Euros
Long battery use period	30%	4	4			Battery must last >1 h in normal use
Short battery charge period	10%	1	1			<1 h
		>750 cm^3; diameter 75 mm; depth 100 mm	>400 cm^3 earth	Max diameter 60 mm; maximum length 90 mm	1bulb; upright; max depth 90 mm	
		How much				

Figure 2.8
Example entries for a partially developed QFD1 matrix.

- The importance of the customer requirements (product attributes) is specified in the column after the customer requirements.
- Quantitative values for the requirements are captured in the "how much" cells at the end of rows and columns, see Figure 2.7.
- The roof of a QFD matrix provides an understanding of the interactions and conflicts between the functions.
- It is noteworthy that functions have to be captured using the verb–noun format.

Example entries for a partially developed QFD1 table for a cordless lawn and garden hand tool with a few row and column entries are given in Figure 2.8. Details on how to complete the matrix are considered in Example 2.3.

A QFD2 matrix represents a systematic approach to translate engineering requirements (or product functions) into product parts. It provides an understanding of the correlation between functions and product parts. The QFD2 method is especially useful when there is need for a rigorous mapping between product functions and parts. A generic QFD2 matrix is illustrated in Figure 2.9.

The three principal steps for completing a QFD2 matrix are as follows:

1. Fill in the rows; start QFD2 by asking what the product must do and how much.
2. Fill in the columns; continue QFD2 by asking how does it accomplish whatever it must do.
3. Establish the relationships; using a numeric notation, identify the relationships between what it must do and how it does it.

Details of how to complete the matrix are considered in Examples 2.3 and 2.4.

QFD, with its emphasis on rigorous requirement management before solution development, represents a step change from a traditional way of approaching product development

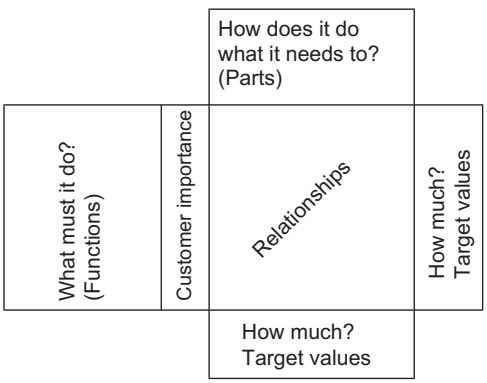


Figure 2.9
Generic QFD2 matrix.

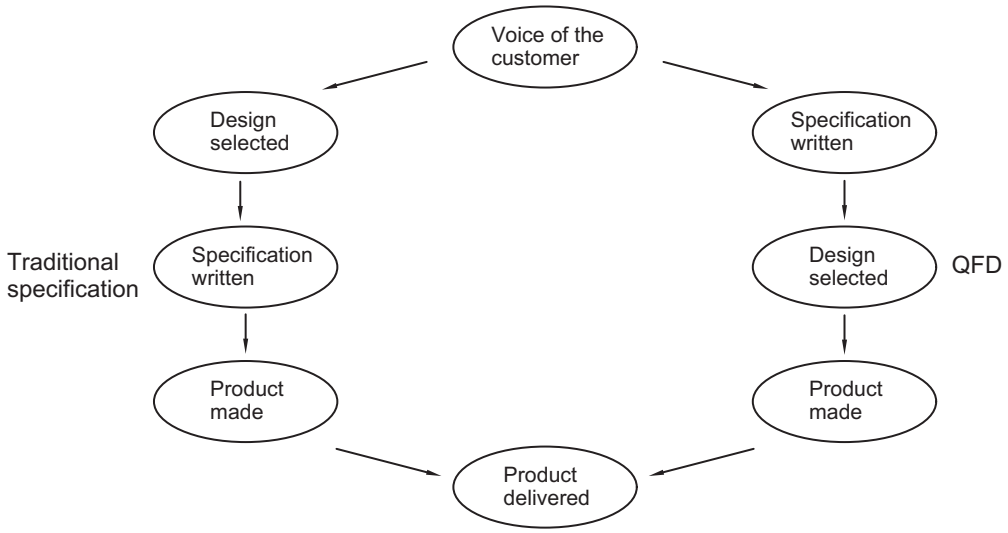


Figure 2.10
QFD versus a traditional industrial approach.

processes. The traditional design approach may have tended to develop solutions and commit to designs early in the product development process. Designers and engineers would not have ignored the VOC completely; but, by choosing designs too early there was an increased risk that the solution offered did not fully meet requirements. A product specification may have been written, but it may have been a retrospective record of the finished design. In this way, products would go into production and be delivered to the customer with relatively high probability of failing to meet customer requirements, i.e. be of low quality, not necessarily because they were manufactured badly but because the design did not provide the functions that the customer required. This "traditional" process is illustrated in Figure 2.10.

Example 2.3

In the 1970s, corrosion of car bodies and frames was widespread worldwide—in many markets you could expect many makes of car to have serious body rust and chassis rust, within a few years after manufacture. Toyota used QFD to resolve this problem in their new vehicle design programs.

The VOC was already loud and clear in this respect. In addition, the car companies were aware of it, but were failing to improve designs. The reason for this was that they were following a traditional engineering approach. "Engineering" knew the problem existed, and no doubt had a strong desire to fix it. The traditional approach, however, led to the early selection of a preferred design, which usually meant using more or less the same body

construction designs and techniques used on previous models. Attempts to address corrosion then became a "sticky plaster" approach, as an afterthought late in the development cycle. Perhaps more paint was specified, or more sealer, but the rust problem remained when the product got to the customer. By using QFD, the problem was firmly recognized at all levels of the company, including top management. In this way, the focus, discipline, and resources needed to solve the problem were generated. The insights from the QFD phases are considered in the following lists.

QFD1/Phase I—Product Planning:

- The problem was clearly understood in customer terms (e.g. what level of corrosion, if any, was acceptable within what time period);
- It was written into the PDS clearly, with both its importance to the customer and specific targets (e.g. types and locations of corrosion that could be accepted, after what period of use); and
- The requirement was embraced by senior management, who then had the commitment to follow through the rest of the process.

QFD2/Phase II—Design Deployment:

- Design concepts had to consider the requirement to address corrosion from the very beginning of design activity;
- Design deployment refers to all aspects of design activity in the product development process. It may be that the only way you are going to eliminate corrosion from designs is by iterations of hardware testing (e.g. both corrosion chamber testing of body components and full vehicle testing). This would be extremely expensive, but the senior management understanding of the importance of the requirement from Phase I means that resources can be justified; and
- Significant design characteristics are to be identified, e.g. material selection, body section/panel designs, paint type and thickness, and metal preparation parameters.

QFD3/Phase III—Process Planning:

- In parallel, the design engineers and process engineers developed the body designs and manufacturing processes (a two-way activity with designs affecting process selection, and vice versa), e.g. material selection, metal forming, metal cleaning, coatings, joining, sealing, and painting; and
- The significant process operations and parameters were defined, e.g. cleaning process durations, cleaning solution concentrations, and cleaning solution temperatures.

QFD4/Phase IV—Production Planning:

- The control requirements are defined, e.g. paint thickness inspection, sealer bead dimensions, and sealant curing hardness; and
- Maintenance requirements are defined, e.g. cleaning solution changeover intervals and spray gun cleaning schedules.

One of the fundamental principles of QFD is that every decision and selection made in the whole product development cycle, from design concepts right through to what things are inspected and controlled in manufacture, can be referred back to the customer requirements.

Example 2.4

An application of the QFD method to the design of a "me-too" hand-held hair dryer product, Figure 2.11, is explored here.

The planning and design of a hair dryer are documented in the QFD1 and QFD2 matrices in Figures 2.12 and 2.13.



Figure 2.11
Generic hand-held hair dryer.

The QFD1 matrix enables the relationship between the following two sets of elements, customer requirements (or product attributes) and engineering requirements (or product functions), to be explored. Customer requirements are captured in the rows of the matrix. In this example, the customer requirements are broken down in a tree structure including three levels. The root concepts are safety, performance, market competitiveness, and ergonomics, see Figure 2.12. Safety, for example, is subdivided into low risk to user and fire protection, overheating protection, and protection from burning. The last column of the matrix captures the "how much" values for the customer requirements. Protection from burning, for example, has a corresponding "how much" cell where it is noted that the maximum allowable temperature for the scalp is 60 °C. Another important feature of QFD1 is the column marked in gray, where the importance of the requirements is set based on discussions

			Importance	Draws air in	Contains rear air flow	Removes debris from air flow	Locates debris filter	Supplies front air flow	Heats front air flow	Guides front air flow	Accelerates front air flow	Lowers static p of front air flow to atmospheric	Insulates front housing	Dries hair	Positions front housing	Positions insulator	Offers holdability	Locates ac neutral supply	Acquires ac voltage	Allows power control	Allows control of a hot or cool burst	Guides switch assembly	Secures most handle and body parts	Allows the device to be hung up	Allows the cord to be stowed	Positions spring	Positions cool shot switch bracket	Positions cord	Protects cord	How much
Safety	Low risk to user	Fire protection	3.8			9			9									4	4										4	Compatible BS/ISO standards numbers …
		Overheating protection	3.8			9			9									4	4											Compatible BS/ISO standards numbers …
		Burning protection	3.8						9				9	9		4														Max temp (scalp): 60 °C
Performance	Quality	Positive switch positions	8.9																	4	4								Switch must engage positively for 99.99% of attempts
		Gentle and even heating	7.9						9																				h < 120 W/m^{2}K
	Efficiency	Dries hair quickly	1.5						9																				Power consumption: 1875 W
	Reliability	Seldom break down (reliable)	7.8					9	9									4	4										Last for 2000 h of drying time
	Robustness	Withstand rough treatment	10.5		9			9	9							9		4	4				9			4	4	4	Resists a drop test from 1.5 m
Market competitiveness	Appearance	Pleasing color-way	7.7																										>80% of focus group must acclaim color-way
		Pleasing shape/form/aesthetic	7		9		9								4	9	9												>80% of focus group must acclaim form
	Price	Low price	6		4		4	4	9						4	4													<£10GBP
Ergonomics design	Usability	Easy storage	3.2																					9	9				Must fit in 0.3 m by 0.2 m by 0.15 m storage volume
		Easy use at distance from the mains	3															9	9										Cord > 3 m
		Low noise	4					9																					Max noise: 72 dB
	Easy controls	Easy one hand use	2.1														9			9	9	4							80% of focus groups to acclaim single handed use
		Choice of air temperature setting	1.5						4												9	1							2 settings; Max *T* (air): 55°C
		Choice of air speed setting	0.8																	9		1							2 speeds; Max speed: 16.44 m/s
	Comfortable to hold	Comfortable hand grip	3.8														9			4	4								80% of focus groups to acclaim hand grip
		Well balanced	3.5					9	4								9												Hand moment < 0.3 N m
	Size and weight	Light weight	7.8	9	4	4		9	9	4	4	4	4				9	4	4	4	4									<0.5 kg
		Appropriate size	1.6		9										9	9	9													<220 mm L × 200 mm H × 80 mm W
		How much	100	MFR (kg/s): 0.05	Leakage minimal (<0.2 g/s)	Screen apertures: 1 mm	±0.2 mm on axial datum	VFR (l/min): 2685	Max T (air): 55°C	Within ±7 mm of axis	Max air speed > 16.44 m/s	101,325 ± 1000 Pa	Casing T < 25°C	Moisture content < 0.1 g per 10 g	±0.2 mm on axial datum	±0.2 mm on axial datum	80% of focus group to acclaim	±0.2 mm on axial datum	Voltage: 220 V	3 positions	2 positions	±0.2 mm on axial datum	±0.2 mm on axial datum	15 to 25 mm hook diameter	1.5 m < cord length < 2.5 m	±0.2 mm on axial datum	±0.2 mm on axial datum	±0.5 mm	To BS/IEC standards	

Figure 2.12
QFD1—product planning matrix for a hair dryer example. NB: All aspects should be quantified if possible in the how much column and row.

		Hair dryer												
		Body					Handle assembly							
	Importance	Debris filter	Front housing	Insulator	Rear housing	Motor casing assembly	Cap	Cord support	Handle casing	Hanger	Spring	Spring bracket	Switch assembly	How much
Draws air in	7					9								MFR (kg/s): 0.05
Contains rear air flow	4				9									Leakage minimal (<0.2 g/s)
Removes debris from air flow	7	9												Screen apertures: 1 mm
Locates debris filter	2				9									±0.2 mm on axial datum
Supplies front air flow	7					9								VFR (l/min): 2685
Heats front air flow	8					9								Max *T* (air): 55 °C
Guides front air flow	2		1	9										Within ±7 mm of axis
Accelerates front air flow	4			9										Max air speed > 16.44 m/s
Lowers static p of front air flow to atmospheric	4			9										101,325 ± 1000 Pa
Insulates front housing	7			9										Casing *T* <25 °C
Dries hair	7					9								Moisture content <0.1 g per 10 g hair
Positions front housing	2				9									±0.2 mm on axial datum
Positions insulator	2		9											±0.2 mm on axial datum
Offers holdability	5								9					80% of focus group to acclaim holdability
Locates ac neutral supply	4												9	±0.2 mm on axial datum
Acquires ac voltage	4												9	Voltage: 220 ± 20 V ac
Allows to control power	4												9	Voltage: 50 to 240 V
Allows to control cool or hot shot	4												9	3 positions
Guides switch assembly	2								9					2 positions
Secures most handle and body parts	2								9					±0.2 mm on axial datum
Allows the device to be hung up	2									9				15 to 25 mm hook diameter
Allows the cord to be stowed	2							9						1.5 m < cord length < 2.5 m
Positions spring	2											4		±0.2 mm on axial datum
Positions cool shot switch bracket	2										4			±0.2 mm on axial datum
Positions cord	2							9						±0.5 mm
Protects cord	2							9						To BS/IEC standards Nos.
How much		One plastic filter	One plastic front housing	One insulator	One plastic rear housing	One motor, etc.	One cap	One cord support	One handle casing	One hanger	One spring	One spring bracket	One switch, etc.	
	100													

Figure 2.13
QFD2—design deployment matrix for a hair dryer. NB: All aspects should be quantified if possible in the how much column and row.

with customers. In this example, the importance is defined distributing 100 points between the 21 customer requirements.

The engineering requirements are captured in the columns of the matrix. Removing debris from air flow, for example, is one of the main functions of a hair dryer. The last row of the matrix captures the "how much" values for the functions. The function removing debris from air flow, for example, has a corresponding "how much" cell where it is noted that the screen apertures have to be 1 mm maximum.

The elements in the rows and columns of a QFD matrix can have relationships of different strength. In QFD, the following three types of relationships are generally used:

- Low (1);
- Medium (4); and
- High (9).

A QFD2 matrix explores the relationship between engineering requirements (or product functions) and product parts. The functions are inherited from the QFD1 matrix, so the first column in Figure 2.13 matches the top row in Figure 2.12. The process to complete this matrix is similar to that illustrated for QFD1. However, the meaning of the matrix is significantly different, as the QFD2 matrix focuses on translating functions into physical components.

Some of the benefits of QFD are as follows:

- Ensures a systematic approach to the design process that is expected to decrease design and manufacturing costs and consequently time to market;
- Shifts the focus of an organization from engineering capabilities to customer needs;
- Makes requirements visible allowing an organization to prioritize them and to deliver them based on customer needs;
- Ensures that an organization captures a complete and accurate set of requirements;
- Allows key requirements to be considered throughout the process;
- Captures design data in a central format;
- Improves communication and sharing of information within a product development team; and
- Allows identification of "holes" in the current knowledge of a design team.

Some of the pitfalls or disadvantages of QFD include:

- Generating the information needed for each matrix is a complex and detailed process which demands commitment and resource; and
- The process of establishing the relationships between the rows and columns of a matrix is generally subjective.

2.4 Conclusions

To date, there is no single proven optimal approach to innovation. Differing innovation strategies have their proponents and detractors. The approach adopted for managing innovation and the development of a commercial enterprise is often dictated by historical precedence within a given industry or organization. The identification of the requirements that a design needs to fulfill is a fundamental aspect of the majority of product innovation

and management processes. This process of identification may involve a number of cycles of detailing with increasing veracity with each iteration as a result of more information becoming available. For some applications, it is not possible to clearly identify the requirements prior to a design concept becoming available for scrutiny. Once some concepts have been produced it is then possible to explore the innovation space and, for example, determine potentially new functionalities that a focus group or management team would not necessarily have envisaged prior to the insights enabled by the new emergent design.

For the majority of industry applications, common practice is to use detailed specification to define the scope for a new product development activity. This specification serves to both inform and limit the activity for the multiple stakeholders involved. A specification enables a design team to know the limit of functionality associated with a particular feature, subsystem, or system. It enables a design team to know when they have delivered sufficiently to fulfill the requirement. It enables management of an innovation process. Critically, if the specification has accurately identified and embodied user requirements and the commercial opportunity, then if a design and development program has met these requirements, a specification provides confidence that the commercial opportunity can actually be realized.

There are a wide range of approaches to specification. Some organizations start with a brief statement of need, which is then subsequently worked up into a formal detailed specification. A principle associated with detailed specification is to ensure that each aspect is quantified addressing the range of functionalities, i.e. technical, aesthetic, economic, and social functions. Detailed specifications can be very large documents occupying hundreds of pages. A specification for a new gas turbine engine for an aviation application might typically be a 500 page document. An alternative to a discursive or tabular-based specification is to use QFD. QFD uses a matrix approach to capture and quantify customer requirements and enable transfer of information through the typical phases associated with product development and delivery.

This chapter has introduced the principles of specification and provided examples of typical pro forma tables used for detailed PDS along with an introduction to QFD. Specification is typically used within a project management process to define given tasks to enable an assessment of whether or not the task has been achieved or completed.

References

Childs, P.R.N., 2004. Mechanical Design, second ed. Elsevier Butterworth Heinemann.
Pugh, S., 1990. Total Design. Addison Wesley.

Further Reading

Ficalora, J., Cohen, L., 2010. Quality Function Deployment and Six Sigma. A QFD Handbook, second ed. Prentice Hall.

Sharing Experience in Engineering Design. Curriculum for Design. Engineering Undergraduate Courses. SEED, 1985.

Web Sites

At the time of going to press the world-wide-web contained useful information relating to this chapter at the following sites:

www.qfdi.org/
www.qfdonline.com/templates

Nomenclature

The following symbols have been used in this chapter:

PDS product design specification
QFD quality function deployment
VOC voice of the customer

CHAPTER 3

Ideation

Chapter Outline

3.1 Introduction 52
3.2 Creative Process 53
3.3 Brainstorming 56
3.3.1 Intellectual Property Rights and Brainstorming 58
3.3.2 Brainstorming Methods 59
3.3.3 Flip Chart 60
3.3.4 Post-it Brainstorming 62
3.3.5 Alphabet Brainstorming 64
3.3.6 Brainwriting 64
3.3.7 Grid Brainstorming 66
3.3.8 Group Dynamics 67
3.3.9 Facilitation 68
3.3.10 Evaluation 69
3.4 Creative Problem Solving 70
3.5 SCAMPER 74
3.6 Create Process 75
3.7 Morphological Analysis 77
3.7.1 Pallet Case Study 79
3.7.2 Applications of Morphological Analysis 82
3.8 Standard Solutions 85
3.8.1 Contradiction and Tradeoff 88
3.8.2 The 40 Principles 91
3.8.3 Contradiction Matrix 106
3.9 Boundary Shifting 113
3.10 The Creativity and Innovation Engine 115
3.11 Conclusions 117
References 118
Further Reading 119

Abstract

Design relies upon the generation of alternative ideas and their effective evaluation in order to ensure that attention is directed in a worthwhile manner. This chapter explores a number of creativity tools that are widely used for the generation of ideas, including flip charts; post-it; alphabet and grid brainstorming; the creative problem solving process, also known as CPS; morphological analysis, a matrix combinational creativity tool; and TRIZ, the theory of inventive problem solving. The chapter includes consideration of group dynamics and the evaluation of ideas as well as approaches for encouraging the realization of value from creativity through innovation.

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00003-4

Copyright © 2014 Elsevier Ltd. All rights reserved.

3.1 Introduction

Ideas mean money. Money means business. These notions are familiar to any entrepreneur and innovator. In order to come up with ideas you either need to embrace creativity directly or surround yourself with people who do. On its own, creativity is not enough to sustain a business. We are familiar with bright, able and creative individuals who are endlessly generating new ideas but are seemingly unable to turn these ideas into reality or make a living out of them. The effective exploitation of an idea is the realm of business, and the skill sets for creativity and commerce are often at odds with each other. If we endlessly generate ideas but do not put in the effort to develop them sufficiently so that an idea is robust—combined with an appropriate business strategy to exploit the idea and turn it into a revenue stream that is sufficient to pay for the development of the product or process concerned as well as the ongoing business costs for its marketing, production, distribution and future development—then the ideas will fail to benefit the originators, employers or sponsors.

The need for robust ideas that have been appropriately developed is significant. If there is a flaw in a product, it is possible that a customer will accept it for a while. We have all put up with inadequate devices, products and processes in our lives; the product martyr is the dream of the warranty department. In a competitive world, however, it is only a matter of time until we are wooed by a competitor's products that appear more reliable and fulfill the desired function to a higher standard. Remember that for every unit of currency spent on marketing to attract a customer for the first time, it can cost 10 times that amount to win back a customer who has become dissatisfied with the quality of a brand Kochan (1996).

It is the blend of methods, work practices, culture and infrastructure that makes an organization succeed. IDEO, the design and innovation consultancy, expounds upon the importance of understanding, observing, visualizing, evaluating, refining and implementing. We are all familiar with the phrase "a picture is worth a thousand words." In idea generation, it is important not just to use pictures but also to get physical and explore ideas in three dimensions where appropriate. In idea generation, it is helpful to leave our performance anxieties behind and, as well as utilizing words and symbols, expand into using imagery and three-dimensional prototyping. This is embodied in the IDEO process of ideation: generating, developing and communicating an idea. As part of this process, the production of prototypes is emphasized. These may be crude representations of the idea, but they provoke discussion and engagement in design; this can lead to significant iterative and step improvements in its implementation. In the case of IDEO, this is combined with capacity, expertise and experience of innovation enabling the fast development and trial of ideas.

Observation is important. If you ask someone what he or she thinks of a product or service, the response may be that it is fine or very good. Part of the reason for this response is that the individual might lack the vocabulary to express an opinion. In response to a question about a

meal, most of us might say that it was very good. If we are enthusiastic cooks at home, we may be able to express our opinion more specifically. It is for this reason that surveys and market-based information need to be carefully assessed or significant information may be lost or never accumulated in such processes. Observation provides key insights into the way customers actually use a product and can reveal particular issues that surveys miss. Observation is a key activity in user-based design, and astute observers tend to produce ideas as a result of their observations because they identify needs and are able to suggest methods to overcome these needs.

Our understanding of creativity stems not just from the insights from successful companies but from the endeavors of individuals throughout history, from specific research into creative processes and people and from the conditions that have enabled or stymied activity. The creative process is introduced in Section 3.2 and a variety of approaches to brainstorming are presented in Section 3.3. A process based on implementation of the principles of brainstorming called Creative Problem Solving (CPS) is described in Section 3.4. A useful ideation tool called SCAMPER is described in Section 3.5. A process developed in industry showing the integration of ideation techniques called the Create Process is introduced in Section 3.6. A combinational creativity tool called morphological analysis is described in Section 3.7. The theory of inventive problem solving, a systematic inventive tool that exploits prior experience, is introduced in Section 3.8. Boundary shifting is described in Section 3.9.

3.2 Creative Process

In their review of research on creativity, Goldenberg and Mazursky (2002) identified that many investigators have used one of three approaches to explain how creativity happens: the creative person, the creative process or the creative idea.

- A creative person is able to choose high-quality ideas from a wider set of ideas. In order to solve problems, a creative person often focuses on finding methods to generate a number of ideas. Some methods are suggested: brainstorming, synectics, random stimulation and lateral thinking.
- Research at the end of the 1980s showed that restricted processes of thinking are more reliable for creativity than unrestricted scopes.
- Some research showed that through the use of the previous creative ideas it is possible to formulate new, successful, creative ideas. For example Genrich Altschuller's TRIZ (Altshuller, 1984) uses a systematic method, based on previous successful inventions, to guide "ordinary" engineers toward creative solutions. After years of research and reviewing more than 200,000 successful patents and invention disclosures, Altshuller and his colleagues defined 40 patterns of inventions that could inspire engineers to generate creative solutions. In some of the modern TRIZ software platforms, this analysis has been extended to over two million patents.

As noted from Wallas' work (1926), the creative process can be modeled in phases:

- Preparation: becoming immersed in a problematic and interesting set of issues;
- Incubation: churning ideas around consciously and unconsciously;
- Illumination or insight;
- Verification or evaluation.

Such phases need to be combined and followed by elaboration in order to work the idea up into something that can be realized and effectively used. It is one thing to have a good idea; it is quite another to turn it into reality. As Edison noted, "Genius is one percent inspiration and 99 percent perspiration." Of course such a model of the creative process is a generalization and simplification of the complex processes involved, which will differ from person to person and topic to topic.

To solve a problem creatively, it is necessary not only to understand the issues but also to use creative thinking. A problem can be a conflict or a tension that needs to be resolved. Some problems are well formulated, with everybody aware of what the issue is, but a solution is missing. Problems can arise from experience, requirements of the domain and social pressure. In some cases stumbling across a new, unexpected problem can lead to a creative discovery without preparation. Examples of this sometimes occur in scientific research where investigation of unusual results provides new insight.

The Design Council (2007) reported a study of the design process in 11 leading companies and identified a four-step design process called the "double diamond" design process model: discover, define, develop and deliver. In Figure 3.1, the divergent and convergent stages of the design process are indicated, showing the different modes of thinking that designers use.

Coming up with new ideas is difficult. This is why creativity is acclaimed and held in high esteem in our societies. Part of the reason that creativity is difficult is that with a population of

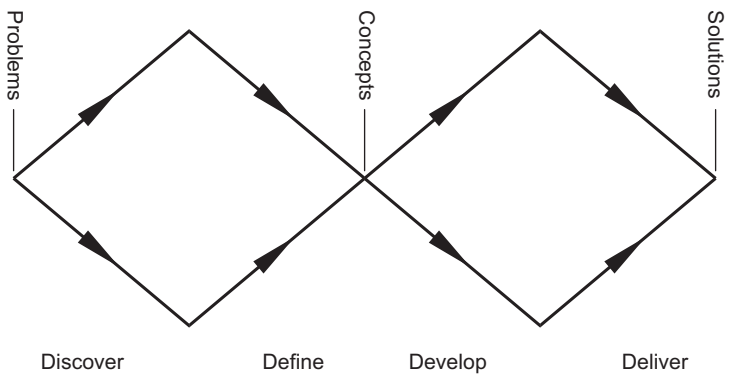


Figure 3.1
Schematic describing the design process. *Design Council (2007).*

billions thinking so much so fast, so much has already been done. Jacob Rabinow (Csikszentmihalyi, 1996) notes that you need three things to be an original thinker: a tremendous amount of information, willingness to come up with ideas and put the effort into the process and the ability to discard the weaker ideas. Creativity tools provide a means to augment innate generative activity. The scope of creativity tools available (e.g. see Lidwell et al., 2003; the Create Project, 2005), with hundreds having been identified, are presented in Table 3.1. Different creativity tools have been found to be suitable for different personality

Table 3.1: Some examples of creativity tools.

"Post-it" and "Grid" brainstorming (Osborn, 1963; Michalko, 2001)	In Post-it, participants record ideas on Post-it notes, which are collected, reviewed and analyzed. In grid, a participant records some solutions to a problem on a grid that is completed by other members of the group. These tools encourage simultaneous generative activity, but some participants may remain reluctant to share an idea for fear of ridicule or loss of ownership. This latter concern may be addressed by implementation of the IPR protocol.
Checklists (Osborn, 1963)	A series of brief questions and statements is used to stimulate creativity when it is proving difficult to think in original ways. Can be used for idea generation *and* evaluation but is a systematic method that may not appeal to all personality traits.
Lateral thinking (de Bono, 1970)	Solutions are proposed by looking at a problem using random associations, provocation, challenging current solutions and divergence. Requires both curiosity and confidence, and encourages a wide range of alternative solutions. Robust evaluation of ideas is necessary to identify worthwhile concepts to develop.
Mind mapping (Buzan and Buzan, 2006)	Connections between associated pieces of information are emphasized by clustering the information on a visual map: this can stimulate creativity. Some people are hesitant to reveal a perceived weakness in sketching, but this can be overcome with practice.
Six hats (de Bono, 1999)	Parallel thinking process in which team members wear colored hats representing data, creativity, positivity, feelings, criticism and control. The approach can minimize conflict as well as encourage participation and consideration of a problem from a wide range of perspectives.
Morphological analysis (Zwicky, 1969; Childs, 2004)	A matrix-based tool in which a problem is broken down into component parts and a range of approaches suggested for each of these elements. Encourages combinations of features that otherwise might not have been considered, but a large quantity of combinations generated means that good ideas can be overlooked. Weighting criteria should therefore be used to guide the selection of solutions.
Synectics (Gordon, 1961)	Exploits our capacity to connect apparently irrelevant elements to spark new ideas and solutions. The approach helps participants break existing mindsets and internalize abstract concepts, but is time-consuming, requiring practice and expert facilitation.
TRIZ (Altshuller, 1984)	Russian acronym for the Theory of Inventive Problem Solving. Provides a framework and toolbox for systematic, inventive problem solving but is sometimes viewed as complicated and difficult to use; this can be overcome by sustained use and practice.

traits and different applications, so care needs to be exercised in the selection of a particular tool (see Yan et al., 2013).

Most creativity tools try to enhance

- Fluency: the quantity of responses;
- Flexibility: ideas that are distinct from each other;
- Originality: the level of uniqueness of the ideas generated.

A creativity tool will not produce ideas. Instead, it can be used to assist in the generative process. Most creativity tools can be used at any stage in a problem solving process and tend to focus mainly on problem exploration, idea generation and concept evaluation as follows:

- Problem exploration: fishbone diagram, question asking, quality function deployment;
- Idea generation: brainstorming, checklists, the gallery method, lateral thinking, morphological analysis, TRIZ and SCAMPER;
- Concept evaluation: SWOT analysis, Taguchi methods, value engineering.

To be effective, a creativity tool needs to influence the thinker at the time of idea generation. Human memory and the process of information retrieval are crucial in this process. Memory can be defined as the ability to store, retain and subsequently access information. Models of memory include the simple multistore memory model of Atkinson and Shriffin (1968). The capacity of short-term memory is limited (see Miller, 1956; Poirier and Saint-Aubin, 1995). Miller (1956) proposed that the capacity of short-term memory is limited to approximately seven items plus or minus two. Research by Poirier and Saint-Aubin (1995) indicated that this figure is influenced by a number of factors; Cowan (2001) has shown that the duration of short-term memory retention is between 2 and 30 s. Regardless, the limited capacity presents a significant challenge to problem solving. If trial and error is used to tackle an issue, the available short-term memory is likely to be dominated by information relating to the problem definition, which is rarely useful for solving it. Most solutions can be characterized by the connection of diverse thoughts from previous experience. Creativity tools function by ensuring that a problem can be understood in relatively simple terms, thereby occupying only a fraction of short-term memory, supplying cues to make the search of long-term memory more efficient and providing cues to ensure refreshing of short-term memory and thereby retention of key information.

3.3 Brainstorming

Brainstorming typically involves a multidisciplinary group meeting to propose and generate ideas to solve a stated problem. Certain forms of brainstorming can also be undertaken individually or with other people with a time lapse between communications. The emphasis

within brainstorming is on quantity rather than quality of ideas, and criticism of another person's idea is strictly forbidden. Following a brainstorming session a review should be undertaken to identify ideas or aspects of the concepts generated that have merit for further consideration. A typical outcome is that aspects of a few of the concepts generated can be combined to form a solution to the problem that gives a better overall solution than any one of the individual concepts from the original session. Brainstorming is a familiar concept within industry and education that is typically associated with a facilitator or scribe standing in front of a group, rapidly writing down ideas as they are called out. However, this description does not necessarily describe either good practice or the scope of brainstorming tools for the effective encouragement of idea generation.

Osborn (1963) observed that results of generative activity improved if there was deferment of judgment and also that quantity breeds quality. These principles have been embedded in a series of rules for brainstorming activities. During a brainstorming session, criticism, evaluation or judgment of an idea is forbidden. This encourages people to come up with and offer ideas without fear of criticism, ridicule or judgment. When sharing an idea it needs to be recognized that a person is sharing something intimate, a set of thoughts. Respect for this is important. In a critical environment people are less likely to offer ideas, or they may limit the ideas they offer to ones that they have evaluated and think reasonable. There have been many ideas that at first airing may not seem viable. But with the benefit of attention from the collective mind of a group it can be built upon, improving the viability of the idea. If the original idea is not shared, this is not possible. It should be noted that individuals will develop coping strategies within a critical environment. Many offices and organizations stumble forward in the midst of a cut-and-thrust, competitive and often highly critical working environment. In such cases it is common to hear comments along the lines of

> *why should I share an idea when all that will happen is:*
> *the idea will be stamped on with person A, for example, saying it has been done before or won't work;*
> *someone else will take or claim the idea and all the credit for it;*
> *not invented here syndrome will be invoked, and the idea will be ignored because it was not suggested by the people in charge (or who matter);*
> *managers won't like it; been there, done that.*

These scenarios are commonplace, and strategies to avoid them include the following:

1. Explaining the rationale and rules of brainstorming prior to the session. This can be achieved by a premeeting to define guidelines for the participants.
2. Using a system to document the originator of an idea such as associating an idea with an individual by including their initials alongside the idea.
3. Using a quasiquantitative technique to evaluate the ideas generated, such as an evaluation matrix (see Section 3.3.10).

4. Explaining to managers the benefits of brainstorming as well as the fact that ideas will occur under the manager's direction and therefore will reflect positively on the manager.

There are several principal rules commonly associated with brainstorming:

- Brainstorming should be a group activity undertaken by an interdisciplinary team.
- There should be no more than 10 people present.
- No criticism or mockery of any idea, statement or individual is allowed.
- Flip charts, Post-its, cards, marker pens, pencils and paper should be provided.
- The design brief should be clearly stated.
- A set period of time should be allocated.
- An individual should be identified to introduce the brief.

Tom Kelley of IDEO suggests that effective brainstorming sessions should not take longer than 60 to 90 min. The IDEO organization uses brainstorming regularly, with each person involved several times a month. Brainstorming is the idea engine for many industries; over 70% of business leaders use it in their organizations (Anderson et al., 2004). There is a substantial body of evidence providing convincing data showing the effectiveness of brainstorming and creative tools based on divergence and convergence when appropriately implemented. Evidence for the effectiveness of brainstorming and associated tools for problem solving, techniques for using them and strategies for their selection and implementation are reviewed by Treffinger et al. (1994), Isaksen and DrSchryver (2000), Torrence (1972) and Rose and Lin (1984).

3.3.1 Intellectual Property Rights and Brainstorming

During a brainstorming session many ideas can be generated and shared. Some of these will be shared during the session as a result of prior consideration over many years. Others may be sparked by interactions within the session itself. Just as in a contractual relationship, it is important to establish what mechanisms will be implemented to deal with any potential intellectual property rights (IPRs). Options include Chatham house rules, direct attribution, equal attribution and company retention of IPR. Regardless of the type of attribution applied within a brainstorming session, care needs to be taken regarding the assertion or claim of a company or organization to ideas developed.

- Chatham house rules: There is no attribution to a particular individual for anything that is said or shared within a discussion.
- Direct attribution: Each participant's initials or moniker are associated with each idea. This can be readily implemented on Post-its, for example where each participant writes their ideas on a single Post-it along with, say, their initials. These can then be scanned following the sessions and embedded within a database.
- Equal attribution: Any IPR reward is divided equally between all participants.

- IPR is retained solely by the organization or client. Such a system may be compatible with the terms of employment.

At the start of a brainstorming session the rules for IPR need to be stated.

3.3.2 Brainstorming Methods

Osborn suggested that it was important that "especially in approaching a creative problem we should give imagination priority over judgment and let it roam around our objective. We might even make a conscious effort to think up the wildest ideas that could possibly apply." The emphasis on quantity in brainstorming increases the number of alternative solutions and hence the potential for a viable and worthwhile end result.

The description of brainstorming given here suggests that there is much more to the process than a simple free-wheeling event where people turn up and share ideas. It is necessary to prepare for the session. Work needs to be undertaken to understand the task at hand or the problem to be solved. This is compatible with the need to ensure that the task can be understood in simple terms so that our limited short-term memory is not occupied by multiple facets of the problem. By freeing up our short-term memory, we enhance our ability to access potential ideas from our long-term memory. In addition, participants need to be forewarned of the task to enable them to develop and rehearse sharing of these ideas prior to a session. Necessary preparation by the organizer prior to the session also includes booking facilities, determining appropriate participants and distributing the brief.

Brainstorming can be used for both individual and group efforts (Isaksen, 2000). A potentially fruitful approach is to generate the core idea by yourself and, subsequently, to leave the development phase to the others.

Rule-breaking and wacky ideas are welcome in brainstorming. It is the rule-breaker who might discover the short cut in a journey by doing something normally forbidden. Although such behavior might rightly be frowned upon in some contexts, the possibilities revealed may provide useful information that had not previously been considered. In idea generation, the concepts of rule-breaking and boundary shifting are useful and should be encouraged. Similarly, wacky ideas should be welcomed. Some suggestions can make a team crack up with laughter and still be amusing several days later. Whether these ideas have merit in terms of the task will be determined at the evaluation phase.

A range of creativity tools embedded with the principles of "deferment of judgment" and "quantity breeds quality" have been developed. Some of these are group based, involving direct verbal interactions between those present; others are silent, known as brainwriting methods. Some are hybrids. Flip chart, Post-it and alphabet brainstorming techniques are described in Sections 3.3.3–3.3.5. Brainwriting and grid methods are described in Sections

3.3.6 and 3.3.7. Although each method is presented in isolation, a typical idea generation session may comprise a sequence of techniques with, for example, a period of flip chart brainstorming, followed by Post-it, and then a warm-up prior to using grid. A warm-up is a mental exercise designed to encourage a particular behavior such as suppressing mental editing.

3.3.3 Flip Chart

The flip chart technique is the poor relation of brainstorming, as it does not encourage simultaneous contributions and therefore does not fully embrace the power of the collective mind. Once the brief has been shared with participants, the technique involves the facilitator inviting suggestions for its solution. Traditionally the facilitator may use a flip chart, whiteboard or large sheet of paper to record the ideas; see Figure 3.2. Alternatively, these can be recorded on a computer system. A typical procedure for flip chart brainstorming follows:

1. Prepare for the session by ensuring that participants are familiar with the principles of brainstorming and have undertaken relevant training as appropriate. Ensure that the brief defines the problem in an accessible manner. Distribute the brief to the group prior to the brainstorming session. Ensure that the room has been booked and the relevant participants have been invited. Make sure that a flip chart, whiteboard or equivalent and relevant markers are available.
2. Organize the group so that they are comfortably able to see the display medium.
3. Remind participants of the brief.
4. The facilitator invites solutions to the brief for a period of time. During this period no criticism of ideas is allowed. If criticism does occur, this needs to be dealt with by the facilitator; see Section 3.3.9.
5. Either the facilitator or a scribe records the suggestions from the group on a flip chart or another recording medium. If it has been agreed that attribution of ideas is to occur, it is



Figure 3.2
Flip chart brainstorming.

important that the facilitator record the moniker of the relevant participant near the idea. If the participants are unknown to the scribe, badges or name tags can be worn to assist in identification. In addition, it is useful if a sequential number is associated with each idea. This provides a means to refer back to a specific idea in a long list.

6. The time for a flip chart session can be until the frequency of ideas being suggested begins to reduce or until it drops to an uncomfortable level for the group. At this point the facilitator can either suggest that it is time to review the material suggested or alternatively use a warm-up, group exercise or series of props to re-invigorate the group prior to another session using the flip chart to record ideas.
7. Once ideas begin to become less frequent, the facilitator can either suggest that it is time to review the material suggested or use a warm-up, group exercise or props to re-invigorate the group prior to another session using the flip chart to record ideas. Groups tend to be capable of determining when it is time to move on.
8. Once a sufficient quantity of ideas have been generated by, say, two 10- to 20-minute sessions, participants can be invited to review the material to see if additional ideas are inspired by, for instance, the combination of two of the suggestions.

Flip chart is a commonly known brainstorming technique. However, a number of factors can combine to reduce its effectiveness as a creativity tool. These include:

- Participants finding it difficult to get a word in;
- Fewer outgoing participants not feeling comfortable or not being able to make suggestions out loud;
- People not being willing to submit ideas within such a session for either selfish reasons, such as "Why should I bother to help this organization?" or because they are not convinced that they will receive due recognition for the idea;
- The facilitator not hearing/taking notice of an idea;
- The facilitator misinterpreting an idea;
- The facilitator failing to pursue the direction of an idea;
- Interruption of each other's thought processes by the constant flood of input to the session.

These factors have been cited as being responsible for the output of brainstorming, in terms of number and quality of ideas, not being as effective as the collective output of individuals working on the task concerned on their own, according to Furnham (2000). In addition, some facilitators choose to implement the technique in such a way that further reduces its effectiveness, such as dominating the session, suggesting too many of their own ideas, dismissing a particular direction of thought and suggestions too early with implied criticism of the ideas, favoring comments from certain individuals and failing to encourage wider participation.

The flip chart approach can serve as a good entry activity to a brainstorming session, but should be used sparingly. It can, for example, be used to determine the important areas of a

problem. The group can "flip chart" brainstorm these areas for several minutes. Following this, a more structured brainstorming technique such as Post-its or grid can be employed, which encourages simultaneous mental activity and contribution to the task for all participants.

3.3.4 Post-it Brainstorming

Post-its are widely used in brainstorming. They provide a readily accessible means to record an idea and then order or collect similar ideas together. The asynchronous nature of flip chart brainstorming can be overcome by asking all participants in a brainstorming session to record their ideas on Post-its, with one idea per Post-it. After a set period of time, say 10—20 min, participants can be invited to consider a stimulus, such as the use of found objects, to inspire ideas or engage in standard mental warm-up exercises. A further session of recording ideas on Post-its with one idea per Post-it can then take place. Following this, all of the ideas generated can be collected together, and an attempt can be made to review and possibly organize them. As Post-its can be peeled off a surface and reapplied several times, different classifications can be explored. It should be noted that an idea generation session using Post-its may not be quick and can take an hour or two.

A typical session is outlined below, but many variants on this are possible:

1. Ensure that the participants are familiar with the principles of brainstorming and have had relevant training.
2. If the task to be tackled is not well understood, hold a focus group to define the brief.
3. Distribute the brief to the participants in advance of the session.
4. Prepare for the session by ensuring relevant materials are available, including sufficient Post-it pads (different colors or shapes can be used for each participant).
5. Organize the group so that they are comfortably able to record their ideas.
6. Remind participants of the brief.
7. The facilitator invites solutions to the brief for a period of time, say 10—20 min, with participants recording one idea per Post-it.
8. After about 20 min, the facilitator can halt recording of ideas and either undertake a review of ideas generated to date or prompt the session with a warm-up or structured thought process to inspire additional ideas.
9. A further idea generation session using Post-its can then take place for perhaps another 20 min.
10. The ideas generated can be reviewed collectively. This can be done by means of asking each member to share a few (two or three) of their ideas with the group. This allows the facilitator to identify some categories into which the different ideas can potentially be placed. The group can then be invited to cluster their different ideas into the categories selected. This can be done by placing the Post-its on whiteboards, flip



Figure 3.3
Random Post-its.

charts or appropriate wall or table surfaces. A camera can be used to record the grouping of ideas.

11. During the review, ideas suggested by the group should be recorded on a flip chart.
12. The review may provoke discussion, and the facilitator can decide whether a further round of Post-its or another technique is appropriate (Figures 3.3 and 3.4).



Figure 3.4
Organized Post-its.

A range of software systems is available to emulate and facilitate group brainstorming sessions. Electronic notice boards and forums can be used for a group to post ideas as well as organize and collate them. Such systems allow groups that cannot physically be present to participate in idea generation and can also enable individuals to feed ideas into a session at disparate times.

3.3.5 Alphabet Brainstorming

In this brainstorming technique, participants are invited to work through the alphabet, letter by letter in sequence, suggesting a few ideas for each letter that are relevant to its potential suggestion. Once a few ideas beginning with the letter A have been suggested and recorded, the group then moves on to consider a few ideas beginning with the letter B and so on, all the way to Z. Alphabet brainstorming serves to encourage consideration of ideas across a wide range of possibilities. At the end of an alphabet brainstorming session, which might take about 45 min to an hour, the group will typically have between 75 and 150 ideas. Alphabet brainstorming is a good method to help a group widen their horizons and develop a large range of options to explore. A typical sequence, in this case for possible solutions to climate change, has been listed in Table 3.2.

Care needs to be taken in introducing alphabet brainstorming to a group. As most of us gave up doing alphabet sequencing in primary school, it is possible that one or more members of a group may ridicule or resent an approach based on such a simple concept. The technique can be introduced by explaining not only the procedure for its implementation but also its rationale.

As with any brainstorming technique, the session should be followed up with a review of the ideas generated. If necessary, this can be done immediately following the session, exploring opportunities to combine ideas and seeing whether any of the suggestions provoke additional avenues for exploration. Following this, an evaluation of the ideas can be done.

3.3.6 Brainwriting

The weakness of flip chart brainstorming with one person's thoughts interrupting another's in a serial manner can be overcome by one of a number of techniques based on parallel activity. Brainwriting is the phrase commonly associated with a series of techniques developed by Horst Geschka and colleagues at the Batelle Institute in Frankfurt. A typical brainwriting session comprises the following phases:

1. The facilitator introduces the session and if necessary the principle of the technique.
2. A discussion of the problem to clarify the issues to be addressed.
3. The participants are grouped in a circle, perhaps around a large table.
4. Each participant in the brainwriting session is given blank cards, such as index cards, and is invited to record their ideas on them, with one idea per card.

Table 3.2: Example of ideas generated in an alphabet brainstorming session. InQbate, University of Sussex 2008, Design Solutions for Climate Change.

Letter	Suggestions
A	Abstinence; air travel reduction; agricultural reform
B	Biofuel; bioenergy; better use of electricity
C	Car pooling; carbon budgets; carbon taxes; combined heat and power; consumption charges
D	Distributed microgeneration; diesel substitutes (biodiesel)
E	Energy efficiency; education; emissions policy; ethics and equity
F	Fuel cells; fuel farming
G	Geothermal energy; global public good
H	Hydropower; hybrid vehicles; hydrogen fuel cells; hydrogen economy
I	Insulation; incentives; ice caps
J	Joined-up thinking; justice
K	Knowledge
L	Light-emitting diodes; low carbon growth; low-energy lifestyles; localization; lichens; lifestyle
M	Methane; many approaches; multidisciplinary; motivation
N	Nuclear power; natural resources; not for profit/payback period
O	Oil; oil substitutes; organic; ocean power
P	Public transport; petrol; packaging; population; power; penalties
Q	Quotas; quality of life; quadruple wind power; quantify product life
R	Resilience; robustness; rate; regeneration; reflectors
S	Solar; sewage gas
T	Technological quantum leap
U	Uncertainty; usage
V	Volcano emissions; vernal pools
W	Waste; wave power
X	Xanthium and xyridales (grow lots); Xerox solar panels
Y	Yew trees; yield enhancement; yachts for transport
Z	Zero emissions; Zambezi hydroelectric power; zonal policy

5. As each member in the group silently generates ideas, they then pass them in one direction, say, to the right, in the group.
6. The intent of the cards is to stimulate ideas. On receipt of a card, a participant reads it to see if it inspires an idea. Each participant can then write down the idea inspired by the stimulus and then pass this new card on to the right.
7. Within a few minutes, several idea cards will be being passed around the group.
8. After about 20–30 min, all of the cards generated can be collected.
9. With guidance from the group, the facilitator can then sort the cards into different categories of ideas, with a generic title given to each category.
10. Each participant in the session can be invited to evaluate the ideas with a traffic light system. Each participant can be given a sheet of dots; they can allocate a set number of these to ideas that they think warrant further consideration. For example, if each person is allowed to use five dots, they can chose to place all five on one idea or divide them between cards.

3.3.7 Grid Brainstorming

The grid method of brainstorming enables all participants to contribute simultaneously. In grid brainstorming each person in the group is given a grid, perhaps a piece of A4-sized paper divided into nine rectangles. The facilitator then explains the brief and invites the participants to develop three potential, partial or full solutions to the problem—one in each sector of the first row. The solutions developed need not fully resolve the problem and can be simply ideas that seem relevant. The solutions can be portrayed using a mixture of sketches and words, an example of which is illustrated in Figure 3.5. Once a row with three part-solutions has been completed, the participant places their sheet of paper in the middle of the group. Another group member, if they have also finished their row, can then pick up a set of solutions that they have not been working on and commence developing solutions in the next row of the grid, based on the ideas developed by the previous person. The method has many similarities to the game Pictionary. In essence, once the participants are working with a set of ideas partially developed by someone else, each person is able to blend at least two sets of thoughts: their own and those of the previous contributor. The grid method allows the participants time to think at their own pace and is helpful for enabling those who are more reticent about participating to contribute.

The different pace at which participants work can present some challenges in running a grid session. Some people may finish their row of ideas very quickly, while others can take longer. A number of strategies can be employed to deal with this. The facilitator can prepare some sheets in advance with the first row completed with their own ideas. If someone has finished very quickly, then such sheets can be handed out in order to keep all participants occupied on the task.



Figure 3.5
Grid brainstorming.

The grid technique can be summarized in the following steps:

- Each member of the group is given a grid divided into, perhaps, nine rectangles.
- The brief is communicated by the facilitator.
- Participants start by sketching three ideas in the top row.
- When finished, each participant places their sheet in the middle for someone else to work on the next row, developing the ideas further.
- Once the sheets have been completed, the ideas generated can be reviewed to see if additional avenues to explore in the session arise.
- Following the session, the sheets can be collected and the ideas evaluated.

Many variations on the grid technique are possible; for example, a 3 by 4 grid with the top row seeded with suggestions; a 1 by 4 grid with the top row seeded with a suggestion.

The grid technique is a particularly powerful brainstorming method. It allows for the collective power of a group's minds to be employed. All participants are able to contribute in simultaneous mental activity applied to the task.

3.3.8 Group Dynamics

The ideal brainstorming group should be derived from diverse disciplines and backgrounds and should include experts and nonexperts. For example, a brainstorming group looking at new marketing ideas could comprise a customer, someone from manufacturing, an engineer, a receptionist, as well as staff members from the marketing department. A brainstorming group for the design of a new product could comprise a chemist, a materials specialist, a manufacturing and a mechanical engineer, a personnel manager and a legal representative as well as a product designer. Participants in a brainstorming session need to regard each other as equals. If a participant feels or is made to feel that his or her opinion is not as valid as someone else's within the group, that person is less likely to contribute effectively. Alternatively, if someone feels their status means that their ideas are more significant than another's, it is possible that this person will draw attention to themselves and become the focus, thereby inhibiting the creative potential of the group. Thus, for the purpose of a brainstorming session, if a person is normally used to having their view prevail in the workplace, this privilege needs to be temporarily surrendered. Similarly, if a person is used to withholding a view or idea or keeping silent, then again this privilege needs to be abandoned for the brainstorming session.

Criticism is a fundamental part of many of our developed and learned thought processes. Some of us have great difficulty in suspending our critical natures during idea generation sessions. In brainstorming, suspension of judgment is very important in order to enable the quantity of ideas to be generated, taking advantage of the collective mind virtually formed by those present in the group. One tactic that can help people adapt their normal behavior is

to avoid saying "But that won't work because…" and instead say "Yes, and another possibility is…"

3.3.9 Facilitation

Facilitation is a fundamental part of brainstorming and many other creative methods. In the absence of facilitation, groups can readily revert to critical and judgmental behavior that normally inhibits generative activity. The facilitator's role is to help the ideas inherently present in a group come to life and flourish.

Preparation is a key aspect of brainstorming. Encouraging participants to devote limited time to any task is a challenge associated with all projects. A tactic that can be employed is to require all participants to submit three ideas at the start of a session as an entry ticket to be allowed into the session (Michalko, 2001). If experience of a particular group suggests that production of ideas may be limited, then an idea quota can be stated to encourage the quantity of ideas. For example, the facilitator may state that the required idea quota is 100 or 200 ideas (see Litchfield, 2008). While some of these ideas may be less valuable than others, interesting suggestions may arise that might not have otherwise been offered.

The facilitator may often take on the role of data recording. For flip chart sessions one person can undertake both the facilitator and scribe roles, or the scribe role can be assigned to someone else. A key function is to capture not only the group activity but also the ideas generated by individuals. The facilitator therefore needs to ensure that Post-its and other materials generated within the session are collected, organized and evaluated following the session. Grids and Post-its, for example, can be scanned and circulated by email or embedded in a database for subsequent retrieval and evaluation.

Facilitation serves the purpose of giving structure to a session, guiding attention to a particular focus and reassurance to the team as appropriate that the process is safe. If brainstorming becomes a natural part of your organization's process, it is possible that a high level of structured facilitation, as described here, is not necessary. Kelley (2001) notes six methods that can kill the productivity of brainstorming:

- Letting the boss speak first, setting an agenda that every idea needs to count or be patentable. This will immediately limit the spontaneity of the session.
- Letting everybody have a turn in sequence. If there are 10 of you in the room, with each taking a minute or two to share an idea, it can take 15 min to get round to you. By that time you may be begging for the world to end and thinking that brainstorming is less effective than even the worst boardroom meeting.
- Stating that only experts should suggest ideas. Even if a domain is particularly complex, valuable ideas can still be offered by nonexperts. Discipline-specific experts may be inspired by the suggestions made.

- Arranging the brainstorming session off-site. If idea generation is important to your organization, brainstorming should become a regular activity in the workplace. If the session is in the workplace, then the buzz of news from the session will rapidly spread through the organization with a net positive benefit.
- Stating that no "silly stuff" is to be suggested. Again, this will limit the ideas offered.
- Note taking. If individuals are focused on writing down everything that is said, their attention to idea generation is reduced. Encourage everyone to focus on idea generation rather than taking notes; in any case, this should be captured by the scribe.

3.3.10 Evaluation

Evaluation is an essential part of the decision process associated with idea generation. If a brainstorming session has generated over 100 possible ideas, then filtering of these to a manageable number for more detailed consideration is necessary. It is important to consider the ideas while people still remember what they are about and while you have the attention of the group. Product development, detailed design and idea implementation teams cannot get going until decisions have been made about which idea or ideas to follow up on. A quasiobjective technique is recommended for evaluation. This can range from use of a voting system, perhaps, using sticky dots to signify a vote against an idea, to an evaluation matrix.

Voting systems can be sophisticated or simple. In a simple system, each person may have a limited number of votes, say 5 or 10, which they are allowed to issue with one or more votes per idea they wish to see considered further. Such voting systems can result in the loss of ideas unless some prior ordering and filtering of ideas has already occurred during the facilitated session. If, for example, a large number of minor variations on a particular idea have been recorded as separate concepts, it is possible that votes will be shared between these; when votes are summed, a rarer but possibly weaker concept may receive more votes because all the votes were concentrated. In order to overcome this potential problem, it is necessary for the facilitator and group to review the ideas generated and to group similar ideas together. In this case, fewer ideas are put forward to the evaluation phase, making the task easier to handle and avoiding dilution of votes.

Votes can be implemented by issuing each person with a set number of dots. The participants are asked to assign their dots to any idea they consider to be among the best for resolving the task. Dots can be assigned with one per idea or in multiple numbers with a participant giving all of their dots or vote to an individual idea. Alternatively, participants can be given a range of colors of dots representing traffic lights: red, amber and green. Each color symbolizes whether the idea is a green for "go" (viable), an amber for "caution" (consideration) or a red for "stop" (no). The dots provide a quick visual indication of where the team members think attention should be focused and can be quickly counted or assessed to see which ideas are favored.

Another method of evaluation is the use of tabular evaluation matrices, as illustrated in Table 3.3. These consist of a series of criteria against which the concepts must be marked. The importance of the criteria can be weighted, if appropriate, and the most suitable concept identified as the one with the highest overall mark. This method provides a structured technique of evaluation and makes it more difficult for individuals within a team to push their own ideas for irrational reasons. The outcome is a quasiquantitative measure of the merits of a specific idea and ranking of concepts.

An example of the use of an evaluation matrix is given in Table 3.4 considering four different concepts for a pallet-moving device. The criteria considered important here are safety, positional accuracy, ease of control, ease of manufacture and durability. These criteria have been weighted, with positional accuracy deemed the most important. The scores are given with a maximum of 10. The marks in the columns for the weighted scores have been summed, and the totals are given at the bottom. The highest weighted score indicates the concept that should be selected and developed.

3.4 Creative Problem Solving

In addition to developing and formulating effective rules for brainstorming, Osborn and Parnes also considered the process of solving a problem (Parnes et al., 1977). They developed an approach for this, comprising 12 steps and six stages, called the CPS process. The process is based on phases of divergent and convergent thinking, illustrated schematically in Figure 3.6, and is one of the oldest structured frameworks for problem solving.

The six stages of the CPS process that feed sequentially into each other are as follows:

- Objective finding: An objective can arise from experience and day-to-day life, requirements of the domain and social pressure.
- Fact finding: Information relevant to the issue that needs to be defined, for example, limits on allowable size or mass, how many users there are or whether there are any legal requirements.
- Problem finding: For any task there are associated challenges that need to be resolved. In the context of CPS, problem finding relates to exploring what the deficiencies of existing solutions really are or which issues or challenges arise in tackling the problem.
- Idea finding: The generation of multiple concepts that may be useful and relevant to the challenge.
- Solution finding: Elaboration is necessary in order to turn any idea into a viable solution. Elaboration may involve exploring design issues in some further detail, providing embodiment to the idea so there is confidence that it will work.

Table 3.3: Generic evaluation matrix.

		Concepts					
		Concept A		Concept B		Concept *N*	
Selection criteria	**Weight**	**Rating**	**Weighted score**	**Rating**	**Weighted score**	**Rating**	**Weighted score**
Aspect 1	Percentage importance of aspect 1	Mark out of 10 for aspect 1 of Concept A	Rating × weight for aspect 1 of Concept A	Mark out of 10 for aspect 1 of Concept B	Rating × weight for aspect 1 of Concept B	Mark out of 10 for aspect 1 of Concept *N*	Rating × weight for aspect 1 of Concept *N*
Aspect 2	Percentage importance of aspect 2	Mark out of 10 for aspect 2 of Concept A	Rating × weight for aspect 2 of Concept A	Mark out of 10 for aspect 2 of Concept B	Rating × weight for aspect 2 of Concept B	Mark out of 10 for aspect 2 of Concept *N*	Rating × weight for aspect 2 of Concept *N*
…							
Aspect *n*	Percentage importance of aspect *n*	Mark out of 10 for aspect *n* of Concept A	Rating × weight for aspect *n* of Concept A	Mark out of 10 for aspect *n* of Concept B	Rating × weight for aspect *n* of Concept B	Mark out of 10 for aspect *n* of Concept *N*	Rating × weight for aspect *n* of Concept *N*
Total weighted score			Sum of weighted score		Sum of weighted score		Sum of weighted score
Ranking							

Table 3.4: Evaluation tabular matrix for a pallet-moving device.

		Concepts							
		Hydraulic		Cable hoist		Linkage mechanism		Rack and pinion	
Selection criteria	Weight	Rating	Weighted score	Rating	Weighted score	Rating	Weighted score	Rating	Weighted score
Safety	20%	2	0.4	3	0.6	4	0.8	2	0.4
Positional accuracy	35%	6	2.1	5	1.75	7	2.45	6	2.1
Ease of control	10%	6	0.6	5	0.5	7	0.7	6	0.6
Ease of manufacture	20%	7	1.4	8	1.6	5	1.0	6	1.2
Durability	15%	7	1.05	6	0.9	5	0.75	6	0.9
Total score			5.55		5.35		5.7		5.2
Rank			2		3		1		4
Continue?			No		No		Develop		No

- Acceptance finding: Clients and peers need to be persuaded that the idea developed is going to work. A bright idea on its own is not enough. Instead, any issues relating to the idea need to be explored in a sufficient and convincing enough a manner that all relevant parties, clients, colleagues, end users and customers can be persuaded that the solution is the right one.

Osborn and Parnes recognized that CPS requires convergence as well as divergence and that each stage of CPS involves these elements. The divergent phase requires creative thinking

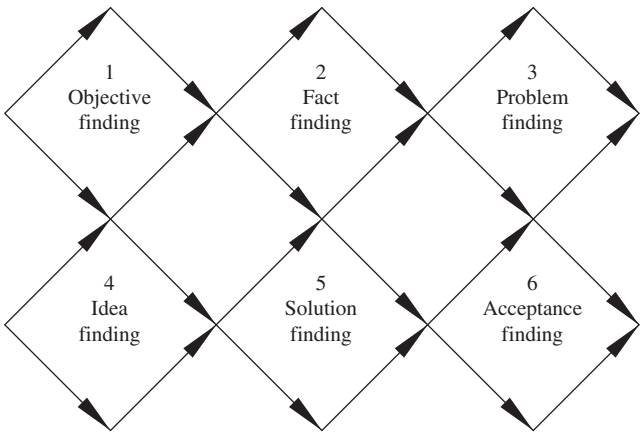


Figure 3.6
The Creative Problem Solving process.

where one encounters paradoxes, opportunities, challenges and concerns, and where searching and generating possible solutions occurs. This may involve generating many new and varied potential solutions and details that may contribute toward a solution. The convergent phase requires critical thinking with examination of the possibilities and focusing of thoughts. This may involve organizing and analyzing the potential solutions and refining them to improve their viability. Potential solutions can be ranked and prioritized so that attention can be concentrated on developing particular ideas. Each stage can involve use of brainstorming with deferment of judgment, production of many ideas and looking for combinations of ideas, prior to sifting and refining these ideas.

Effective tools are required to help evaluate competing ideas in order to overcome the temptation to slam an idea as unworkable at too early a stage in its development. Tools for focusing options recommended by Treffinger et al. (2006) include the following:

- Hits and hotspots: Identifying promising possibilities and clustering and organizing them to see if meaningful insights can be identified;
- ALoU (Advantages, Limitations and Unique features): Consider the advantages, limitations and possible methods to overcome them as well as the unique features of an idea in a constructive manner with a view toward strengthening the idea;
- PCA (paired comparison analysis);
- Sequencing: short-, medium- and long-term actions;
- Evaluation matrices.

In application of CPS or any creative method, considering the magnitude of the challenge is as important as assessing whether there is a high level of tension or conflict in the group of personnel involved. If an initial exploration of the available data reveals that the task to be undertaken is large, subdivision of the activity may be appropriate, with delegation of duties to specific subteams. If there is already a high level of tension or conflict in a team, conflict resolution or team-building techniques will need to be employed.

CPS can be used when working individually or in a team (Isaksen, 2000). If working in a team, this will require more planning and resourcing than if working alone as it will require coordination of several people and provision of resources.

The six stages of CPS can be grouped into phases:

- Identifying and clarifying the challenge: stages 1−3;
- Finding solutions and evaluation: stages 4 and 5;
- Putting the solutions into action: stage 6.

A statement of opportunity does not represent a statement of a problem. The purpose of identifying and clarifying the challenge, stages 1−3, is to enable definition of the problem you need to resolve and to give a direction to the effort. Question asking using who, what,

where, when, why and how can be useful in identifying key data relevant to the problem. In addition, considering a future desired state in the form of an ideal final result can inspire formulation of the problem statement to enable one to get from the current status to fulfillment of the objective.

Problems sometimes arise as a result of dire prospects or aspirations and wishes. Two facilitating constructs used in CPS to explore these situations are WIBAI and WIBNI. These abbreviations represent, respectively, "Wouldn't it be awful if…" and "Wouldn't it be nice if…" These constructs can be used to explore and develop an opportunity. An issue may begin with WIBAI and be turned into a constructive opportunity by WIBNI.

Experience in using the CPS technique has resulted in the use of a series of facilitating open-ended problem statements that invite ideas, including

How might we…? or How might I…? HMW, HMI
In what ways might we…? or In what ways might I…? IWWMW, IWWMI
How to…? H2

These constructs can be combined with a verb and an objective and used to explore and construct an opportunity. Examples of this are "How might we extend the useful life of this product?" or "In what ways might we increase the chance of winning this bid?"

Constructs for encouraging divergent thinking include consideration of what options and alternatives there are or might be. Convergent thinking involves focusing attention on the hits or most appealing alternatives in an extensive list of ideas.

Once an idea has been explored and selected as the best option, it will be necessary to persuade other stakeholders, such as managers and colleagues, that it is the right idea. It is helpful to identify people and factors that can assist this. The 5Ws (who, what, when, where and why) can be used to consider, for example, who can help, what resources are necessary for successful implementation of the idea, when is the best time to implement the plans, where the ideas can be developed, and why the idea should be taken forward. In a similar fashion, detractors or resistance to an idea can be explored in order to mitigate things that could go wrong. Again the 5W questions can be used to identify who might criticize the ideas and potential responses for them, what resources might be missing or unavailable, when might the worst possible time be to implement or reveal the idea, where might the worst place or factory be and what are the least persuasive aspects of the proposition.

3.5 SCAMPER

Osborn developed sets of words and phrases called idea-spurring questions. Bob Eberle subsequently developed these ideas into a memorable acronym called SCAMPER, which stands for

- Substitute
- Combine
- Adapt
- Modify/Magnify/Minimize
- Put to other uses
- Eliminate
- Reverse/Rearrange

Each letter can be used to remind us of a word or phrase, and this can be used to inspire and provoke related ideas. SCAMPER can be particularly useful when we feel stuck or uninspired; it can be employed to get ideas flowing or to provoke thinking from another perspective. Some possible provocations for SCAMPER are given in Table 3.5.

SCAMPER can be applied to almost any domain. We do this almost instinctively in addressing an issue.

3.6 Create Process

The Create Project offers a framework, the Create Process, for generative activity developed in association with industry and academia. The Create Process comprises phases on predisposition, external mapping, internal mapping, idea generation and evaluation. This process is summarized in Table 3.6 and provides a validated framework within which to undertake the design process. The Create Process can be implemented using a series of practices readily recognizable and familiar to the practicing engineer.

Possible techniques suggested for implementation of each phase of the Create Process are listed below (Childs, 2006a,b).

- Predisposition: Providing a work environment that allows expression of individual creativity, promotes a creative culture and identifies leaders and facilitators. This can be achieved by training in creativity and innovation, and change management processes.
- External mapping: Analyzing the environment outside the organization to identify, for example, new needs and opportunities, talented individuals and the economic implications of entering a market. A questionnaire survey and analysis known as the Attribute Value Chain can be used to produce a map to stimulate the next phase of idea generation.
- Internal mapping: Analyze the organization concerned using, for example, SWOT analysis to identify the Strengths, Weaknesses, Opportunities and Threats.
- Idea generation: Augment with, for example, brainstorming, morphological analysis and TRIZ.
- Evaluation: Select the best idea(s) using, for example, the criteria matrix and six thinking hats tools.

Table 3.5: SCAMPER provocations.

Substitute	What might you use instead? What can be done instead? Who else? Can the rules be altered? Alternative materials? Another place? Other processes or procedures? Another approach?
Combine	What ideas can be combined? What materials can be combined? What methods can be combined? What procedures or tasks can be combined? What functions can be combined? What can be blended? What can be synthesized? What can be mixed? Can an assortment fulfill the requirements?
Adapt	What might be changed? What might be used in a different way? How can this be adapted for different customers? What other ideas does this suggest? What else is like this? What can be copied to produce this? Who could be emulated? What different contexts can this concept be applied to? What ideas from other disciplines can be applied?
Magnify/Modify	Can the idea or object be scaled? What can be made larger? What can be made smaller? Can it be multiplied? Can the color be changed? Can the shape or/and form be changed? Can the sound be altered? Can the scent be changed? What changes can be made elsewhere in the system?
Put to other uses	What else can it be used for?
Eliminate	What can be omitted? Can it be divided? Can it be separated? What can be trimmed? What can we do without? Can it be streamlined to reduce losses? Can it be miniaturized? Can something be subtracted?

Table 3.5: SCAMPER provocations.—cont'd

	Can something be deleted or removed? What is unnecessary?
Reverse/Rearrange	What can be made opposite? What can be mirrored? Can it be turned inside out? Can roles be reversed? What are the negatives? What can be restructured?

3.7 Morphological Analysis

Morphological analysis is a technique that can be used to generate additional ideas for product concepts that would not normally spring to mind. The technique involves considering the function of a generic solution to a problem and breaking it down into a number of systems or subfunctions. The next step is to generate a variety of means to fulfill each of these systems or subfunctions. The subfunctions and potential means of fulfilling each of these subfunctions

Table 3.6: The Create Process phases.

Phase	Summary
Predisposition	This concerns the creation of those essential circumstances in an organization that are the preliminary steps to the development of creativity: • company atmosphere; • leadership style; • culture; • organization structure and systems.
External mapping	In this phase a series of activities are included, such as • the search for entrepreneurial ideas linked to the environment outside a company; • identification of new and unexpressed needs; • utilization of talents; • new opportunities, and abilities; • evaluation of the economic implications of possible changes in competition.
Internal mapping	This phase concerns the identification and evaluation of all the company resources, in order to promote the development of new opportunities/business concepts by making the most of its potential. It also includes an internal study to identify malfunctions, processes and products that could be improved.
Idea generation	This phase concerns the production of ideas that can occur at an individual and joint level.
Evaluation	This involves identification and selection of the best ideas according to the organization's, group's or company's own internal assessment criteria.

Source: The Create Project (2005).

Table 3.7: Generic morphological analysis chart.

Subsystem	Means			
Subsystem 1	Method 1 of fulfilling subsystem 1	Method 2 of fulfilling subsystem 1	Method 3 of fulfilling subsystem 1	Method *n* of fulfilling subsystem 1
Subsystem 2	Method 1 of fulfilling subsystem 2	Method 2 of fulfilling subsystem 2	Method 3 of fulfilling subsystem 2	Method *n* of fulfilling subsystem 2
Subsystem 3	Method 1 of fulfilling subsystem 3	Method 2 of fulfilling subsystem 3	Method 3 of fulfilling subsystem 3	Method *n* of fulfilling subsystem 3
Subsystem 4	Method 1 of fulfilling subsystem 4	Method 2 of fulfilling subsystem 4	Method 3 of fulfilling subsystem 4	Method *n* of fulfilling subsystem 4
Subsystem 5	Method 1 of fulfilling subsystem 5	Method 2 of fulfilling subsystem 5	Method 3 of fulfilling subsystem 5	Method *n* of fulfilling subsystem 5

are arranged in a grid. An overall solution is then formulated by selecting one means for each subfunction, and the combination of these forms the overall solution. A generic morphological analysis table is illustrated in Table 3.7. The grid can be populated by text or sketches depicting the potential means of fulfilling the subsystem requirement (Childs, 2004). The technique relies upon the user's selection of subfunction options to synthesize an overall solution or set of possibilities. The user can explore the design space systematically, using their experience to guide the outcome, or experimentally. The matrix given in Table 3.7 is abstract and generic in its formulation. Care needs to be applied in defining the functional requirements in order to avoid the specification of functions and subfunctions with a preconceived solution. Specific examples arising from practical implementation are given in Sections 3.7.1 and 3.7.2.

Morphological analysis is a phrase common to a number of subject disciplines. For example, in the field of linguistics, morphology is the study of the internal structure of words and patterns of word formation within and across languages. Morphology in this context involves formulation of rules that model the knowledge of the speakers of those languages, and morphological analysis is the application of procedures and principles to individual words and phrases (see, for example, Bauer, 2003). In biochemistry, morphological analysis is the study of the cell and compound structure. In this chapter, however, the term morphological analysis is used to refer to a particular creativity tool for the generation of alternative concepts by means of development of generic subfunctions for a product or process and consideration of alternative means for implementation of each subfunction.

Morphological analysis was originally developed and applied to jet engine technology following the Second World War by F Zwicky (see, for example, Zwicky, 1969 and the Swedish Morphological Society), although Allen calls his 1962 book *The Allen Morphologizer*. Zwicky identified the important parameters of aviation engine technology, including the thrust mechanism, oxidizer and fuel type (Allen, 1962). He then broke each of these functions down into constituent parts under each parameter or subsystem heading. He then assembled all possible permutations, such as a ramjet that used, for example, atmospheric oxygen and a solid fuel. For some of the permutations produced, an existing product and solution using this approach already existed. The new or original permutations were viewed by Zwicky as stimuli for creativity and worthy of the question, "Why not?" Morphological analysis is a proven ideation method that results in organized invention and has been demonstrated to be appropriate for new product development, services, patent definition and value management. The technique comprises two key elements:

- A systematic analysis of the current and future structure of a domain as well as identification of key gaps in that structure;
- A strong stimulus for the invention of new alternatives that fill these gaps and meet any imposed requirements.

A typical morphological field can contain thousands to millions of formal configurations. This number of permutations is far too many to inspect by hand. In order to overcome this, the internal relationships between the field parameters can be examined and the field reduced by weeding out all mutually contradictory conditions. In addition, increased levels of sophistication can be applied to morphological analysis, such as the weighting of ideas and the addition of cost information in order to guide users in the impact of the individual selection of each subfunctional option. In the absence of coherent strategies to identify preferred solutions, it has been observed that once a few sets of plausible alternative selections have been tried, searching for a better solution gets abandoned (see Jones, 1970). Whiting (1958) notes that in practice too many possibilities can be developed, leading to problems in evaluation of the alternative concepts effectively.

In order to illustrate the technique, a case study for a pallet-moving device is considered in Section 3.7.1, prior to the introduction of case studies across diverse applications in Section 3.7.2.

3.7.1 Pallet Case Study

Following some initial market assessments it is assumed that the Board of a plant machinery company has decided to proceed with the design of a new product for transporting pallets around factories.

Table 3.8: Morphological chart for a pallet-moving device.

Feature	Means				
Support	Track	Wheels	Air cushion	Slides	Pedipulators
Propulsion	Driven wheels	Air thrust	Moving cable	Linear induction	Toothed wheel and rack
Power	Electric	Diesel	Petrol	Bottled gas	Steam
Transmission	Belts	Chains	Gears and shafts	Hydraulics	Flexible cable
Steering	Turning wheels	Air thrust	Rails	Magnetism	Drag
Stopping	Brakes	Reverse thrust	Ratchet	Magnetism	Anchor
Lifting	Hydraulic ram	Rack and pinion	Screw	Chain or rope hoist	Telescopic linkage
Operator	Standing	Walking	Seated at front	Seated at rear	Remote control

The use of morphological charts requires the designer to consider the function of components rather than their specific details. A pallet-moving device would probably comprise support, propulsion, power, transmission, steering, stopping and lifting systems. Within each of these criteria all plausible options should be listed, as indicated in Table 3.8. In this table only a selection of possible means for fulfilling each subfunction have been included in order to illustrate the technique. In practice such tables can occupy large sheets of paper or many columns in a spreadsheet. The task of the designer is to look at this chart and select a justifiable option for each of the criteria, producing the makeup of the pallet-moving device. In this case there would not necessarily be any surprises in the final makeup of the product, and selection of wheels, driven wheels, petrol, gears and shafts, turning wheels, brakes, hydraulic ram, and seated at the front would result in the traditional forklift truck epitomized by the E12 to E20 Linde trucks. However, the use of this method can assist in the production of alternatives.

In a regular exercise, engineering and design students are challenged to use this table to produce and then sketch three radically different alternatives for consideration. For example, air cushion for support, air thrust for propulsion, bottled gas for power, gears and shafts for transmission, air thrust for steering, reverse thrust for stopping, hydraulics for lifting, and seated for the operation could be selected for each of the subsystems, respectively. Images from this exercise are illustrated in Figure 3.7. While some aspects of the images given in Figure 3.7 may provoke some alarm among experienced plant designers and health and safety experts, the intent of the technique is to provide alternatives for consideration. Morphological analysis serves to augment the range of possibilities to be considered and does not negate the responsibilities of a professional in

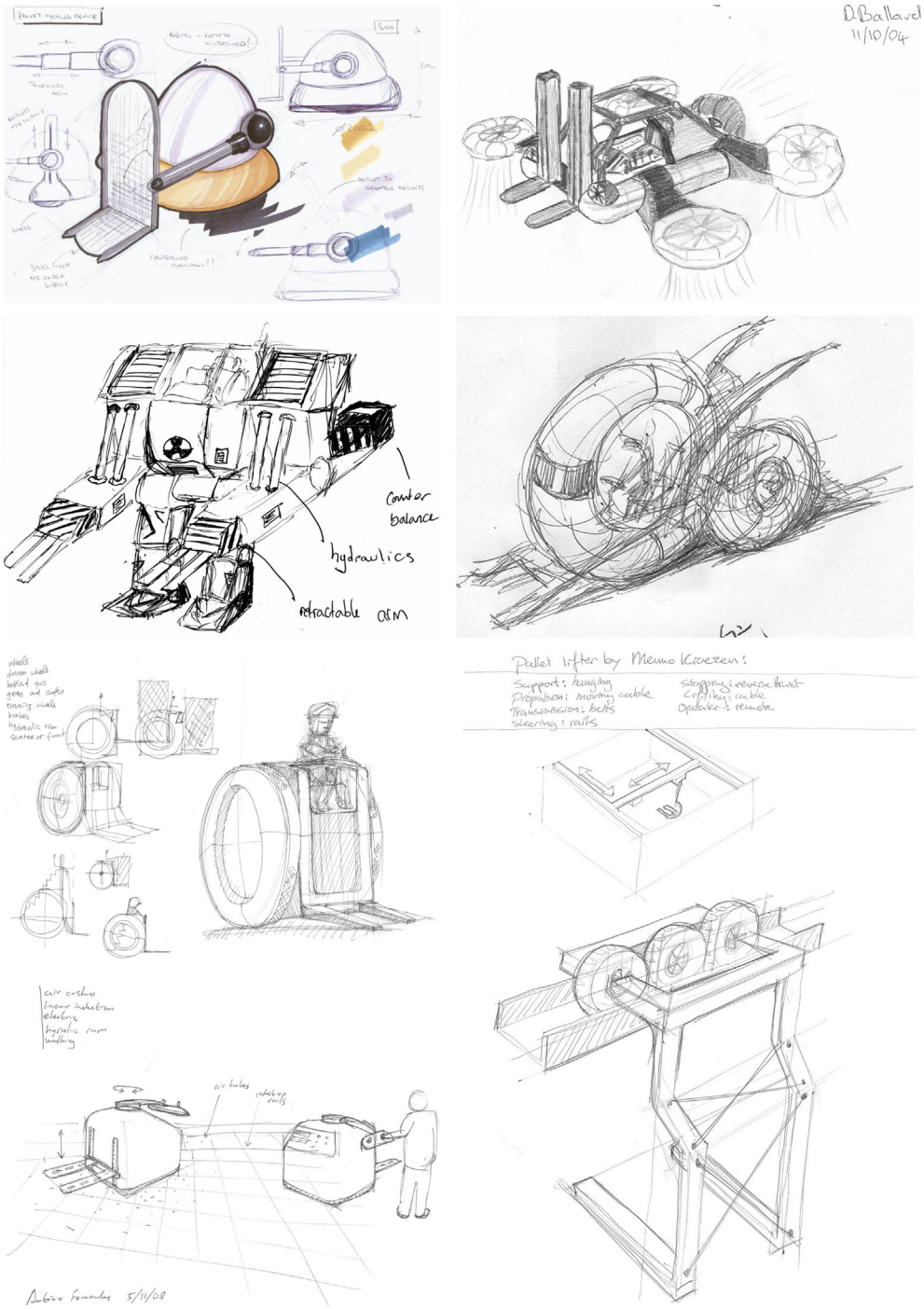


Figure 3.7
Sketches following selection of alternative concepts. *University of Sussex (2004), Beihang University (2008), RCA (2008).*

Table 3.9: Morphological chart for a pallet-moving device with choices identified.

Feature	Means				
Support	Track	Wheels	Air cushion	Slides	Pedipulators
Propulsion	Driven wheels	Air thrust	Moving cable	Linear induction	Toothed wheel and rack
Power	Electric	Diesel	Petrol	Bottled gas	Steam
Transmission	Belts	Chains	Gears and shafts	Hydraulics	Flexible cable
Steering	Turning wheels	Air thrust	Rails	Magnetism	Drag
Stopping	Brakes	Reverse thrust	Ratchet	Magnetism	Anchor
Lifting	Hydraulic ram	Rack and pinion	Screw	Chain or rope hoist	Telescopic linkage
Operator	Standing	Walking	Seated at front	Seated at rear	Remote control

carrying out their task responsibly and utilizing their detailed analytical skills. The morphological analysis chart repeated in Table 3.9 shows the selection of a telescopic-based lifting method; the implementation of this is illustrated in Figure 3.8.

3.7.2 Applications of Morphological Analysis

Although originally developed to tackle technological challenges in the US aerospace industry and military sector, the formulation of morphological analysis does not limit it to a particular discipline. As examples, Tables 3.10 and 3.11 show the development of



Figure 3.8
The Teletruk concept by JCB. *Courtesy of J.C. Bamford Excavators Ltd.*

Table 3.10: Morphological chart for an image-recording device.

Subfunction	Means							
Image capture	Lens	Detector array	Static charge	Photosensitive bacteria	Chemical			
Image storage	Chemical film	Chemical bath	Digital memory	Static charge	Photosensitive bacteria	Instant processing		
Actuation	Tilt button	Joystick	Push button	Programmable	Timer	Tilt	Light activated	
Image display	Project image	Screen	Lens	Printed image	None	Liquid crystal display	Use computer or phone screen	Hologram projection
Storage	Clamshell mold	Compliant material case	None	Metal case	Contained in another product			
Image development	Chemical processing	Memory manipulation	None	Printing				
Electromagnetic waves	Ultrasonic							
Communications	Bluetooth	Infrared	Cable digital	Cable analog	Physical transfer			
Protection of subsystems	Molded casing	Bag	Frame	C-shell	Compliant material case	Shell		
Power	Manual	None	Biological	Battery	AC mains	Clockwork	Solar	Chemical
Target illumination	Flash	None	Infrared	Spot lamp	Reflector	Laser	Natural light	
Added functionality	Video	Zoom	Sound recording	Image manipulation	Lock	Time reference	Manual focus	Automatic focus

Table 3.11: Morphological chart for vehicle design.

Subsystem	Means									
Stopping	Disc brake	External drum brake	Internal drum brake	Parachute	Air jet	Momentum exchange	Magnetic induction	Anchor		
Steering	Steering wheel	Joystick	Handle bars	Automatic	Reins	Remote control	Air jet	Levers	Prosthetic implant on wrists and static sensors	Magnetic
Styling	Futuristic	Space age	Old school	Retro 1960s	Retro 1970s	Sporty	Aerodynamic	Functional	Chunky	Off road
Prime mover	4 stroke	2 stroke	GTE	Human powered	Electric motor	Electromagnetic induction	Hybrid	Clockwork	Rocket	Jet
Transmission	Gears	Belts	Chains	Hydraulics	Pneumatics					
Propulsion	Wheels	Balls	Tracks	Air cushion	Glide on smooth surface	Electromagnetic induction	Rails	Rollers		
Accessory pack	GPS, e-windows, air con	MP3 stereo, back seat speaker	Sun roof, alloys, blazer hook	Two bar, caravan mirror	DVD players	Underpan neon lighting				
Entry	Teleport	Electric ramp	Door	Hole	Gull-wing	Cockpit entry	Curtain			
Storage	Boot	Bonnet	Top box	Under floor	Overhead storage	Panniers	Roof rack			
Purpose	Family	Commercial	Sport	Professional female	City	Off road	Racing	Business	Wideboy	Manual worker
Seating	Standing	2 + 2	2 + 3	Bench seat	2 front + bed	Single seater	None, commercial only	Kneeling pads	Double decker	
Fuel	Cow manure	Diesel	Petrol	Cooking oil	H_2	Compost	Steam	Electricity	Nuclear	Biofuel
Performance	Sports	Efficient	High octane	All round	High load capacity	Aid	Mobility +	MPV	Camping	

morphological analysis charts to an image-recording device and vehicle design. The method has also been applied to applications such as protein identification, interior design and creative writing. The diverse nature of these examples indicates the potential for this technique.

Morphological analysis has a long history of application in industry and in education. The technique involves formulation of the design space in terms of subfunctions and potential options for each of the subfunctions. The technique relies upon the user's selection of subfunction options to synthesize an overall solution or set of possibilities. The user can explore the design space systematically, using their experience to guide the outcome, or experimentally. This chapter has illustrated the use of morphological analysis across a wide range of disciplines focusing on technology, furniture, biotechnology, fixings and environment and literature. The case studies highlighted demonstrate that the technique is accessible across a range of disciplines and assists in provoking consideration of alternatives. The process of selection and generation of a quick sketch or outline provides a prompt for dealing with the integration of the subfunctions into a holistic proposal. Some concerns remain that too many possibilities can be developed, leading to problems in evaluating the alternative concepts effectively. This can be offset by means of weighted functions relating to, for example, cost of implementation or marketing advantage. A structured approach using morphological analysis is reported by Garvey and Childs (2013). The diverse case studies presented reveal that the technique can be accepted and readily used by heterogeneous groupings of individuals. It is proposed that morphological analysis is valuable in producing additional conceptual solutions and ideas for consideration, but it is not a substitute for the application of experience, use of skills and innate talent.

3.8 Standard Solutions

Many organizations rely on new products and services to sustain their activity and grow. Examples include Procter & Gamble, Unilever, Sony, Motorola, Hewlett-Packard and 3M, where more than 30% of their revenue is due to products introduced in the previous two years. Sony introduces about 50 new products every year. These examples show the importance of the realization of new ideas in a form that can generate revenue. In the UK, the Cox review (Cox, 2005) confirmed this by stating that 75% of company turnover stems from products developed within the previous five years.

The requirement for new products and services is embodied in the sigmoid S curve, Figure 3.9. Many processes that involve growth, such as scientific progress and sales, tend to follow this S curve. The process is characterized by an initial period of low growth where progress is slow. This is followed typically by a period of rapid progress and growth until a point where the process starts to saturate with a reduction in the rate of growth and the

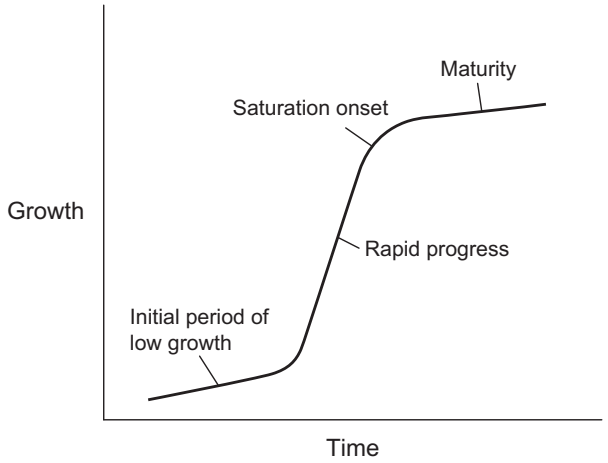


Figure 3.9
A sigmoid S curve showing infancy in growth or progress followed by rapid growth, onset of saturation and maturity.

progress begins to level off. These periods can be classed as infancy, rapid progress, saturation onset and maturity.

However, this is not the end of the story. The S curve nature of many products and services is subject to intervention and deviations as a result of competitive and collaborative factors. Companies within a market sector during a period of high sales and rapid growth will be aware that at some stage the sales successes are likely to level off, with most consumers likely to want to purchase the item having considered it or bought it. In the case of scientific development the "easy" hits of knowledge and capability may have been obtained, and advances level off as a result of the law of diminishing returns where more and more effort is required for less and less progress. As a result of intervention or simultaneous action, development may start on a new related area within the field. Initially the performance in terms of output in this new area may be below that of the existing approach. However as effort is applied progress begins to take off and the new approach goes into a period of rapid growth with performance exceeding that of the previous idea. This process can then be repeated as is illustrated in Figure 3.10 as a series of S curves.

The repeated experience of trends following a family of S curves for a wide range of products and processes demonstrates the need for realization of new ideas in order to sustain an organization. Changing from making proven products that a company is familiar and feels safe with, to innovative new products is not easy. Things we might have cherished in the past and found security in, such as specialist skills, habits, paradigms, or working environment, can nourish mental blocks that constrain our innovative thinking. Such constraints can act as psychological inertia, preventing a domain from moving forward.

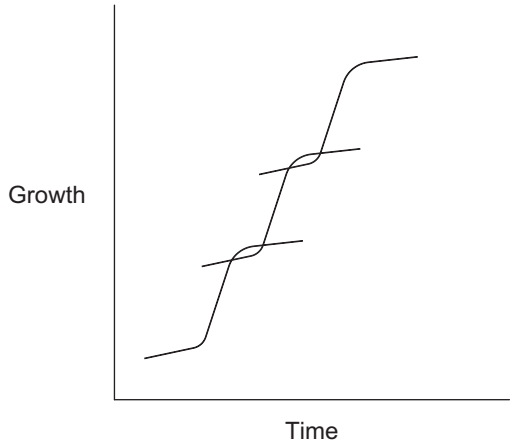


Figure 3.10
Family of S curves showing progress in a new area beginning to occur as the previous curve begins to saturate.

TRIZ is a technology-based systematic methodology that aims to overcome "psychological inertia" and generate a large range of solution concepts. The emphasis is on finding innovative solutions and concepts from other fields that use available resources. TRIZ is a Russian acronym for *Teoria Resheneya Isobretatelskih Zadach*. This can be translated as the "Theory of Inventive Problem Solving," which would give the acronym TIPS. Nearly all practitioners, however, use the original acronym TRIZ.

TRIZ was originally developed by Genrich Altshuller. In 1946, Genrich Altshuller, aged 20, was a reviewer for the registry of inventions/patents at the Russian naval patent office. In reviewing inventions being submitted he became convinced that there were definite patterns to the way innovation was occurring in technical systems. Altshuller and his collaborators based their initial findings on an analysis of 46,000 registrations of inventions/patents. This was then expanded to a study of 200,000 patents. Since then, analysis of the worldwide patent base has continued, and some of the modern TRIZ databases rely on analysis of over 2 million patents. Altshuller and his collaborators found that the same fundamental solutions were used over and over again. A series of tools (Altshuller, 1984, 1994, 2005) that pulled this experience together and could be applied to solving new problems were developed.

TRIZ represents a shortcut to experience. You could spend your life studying, reading, going around museums looking at iconic designs, taking note of the latest scientific breakthroughs and generally expanding your knowledge base. This will, combined with your own activities in a domain, provide a valuable set of experiences with which you can tackle problems. Each time you see how someone else has solved a problem you are likely to internalize the experience and hopefully be able to apply the principle when needed. This gathering of



Figure 3.11
The TRIZ process.

experience takes time. The principles embodied in TRIZ provide a faster route to the toolbox of lifetimes of experience.

TRIZ involves the transformation of a specific problem into that of a general problem. If the problem is made general, it is possible to take advantage of the knowledge available from the solution of similar problems. The general solutions can then be transformed into specific ideas to resolve the specific problem, Figure 3.11.

3.8.1 Contradiction and Tradeoff

A common characteristic of problem solving is that in trying to improve one desirable parameter or property, another desirable property degrades. This characteristic of problem solving can be viewed as a tradeoff or contradiction. Examples of contradictions in tackling a problem can be observed in the following examples:

- Bin design: A large bin is desirable so that it does need to be emptied often, but a small one is desirable so that it does not occupy too much space.
- The forming of a horseshoe: A horseshoe needs to be hot so that it is easier to alter the shape of the material, but it is also desirable for the shoe to be cold so that it is easy to handle.
- Axe design: If you make the blade heavier it can strike a more effective blow with better splitting power, but it becomes more awkward to handle.

Conventional problem solving generally leads to a compromise solution. Altshuller noted that the most inventive solution is obtained when a technical problem containing a contradiction is solved by completely eliminating the contradiction.

From his initial research, Altshuller found that there were only 39 generic parameters that either improve or degrade. Examples of these parameters include items such as the weight of a moving object, speed, force, temperature, accuracy of measurement and ease of repair. In the development of the list of generic parameters, specific meaning was given to moving versus stationary objects. Objects that can easily change position in space, either on their own or as a result of external forces, are classed as moving objects. Objects or vehicles that are designed to be portable are the basic members of this class. Objects that do not change position in space, either on their own or as a result of external forces, are classed as stationary objects. The 39 generic parameters and a brief explanation are given in Table 3.12.

Table 3.12: The 39 parameters of TRIZ.

No	Title	Explanation
1	Weight of moving object	The mass of the object, in a gravitational field. The force that the body exerts on its support or suspension.
2	Weight of stationary object	The mass of the object, in a gravitational field. The force that the body exerts on its support or suspension, or on the surface on which it rests.
3	Length of moving object	Any one linear dimension, not necessarily the longest, is considered a length.
4	Length of stationary object	Any one linear dimension, not necessarily the longest, is considered a length.
5	Area of moving object	A geometrical characteristic described by the part of a plane enclosed by a line. The part of a surface occupied by the object. Or the square measure of the surface, either internal or external, of an object.
6	Area of stationary object	A geometrical characteristic described by the part of a plane enclosed by a line. The part of a surface occupied by the object. Or the square measure of the surface, either internal or external, of an object.
7	Volume of moving object	The cubic measure of space occupied by the object. Length × width × height for a rectangular object, height × area for a cylinder, etc.
8	Volume of stationary object	The cubic measure of space occupied by the object. Length × width × height for a rectangular object, height × area for a cylinder, etc.
9	Speed	The velocity of an object; the rate of a process or action in time.
10	Force	Force measures the interaction between systems. In Newtonian physics, force = mass × acceleration. In TRIZ, force is any interaction that is intended to change an object's condition.
11	Stress or pressure	Force per unit area. Also, tension.
12	Shape	The external contours, appearance of a system.
13	Stability of the object's composition	The wholeness or integrity of the system; the relationship of the system's constituent elements. Wear, chemical decomposition and disassembly are all decreases in stability. Increasing entropy is decreasing stability.
14	Strength	The extent to which the object is able to resist changing in response to force. Resistance to breaking.
15	Duration of action by a moving object	The time that the object can perform the action. Service life. Mean time between failures is a measure of the duration of action. Also, durability.
16	Duration of action by a stationary object	The time that the object can perform the action. Service life. Mean time between failures is a measure of the duration of action. Also, durability.
17	Temperature	The thermal condition of the object or system. Loosely includes other thermal parameters, such as heat capacity, that affect the rate of change of temperature.
18	Illumination intensity	Light flux per unit area, also any other illumination characteristics of the system such as brightness, light quality, etc.
19	Use of energy by moving object	The measure of the object's capacity for doing work. In classical mechanics, energy is the product of force × distance. This includes the use of energy provided by the supersystem (such as electrical energy or heat). Energy required doing a particular job.
20	Use of energy by stationary object	The measure of the object's capacity for doing work. In classical mechanics, energy is the product of force × distance. This includes the use of energy provided by the supersystem (such as electrical energy or heat.) Energy required doing a particular job.

Continued

Table 3.12: The 39 parameters of TRIZ.—cont'd

No	Title	Explanation
21	Power	The time rate at which work is performed. The rate of use of energy.
22	Loss of energy	Use of energy that does not contribute to the job being done. See 19. Reducing the loss of energy sometimes requires different techniques from improving the use of energy, which is why this is a separate category.
23	Loss of substance	Partial or complete, permanent or temporary, loss of some of a system's materials, substances, parts or subsystems.
24	Loss of information	Partial or complete, permanent or temporary, loss of data or access to data in or by a system. Frequently includes sensory data such as aroma, texture, etc.
25	Loss of time	Time is the duration of an activity. Improving the loss of time means reducing the time taken for the activity. "Cycle time reduction" is a common term.
26	Quantity of substance/the matter	The number or amount of a system's materials, substances, parts or subsystems which might be changed fully or partially, permanently or temporarily.
27	Reliability	A system's ability to perform its intended functions in predictable ways and conditions.
28	Measurement accuracy	The closeness of the measured value to the actual value of a property of a system. Reducing the error in a measurement increases the accuracy of the measurement.
29	Manufacturing precision	The extent to which the actual characteristics of the system or object match the specified or required characteristics.
30	External harm affects the object	Susceptibility of a system to externally generated (harmful) effects.
31	Object generated harmful factors	A harmful effect is one that reduces the efficiency or quality of the functioning of the object or system. These harmful effects are generated by the object or system, as part of its operation.
32	Ease of manufacture	The degree of facility, comfort or effortlessness in manufacturing or fabricating the object/system.
33	Ease of operation	Simplicity: The process is NOT easy if it requires a large number of people, large number of steps in the operation, needs special tools, etc. "Hard" processes have low yield and "easy" process have high yield; they are easy to do right.
34	Ease of repair	Quality characteristics such as convenience, comfort, simplicity, and time to repair faults, failures or defects in a system.
35	Adaptability or versatility	The extent to which a system/object positively responds to external changes. Also, a system that can be used in multiple ways in a variety of circumstances.
36	Device complexity	The number and diversity of elements and element interrelationships within a system. The user may be an element of the system that increases the complexity. The difficulty of mastering the system is a measure of its complexity.
37	Difficulty of detecting and measuring	Measuring or monitoring systems that are complex, costly, require much time and labor to set up and use or that have complex relationships between components or components that interfere with each other all demonstrate "difficulty of detecting and measuring." Increasing cost of measuring to a satisfactory error is also a sign of increased difficulty of measuring.

Table 3.12: The 39 parameters of TRIZ.—cont'd

No	Title	Explanation
38	Extent of automation	The extent to which a system or object performs its functions without human interface. The lowest level of automation is the use of a manually operated tool. For intermediate levels, humans program the tool, observe its operation and interrupt or reprogram as needed. For the highest level, the machine senses the operation needed, programs itself and monitors its own operations.
39	Productivity	The number of functions or operations performed by a system per unit time. The time for a unit operation. The output per unit time, or the cost per unit output.
	Moving objects	Objects that can easily change position in space, either on their own or as a result of external forces. Vehicles and objects designed to be portable are the basic members of this class.
	Stationary objects	Objects that do not change position in space, either on their own or as a result of external forces. Consider the conditions under which the object is being used.

Source: Adapted from Domb (1998).

If the concept of contradiction in problem solving is accepted, then every problem can be described as a conflict between a pair of parameters. Altshuller and his collaborators observed that many patents had, in the past, resolved these individual conflicts in several different fields. He also found that the principles used to resolve these contradictions fully, not as a tradeoff or compromise solution, could be described in a list of just 40 principles. These 40 principles are listed in Table 3.13. TRIZ has developed over several decades, and the specific names and numbering for the principles vary according to author and translation. Along with the names for the principles used in this text, a list of other commonly used names are given in the table.

3.8.2 The 40 Principles

The 40 principles are powerful tools on their own. Each one represents an approach to solving a challenge. Some principles are found to be generally more effective than others; this section concentrates on describing these along with relevant examples.

Principle 1: Segmentation—The segmentation principle encourages consideration of dividing an object into smaller independent parts, making an object sectional, making an object easy to assemble or disassemble, increasing the degree of an object's divisibility or fragmentation. Examples of segmentation include multistory housing, dividing an organization into different product or process centers, franchise outlets, modular furniture, modular computer components, providing separate bins for recycling different materials, using office cubicles to

Table 3.13: The 40 principles of TRIZ. Principles for resolving design contradictions.

Number	Principle	Alternative names
1	Segmentation	Fragmentation; segmentation
2	Extraction	Separation; take out
3	Local quality	Local property
4	Asymmetry	Symmetry change
5	Combining	Merging; combination; consolidation; unite
6	Universality	Multifunctionality
7	Nesting	Nested doll; matryoshka
8	Counterweight	Antiweight; weight-compensation
9	Prior counteraction	Preliminary antiaction; preliminary counteraction
10	Prior action	Preliminary action; do it in advance
11	Cushion in advance	Beforehand cushioning; beforehand compensation; previously installed cushions
12	Equipotentiality	
13	Inversion	The other way round; inverse action
14	Spheroidality	Curvature
15	Dynamics	Dynamicity; dynamism
16	Partial or excessive action	
17	Transition into another dimension	Dimensionality; another dimension
18	Mechanical vibration	Use of mechanical oscillations
19	Periodic action	
20	Continuity of useful action	Uninterrupted useful function
21	Rushing through	Skipping; quick jump
22	Convert harm into benefit	Blessing in disguise; transform damage into use; lemons to lemonade
23	Feedback	
24	Mediator	Intermediary
25	Self-service	
26	Copying	
27	Inexpensive short-life	Cheap disposables; cheap short living
28	Mechanical substitution	Another sense; replacement of a mechanical system; use of fields; replacement of mechanical matter
29	Pneumatics and hydraulic construction	Pneumatics and hydraulics
30	Flexible membranes and thin films	Flexible shells and thin films
31	Porous materials	
32	Color change	Optical property changes
33	Homogeneity	
34	Discard and renewal	Rejecting and regenerating parts; discarding and recovering
35	Transforming the physical or chemical state of an object	Parameter change; transforming physical or chemical states; transformation of properties; change in the aggregate state of an object
36	Phase change	Phase transition
37	Thermal expansion	
38	Strong oxidants	Accelerated oxidization
39	Inert environment	Inert atmosphere
40	Composite materials	

enable the layout to be easily changed, use of temporary workers on short-term projects, a folding wooden ruler, garden hose sections that can be joined together to give the total length required, Venetian blinds and multipane windows. An advantage of separation is that each part can be treated differently. Parts can be made of different materials or have different material treatments or different shapes.

Principle 2: Extraction—Extraction involves removing or separating an undesirable or disturbing part or property from an object or system. Alternatively it may involve extracting the desirable part or property of a body or isolating the critical part of a system. An example is the location of noisy equipment such as an air conditioning unit away from the room being cooled so that the occupants are not disturbed by noise. Another example illustrating this principle is embodied in water mist fire suppression systems where pumps are located away from the potential fire zone, thereby protecting the pumps, and only small-diameter pipes are necessary to convey the water to the spray nozzles.

Principle 3: Local quality—It need not be assumed that the current use or location of parts in a system cannot be changed. Local quality involves identifying specific parts and changing or moving them so that they can operate at optimal conditions. This may involve changing an object's structure from uniform to nonuniform, or making each part of a system fulfill a different or complementary function. An example of local quality can be found in coal mining to combat dust where a fine mist of water in a conical form is sprayed on parts of the drilling and loading machinery. The smaller the droplets, the greater the effect in suppressing the dust, but fine mist can hinder work. A solution to this is to develop a layer of coarse mist around the cone of fine mist. Another example of local quality is the combination pencil and eraser in the same unit, whereby additional local functionality is provided.

Principle 4: Asymmetry—Symmetry is a powerful organizing factor. We tend to find symmetric products more aesthetically pleasing. Symmetric objects also tend to be easier to form and manufacture. However, a symmetric object may not be the easiest to handle or organize. Asymmetry involves replacing a symmetrical form with an asymmetrical form. Alternatively, if an object is already asymmetrical, the degree of asymmetry can be increased. Asymmetric stirrers and paddles mix more effectively than symmetric ones. Asymmetric scissors handles provide increased ease of use. Asymmetry is widely used in fashion to produce interesting and engaging styles. As the asymmetry principle involves changing the geometric form, it is often an easy method for exploiting the available resource inherent in the geometric form of an object.

Principle 5: Combining—Combining involves bringing together or uniting things that happen at the same time or in the same place. The combination may be permanent or temporary. An example is a combination washing machine and tumble dryer that provides added functionality within the same spatial footprint. The principle of combining is embodied in networked computers and parallel computing, providing added functionality and increased

processing power. Other examples include double glazing and mixer taps. Fragile glass can be combined with a frame to provide increased strength. An example of temporary union is the use of booster rockets for space payloads where the boosters are fixed to the craft for the first stage of a launch and then discarded.

Principle 6: Universality—The principle of universality refers to making an object perform multiple functions, thereby eliminating the need for some other object or objects. An example is a sofa that can convert into a bed or a vehicle seat that can be adjusted to provide sleeping or cargo-carrying capacity. The principle is embodied in the classic Swiss army knife and other multifunction tools. The principle can also be applied to the workplace where an individual in a manufacturing cell is multiskilled so that he/she can attend to several tasks. Roof lights can be designed to provide the multiple functions of illumination, weather proofing, ventilation and thermal and sound insulation.

Principle 7: Nesting—Nesting involves putting one object inside another which, in turn, is placed inside a third. In Russian, the principle is called matryoshka after the well-known Russian dolls. An object that is inside another is protected and reduces the overall volume occupied by the components. Alternatively, nesting refers to passing an object through a cavity of another object. Examples of nesting include the telescope where the casing provides protection for the optical components and the telescopic assembly reduces the total volume for storage and transport, telescopic antennae, nested measuring cups, chairs that can be stacked on each other to facilitate storage, stuffing a chicken or turkey so that two foodstuffs are cooked at once and the overall flavors are mutually enhanced and the mechanical pencil with a series of leads stored within its barrel.

Principle 8: Counterweight—The counterweight principle involves compensating for an undesirable force in one direction by introducing a force in the opposite direction. Counterweights are used, for example, in sash windows with a heavy mass slung over a pulley wheel connected to the window frame so that force required to open or close the window is reduced. Alternatively, the counterweight principle involves compensating for the weight of an object by interaction with an environment providing aerodynamic or hydrodynamic lift or drag or buoyancy force. An example of this principle is the inverted rear wing in racing cars, which increases the downward force on the car. An example of weight compensation is the use of helium or hot air blimps for advertising. A business example of the counterweight principle is to increase falling sales by associating a product with a rising symbol such as a new movie or trendy star.

Principle 9: Prior counteraction—Prior counteraction involves performing a mitigating task in advance. If you know that something is going to happen, preparations can be made or action taken in advance to either prevent it from happening or reduce the impact of the event occurring. Examples of prior counteraction are the use of reinforced concrete for a column or floor slab, prestressed bolts, and dust removal systems for power tools so that a workpiece can

be seen and so that harmful dust does not block cooling vents. The principle of prior counteraction is addressed in sustainable design where the potentially harmful impacts of a product are considered and either minimized or eliminated.

Principle 10: Prior action—Prior action involves carrying out all or part of a required action in advance or arranging objects so they can go into action in a timely matter and from a convenient location. An example is embodied in modular assembly, where modules are manufactured in advance and then joined together, such as prefabricated windows for buildings, prefabricated building sections and gearboxes for installation in a transmission. Another example is the notching of a craft knife enabling a used blunt section to be snapped off to reveal a new sharp edge. Blanks for machining can be produced to resemble the final component thereby requiring only partial or finish machining. The Kanban principles for just-in-time supply in factory production involve prior action with an order being made for resupply of components as soon as the level in a parts bin drops below a critical value.

Principle 11: Cushion in advance—Cushion in advance involves preparing measures in advance to compensate for the relatively low reliability of an object or system. Examples include uninterruptible power supplies to protect key equipment in case of electrical supply failure, airbags, lifeboats and lifejackets, emergency exits with inflatable ramps on aircrafts, runoff strips on hills in case of brake failure and fire extinguishers that are conveniently located and easy to use. Emergency instructions and contingency planning are also examples of cushioning in advance of an event. A domestic example is the use of an aerator in a tap to make water feel "softer."

Principle 12: Equipotentiality—Many tasks involve lifting and lowering objects. Equipotentiality involves altering working conditions so that the lifting or lowering of an object is avoided. Chests of drawers, for example, negate the need to lift everything out of a chest in order to access objects at a lower level. Another example is the use of a pit to facilitate access to an automobile engine. The principle can be applied to a gravitational field or any other potential field. Grounding straps can be used to reduce the risk of static shocks on key equipment in factories.

Principle 13: Inversion—Inversion involves deliberately implementing a solution that is opposite to that initially specified or envisaged. Examples include heating an object instead of cooling it. Inversion can be achieved in a number of ways:

1. Instead of an action dictated by the specifications of the problem, deliberately implement an opposite action.
2. Make a moving part of the object or the outside environment fixed and the fixed part movable. An example of this is embodied in a fitness running machine where the runner remains stationary in the room but the treadmill belt moves.

3. Turn the object upside down or around. For example, a chiseling tool in a process can be applied from below the workpiece so that chips of waste material fall away from the item being machined.

Other examples of inversion include slow food instead of fast food, such as a slowly cooked casserole; working at home to avoid traveling; problem-based learning where students explore the material to be learned rather than being given lectures; broadcasting events such as concerts directly into homes so that people can participate in the experience without having to travel; using a small screen or display in front of an eye as part of a head-up display system instead of using a large screen. Another example of inversion includes expanding instead of contracting a business during a recession or downturn and preparing products for an assumed or engineered upturn.

Principle 14: Spheroidality—Many objects are designed to use or be used on flat surfaces. The principle of spheroidality involves considering using curvature or rotation; it can be implemented by a number of means:

1. Changing linear parts or flat surfaces to curved ones; changing cube and parallelepiped features to spherical shapes. A historic example of this principle is the use of arches and domes in place of lintels for increased structural strength. In order to reduce stress concentrations in formed products, a fillet radius can be introduced between features at different angles. Curved furniture can be used for playrooms to mitigate injuries.
2. Using rollers, balls, spirals or springs. Rolling element bearings reduce friction by providing reduced surface-to-surface contact and extra freedom of movement. A spiral can be used for the filament of a classic filament lightbulb in order to enable a longer length of wire to be located in a given space and reduce stresses.
3. Replacing a linear motion with rotating movement; using centrifugal effects. A rotating cylinder with brushes can be used to disturb the surface of a carpet so that it is easier for dirt particles to become airborne and get carried into a vacuum cleaner. Centripetal acceleration is exploited in cyclone separators so that heavy particles are separated from lighter ones. A further example is the use of multiaxis rotation in rotational molding.

Principle 15: Dynamics—Objects or systems that are rigidly connected together are less able to respond adaptively to changes in conditions than those with some degree of freedom. Dynamics involves designing a system so that it can adjust readily to an external influence. The principle of dynamism can be implemented in a number of ways:

1. Make an object or its environment adjust automatically so that performance is optimized at each stage of operation. An example is the use of an adjustable swept wing as on the Tornado, F14 Tomcat, Su 17 and Tu 160 to enable efficient aerodynamic performance of an aircraft for both subsonic and supersonic flight. Other examples include photo and thermochromic glass.

2. Divide an object into subelements that can change position relative to each other. An example to prevent illegal parking is to use posts that are hinged at the base with a lock to set the position of the post in its vertical or horizontal position. Other examples embodying this principle include roll-on and roll-off ferries with a hinged bow to enable access, and the Beluga Airbus transportation aircraft with its hinged cabin section that provides efficient aerodynamics in its closed position but facilitates the transportation of aircraft wings between factories.
3. Make a fixed object moveable or interchangeable. For example, photovoltaic solar panels that track the sun.

Repeated use of the dynamism principle in combination with segmentation is characterized by an increase in interactions and transition to a microlevel. This pattern can be illustrated by the following sequence: rigid system, single hinge, multiple hinges, elastic system, a field in place of a physical system. An example illustrating such development can be found in plumbing, where fixed hard-soldered pipework has been substituted by a system of pipes and compression joints, which in turn has been replaced by flexible pipes and hoses. A similar trend can be found in desk lamps, where the functionality of a standard lamp is improved by an anglepoise lamp and, in turn, by articulated joints.

Principle 16: Partial or excessive action—If it is difficult to obtain 100% of a desired effect, consider whether achieving a bit more or a bit less makes the problem easier. This is the principle embodied in partial or excessive action. An example is to apply excess paint to a component by dipping it in a vat of paint and then removing the excess paint by spinning the component to throw off the excess by centrifugal effects. Another example is continuously filling a hopper, or constant head device, so that the force of the material above causes the material to flow out of a discharge point at an even rate. By removing some material in advance, perforations in packaging make it easier to open.

Principle 17: Transition to a new dimension—Transition to a new dimension involves a shift in approach to another plane. This can involve consideration of another physical dimension, say changing linear to planar motion, or going upwards or around a corner. The principle of transition to a new dimension can be formalized as follows:

1. Move an object or system in two-dimensional or three-dimensional space.
2. Change the motion of an object from linear to movement in a plane (two dimensions). Change two-dimensional planar motion to three-dimensional motion.
3. Use a multilayered assembly of objects or components instead of a single layer. Examples include multistory car parks, underground rail systems running in tunnels on different levels and apartments in multistory buildings.
4. Incline an object or rotate it. A square mirror tile can be hung or supported using a corner rather than its edge. A drawing or painting surface can be inclined to the user to facilitate

use. The side or inside of packaging can be used to list the instructions for a game. Inclined reflectors are used in some greenhouses and biospheres to improve insolation and hours of illumination by daylight in housing.

Principle 18: Mechanical vibration—Mechanical vibration can be used to put energy into a system. This energy can then be used to excite an object or system so that, for example, it can more easily be broken, moved or mixed. Mechanical vibration can be achieved by a number of means, including shaking, sound waves and ultrasonics. Means of implementing the principle are as follows:

1. Cause an object to vibrate. Vibration instead of noise can be used as an alert for a message. In order to remove a cast from a limb, a vibrating knife can be used in place of a hand saw. Vibration can be used to improve the flow and structural properties of a casting in a mold.
2. If an object is already vibrating or oscillating, raise the frequency of oscillation. Consider ultrahigh frequency vibration. An ultrasonic bath can be used to clean bearings by exciting dirt and grime so that it detaches from the bearing's surfaces.
3. Use the resonant frequency to excite an object. In order to break up stones in gallbladders and kidneys, the stones can be excited at their resonant frequency using ultrasound, negating the need for invasive surgery.
4. Instead of mechanical vibrations, use piezovibrators.
5. Use ultrasonic vibrations in conjunction with an electromagnetic field.

Principle 19: Periodic action—The principle of periodic action involves introducing gaps into an action. This principle can be implemented in a number of ways:

1. Replace a continuous with a periodic or pulsed action. A warning lamp, such as bicycle lights, can be made to flash so they are more noticeable and require less power than if powered continuously. In a hammer action drill the continuous rotation associated with a twist drill is replaced by a periodic reciprocating motion, superimposed on the rotating motion so that the drill imparts an impulse of energy to the work area.
2. If an action is already periodic, alter its frequency. A pulsed siren can be replaced by a siren with variations in both amplitude and frequency.
3. Use the period between impulses to provide an additional action. If the periods of demand from several devices are synchronized, a single limited energy storage device such as a capacitor, battery or flywheel can be used to provide energy to them all. If two propellers are used on a boat or ship, their blades can be synchronized so that the impulse of one blade provides an impulse while its equivalent has moved on. The period between demands for energy can be used to store energy in, say, a capacitor so that energy can be quickly supplied if required to give greater power.

Principle 20: Continuity of a useful action—Some actions, components or processes do not provide a useful function throughout their entire period of operation. The principle of continuity of a useful action involves making all parts of an action provide a useful function throughout the operation. The principle can be subdivided as follows:

1. Carry out an action continuously, without interruptions, where all parts of an object operate at full capacity. Lean and just-in-time manufacturing, continuous casting of metals and continuously variable transmission are examples of this principle.
2. Remove idle motions and interruptions. This principle is used in self-emptying filters that dump contaminants when the load is low.

It should be noted that the principles of continuity of useful action and periodic action contradict each other. If all intermittent actions are eliminated, there will not be any gaps to exploit. As with any guiding principle, application of common sense and judgment is necessary in their use.

Principle 21: Rushing through—Rushing through a task can reduce the time during which problems can occur. This principle involves performing harmful or hazardous operations at high speed. When cutting a soft material slowly it can deform, making the task difficult and inaccurate. By speeding up the cutting process it is sometimes possible to complete the task before the material has time to distort significantly. For example, when cutting plastic pipes, by performing the task quickly heat from the cutting process does not propagate significantly within the surrounding material to cause distortion. Another example of the rushing through principle is harvesting, as a crop remains ripe for a very short period of time and leaving it for too long increases the risk of overripening and damage by weather and other environmental factors.

Principle 22: Convert harm into benefit—Sometimes undesirable effects such as the generation of waste result from a process. The convert harm into benefit principle encourages conversion of an undesirable effect into a benefit. Many industries and commercial ventures have developed out of developing inventive means to treat or convert waste products into something harmless or useful. The following examples illustrate this principle:

1. Use damaging factors or harmful environmental effects to obtain a positive result. An example is the conversion of methane emitted from decomposing rubbish into useful power.
2. Remove a harmful factor by combining it with other harmful factors. For example toxic chemicals can be used to treat timber to prevent or limit rot and infestation.
3. Increase the amount of harmful action or tolerate it until it ceases to be harmful. Each time a computer system is exposed to a virus attack, the systems developed to counteract this increase the protection available for the system.

Principle 23: Feedback—Feedback involves taking a signal from a system and using this to control the system itself. Feedback can be used to exaggerate an effect or to cancel out something undesirable. Feedback is used in many heating systems. A thermostat measures the temperature of the system and when the temperature of the thermostat exceeds a pre-set value the thermostat causes a heating element to turn off or reduce its output. Feedback can take various forms. If the feedback signal causes the system to respond such that the feedback then increases, it is known as positive feedback. If the feedback signal causes the system to respond in such a way that the feedback decreases, it is known as negative feedback. If the feedback signal does not cause the system to change, then the system is in equilibrium. If feedback is not present in a system it can be introduced in order to improve control. If feedback already exists in a system then options for changing it can be considered. Other examples of the use of feedback include statistical process control, motion sensitive flush, and tap systems. An example of adjusting the feedback in a system is the use of variable sensitivity for a thermostat so that it responds to rising temperature in a different way to falling temperature in order to improve comfort by reducing temperature fluctuations or reduce energy consumption.

Principle 24: Mediator—A task may need to be undertaken that cannot readily be performed by an existing system. The mediator principle involves introducing a new object to transfer or transmit an action, or temporarily introducing an object to perform a function that can subsequently be removed. Examples of the mediator principle include coating of weights in fitness machines with a sound-deadening material such as rubber; fixtures and jigs that make positioning and processing of parts easier; molds for forming shapes which are removed after the shape has been formed; using coatings to protect a surface from corrosion; and introducing abrasive particles to a water jet, to enhance water jet cutting.

Principle 25: Self-service—The self-service principle involves configuring the object or system so that it services itself and carries out supplementary and repair operations, and also considering making use of waste energy and material. Examples include a tyre that automatically repairs itself by releasing fluid from an integrated container filled with repair compound; if a surface is subject to wear or corrosion, the prevention of wear or deterioration by use of a layer of the abrasive or corrosive material itself; heating of a vehicle using fluid that transfers harmful heat away from the engine; designating customers as the waiters so that they can serve themselves and clear away their mess in a self-service, carvery or fast-food restaurant; advancing the rod in a welding gun using a solenoid. Halogen lamps invoke the self-service principle, regenerating the filament by redepositing material on the filament that has evaporated off. Parasitic power generation systems exploit energy available from the host. Examples include quartz watches powered by the wearer's movements, bicycle hub, heel strike and light generators.

Principle 26: Copying—The copying principle suggests

1. Using a simple and inexpensive copy in place of an object that is complex, expensive, fragile or inconvenient to operate;
2. Replacing an object or system of objects by an optical copy or image. Scaling can be used to reduce or enlarge the image;
3. If visible copies are used already, replacing them with infrared or ultraviolet copies.

Examples include using 2D cutouts for shop window dummies to display clothes; holographic projections to display valuable sculptures, manuscripts or paintings; video conferencing instead of traveling; a virtual reality model to explore or test an environment, or train on it in advance of the environment being available. Measurements can be made from images rather than directly from the original item. Examples of this approach include measurements for maps and surveys based on satellite images and ultrasonic images of fetuses in the womb. The height of tall objects can be measured by means of their shadow.

Principle 27: Inexpensive short-life—The inexpensive short-life principle involves replacement of an expensive object by a series of inexpensive ones, possibly forgoing some, or all, of the original object's attributes such as longevity. Examples include use of recyclable paper cups in place of ceramic crockery in cafes in order to avoid the cost of cleaning and storing durable objects; using disposable diapers in place of reusable ones; the use of fuses to protect expensive electrical hardware; medical swabs, poultices and sample containers. Numerical simulation, such as computational fluid dynamics, can be used to model the effect of a change in conditions on a system without having to go to the expense of testing the actual system.

Principle 28: Mechanical substitution—Many mechanical systems, due to their reliance on physical parameters, are inherently bulky and expensive. The mechanical substitution principle involves replacement of the mechanical system by a sensory system or field. The principle can be invoked in a number of ways:

1. Replace a mechanical system by a sensory (optical, acoustic or olfactory) system. An example of this is the inclusion of an additional gas with a distinct odor in natural gas supplies in order to alert customers of leaks. Require customers to enter data by means of a touch screen instead of filling out a form that must be keyed in by employees. Replace a physical fence for confining animals such as dogs with an acoustic "fence" (where the signal is audible to the animal). Using motion or infrared sensors to avoid the user having to locate or touch a control, such as hands-free taps and dryers.
2. Use electrical, magnetic or electromagnetic fields to interact with objects. An example embodying this principle are electronic swipe keys and "oyster" cards, GPS for monitoring fleet vehicle movement and the use of electromagnet stays to hold open fire doors until released by the alarm.

3. Replace static fields with dynamic fields, replace temporarily fixed fields with flexible ones, change unstructured fields to structured, e.g. load-based traffic and zonal heating.
4. Use a field in conjunction with field-activated (e.g. ferromagnetic) particles. Examples include intelligent tidal traffic flow control (using, for example, road cameras), integrating a ferromagnetic material in a sample in order to allow it to be manipulated or sorted.

Principle 29: Pneumatic or hydraulic construction—The pneumatic or hydraulic construction principle involves replacing solid parts of an object by a gas or a liquid that can be more easily channeled and have different properties that may be useful such as compressibility. This may enable parts to be inflated, filled, supported or separated using hydrodynamic effects and hydrostatics as in bearings. Examples also include air tents for exhibitions, inflatable boats, play-castles, pneumatic and hydraulic clamps with an elastic cushion that holds a part with evenly distributed pressure, the use of bubble wrap to protect fragile goods. Fluid pressure can be used to produce mechanical advantage by altering the bore of pipes in a system, as in pneumatic and hydraulic rams and jacks with a small force but large motion resulting in a large force but small motion (see Chapter 18).

Principle 30: Flexible membranes or thin films—Thin films and membranes offer potential advantages in terms of cost, flexibility, space usage and separation. The flexible membranes or thin films principle involves using flexible membranes or thin film to replace traditional construction, or isolating an object from its environment using a flexible membrane or thin film. Examples include the use of sprayed or painted coatings to protect a surface and the thin walled structures of greenhouses. Other examples include webbed structures, bubble wrap and egg boxes. In order to enable a drug to release over a period of time, a pill can be formed with layers of material that dissolve at different rates, encapsulating the active drug.

The thin film principle can be combined with hydraulics or pneumatics to produce strength in the structure as in aluminum drink cans where a very thin lightweight vessel is pressurized, giving it increased structural properties.

Principle 31: Porous materials—A porous material allows some substances to pass through it while filtering out or blocking others. The porous materials principle can be useful for separating or filtering out undesirable items or substances and involves making an object porous or adding porous elements (inserts, covers, etc.), or, if an object is already porous and this is undesirable, filling the pores in advance with some kind of substance. An example of this principle includes drilling or forming holes in a structure to make it lighter without impacting significantly on the structure properties. Cinder and breeze blocks formed by coal and coke ash bonded with cement provide a lightweight construction material as much of the volume of the block is air. Microfibers are widely used in sports clothing, with the small pores of the fibers preventing moisture from passing through it as liquid, but still allowing moisture to evaporate. Semipermeable films are used for some adhesive skin plasters that allow

moisture to evaporate but exclude water, dirt and bacteria. In order to avoid installing equipment for pumping coolant around a system, for a short working life, the system can be prefilled with just enough coolant to operate for the period concerned.

Principle 32: Color change—Color influences our aesthetic judgments and can also be used for providing information. The color change principle can be implemented in several ways:

1. Changing the color of an object or its surroundings. Many warning lights involve a change of a color to signify a status. The six hats thinking technique provokes changes in viewpoint by adoption of different colored hats.
2. Changing the level of transparency of an object or its environment. A transparent bandage enables a wound to be inspected without removing the dressing.
3. Using color additives to aid observation of objects or processes that are difficult to see. Fluorescent strips can be used on cycle jackets to aid visibility to other road users. Colored threads and strips are used to aid the security of bank notes and bonds. Another example is the addition of particles to a flow of fluid in order to aid observation of its detailed flow paths.
4. If such additives are already used, adding lighting or employing luminescent traces or tracer elements.

Principle 33: Homogeneity—A homogeneous material has the same physical properties throughout its structure. The homogeneity principle involves making objects that interact with a primary object out of the same material or out of one with similar properties. If all the components are made from the same material, this may result in cheaper material sourcing because of economies of scale and less corrosion due to chemical or thermochemical reactions between components. For example, the contacts in an electric circuit are usually made of the same material as the circuit in order to avoid corrosion problems. If the circuit is made from copper, gold can be used for the small surfaces associated with the contacts as it offers similar electrical properties to copper, has a low resistance and good surface corrosion resistance. The homogeneity principle can also be applied to components. All connectors in a design can be chosen so that they are of the same kind. In building design, the same type of motor could be specified for a wide range of different applications. In order to avoid distortion of assemblies, components can be made from materials with similar coefficients of linear expansion.

Principle 34: Discard and renewal—Once a component has served a useful purpose it may not be required again or it may need to be replaced if its functionality has been impaired. The discard and renewal principle can be considered in two different ways:

1. After a component has fulfilled its function or become useless, discard it. This can be achieved by, for example, ejecting, dissolving or causing it to evaporate. Biodegradable components used in surgery such as polylactide pins and screws dissolve slowly in body

fluids providing a temporary fastening while the body heals. An example of discard after use is the separation and release of rocket boosters after firing.
2. As soon as a component has become worn or depleted, replace it immediately. An example of this is the ejection of a bullet from a gun after firing, then its replacement from a magazine. Similarly, many machine tools have the facility to replace a broken or worn cutting tool by a new one from a magazine. Some business organizations follow this principle in their selection of which projects to support. If, say, a new type of snack has been launched with a shelf life of 12 weeks, then this period can be used to ascertain whether it should be supported into the future based on the sales figures over the 12-week period in a test stall or supermarket. If sales are disappointing, the snack line can be discarded and replaced by a new idea. Other examples of this principle include gray water recycling and heat exchangers.

Principle 35: Transforming the physical or chemical state of an object—Altering an object in some way, such as changing its temperature, the concentration of compounds or its density, can sometimes provide overall benefits. Transforming the physical or chemical state of an object involves such considerations and can be implemented in several ways:

1. Change an object's physical state. Examples include use of adhesives in place of mechanical fastening and virtual prototyping in place of physical prototyping.
2. Change concentration or consistency. Washing powders can be concentrated to reduce packaging and transportation costs. Powdered paint that can be made up with water, offering reduced packaging costs and longer storage life.
3. Change the degree of flexibility. Software can be offered with a range of options or levels, such as novice to expert user, with increasing levels of sophistication.
4. Change the temperature or pressure. Examples include climate control and laminar flow curtains used for some shop doorways, and open refrigerators.
5. Change other parameters.
6. Use the transition to or through a pseudostate, such as a pseudoliquid state, or use a transitionary phase, such as plastic deformation.

Principle 36: Phase change—It may be advantageous to exploit the phenomena that alter during the phase change of a substance in order to produce a desired effect. Physical properties that change during phase transitions include volume, heat release and heat absorption. When exploiting phase change it is important to consider how the process is started and how it will stop. The most common phase changes are melting from a solid to a liquid phase, boiling and evaporation, freezing and condensation for solids, liquids, gases and plasmas. Other transitional phenomena include paramagnetic to ferromagnetic, conductor to superconductor, crystallographic changes and superfluidity. An example is the use of sodium acetate for heat pads, which contain a supersaturated solution. Crystallization of the fluid can be triggered by releasing crystals into the solution, which act as nucleation sites and cause the

solution to crystallize, suddenly thereby releasing crystal lattice energy, which is manifested by a temperature rise.

Principle 37: Thermal expansion—When an object is heated, it tends to expand. Thermal expansion of materials can be exploited in tackling a range of problems. It should, however, be noted that there are some exceptions to this tendency, with water in its solid phase being less dense than as a liquid (this is why lumps of ice tend to float albeit with only a small proportion of their surface exposed at the free surface). Another substance where expansion is minimal with increased temperature is Invar, which has a near zero coefficient of linear expansion. Thermal expansion is used in bimetallic thermometers and thermostats. In these devices two strips of dissimilar materials such as brass and steel are welded together (Childs, 2001). As the materials have different coefficients of linear expansion, with brass having the higher value, if the temperature rises then the brass will expand more than the steel, causing the assembly to bend. The magnitude of the distortion of the strip can be calibrated to give an indication of temperature and the strip either connected to a dial indicator or the strips used to close an electric circuit for control purposes. Another example of the use of thermal expansion is in interference fits (see Chapter 19), where the diameter of two components overlap and assembly is achieved by either forcing the components together or by heating up the outer component and cooling the inner component so that the two components can readily slide together. With subsequent heat transfer, the temperature of both components will tend to equalize, with the outer component cooling and the inner component heating up causing the outer component's inner dimension to reduce, thereby clamping onto the inner.

Principle 38: Strong oxidants—Oxygen reacts readily with many elements and compounds. The strong oxidants principle involves enhancing this effect by using materials that combine with oxygen readily, or by adding more oxygen to the system. Examples of the principle include

1. Replacing normal air with enriched air with a higher concentration of oxygen;
2. Replacing enriched air with oxygen;
3. Treating an object in air or in oxygen with ionizing radiation;
4. Using ozone in combination with oxygen;
5. Using ionized oxygen.

For example, stainless steel can be cut with plasma arcs in a stream of oxygen.

Principle 39: Inert environment—If chemical reactions in an environment are a problem, an option is to remove the chemicals. The inert environment principle involves replacing the normal medium or environment with an inert one or carrying out the process in a vacuum. For example, to reduce the risk of cotton from catching fire in, say, a warehouse, inert gas can be used to purge it of air while it is being transported from one location to another. Some welding processes are undertaken with the workpiece held in a purge flow of inert gas. Some

lightbulbs are partially evacuated and then filled with an inert gas in order to reduce oxidation of the light element.

Principle 40: Use composite materials—By using materials that have complementary or synergistic properties, it is sometimes possible to construct an assembly that has the desirable properties of the constituent elements. The use of composite materials principle involves replacing a homogeneous material with a composite or combination. Examples include rebar reenforced concrete, the reenforcement of wedge belts with hoop wires (see Chapter 12) and Kevlar. Some military aircraft have wings made of composites of plastics and carbon fibers for high strength and low weight.

The 40 principles represent a toolkit for problem solving in their own right. The list can be browsed to provoke ideas. Another strategy for using the principles is to select them based on tradeoffs. Principles that negate tradeoffs can be selected and explored. Alternatively, use can be made of the contradiction matrix (see Section 3.8.3) to select the principles that are most likely to be useful based on experience to resolve a contradiction between two features.

The examples presented for each of the principles have been sourced from a wide range of applications; their inclusion here does not necessarily mean that the originator used or was even aware of TRIZ. The examples nevertheless show that the principles are wide ranging in their application to technology and business. TRIZ comprises various tools that can be used in isolation to assist in CPS. Tools such as the 40 principles can be applied within a brainstorming session in order to provoke ideas. The TRIZ tools, with their built-in knowledge of generic solutions, represent a short cut to experience, giving an extremely powerful tool. It is used widely in industry and embedded in various inventive problem-solving software packages. Further information on TRIZ can be found in Gadd (2011), The TRIZ Journal and in Vaneker and Lutters (2008).

3.8.3 Contradiction Matrix

As noted earlier, while trying to improve one desirable property, another desirable property may deteriorate. Conventional problem solving generally leads to a compromise solution. The most inventive solution is obtained when a technical problem containing a contradiction is solved by completely eliminating the contradiction where one property improves and another degrades. Altshuller, from his research on over 40,000 inventive patents, found that there are only 39 features, defined in Table 3.12, that either improve or degrade. As a result, every problem can be described as a conflict between a pair of parameters (2 out of 39 parameters). Altshuller found that many patents and invention disclosures had, in the past, resolved these individual conflicts in several different fields. He concluded that only 40 inventive principles, as defined in Table 3.13, were used to resolve these contradictions fully,

without a tradeoff or compromise. He argued that if these results were known earlier, the problems being tackled in the original patents would have been solved with more ease.

From knowledge of the 40 principles, combined with a method to identify the similarity of a new problem to previous problems, it is possible to tackle an issue with greater ease. One of the processes Altshuller developed was based on analysis of contradiction that was seen to have been resolved in a patent. Altshuller categorized these, along with the principles used to resolve the contradiction, in a tool called the contradiction matrix. This gives an indication of which of the 40 inventive principles are most likely to be useful in resolving the contradiction. This tool has been found to be very powerful and has since been embodied in a wide range of TRIZ software packages.

The matrix is organized in the form of 39 improving parameters and 39 worsening parameters (a 39 by 39 matrix), with each cell entry giving the most often used inventive principles that may be used to eliminate the contradiction. This matrix is known as the contradiction matrix and is one of the main TRIZ tools. The contradiction matrix consists of a list of parameters so that the parameter that is getting better can be identified. In addition, the corresponding parameter that is getting worse can also be identified. The improving parameters are listed in the first column and the worsening parameters in the first row. In the corresponding intersection between the improving and worsening parameter, the table gives a list of principles that can be explored in order to resolve the contradiction. A full version of the matrix is shown in Table 3.14.

A four-step process for using the contradiction matrix can be followed:

1. Use the 39 parameters to identify critical features in the problem.
2. Identify the contradictions between the parameters where one causes problems with another.
3. Use the contradiction matrix to identify principles that can be used to resolve the contradictions.
4. Use the numbers from the contradiction matrix to look up resolution principles, and use these principles to find solutions to the problem.

It should be noted that the principles in the table for a given entry are listed in order of priority based on analysis of previous solutions. The principle most likely, from experience, to yield a helpful solution is listed first. If exploring this principle proves unhelpful, then the next principle in the list can be tried and so on, until a solution is apparent or options have been exhausted.

Example 3.1

The example of contradiction in the design of conventional axes for splitting wood was introduced earlier. If the blade is made heavier, it can strike a more effective blow with better splitting power. However, the quantity of material used increases, making the device more awkward to handle.

Table 3.14: TRIZ contradiction matrix.

Worsening parameter → / ↓Improving parameter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1 Weight of moving object		All	15 8 29 34	All	29 17 38 34	All	29 2 40 28	All	2 8 15 38	8 10 18 37	10 36 37 40	10 14 35 40	1 35 19 39	28 27 18 40	5 34 31 35	All	6 29 4 38	19 1 32	35 12 34 31	All	12 36 18 31	6 2 34 19	5 35 3 31	10 24 35	10 35 20 28	3 26 18 31	1 3 11 27	28 27 35 26	28 35 26 18	22 21 18 27	22 35 31 39	27 28 1 36	35 3 2 24	2 27 28 11	29 5 15 8	26 30 36 34	28 29 26 32	26 35 18 19	35 3 24 37
2 Weight of stationary object	All		All	10 1 29 35	All	35 30 13 2	All	5 35 14 2	All	8 10 19 35	13 29 10 18	13 10 29 14	26 39 1 40	28 2 10 27	All	2 27 19 6	28 19 32 35	19 32	All	18 19 28 1	15 19 18 22	18 19 28 15	5 8 13 30	10 15 20 35	10 20 35 26	19 6 18 26	10 28 8 3	18 26 1 28	10 1 35	2 19 22 1	35 22 1 17	28 1 9	6 13 1 32	2 27 28 11	19 15 29	1 10 26 39	25 28 17	2 26 35	1 28 15 35
3 Length of moving object	8 15 29 34	All		All	15 17 4	All	7 17 4 35	All	13 4 8	17 10 4	1 8 35	1 8 10 29	1 8 15 34	8 35 29 34	19	All	10 15 19	32	8 35 24	All	1 35	7 2 35 39	4 29 23 10	1 24	15 2 29	29 35	10 14 29 40	28 32 4	10 28 29 37	1 15 17 24	17 15	1 29 17	15 29 35 4	1 28 10	14 15 1 16	1 19 26 24	35 1 26 24	17 24 4	14 4 28 29
4 Length of stationary object	All	35 28 40 29	All		All	17 7 10 40	All	35 8 2 14	All	28 10	1 14 35	13 14 15 7	39 37 35	15 14 28 26	All	1 10 35	3 35 38 18	3 25	All	All	12 8	6 28	10 28 24 35	24 26	30 29 14	All	15 29 28 3	32 28 32 10	2 1 18	All	15 17 27	2 25	3	1 35	1 26	26	All	30 14 7 26
5 Area of moving object	2 17 29 4	All	14 15 18 4	All		All	7 14 17 4	All	29 30 4 34	19 30 35 2	10 15 36 28	5 34 29 4	11 2 13 39	3 15 40 14	6 3	All	2 15 16	15 32 19 13	19 32	All	19 10 32 18	15 17 30 26	10 35 2 39	30 26	26 4	29 30 6 13	29 9	26 28 32 3	2 32	22 33 28 1	17 2 18 39	13 1 26 24	15 17 13 16	15 13 10 1	15 30	14 1 13	2 36 26 18	14 30 28 23	10 26 34 2
6 Area of stationary object	All	30 2 14 18	All	26 7 9 39	All		All	All	All	1 18 35 36	15 36 37	All	2 38	40	All	2 10 19 30	35 39 38	All	All	All	17 32	17 7 30	10 14 18 39	30 16	10 35 4 18	2 18 40 4	32 35 40 4	26 28 32 3	2 29 18 36	27 2 39 35	22 1 40	40 16 4	16	15 16	1 18 36	2 35 30 18	23	10 15 17 7
7 Volume of moving object	2 26 29 40	All	1 7 4 35	All	1 7 4 17	All		All	29 4 38 34	15 35 36 37	6 35 36 37	1 15 29 4	28 10 1 39	9 14 15 7	6 35 4	All	34 39 10 18	2 13 10	35	All	35 6 13 18	7 15 13 16	36 39 34 10	2 22	2 6 34 10	29 30 7	14 1 40 11	25 26 28 2	25 28 21 27	22 21 2 40 1	17 2 1 40	29 1 40 30 12	15 13	10	15 29	26 1	29 26 4	35 34 16 24	10 6 2 34
8 Volume of stationary object	All	35 10 19 14	19 14	35 8 2 14	All	All	All		All	2 18 37	24 35	7 2 35	34 28 35 40	9 14 17 15	All	35 34 38	35 6 4	All	All	All	30 6	All	10 39 35 34	All	35 16 32 18	35 3	2 35 16	All	35 10 25	34 39 19 27	30 18 35 4	35	All	All	1 31	2 17 26		35 37 10 2
9 Speed	2 28 13 38	All	13 14 8	All	29 30 34	All	7 29 34	All		13 28 15 19	6 18 38 40	35 15 18 34	28 33 1 18	8 3 26 14	3 19 35 5	All	28 30 36 2	10 13 19	8 15 35 38	All	19 35 38 2	14 20 19 35	10 13 28	13 26	All	10 19 29 38	11 35 27 28	28 32 1 24	10 28 32 25	1 28 35 23	2 24 35 21	35 13 8 1	34 28 2 12	15 10 28 27	10 28 4 34	3 34 27 16	10 18	All
10 Force (intensity)	8 1 37 18	18 13 1 28	17 19 9 36	28 10	19 10 15	1 18 36 37	15 9 12 37	2 36 18 37	13 28 15 12		18 21 11	10 35 40 34	35 10 21	35 10 14 27	19 2	All	35 10 21	All	19 17 10	1 16 36 37	19 16 35 18 37	14 15	8 35 40 5	All	10 37 36	14 29 18 36	3 35 13 21	35 10 23 24	28 29 37 36	1 35 40 18	13 3 37 24	15 1 1	15 28 1 25	15 1 11	26 35 18 20	36 37 10 19	2 35	3 28 35 37

11	Stress or pressure	10 36 37 40	13 29 10 18	35 10 36	35 1 14 16	10 15 36 28	10 15 36 37	6 35 10	35 24	6 35 36	36 35 21		35 4 15 10	35 33 2 40	9 18 3 40	19 3 27	All	35 39 19 2	All	14 24 10 37	All	10 35 14	2 36 25	10 36 3 37	All	37 36 4	10 14 36	10 13 19 35	6 28 25	3 35	22 2 37	2 33 27 18	1 35 16	11	2	35	19 1 35	2 36 37	35 24	10 14 35 37
12	Shape	8 10 29 40	15 10 26 3	29 34 5 4	13 14 10 7	5 34 4 10	All	14 4 15 22	7 2 35	35 15 34 18	35 10 37 40	34 15 10 14		33 1 18 4	30 14 10 40	14 26 9 25	All	22 14 19 32	13 15 32	2 6 34 14	All	4 6 2	14	35 29 3 5	All	14 10 34 17	36 22	10 40 16	28 32 1	32 30 40	22 1 2 35	35 1	1 32 17 28	32 15 26	2 13 1	1 15 29	16 29 1 28	15 13 39	15 1 32	17 26 34 10
13	Stability of object composition	21 35 2 39	26 39 1 40	13 15 1 28	37	2 11 13	39	28 10 19 39	34 28 35 40	33 15 28 18	10 35 21 16	2 35 40	22 1 18 4		17 9 15	13 27 10 35	39 3 35 23	35 1 32	32 3 27 16	13 19	27 4 29 18	32 35 27 31	14 2 39 6	2 14 30 40	All	35 27	15 32 35	All	13	18	35 24 30 18	35 40 27 39	35 19	32 35 30	2 35 10 16	35 30 34 2	2 35 22 26	35 22 39 23	1 8 35	23 35 40 3
14	Strength	1 8 40 15	40 26 27 1	1 15 8 35	15 14 28 26	3 34 40 29	9 40 28	10 15 14 7	9 14 17 15	8 13 26 14	10 18 3 14	10 3 18 40	10 30 35 40	13 17 35		27 3 26	All	30 10 40	35 19	19 35 10	35	10 26 35 28	35	35 28 31 40	All	29 3 28 10	29 10 27	11 3	3 27 16	3 27	18 35 37 1	15 35 22 2	11 3 10 32	32 40 25 2	27 11 3	15 3 32	2 13 25 28	27 3 15 40	15	29 35 10 14
15	Duration of action by moving object	19 5 34 31	All	2 19 9	All	3 17 19	All	10 2 19 30	All	3 35 5	19 2 16	19 3 27	14 26 28 25	13 3 35	27 3 10		All	19 35 39	2 19 4 35	28 6 35 18	All	19 10 35 38	All	28 27 3 18	10	20 10 28 18	3 35 10 40	11 2 13	3	3 27 16 40	22 15 33 28	21 39 16 22	27 1 4	12 27	29 10 27	1 35 13	10 4 29 15	19 29 39 35	6 10	35 17 14 19
16	Duration of action by stationary object	All	6 27 19 16	All	1 40 35	All	All	All	35 34 38	All	All	All	All	39 3 35 23	All	All		19 18 36 40	All	All	All	16	All	27 16 18 38	10	28 20 10 16	3 35 31	34 27 6 40	10 26 24	All	17 1 40 33	22	35 10	1	1	2	All	25 34 6 35	1	20 10 16 38
17	Temperature	36 22 6 38	22 35 32	15 19 9	15 19 9	3 35 39 18	35 38	34 39 40 18	35 6 4	2 28 36 30	35 10 3 21	35 39 19 2	14 22 19 32	1 35 32	10 30 22 40	19 13 39	19 18 36 40		32 30 21 16	19 15 3 17	All 2 14 17 25	21 17 35 38	21 36 29 31	All	35 28 21 18	3 17 30 39	19 35 3 10	32 19 24	24	22 33 35 2	22 35 2 24	26	26 27	4 10 16	2 18 27	2 17 16	3 27 35 31	26 2 19 16	15 28 35
18	Illumination intensity	19 1 32	2 35 32	19 32 16	All	19 32 26	All	2 13 10	All	10 13 19	26 19 6	All	32 30	32 3 27	35 19	2 19 6	All	32 35 19		32 1 19	32 35 1 15	32	13 16 1 6	13 1	1 6	19 1 26 17	1 19	All	11 15 32	3 32	15 19	35 19 32 39	19 35 28 26	28 26 19	15 17 13 16	15 1 19	6 32 13	32 15	2 26 10	2 25 16
19	Use of energy by moving object	12 18 28 31	All	12 28	All	15 19 25	All	35 13 18	All	8 15 35	16 26 21 2	23 14 25	12 2 29	19 13 17 24	5 19 9 35	28 35 6 18	All	19 24 3 14	2 15 19		All	6 19 37 18	12 22 15 24	35 24 18 5	All	35 38 19 18	34 23 16 18	19 21 11 27	3 1 32	All	1 35 6 27	2 35 6	28 26 30	19 35	1 15 17 28	15 17 13 16	2 29 27 28	35 38	32 2	12 28 35
20	Use of energy by stationary object	All	19 9 6 27	All	All	All	All	All	All	All	36 37	All	All	27 4 29 18	35	All	All	All	19 2 35 32	All		All	All	28 27 18 31	All	All	3 35 31	10 36 23	All	All	10 2 22 37	19 22 18	1 4	All	All	All	All	19 35 16 25	All	1 6
21	Power	8 36 38 31	19 26 17 27	1 10 35 37		19 38	17 32 13 38	35 6 38	30 6 25	15 35 2	26 2 36 35	22 10 35	29 14 2 40	35 32 15 31	26 10 28	19 35 10 38	16	2 14 17 25	16 6 19	16 6 19 37	All		10 35 38	28 27 18 38	10 19	35 20 10 6	4 34 19	19 24 26 31	32 15 2	32 2	19 22 31 2	2 35 18	26 10 34	26 35 10	35 2 10 34	19 17 34	20 19 30 34	19 35 16	28 2 17	28 35 34

Continued

Table 3.14: TRIZ contradiction matrix.—cont'd

	Worsening parameter→ / ↓Improving parameter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
22	Loss of energy	15 6 19 28	19 6 18 9	7 2 6 13	6 38 7	15 26 17 30	17 7 30 18	7 18 23	7	16 35 38	36 38	All	All	14 2 39 6	26	All	All	19 38 7	1 13 32 15	All	All	3 38		35 27 2 37	19 10	10 18 32 7	7 18 25	11 10 35	32	All	21 22 35 2	21 35 2 22	All	35 32 1	2 19	All	7 23	35 3 15 23	2	28 10 29 35
23	Loss of substance	35 6 23 40	35 6 22 32	14 29 10 39	10 28 24	35 2 10 31	10 18 39 31	1 29 30 36	3 39 18 31	10 13 28 38	14 15 18 40	3 36 37 10	29 35 3 5	2 14 30 40	35 28 31 40	28 27 3 18	27 16 18 38	21 36 39 31	1 6 13	35 18 24 5	28 27 12 31	28 27 18 38	35 27 2 31		All	15 18 35 10	6 3 10 24	10 29 39 35	16 34 31 28	35 10 24 31	33 22 30 40	10 1 34 29	15 34 33	32 28 2 24	2 35 34 27	15 10 2	35 10 28 24	35 10 18 13	35 18 10 23	28 35 10 23
24	Loss of information	10 24 35	10 35 5	1 26	26	30 26	30 16		2 22	26 32		All	All	All	All	10	10	All	19	All	All	10 19	19 10	All		24 26 28 32	24 28 35	10 28 23	All	All	22 10 1	10 21 22	32	27 22	All	All	All	35 33	35	13 23 15
25	Loss of time	10 20 37 35	10 20 26 5	15 2 29	30 24 14 5	26 4 5 16	10 35 17 4	2 5 34 10	35 16 32 18	All	10 37 36 5	37 36 4	4 10 34 17	35 3 22 5	29 3 28 18	20 10 28 18	28 20 10 16	35 29 21 18	1 19 26 17	35 38 19 18	1	35 20 10 6	10 5 18 32	35 18 10 39	24 26 28 32		35 38 18 16	10 30 4	24 34 28 32	24 26 28 18	35 18 34 18	35 22 18 39	35 28 34 4	4 28 10 34	32 1 10	35 28	6 29	18 28 32 10	24 28 35 30	All
26	Quantity of substance	35 6 18 31	27 26 18 35	29 14 35 18	All	15 14 29	2 18 40 4	15 20 29	All	35 29 34 28	35 14 3	10 36 14 3	35 14	15 2 17 40	14 35 34 10	3 35 10 40	3 35 31	3 17 39	All	34 29 16 18	3 35 31	35	7 18 25	6 3 10 24	24 28 35	35 38 18 16		18 3 28 40	13 2 28	33 30	35 33 29 31	3 35 40 39	29 1 35 27	35 29 25 10	2 32 10 25	15 3 29	3 13 29 10	3 27 29 18	8 35 3 27	13 29 27
27	Reliability	3 8 10 40	3 10 8 28	15 9 14 4	15 29 28 11	17 10 14 16	32 35 40 4	3 10 14 24	2 35 24	21 35 11 28	8 28 10 3	10 24 35 19	35 1 16 11	All	11 28	2 35 3 25	34 27 6 40	3 35 10	11 32 13	21 11 27 19	36 23	21 11 26 31	10 11 35 29	10 35 29 39	10 28	10 30 4	21 28 40 3		32 3 11 23	11 32 1	27 35 2 40	35 2 40 26	All	27 17 40	1 11	13 35 8 24	13 35 1	27 40 28	11 13 27	1 35 29 38
28	Measurement accuracy	32 35 26 28	28 35 25 26	28 26 5 16	32 28 3 16	26 28 32 3	26 28 32 3	32 13 6	All	28 13 32 24	32 2	6 28 32	6 28 32	32 35 13	28 6 32	28 6 32	10 26 24	6 19 28 24	6 1 32	3 6 32	All	3 6 32	26 32 27	10 16 31 28	All	24 34 28 32	2 6 32	5 11 1 23		All	28 24 22 26	3 33 39 10	6 35 25 18	1 13 17 34	1 32 13 11	13 35 2	27 35 10 34	26 24 32 28	28 2 10 34	10 34 28 32
29	Manufacturing precision	28 32 13 18	28 35 27 9	10 28 29 37	2 32 10	28 33 29 32	2 29 18 36	32 23 2	25 10 35	10 28 32	28 19 34 36	3 35	32 30 40	30 18	3 27	3 27 40	All	19 26	3 32	32 2	All	32 2	13 32 2	35 31 10 24	All	32 26 28 18	32 30	11 32 1	All		26 28 10 36	4 17 34 26	All	1 32 35 23	25 10	All	26 2 18	All	26 28 18 23	10 18 32 39
30	Object affected harmful factors	22 21 27 39	2 22 13 24	17 1 39 4	1 18	22 1 33 28	27 2 39 35	22 23 37 35	34 39 19 27	21 22 35 28	13 35 39 18	22 2 37	22 1 3 35	35 24 30 18	18 35 37 1	22 15 33 28	17 1 40 33	22 33 35 2	1 19 32 13	1 24 6 27	10 2 22 37	19 22 31 2	21 22 35 2	33 22 19 40	22 10 2	35 18 34	35 33 29 31	27 24 2 40	28 33 23 26	26 28 10 18		All 35 2	24 2 25 28 39	35 10 2	35 11 22 31	22 19 29 40	22 19 29 40	33 3 34	22 35 13 24

31	Object generated harmful factors	19 22 15 39	35 22 1 39	17 15 16 22	All	17 2 18 39	22 1 40	17 2 40	30 18 35 4	35 28 3 23	35 28 1 40	2 33 27 18	35 1	35 40 27 39	15 35 22 2	15 22 33 31	21 39 16 22	22 35 2 24	19 24 39 32	2 35 6	19 22 18	2 35 18	21 35 2 22	10 1 34	10 21 29	1 22	3 24 39 1	24 2 40 39	3 33 26	4 17 34 26	All		All	All	All	All	19 1 31	2 21 27 1	2	22 35 18 39
32	Ease of manufacture	28 29 15 16	1 27 36 13	1 29 13 17	15 17 27	13 1 26 12	16 40	13 29 1 40	35	35 13 8 1	35 12	35 19 1 37	1 28 13 27	11 13 1	1 3 10 32	27 1 4	35 16	27 26 18	28 24 27 1	28 26 27 1	1 4	27 1 12 24	19 35	15 34 33	32 24 18 16	35 28 34 4	35 23 1 24	1 35 12 18	All	24 2	All		2 5 13 16	35 1 11 9	2 13 15	27 26 1 1	6 28 11 1	8 28 1	35 1 10 28
33	Convenience of use	25 2 13 15	6 13 1 25	1 17 13 12	All	1 17 13 16	18 16 15 39	1 16 35 15	4 18 39 31	18 13 34	28 13 35	2 32 12	15 34 29 28	32 35 30	32 40 3 28	29 3 8 25	1 16 25	26 27 13	13 17 1 24	1 13 24	All	35 34 2 10	2 19 13	28 32 2 24	4 10 27 22	4 28 10 34	12 35 8 40	17 27 2 34	25 13 35 23	1 32 28 39	2 25	All	2 5 12	12 26 1 32	15 34 1 16	32 26 12 17	All	1 34 12 3	15 1 28
34	Ease of repair	2 27 35 11	2 27 35 11	1 28 10 25	3 18 31	15 13 32	16 25	25 2 35 11	1	34 9	1 11 10	13	1 13 2 4	2 35	11 1 2 9	11 29 28 27	1	4 10	15 1 13	15 1 28 16	All	15 10 32 2	15 1 32 19	2 35 32 27	All	32 1 10 25	2 28 10 25	11 10 1 16	10 2 13	25 10	35 10 2 16	All	1 35 11 10	1 12 26 15		7 1 4 16	35 1 13 11	All	34 35 7 13	1 32 10
35	Adaptability or versatility	1 6 15 8	19 15 29 16	35 1 29 2	1 35 16	35 30 29 7	15 16	15 35 29	All	35 10 14	15 17 20	35 16	15 37 1 8	35 30 14	35 3 32 6	13 1 35	2 16	27 2 3 35	6 22 26 1	19 35 29 13	All	19 1 29	18 15 1	15 10 2 13	All	35 28	3 35 15	35 13 8 24	35 5 1 10	All	35 11 32 31	All	1 13 31	15 34 1 16	1 16 7 4		15 29 37 28	1	27 34 35 28	35 28 6 37
36	Device complexity	26 30 34 36	2 26 35 39	1 19 26 24	26	14 1 13 16	6 36	34 26 6	1 16	34 10 28	26 16	19 1 35	29 13 28 15	2 22 17 19	2 13 28	10 4 28 15	All	2 17 13	24 17 13	27 2 29 28	All	20 19 30 34	10 35 13 2	35 10 28 29	All	6 29	13 3 27 10	13 35 1	2 26 10 34	26 24 32	22 19 29 40	19 1	27 26 1 13	27 9 26 24	1 13	29 15 28 37		15 10 37 28	15 1 24	12 17 28
37	Difficulties of detecting and measuring	27 26 28 13	6 13 28 1	16 17 26 24	26	2 13 18 17	2 39 30 16	29 1 4 16	2 18 26 31	3 4 16 35	30 28 40 19	35 36 37 32	27 13 1 39	11 22 39 30	27 3 15 28	19 29 39 25	25 34 6 35	3 27 35 16	2 24 26	35 38	19 35 16	18 1 16 10	35 3 15 19	1 18 10 24	35 33 27 22	18 28 32 9	3 27 29 18	27 40 28 8	26 24 32 28	All	22 19 29 28	2 21	5 28 11 29	2 5	12 26	1 15	15 10 37 28		34 21	35 18
38	Extent of automation	28 26 18 35	28 26 35 10	14 13 17 28	23	17 14 13	All	35 13 16	All	28 10	2 35	13 35	15 32 1 13	18 1	25 13	6 9	All	26 2 19	8 32 19	2 32 13	All	28 2 27	23 28	35 10 18 5	35 33	24 28 35 30	35 13	11 27 32	28 26 10 34	28 26 18 23	2 33	2	1 26 13	1 12 34 3	1 35 13	27 4 1 35	15 24 10	34 27 25		5 12 35 26
39	Productivity	35 26 24 37	28 27 15 3	18 4 28 38	30 7 14 26	10 26 34 31	10 35 17 7	2 6 34 10	35 37 10 2	All	28 15 10 36	10 37 14	14 10 34 40	35 3 22 39	29 28 10 18	35 10 2 18	20 10 16 38	35 21 28 10	26 17 19 1	35 10 38 19	1	35 20 10	28 10 29 35	28 10 35 23	13 15 23	All	35 38	1 35 10 38	1 10 34 28	18 10 32 1	22 35 13 24	35 22 18 39	35 28 2 24	1 28 7 10	1 32 10 25	1 35 28 37	12 17 27 24	35 18 35 2	5 12 35 26	

A blank means no principle suggested; try exploring a different contradiction.
Source: Adapted from Altshuller (2005).

Solution

The improving feature in this case is force, and the worsening feature is the quantity of material. The corresponding feature numbers are 10 and 26, respectively. Examination can be made of Table 3.14 to identify which of the 40 principles are recommended. By identifying row 10 for force and column 26 for quantity of substance, the recommended principles are seen to be 14, 29, 18, and 36, corresponding to curvature, pneumatics and hydraulics, mechanical vibration and phase transition, respectively. The relevant part of the contradiction matrix is illustrated below in Table 3.15. The axe developed for Fiskars (Figure 3.12) makes use of a hollow handle to resolve the issue using the pneumatics and hydraulics principle. By using a hollow handle, i.e. replacing solid parts of an object with, in this case, air, the center of gravity is moved forward and the effective blow from the axe becomes more powerful from a lighter tool.

Example 3.2

In competitive high diving, it is important for the diver to engage in much practice. But repetitive diving involves multiple impacts between the diver and the water, potentially leading to injuries. Here the improving feature is speed (the rate of action in time), and the worsening feature is reliability (the ability of the body to perform healthily as intended).

Solution

Examination can be made of Table 3.14 to identify which of the 40 principles are recommended. By identifying row 9 for speed and column 27 for reliability, the recommended principles are 11, 35, 27 and 28, corresponding to cushion in advance, parameter changes, cheap short living and mechanics substitution, respectively. The cushion in advance principle has been exploited in one solution by using aeration of the water to make it softer during diving practice, enabling more dives to be undertaken.

Additional examples of contradiction and identification of possible parameters to explore with the contradiction matrix shown in the square brackets include

Table 3.15: Extract from the contradiction matrix showing the principles recommended for resolving a contradiction between force and weight of the moving object.

	Worsening feature Improving feature	25	26	27
9	Speed	All	10 19 29 38	11 35 27 28
10	Force	10 37 36	14 29 18 36	3 35 13 21



Figure 3.12
Fiskars axe using a hollow handle to make the tool lighter for handling. Image courtesy of Fiskars.

- Horseshoes (hot to work but cold to handle) [12; 17];
- Icebreaker (fast but low power) [9; 21];
- Flag (big but quiet) [6; 31];
- Pipe connection (fast but low force) [9; 10];
- Training (lots but short) [16; 25];
- Smuggling (heavy to sink but light to float) [2; 16];
- Bin (big but small) [8; 26].

3.9 Boundary Shifting

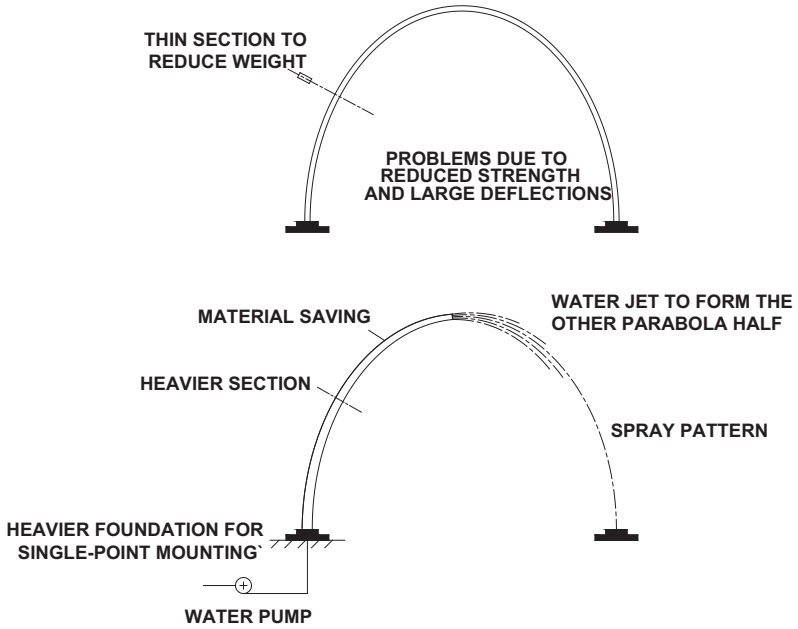
Boundary shifting involves challenging the constraints defined in the problem design specification (PDS) to identify whether they are necessary. For example, the PDS may define that steel should be used for a component. Boundary shifting challenges this specification to see whether it is appropriate and, if not, which other materials could be considered.

Example 3.3

The mayor of Stockholm has outlined funding for a sculpture, based upon an architect's sketches, to span the entrance of Stockholm harbor. As a leading engineering company, you have been presented with the contract to design and construct the sculpture (see Matthews, 1998).

Solution

There are often problems associated with designing something new, and this sculpture is not an exception. Consideration of fatigue due to wind loading is critical for a structure of this sort. In this case, the designer challenged the original specification for a sculpture that spanned the harbor and in place proposed a half-arch structure with the remainder of the geometry completed by a stream of water, Figure 3.13. This solution was acceptable to the mayor.



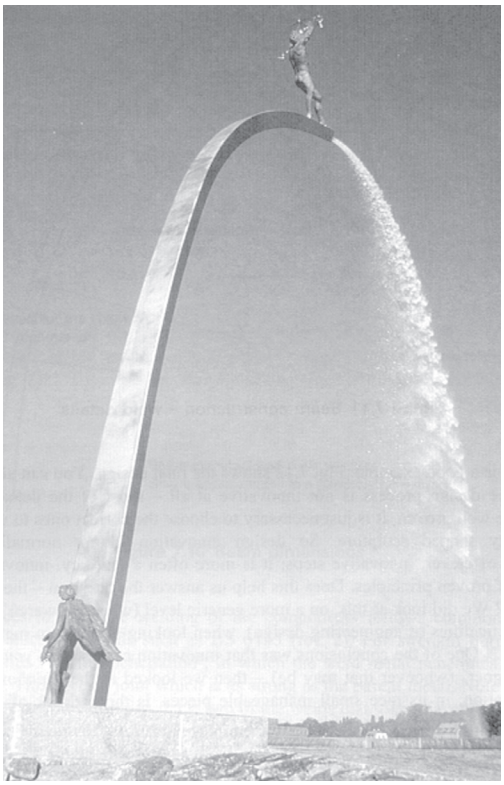


Figure 3.13
Conceptual and final solutions. *Matthews (1998)*.

3.10 The Creativity and Innovation Engine

In essence, creativity represents a significant risk. Yet in a fast-moving business climate, "no creativity" is even riskier. The definition and embodiment of details that enable an idea to be realized in practice is essential for realizing financial or societal value from an idea. The tensions involved with creativity, the value and use of creativity tools and the importance of design in realizing commercial potential can be embodied in the moniker "commercivity"—the commercial exploitation of creativity through the implementation of game-changing and sustainable ideas (see Childs and Fountain, 2011).

Creativity tools provide a means to augment innate generative activity. Most creativity tools can be used at any stage in a problem-solving process and tend to be focused on problem exploration, idea generation and concept evaluation. Creativity tools in expert hands are as important to commercivity as the surgeon's tools to the saving of a life in a medical procedure. Used recklessly they can be dangerous and counterproductive, yet without them there is scant chance of success. Deciding on the appropriate tool and then managing its use with great care is one of the defining skills of creativity.

Creativity tools can certainly be of assistance in developing the quantity of ideas, and if a process of convergence and divergence is followed, as in the CPS process, it will involve evaluation, refinement of the ideas considered as a means to improve quality. It is questionable whether the tools will provoke game-changing concepts. More likely, game-changing ideas that lead to new commercial opportunities will arise from sustained effort and an environment that encourages and permits risk taking. Creativity tools may, however, be of significant value in enabling the design space to be explored and for developing the details for each step of the idea in order to overcome challenges and issues inherent in idea realization.

The conditions for creativity to occur include the combination of expertise, sponsorship, motivation and good communication skills. Creativity can be enhanced by ensuring that our short-term memory is free enough to be able to consider memories. This can be achieved in a number of ways, including the use of creative tools by ensuring that a problem can be understood in relatively simple terms; supplying cues to make the search of long-term memory more efficient; providing cues to refresh short-term memory and thereby retain key information as appropriate. The combination of these factors for encouraging creativity is illustrated in the concept of an idea engine as shown in Figure 3.14, which uses the analogy of a jet engine with the multiple inputs of motivation, expertise, sponsorship and communication skills used for air, fuel, oil and control, the short-term memory modeled by a compressor connected to the powerhouse of the long-term memory or turbine. In an engine, the compressor is usually directly connected to the turbine; this connectivity is preserved in the analogy with the short-term memory fed with ideas by the long-term memory. The "refresh"

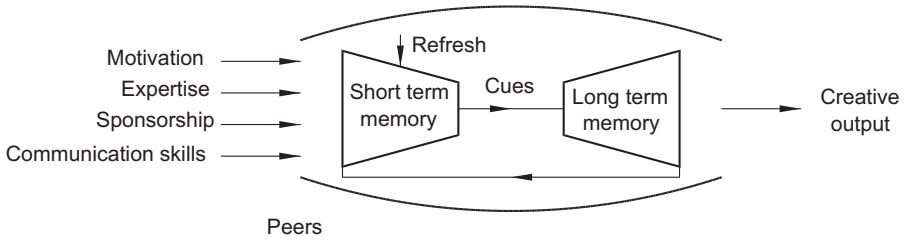


Figure 3.14
The idea engine. © *Peter Childs*.

for ensuring retention of an idea in working memory is taken as analogous to a glow or spark plug.

In order to take an idea forward, the developed knowledge in the field of innovation and experience in commercial activities can be applied. Creativity and innovation are integrally linked by design, where embodiment is given to the idea with refinement and elaboration of all of the details required to enable the idea to be detailed. Innovation, the realization of value from creativity, has been extensively studied and can be represented by a simple model in an extension to the idea engine shown in Figure 3.15 comprising the steps of financing and intellectual property considerations, development of a functional prototype or system, winning initial customers and thereby securing the finance for either licensing or winning more customers. The extension to the idea engine uses the analogy of an afterburner. The innovation engine encompasses the notions of testing whether the venture is concerned with a physical product or artifact or a process or system. Just as with a physical prototype, an essential part of the development process is to explore customer reaction and to explore its functions, so too with a process or system. The creativity and innovation engine also promotes a systems approach where there needs to be an incentive for engagement in the commercial activity.

There are many ways to enable the processes indicated in Figure 3.15. The Create Project process (2005) could be adopted to provide an overall framework for the activity, with creativity tools such as morphological analysis, Post-it and grid brainstorming used to

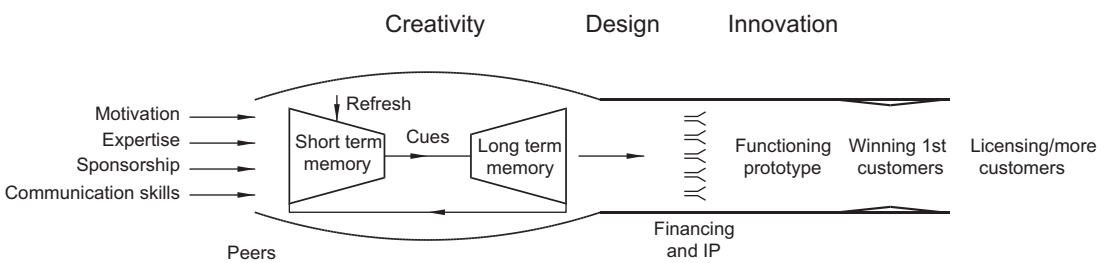


Figure 3.15
The creativity and innovation engine. © *Peter Childs*.

facilitate the generation of ideas. A standard design process could be applied and the ideas taken to market through the processes of securing finance, prototyping and marketing to initial customers prior to enable the generation of sufficient finance in order to take the idea forward to wider markets. The innovation project process of building a star team, agitating to generate key insights and breakthroughs, embodying and detailing ideas, piloting and roll-out (Neill, 2010) has been demonstrated as a highly effective, albeit linear approach to turning ideas to reality.

Turning ideas into reality generally requires small, balanced, empowered and well-resourced teams of people or a single person fulfilling multiple roles. A single project champion for the duration of the project provides many advantages, including ensuring that the vision is preserved. A challenging but uncomplicated project plan ensures focus without diverting attention to process. Early conceptualization of ideas and prototyping provides clear advantages in building confidence in ideas and demonstrating the potential viability of a concept. Iteration and product or system optimization enables cycles of improvements to be made to an idea. Facilitation of problem solving at decision gates assists in ensuring that ideas are addressed and rationale used in the judgment of issues.

Design thinking offers valuable insight into commercivity. Designing can be used as a verb or noun. If you are designing, as a verb, then you are designing an experience, whereas if you are designing, as a noun, then you are designing "stuff." With about 80% of the Western economy currently based on services as well as an increasing proportion of China's and India's economies (currently about 40% and 50%, respectively), much greater opportunities for commercial activity reside with services and the associated user experience. By addressing the design of a sociocultural system rather than just concentrating on a technical system, you are more likely to be able to develop game-changing and disruptive innovations that serve to raise the market capitalization and value of the business or organization concerned.

3.11 Conclusions

There are a wide number of creativity tools that can be used to aid in the generation of ideas. For example, some of the brainstorming tools such as the Post-it, alphabet and grid processes can be useful in the early phases of a project and whenever progress is blocked. Some organizations are serial users of brainstorming, such as IDEO (Kelley, 2001), and over 70% of businesspeople use them in their organizations (Anderson et al., 2004). The creativity problem-solving process has been validated extensively and provides a valuable framework for tackling a challenge and developing ideas. Other tools such as morphological analysis can be used to aid the development of ideas based on functional analysis of the subsystems. The theory of inventive problem solving provides a shortcut to experience. It is suggested that regular use of these tools within the engineering design activity can aid significantly in the generation of ideas.

References

Allen, M.S., 1962. The Allen Morphologizer. Prentice-Hall.

Altshuller, G.S., 1984. Creativity as an Exact Science — The Theory of the Solution of Inventive Problems. New York.

Altshuller, G., 1994. And Suddenly the Inventor Appeared: TRIZ, the Theory of Inventive Problem Solving (L. Shulyak, Trans.). Technical Innovation Center, Inc.

Altshuller, G., 2005. 40 Principles Extended Edition: TRIZ Keys to Technical Innovation. Technical Innovation Center.

Anderson, A., Heibeler, R., Kelley, T., Ketteman, C., 2004. Best Practices Building Your Business with Customer Focussed Solutions, Kindle ed. Touchstone.

Atkinson, R.C., Shriffin, R.M., 1968. Human memory: a proposed system and its control processes. In: Spence, W.K., Spence, J.T. (Eds.), 1968. The Psychology of Learning and Innovation: Advances in Research and Theory, vol. 2. Academic Press, pp. 89−195.

Bauer, L., 2003. Introducing Linguistic Morphology, second ed. Georgetown University Press.

Buzan, T., Buzan, B., 2006. The Mind Map Book. BBC Active.

Childs, P.R.N., 2006a. CETL in Creativity. Annual Report 2005−2006. InQbate, University of Sussex.

Childs, P.R.N., 2004. Mechanical Design, second ed. Butterworth Heinemann.

Childs, P.R.N., 2001. Practical Temperature Measurement. Butterworth Heinemann.

Childs, P.R.N., 11 December, 2006b. The Mercenary Use of Creative Methods. HEEG, University of Sussex, UK.

Childs, P.R.N., Fountain, R., 2011. Commercivity. In: Kovacevic, A., Ion, W., McMahon, C., Buck, L., Hogarth, P. (Eds.), E&PDE11, 2011. Design Education for Creativity and Business Innovation. 13th International Conference on Engineering and Product Design Education. Design Society, DS69-1, pp. 3−8.

Cowan, N., 2001. The magical number 4 in short term memory: a reconsideration of mental storage capacity. Behav. Brain Sci. 24, 1−185.

Cox, G., 2005. Cox Review of Creativity in Business: Building on the UK's Strengths. HM Treasury.

Create Project, 2005. www.diegm.uniud.it/create/ (accessed 10.10.08).

Csikszentmihalyi, M., 1996. Creativity — Flow and the Psychology of Discovery and Invention. Harper-Perennial.

de Bono, E., 1970. Lateral Thinking. Penguin.

de Bono, E., 1999. Six Thinking Hats. Penguin.

Design Council, 2007. The double diamond design process model. www.designcouncil.org.uk/en/About-Design/managingdesign/The-Study-of-the-Design-Process/ (accessed December 2007).

Domb, E., 1998. The 39 features of Altshuller's contradiction matrix. TRIZ J.

Furnham, A., 2000. The brainstorming myth. Bus. Strat. Rev. 11, 21−28.

Goldenberg, J., Mazursky, D., 2002. Creativity in Product Innovation. Cambridge University Press.

Gadd, K., 2011. TRIZ for Engineers. Wiley.

Garvey, B., Childs, P.R.N., 2013. Applying problem structuring methods to the design process for safety helmets. Paper HPD-2013-12, Helmet Performance and Design.

Gordon, W.J.J., 1961. Synectics: The Development of Creative Capacity. Harper & Row. pp. 3−7 and pp. 33−56.

Isaksen, S.G. (Ed.), 2000. Facilitative Leadership: Making a Difference with CPS. Kendall-Hunt.

Isaksen, S.G., De Schryver, L., 2000. Making a difference with CPS: a summary of the evidence. In: Isaksen, S.G. (Ed.), Facilitative Leadership: Making a Difference with CPS. Kendall-Hunt, pp. 187−248.

Jones, J.C., 1970. Design Methods: Seeds for Human Futures. John Wiley.

Kelley, T., 2001. The Art of Innovation. Profile Books.

Kochan, N. (Ed.), 1996. The world's greatest brands. Palgrave Macmillan.

Lidwell, W., Holden, K., Butler, J., 2003. Universal Principles of Design. Rockport.

Litchfield, R.C., 2008. Brainstorming reconsidered: a goal based view. Acad. Manage. Rev. 33, 649−668.

Matthews, C., 1998. Case Studies in Engineering Design. Arnold.

Michalko, M., 2001. Cracking Creativity. Ten Speed Press.

Miller, G., 1956. The magical number seven, plus or minus two: some limits on our capacity for processing information. Psychol. Rev. 63, 81–97.
Neill, R., 2010. Per Diem. Private communication.
Osborn, A.F., 1963. Applied Imagination: Principles and Procedures of Creative Problem-Solving, third ed. Scribner's.
Parnes, S.J., Noller, R.B., Biondi, A.M., 1977. Guide to Creative Action. Scribner's.
Poirier, M., Saint-Aubin, J., 1995. Memory for related and unrelated words: further evidence on the influence of semantic factors in immediate serial recall. Q. J. Exp. Psychol. 48A, 384–404.
Rose, L.H., Lin, H.T., 1984. A meta-analysis of long-term creativity training programs. J. Creat. Behav. 18, 11–22.
Swedish Morphological Society. www.swemorph.com (accessed 18.10.08).
The TRIZ Journal. www.triz-journal.com/.
Torrence, E.P., 1972. Can we teach children to think creatively? J. Creat. Behav. 6, 114–143.
Treffinger, D.J., Isaksen, S.G., Dorval, K.B., 1994. Creative learning and problem solving: an overview. In: Runco, M. (Ed.), Problem Finding, Problem Solving, and Creativity. Ablex, pp. 223–236.
Treffinger, D.J., Isaksen, S.G., Stead-Dorval, K.B., 2006. Creative Problem Solving: an Introduction, fourth ed. Prufrock Press.
Vaneker, T., Lutters, E., 2008. TRIZ-Future Conference '08 Synthesis of Innovation. Ipskamp.
Wallas, G., 1926. The Art of Thought. Jonathan Cape.
Whiting, C.S., 1958. Creative Thinking. Van Nostrand.
Yan, Y., Childs, P.R.N., Hall, A., 2013. An assessment of personality traits and their implication for creativity amongst Innovation Design Engineering masters students using the MBTI and KTS instruments. 19th International Conference on Engineering Design.
Zwicky, F., 1969. Discovery, Invention, Research through the Morphological Analysis. The Macmillan Company.

Further Reading

Boden, M.A., 1990. The Creative Mind: Myths and Mechanisms. Weidenfeld, London.
Childs, P.R.N., Hamilton, T., Morris, R.D., Johnston, G., 7–8 September, 2006. Center for technology enabled creativity. In: Rothbucher, B., Kolar, M., Ion, B., Clarke, A. (Eds.), Proceedings of the 4th Engineering and Product Design Education International Conference. Hadleys, Salzburg, Austria, pp. 367–372.
Eberle, B., 1997. Scamper: Creative Games and Activities for Imagination Development. Prufrock Press.
Guildford, J.P., 1967. Intellectual factors in productive thinking. In: Moonly, R.L., Razik, T.A. (Eds.), Explorations in Creativity. Harper & Row, New York, Evanston, and London, pp. 95–106.
Harris, R., 1998. Introduction to creative thinking. www.virtualsalt.com/crebook1.htm (accessed July 2006).
Isaksen, S.G., Treffinger, D.J., 1985. Creative Problem Solving: the Basic Course. Bearly Limited, Buffalo, New York.
Isaksen, S.G., Dorval, K.B., Treffinger, D.J., 1993. Creative Approaches to Problem Solving. Third Test Edition, Buffalo, New York.
Rantanen, K., Domb, E., 2002. Simplified TRIZ. St Lucie Press.
Thompson, G., Lordan, M., 1999. A review of creative principles applied to engineering design. Proc. Inst. Mech. Eng. 213 (Part E), 17–31.
Vidal, R.V.V., 2005. Creativity for operational researchers. Investigacao Operational 25 (1), 1–24.
Zusman, A., Zlotin, B., 1999. Overview of creative methods. TRIZ J. 7.

CHAPTER 4

Machine Elements

Chapter Outline

4.1 Introduction 121
4.2 Bearings 126
4.3 Gears, Belts, and Chains 128
4.4 Clutches and Brakes 130
4.5 Seals 132
4.6 Springs 133
4.7 Fasteners 133
4.8 Wire Rope 134
4.9 Pneumatics and Hydraulics 134
4.10 Enclosures 134
4.11 Conclusions 136
References 136
Further Reading 136
Nomenclature 137

Abstract

Many machine elements are available as stock items from a wide range of suppliers, while others represent established technology with defined rules and approaches for their design and selection. Machine elements, properly selected and specified, can be used as building blocks within more sophisticated machine designs enabling concentration of effort on the machine, rather than developing each and every machine element from scratch. The aim of this chapter is to introduce the scope of machine elements and aspects of their integration in machine design and mechanical engineering.

4.1 Introduction

Design activity involves endowing form and ensuring functionality of a given product. The function of many products depends on devices that modify force or motion and consist of a number of interrelated units. An example is a gear box that modifies the torque speed characteristic of a prime mover to match that of the load, and comprises gears, shafts, bearings, and seals. Such devices that modify force or motion are called machines, and the interrelated units are sometimes known as machine elements. The technology base has developed into a mature status in this area, providing a number of such devices that can be used, in some cases, as is or can be designed as fit-for-purpose.

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00004-6
Copyright © 2014 Elsevier Ltd. All rights reserved.

In the design of a machine and its machine elements, it is the designer's task to determine the motion, forces, and changes in energy involved so that sizes, shapes, and materials for each machine element making up the whole machine can be determined. Although design work may involve concentrating on one component at a given time, it is essential to consider its interrelationship with the whole product, following the total design philosophy and taking into account the market requirement, detailed specifications, the conceptual design, the design of other components, and the manufacturing requirements.

The goal in machine design is to determine the size, shape, and selection of materials and manufacturing processes for each of the parts of the machine, so that the machine will perform its intended function without failure. This requires the designer to model and predict the performance and operation of each component and overall assemblies, studying the mode of failure for each and ensuring that conditions expected in service for the product are met. This may involve determining stresses, deflections, temperatures, and material degradation for each and every component and overall assembly, as well as its influence within the total design. As stresses are a function of forces, moments, and torques, consideration of the dynamics of the system will often be necessary before a detailed stress analysis can be undertaken. Furthermore, as the strength of materials is a function of temperature, a thermal analysis may also be necessary in order to determine operating temperatures throughout the performance cycle of the machine.

If a machine has no moving parts, then the tasks for the designer are simpler because only static force analysis and thermal analysis are necessary. However, structures are subject to loading from external loads such as weather conditions of wind and rain, earth tremors, and traffic. As such, consideration of the environment into which the machine will be installed is warranted and appropriate failure modes mitigated against. Mitigation involves considering what may go wrong and determining what action to take—to either prevent it from going wrong in the first place or, if it does go wrong, what is going to be done about it.

If the motions involved in a machine are slow then it is likely that a static force analysis will be sufficient. In a static structure, such as the floor of a casing designed to support a particular weight, the payload that the structure can support can be increased by adding appropriately distributed weight (dead weight) to its structure. If, however, the components of a machine have significant accelerations, a dynamic force analysis will be necessary. Adding additional mass to a moving component increases the forces involved, and it is possible for such components to become victims of their own mass. This is because some of the loading causing stress in a moving component is due to inertial forces modeled by Newton's second law of motion, embodied in the following equation:

$$F = ma \tag{4.1}$$

Although the addition of mass may increase the strength of a component, the benefit may be reduced by the increases in inertial forces.

To meet a design requirement, the designer needs to conceive a form and express the connectivity of its parts in order to meet the desired goal. A proportion of the mental sculpting involved is geometric in nature because shape often permits the achievement of a given function. The design procedures presented here follow this approach, with an initial proposal for the geometric configuration followed by analysis to justify the proposal or provide guidance for its modification. Boden (1990) discusses creativity in terms of the connection of conceptual spaces in the mind. In order to create a design, it is often necessary to combine ideas from different fields or disciplines. This may or may not take place as part of a structured design procedure or in a prescribed time period. Over-reliance on procedure should, in some cases, actually be guarded against, as this may stymie the activity.

The variety of machine elements and technologies already available to the designer as a result of years of development is extensive, as indicated in Table 1.3 in Chapter 1. Machine elements considered in this book are those more commonly found in machines transmitting power by means of moving components, such as motors and engines. The diagram given in Figure 4.1 illustrates the range of machine elements involved in the design of a gear box for a

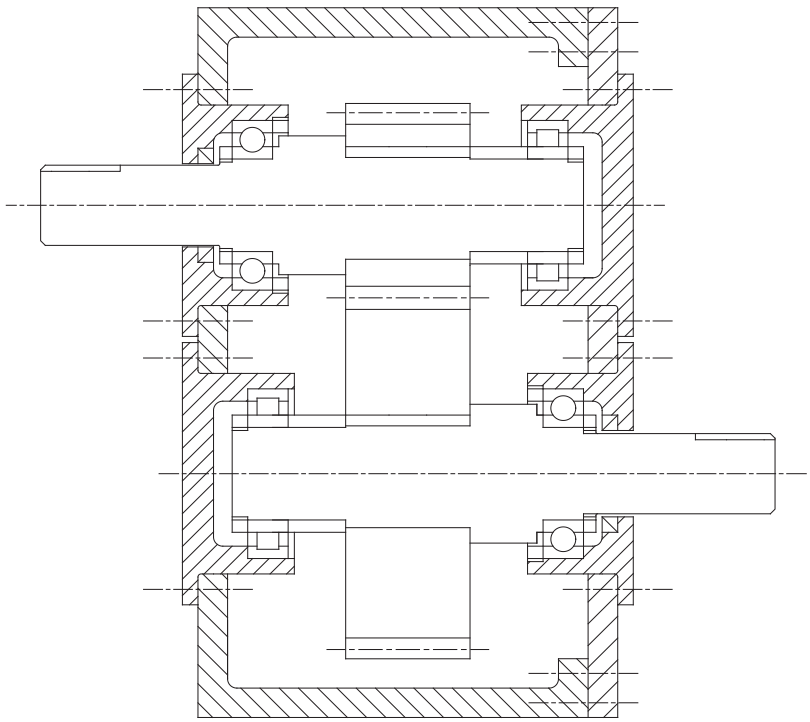


Figure 4.1
Transmission for an engine-driven compressor.

single cylinder air compressor and comprises shafts, bearings, gears, seals, and enclosure. The machine elements considered in this book provide information about the design of such applications and cover bearings, shafts, gears, belts and chains, seals, clutches and brakes, springs, fasteners, and enclosures. These subjects are introduced in Sections 4.3–4.9 prior to more thorough consideration in the relevant chapters presented later in this book.

A fundamental element for many machines is the lever, and the principle of leverage is used in many machine elements as well as machines. The fixed point of the lever, about which it moves, is known as the fulcrum. A lever can be used with a fulcrum (also known as a pivot) to allow a small force moving over a large distance to create a large force moving over a short distance. The positions of the force and the load are interchangeable; by moving them to different points on the lever, different effects can be produced. There are three generic classes of levers: first-order lever, second-order lever, and third-order levers. An example of a first-order lever is a seesaw, or balance, with the load and the force separated by the fulcrum. As one side moves up, the other side moves down. The amount and the strength of the movement are proportional to the distance from the fulcrum. If L_2 is $10 \times L_1$, then F_1 will be $10 \times F_2$.

$$F_1 = F_2 \frac{L_2}{L_1} \tag{4.2}$$

An example of a second-order lever is a wheel barrow. Here, the load is between the force and the fulcrum. This uses mechanical advantage to ease lifting of a large weight. In a third-order lever, the force is between the fulcrum and the load. Mechanical advantage is reduced, but the movement at the load point is increased. An example of a third-order lever is a tweezer.

A further fundamental building block in machine design and many mechanisms is the linkage. A linkage is a system of links, rods, or spars connected at joints with rotary or linear bearings. Linkages can be used to change direction, alter speed, and vary the timing of moving parts. An example of the use of linkages is shown in Figure 4.2 where the articulation of the grab bucket is enabled by a four-bar linkage, giving prescribed rotation in response to the hydraulic ram actuation. A four-bar linkage is also used to provide mechanical advantage for grasping a workpiece in mole grips.

In the example shown in Figure 4.3, two linked linkages are used to convert the small linear movement of the drive shaft (bottom left); first into a rotational body movement and second into a fast hammer movement. In this case, the speed of the input drive shaft is amplified by the use of the linkages providing high-speed hammer action.

Many machine elements involve relative motion between components and associated friction and wear. Tribology is the study of friction, lubrication, and wear of surfaces in relative motion. Because of the desire to produce efficient machines, where the loss of energy due to friction is minimal, extensive research and effort have been put into studying tribology. As a subject, it has considerable interrelationships and cross-over with materials and fluid



Figure 4.2
Multiple uses of linkages in construction equipment, JCB JS210. *www.jcb.com/products/MachineProduct.aspx?PID=11&RID=2*.

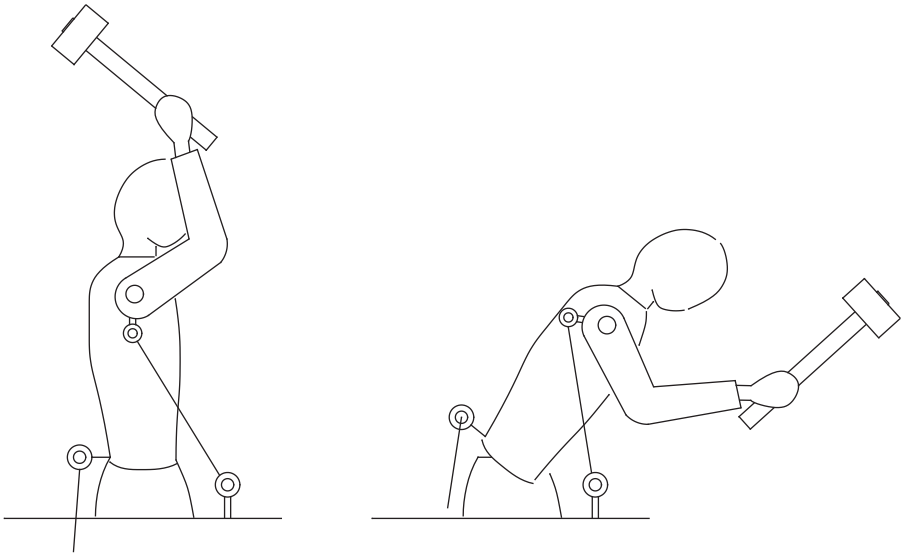


Figure 4.3
Two linked linkages.

mechanics. The study and reduction of friction has been of concern to humans for as long as machines have been in use. The subject of tribology includes consideration of machine elements such as bearings, gears, belts, chains, clutches, brakes, and seals. Tribology, as an engineering science, is now a well-established subject and has resulted in significant breakthroughs. For example, in bearing technology, some companies are now able to offer high efficiency bearings with significantly reduced friction in comparison with previous products.

4.2 Bearings

The term bearing, in its general sense, refers to the assembly formed by two surfaces that has the capacity for relative motion. A wide range of bearings has been developed; some involve rolling motion, sliding motion, and both rolling and sliding motions. Lubrication of a bearing is frequently used to reduce friction and, therefore, the wear and power absorbed by the bearing, as well as to remove heat. This ensures the operation of the bearing assembly at temperatures compatible with the materials and lubricant used.

Typical examples of bearings are shown in Figures 4.4 and 4.5. Figure 4.4 illustrates rolling bearings. These come in many forms, each designed for a given set of performance characteristics, such as speed and load capability. Rolling element bearings usually need to be lubricated by grease or oil with the choice depending upon the speed and temperature of operation. Rolling element bearings are typically available as stock items. This is of significant potential advantage to a machine designer, as the task becomes that of bearing selection rather than bearing design. Bearing design is a significant undertaking, involving consideration of very high point contact stresses and use of special materials. The technology involved relies heavily on the science of tribology and there are a number of worldwide companies already operating in this market. A designer needs to carefully consider the issue of whether time is better spent either on producing a new design for a bearing that is already available from an existing supplier or on designing and adding value to the product in hand.



Figure 4.4
Examples of rolling element bearings available from existing suppliers.



Figure 4.5
Fixed geometry journal (multilobe) bearings. *Image courtesy of John Crane, http://www.johncranetoday.com/tasks/sites/johncranetoday/assets/Image/Bearing_Advantage_Pivot_Shoe_small.jpg.*

Plain bearings rely on either rubbing contact between the bearing surfaces or pressure to separate the two surfaces. The pressure required can be induced in a lubricant by the movement of one component relative to another or by the external supply of the lubricant under the required pressure. Plain bearings are also available from specialist bearing suppliers. In principle, they can be significantly less expensive than rolling element bearings because they involve fewer moving components. The selection of an appropriate bearing for a given task, however, is an involved activity, which needs to take into account, among other factors:

- Load,
- Speed,
- Location,
- Size,
- Cost,
- Starting torque,
- Noise, and
- Lubrication supply.

The design of plain bearings and the selection of rolling element bearings as stock items from bearing supplies are considered more fully in Chapters 5 and 6. The related topic of shafts is

introduced in Chapter 7. A consequence of bearings being supplied by a select number of specialist manufacturers is that they have been subject to standardization and are usually only available in a limited number of standard sizes. Standardization has an advantage in that the parts from one original equipment manufacturer (OEM) will be interchangeable with another in terms of their geometry. This does not, however, mean that the performance of two similarly sized components from different manufacturers will be the same.

4.3 Gears, Belts, and Chains

Gears, belts, and chains can be used to transmit power from one shaft to another. Typical examples are illustrated in Figures 4.6–4.8.

Many power producing machines, or prime movers, such as internal combustion engines, gas turbines, and electric motors, produce power in the form of rotary motion. The operating torque versus speed characteristics of prime movers vary according to their type and size, as do the driven loads. It is common for the torque versus speed characteristics to be mismatched, requiring the need for gearing and, perhaps also, a clutch to enable the prime mover to attain sufficient momentum prior to engaging a load.

When transmitting power from a source to the required point of application, a series of devices are available including belts, pulleys, chains, hydraulic and electrical systems, and gears. Generally, if the distances of power transmission are large, gears are not suitable and chains and belts can be considered. However, when a compact, efficient, or high-speed drive is required, gear trains offer a competitive and suitable solution.

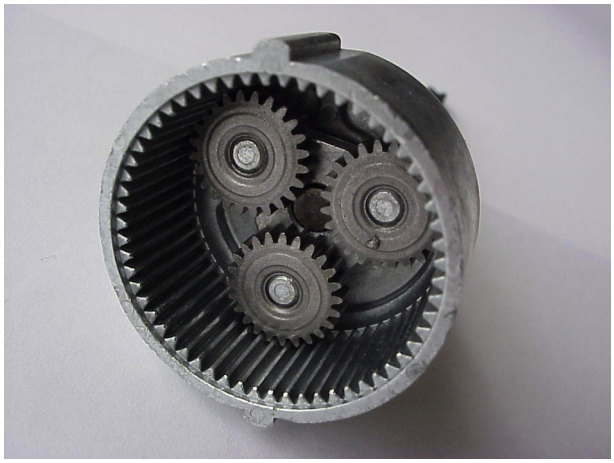


Figure 4.6
Spur gears incorporated within an epicyclic gear box for a cordless hand-tool transmission.

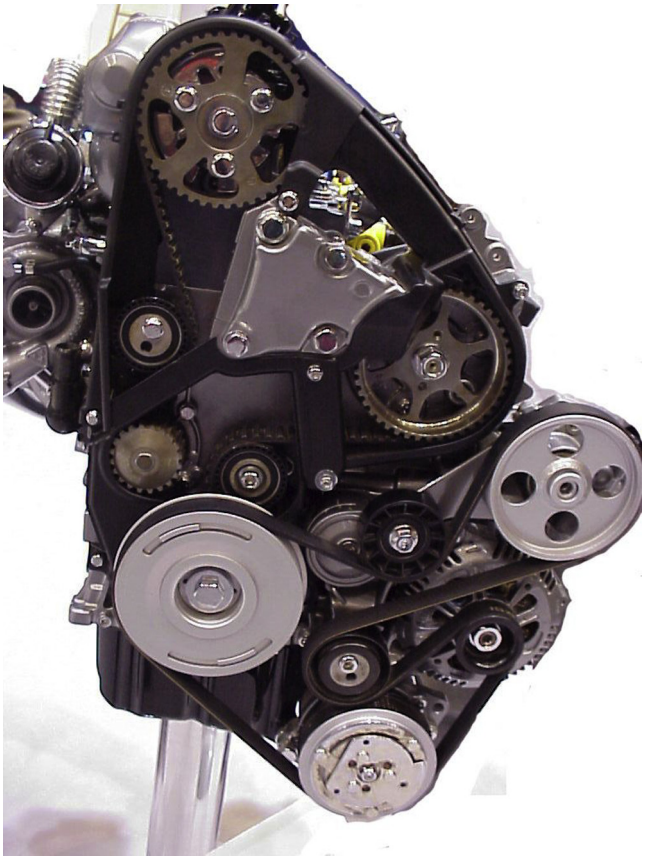


Figure 4.7
Flat belt and synchronous belt drives driving multiple devices from a single belt.

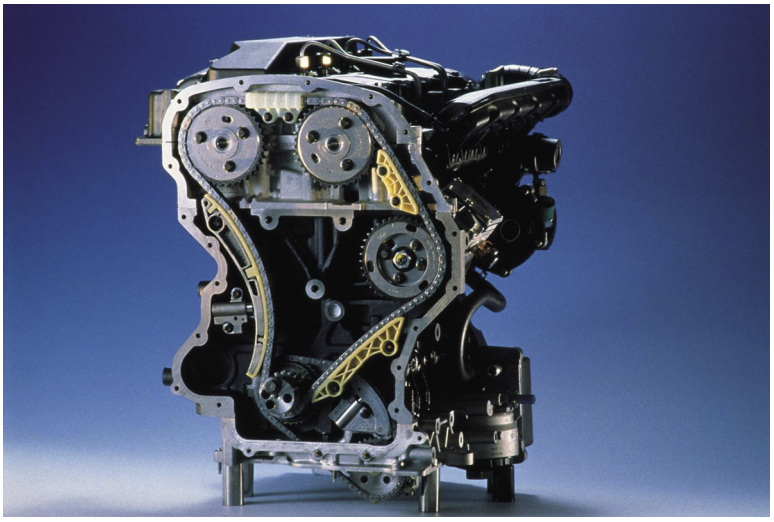


Figure 4.8
Chain drive for powering multiple elements.

For example, for a small electric motor-driven air compressor, options that might be possible for the transmission include: belt drive; chain drive; gear drive; belt and chain; belt and gear; and gear and chain. Three different types of machine element drives along with direct drives are all potential solutions that could be viable. The different concepts are illustrated as general arrangements in Figure 4.9.

Gears, belts, and chains are all available from specialist suppliers. The manufacture of gears requires specialist machinery, and the stresses in the region of contact between gears in mesh can be exceptionally high requiring the use of specialist materials. Similarly, the design of roller chain and associated sprockets involves high contact stresses and the design of an articulated joint and associated bearings. For these reasons, combined with financial expediency, leaving the cost of specialist manufacture infrastructure to other companies, gear, chain, and belt manufacture is usually best left to the specialist. As such, the task of the machine designer becomes that of selecting and specifying appropriate components from the specialist stock suppliers and original equipment manufacturers. Gears are explained in Chapters 8–11 and belts and chains are explained in Chapter 12.

4.4 Clutches and Brakes

Clutches and brakes provide frictional, magnetic, hydraulic, or mechanical connection between two machine elements, usually shafts. There are significant similarities between clutches and brakes. If both shafts rotate, then the machine element will be classed as a clutch; the usual function is to connect or disconnect a driven load from a driving shaft. If one shaft rotates and the other is stationary, then the machine element is classed as a brake; the likely function is to decelerate a shaft. In reality, however, the same device can function as a brake or clutch by fixing its output element to either a shaft that can rotate or to ground, respectively.

Brakes and clutches are familiar devices from their use in automotive applications. They are, however, also used extensively in production machinery. Clutches allow a high inertia load to be rotated with a smaller electric motor than would otherwise be required if it was directly connected. Clutches are also used to maintain a constant torque on a shaft for tensioning of webs and filaments. They can be used, in emergencies, to disconnect a driven machine from a motor if machinery should become jammed. In such cases, a brake may also be installed in order to bring the machinery to a rapid stop.

Complete clutches and brakes are available from specialist manufacturers. In addition, key components such as disks and hydraulic and pneumatic actuators are also available, enabling the designer to opt, if appropriate, to design the overall configuration of the clutch or brake and buy in the frictional lining surface. Clutches and brakes are presented in more detail in Chapter 13.

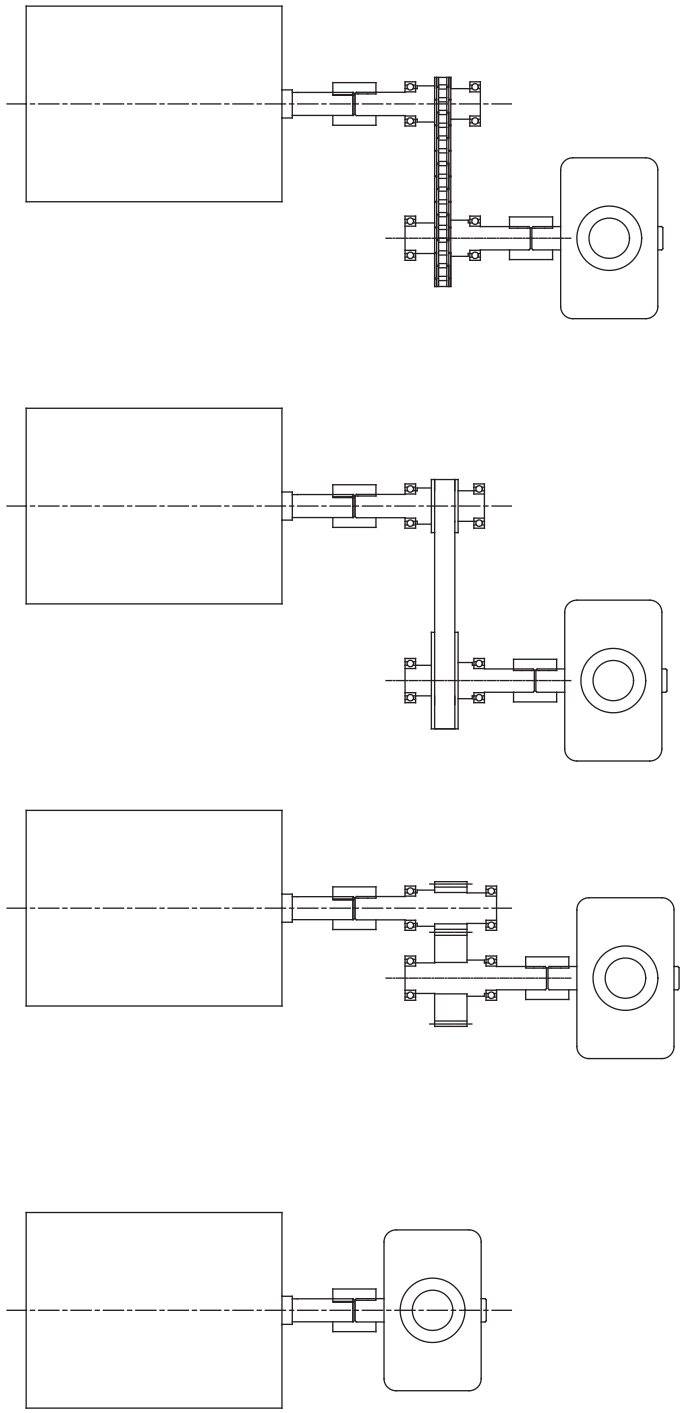


Figure 4.9
Conceptual arrangements for a transmission.

4.5 Seals

Seals are devices used to reduce or eliminate the flow of fluid or particulates between two locations. Seals are an important part of a machine design where fluids, under relatively high differential pressures, must be contained within a particular region. An example is the fluid within the cylinder of an internal combustion engine. Here, the working fluid needs to be contained in the cylinder for part of the cycle and excluded, in as much as possible, from contaminating the lubricating oil for the crankshaft bearings. This is conventionally achieved by means of piston rings, Figure 4.10.

There are many types of seals, but they can generally be classified as either static or dynamic depending on whether relative movement is involved. Many static seals involve the use of a gasket or sealing material, such as an elastomer. These are widely available from specialist suppliers, and the task facing the designer is the selection of appropriate materials for the application. Some dynamic seals are also commercially available, such as radial lip seals and mechanical face seals. However, there is not the level of standardization in shaft design. In addition, the range of possible applications is seemingly infinite with differing pressure ratios and fluids being sealed. As a result, the range of seal geometries required is too extensive to warrant an original equipment manufacturer designing or stocking a wide number of seal sizes and types. Instead, many dynamic seals are designed fit-for-purpose. Typical considerations in the selection and design of seals include the nature of the fluid or particulates to be contained, the differential pressure, the nature of any relative motion between components, the level of sealing required, operating conditions, life expectancy, and total costs. Seals are presented in greater detail in Chapter 14.



Figure 4.10
Piston ring seals.

4.6 Springs

Springs are flexible elements used to exert force or torque and store energy. The force exerted by a spring can be a linear push or pull, or a radial force similar to that of a rubber band around a roll of paper. Alternatively, a spring can be configured to produce a torque with applications including door closers. Springs store energy when they are deformed and return energy when the force that is causing the deflection is removed. A further type of spring is the power spring, or spring motor, which once wound up can dissipate energy at a steady pace.

Springs can be made from round or rectangular wire bent into a suitable form such as a coil, or made from flat stock and loaded as a cantilevered beam. Many standard size spring configurations are available as stock items from specialist manufacturers and suppliers, and it is usually expedient to use these where possible. It is important to focus on both the performance characteristics of the spring and the spring's ends or terminations. Many options are available such as twist loops or hooks, side loops, extended hooks, etc. Some applications require a custom spring design. Spring design involves consideration of many variables such as length, wire diameter, forces, spring rate, spring index, number of coils, pitch, pitch angle, installation considerations, coil clearance, materials, types of loading, and allowable stresses. The selection and design of springs is presented in Chapter 15.

4.7 Fasteners

Fasteners connect or join two or more components. It has been estimated that there are over 2.5 million fasteners in a Boeing jumbo jet. For more modern airliners, such as the Airbus Industrie A340, this amount of fasteners has been dramatically reduced, but the total is still significant. This example illustrates the importance of fasteners to the designer.

There are thousands of types of fasteners commercially available from specialist suppliers. The most common include threaded fasteners such as bolts, screws, nuts, studs and set screws, rivets, fast operating fasteners requiring a quarter turn to connect or disconnect, and snap joints commonly used in plastic component assembly. In addition, there is the range of permanent fastening techniques by means of welding, brazing, soldering, and using adhesives.

Given the wide range of fastening techniques available and the costs involved in sourcing, stocking, and assembly, the correct choice of fastener is very important. Chapter 16 presents information on a variety of types of fasteners and fastenings, and their selection based on appropriate analysis and consideration of the intended function.

4.8 Wire Rope

Wire rope is sometimes used in hoisting, haulage, and conveyer applications. By using many small diameter wires, twisted around a central core, some flexibility in the wire rope can be achieved. It is possible for the wire rope to articulate drums and other radial segments. More details on wire rope are presented in Chapter 17.

4.9 Pneumatics and Hydraulics

Significant mechanical advantage can be achieved by exposing a surface area to a pressurized fluid. This principle is exploited in many hydraulic and pneumatic devices, which have been used extensively in machine design and are available as standard stock items. Information on pneumatics and hydraulics is presented in Chapter 18.

4.10 Enclosures

Individual machine elements such as shafts on bearings must be supported, and this is the function of an enclosure or frame. The total design model requires attention to all other phases of the design process while undertaking any one task. As such, the design of any given machine element should not occur in isolation; but instead give due consideration to how it will fit in and interact with the overall assembly. The enclosure for a machine provides both the support for the machine and the form to a product. Visual attractiveness is not always given the attention it warrants by designers of machine elements; however, we produce our components to be purchased. Current thinking indicates that visual perception is dominant among the senses, with over one-half the cerebral cortex dedicated to visual processing. Therefore, the shapes of our products are important, and we should design accordingly.

Enclosures are usually unique to a given class of machine, as each machine tends to be different in terms of the number, size, and function of its constituent components. It is not practical, therefore, to have a completely general approach to enclosure and frame design. Common examples of the variations of frames and enclosures are those for vehicles and toys. Toys must be safe, functional, and fun; yet the material and personnel effort in producing the toy should minimize the production costs and maximize profits.

The design of an enclosure or frame is an art to some extent, in that the constituent machine components must be accommodated. There may be constraints on the location of supports in a machine such that they facilitate the function and operation of the machine and one can access particular components. The frame for the bus shown in

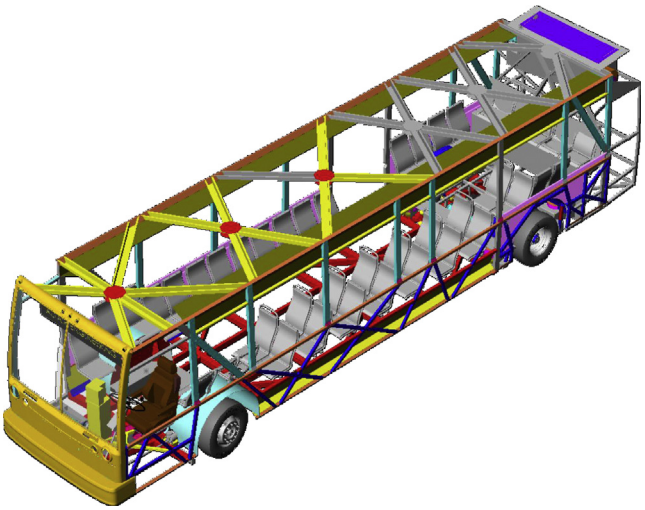


Figure 4.11
Bus frame design. *Allin and Simon (2002)*.

Figure 4.11 illustrates this point in that access for doors, windows, and occupants must be provided.

Some of the more important design parameters to consider for an enclosure or frame include: strength, size, assembly, appearance, corrosion resistance, stiffness, vibration, weight, noise, production costs, maintenance costs, and sustainability. A list such as this may be too all-encompassing to be of much use at the start of a design process. The following list of factors may be more helpful to consider as the onset for a frame or enclosure:

- Forces exerted by the machine components through mounting points such as bearings, pivots, brackets, and feet,
- Requirements to support the frame itself,
- Allowable deflections of constituent machine elements,
- Cooling requirements,
- Environments into which the machine will be transported and installed in relationship to other machines and infrastructure,
- Quantity required,
- Production facilities available, and
- Expertise available for the design task.

The complexity involved when considering more that one machine element is evident. Due to the need to "keep the bigger picture in mind," models for the design process such as total design are useful.

4.11 Conclusions

The technology base available to us includes a wide range of machine elements such as bearings, shafts, gears, chains, belts, seals, clutches, brakes, and fasteners. This chapter has served to introduce these items, many of which are available as stock items from specialist manufacturers. This means that the tasks of the designer become that of selecting and specifying the appropriate item, which eliminates the task of the detailed design of, for instance, a roller chain. Subsequently, greater focus can be applied to the design of the overall machine.

It is most appropriate to take the time to obtain an overview of the technologies available to the designer and to be cognizant of the interrelationships between individual machine elements. This is consistent with the total design model, where each phase takes into account all other phases and aspects of the design process.

References

Allin, K., Simon, D., 2002. A new approach to vehicle architecture design. In: Stobart, R.K., Childs, P.R.N. (Eds.), Total Vehicle Technology, How Do We Get the Innovation Back Into Vehicle Design. Professional Engineering Publishing, pp. 49–56.

Boden, M.A., 1990. The Creative Mind: Myths and Mechanisms. Weidenfeld and Nicolson.

Further Reading

Web Sites

At the time of going to press the world-wide-web contained useful information relating to this chapter at the following sites.

jolisfukyu.tokai-sc.jaea.go.jp/fukyu/tayu/ACT01E/02/0202.htm
ocw.mit.edu/OcwWeb/Mechanical-Engineering/2-007Spring-2005/LectureNotes/index.htm
science.howstuffworks.com/gear7.htm
video_demos.colostate.edu/mechanisms/index.html
www.acmcf.org.tw/model/page/simulation/ncku/D01.htm
www.coinopgamemuseum.com/mechanisms/photoboothcamera/photoboothcamera.html
www.cs.cmu.edu/~rapidproto/mechanisms/tablecontents.html
www.flying-pig.co.uk/mechanisms/
www.howstuffworks.com/transmission.htm
www.ijn.dreamhost.com/Optics/Optics%20-%2012%20cm%20binoc%20model%205.htm
www.jcb.com/products/MachineProduct.aspx?PID=7&;RID=2
www.mechanisms101.com/theo_jansen.shtml
www.mystery-productions.com/hyper/Hypermedia_2003/Muirhead/website/index.htm
www.precisionmotion.co.uk/page3.htm
www.sandsmuseum.com/
www.stanleyworks.co.uk/Clamps+and+Pliers/Mole+Grips.ctlg
www.tc.gc.ca/roadsafety/safevehicles/mvstm_tsd/tsd/2060rev0_e.htm
www.technologystudent.com/cams/cam1.htm
www.ul.ie/~kirwanp/linkstoanimations.htm

Nomenclature

The following symbols have been used in this chapter. Note that, generally, preferred SI units have been stated.

a	acceleration (m/s^2)
F	force (N)
m	mass (kg)
OEM	original equipment manufacturer

CHAPTER 5

Journal Bearings

Chapter Outline

5.1 Introduction 139
5.2 Sliding Bearings 142
5.2.1 Lubricants 145
5.3 Design of Boundary-Lubricated Bearings 146
5.4 Design of Full-Film Hydrodynamic Bearings 150
5.4.1 Reynolds Equation Derivation 153
5.4.2 Design Charts for Full-Film Hydrodynamic Bearings 168
5.4.3 Alternative Method for the Design of Full-Film Hydrodynamic Bearings 189
5.5 Conclusions 196
References 196
Further Reading 197
Nomenclature 198

Abstract

The purpose of a bearing is to support a load, typically applied to a shaft, whilst allowing relative motion between two elements of a machine. The two general classes of bearings are journal bearings, also known as sliding or plain surface bearings, and rolling element bearings. The aims of this chapter are to describe the range of bearing technology, to outline the identification of which type of bearing to use for a given application, and to introduce journal bearing design with specific attention to boundary lubricated bearings and full-film hydrodynamic bearings. The selection of rolling element bearings is considered in Chapter 6.

5.1 Introduction

The term "bearing" typically refers to contacting surfaces through which a load is transmitted. Bearings may roll or slide or do both simultaneously. The range of bearing types available is extensive, although they can be broadly split into two categories: sliding bearings, also known as journal or plain surface bearings, where the motion is facilitated by a thin layer or film of lubricant, and rolling element bearings, where the motion is aided by a combination of rolling motion and lubrication. Lubrication is often required in a bearing to reduce friction between surfaces and to remove heat. Figure 5.1 illustrates two of the more commonly known bearings: a journal bearing and a deep groove ball bearing. A general classification scheme for the distinction of bearings is given in Figure 5.2.

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00005-8
Copyright © 2014 Elsevier Ltd. All rights reserved.

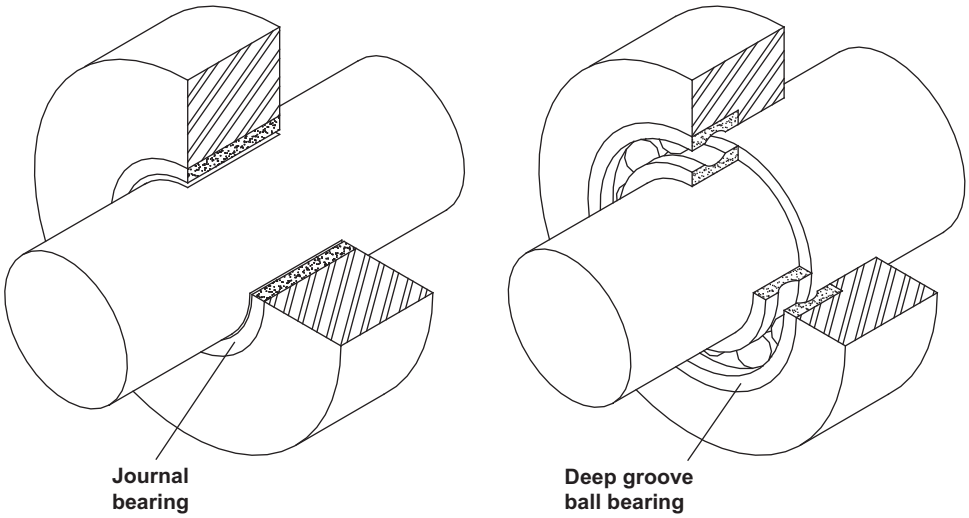


Figure 5.1
A journal bearing and a deep groove ball bearing.

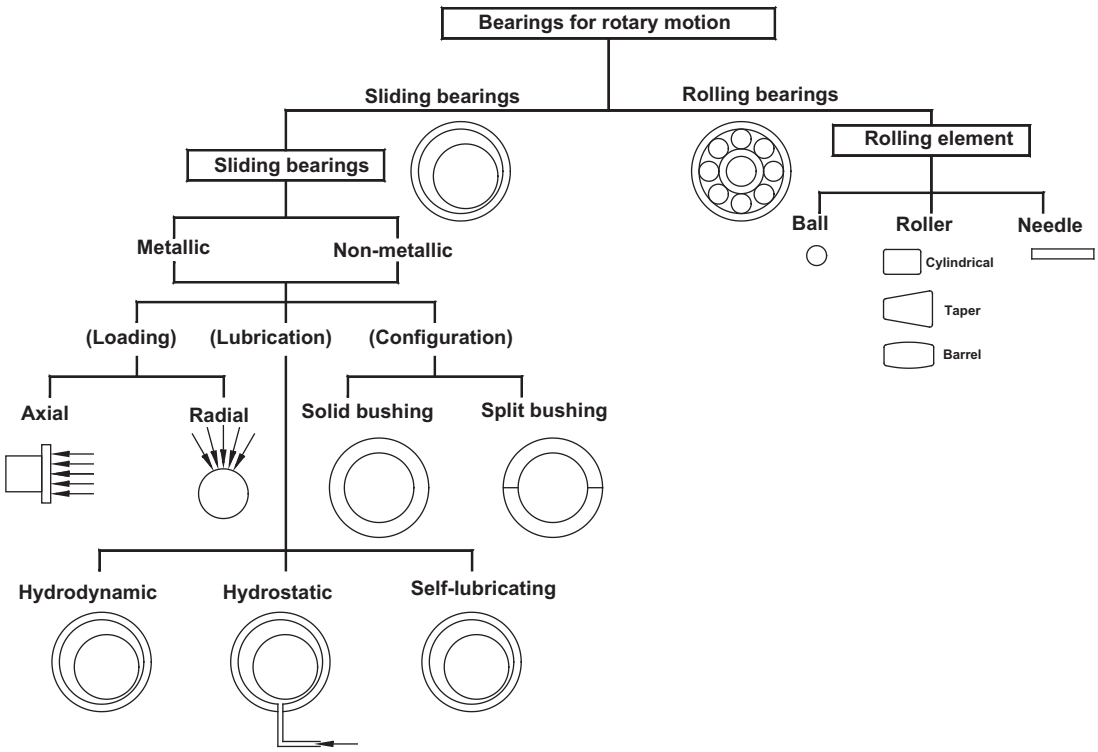


Figure 5.2
Bearing classification. *Reproduced with adaptations from Hindhede (1983).*

As can be seen from Figure 5.2, the scope of choice for a bearing is extensive. For a given application it may be possible to use different bearing types. For example, in a small gas turbine engine rotating at say 50,000 rpm, either rolling bearings or journal bearings could typically be used, although the optimal choice will depend on a number of factors such as life, cost, and size. The texts by Harris (2001) and Brandlein et al. (1999) provide an extensive overview of bearing technologies. Figure 5.3 can be used to give guidance for which kind of bearing has the maximum load capacity at a given speed and shaft size, and Table 5.1 gives an indication of the performance of the various bearing types for some criteria other than load capacity.

Lubricant film sliding bearings are introduced in Section 5.2, the design of boundary lubricated bearings which are typically used for low speed applications are considered in Section 5.3, and the design of full-film hydrodynamically lubricated bearings is described in Section 5.4.

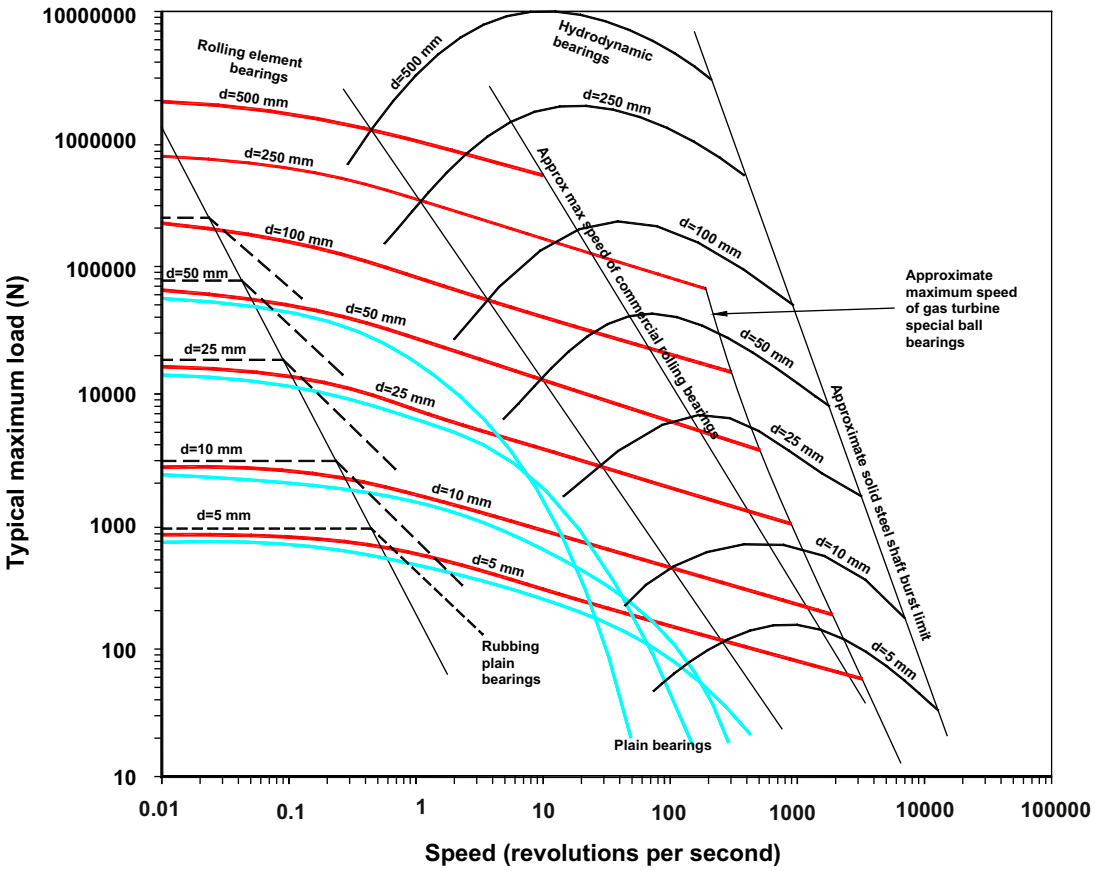


Figure 5.3

Bearing type selection by load capacity and speed. *Reproduced courtesy of Neale (1995).*

Table 5.1: Comparison of bearing performance for continuous rotation.

Bearing type	Accurate radial location	Combined axial and radial load capability	Low starting torque capability	Silent running	Standard parts available	Lubrication simplicity
Rubbing plain bearings (nonmetallic)	Poor	Some in most cases	Poor	Fair	Some	Excellent
Porous metal plain bearings oil impregnated	Good	Some	Good	Excellent	Yes	Excellent
Fluid film hydrodynamic bearings	Fair	No; separate thrust bearing needed	Good	Excellent	Some	Usually requires a recirculation system
Hydrostatic bearings	Excellent	No; separate thrust bearing needed	Excellent	Excellent	No	Poor special system needed
Rolling bearings	Good	Yes in most cases	Very good	Usually satisfactory	Yes	Good when grease lubricated

Reproduced courtesy of Neale (1995).

5.2 Sliding Bearings

The term "sliding bearing" refers to bearings for which two surfaces move relative to each other without the benefit of rolling contact. The two surfaces slide over each other, and this motion can be facilitated by means of a lubricant which gets squeezed by the motion of the components and can generate sufficient pressure to separate them, thereby reducing frictional contact and wear.

A typical application of sliding bearings is to allow rotation of a load-carrying shaft. The portion of the shaft at the bearing is referred to as the journal, and the stationary part, which supports the load, is called the bearing (see Figure 5.4). For this reason, sliding bearings are often collectively referred to as journal bearings, although this term ignores the existence of sliding bearings that support a linear translation of components. Another common term is plain surface bearings. This section is principally concerned with bearings for rotary motion, and the terms "journal" and "sliding bearing" are used interchangeably.

There are three regimes of lubrication for sliding bearings:

1. Boundary lubrication,
2. Mixed-film lubrication,
3. Full-film lubrication.

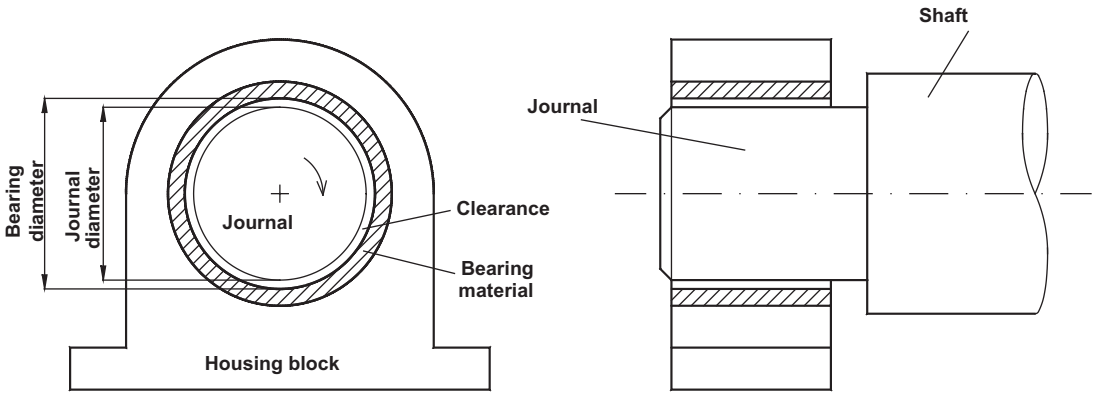


Figure 5.4
A plain surface, sliding or journal bearing.

Boundary lubrication typically occurs at low relative velocities between the journal and the bearing surfaces and is characterized by actual physical contact. The surfaces, even if ground to a low value of surface roughness, will still consist of a series of peaks and troughs as illustrated schematically in Figure 5.5. Although some lubricant may be present, the pressures generated within it are not significant and the relative motion of the surfaces brings the corresponding peaks periodically into contact. Mixed film lubrication occurs when the relative motion between the surfaces is sufficient to generate high enough pressures in the lubricant film, which can partially separate the surfaces for periods of time. There is still contact in places around the circumference between the two components. Full-film lubrication occurs at higher relative velocities. Here the motion of the surface generates high pressures in the lubricant, which separate the two components and the journal can "ride" on a

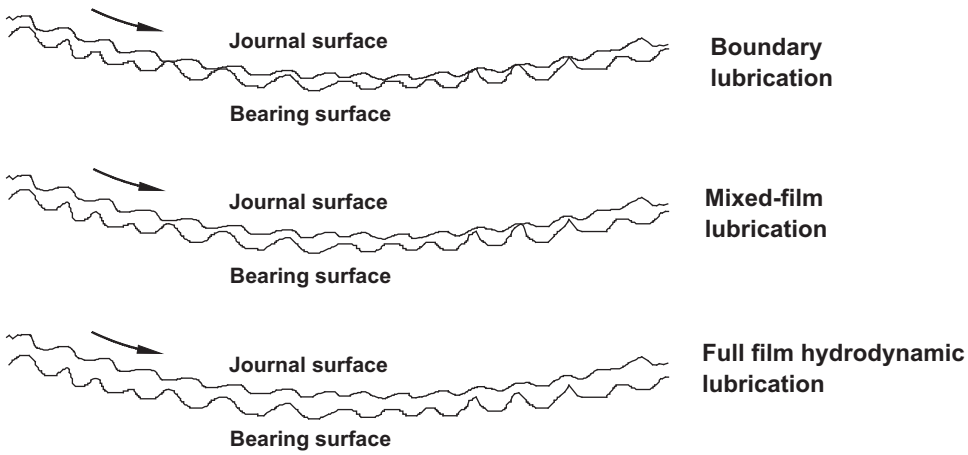


Figure 5.5
A schematic representation of the surface roughness for sliding bearings and the relative position depending on the type of lubrication occurring.

wedge of fluid. All of these types of lubrication can be encountered in a bearing without external pressuring of the bearing. If lubricant under high enough pressure is supplied to the bearing to separate the two surfaces, it is called a hydrostatic bearing.

The performance of a sliding bearing differs markedly depending on which type of lubrication is physically occurring. This is illustrated in Figure 5.6, which shows the variation of the coefficient of friction with a group of variables called the bearing parameter, which is defined by

$$\frac{\mu N}{P} \tag{5.1}$$

where $\mu =$ the viscosity of the lubricant (Pa s), $N =$ speed (for this definition, normally in rpm), $P =$ load capacity (N/m^2) given by

$$P = \frac{W}{LD} \tag{5.2}$$

$W =$ applied load (N), $L =$ bearing length (m), and $D =$ journal diameter (m).

The bearing parameter, $\mu N/P$, groups several of the bearing design variables into one number. Normally, of course, a low coefficient of friction is desirable. In general, boundary lubrication is used for slow speed applications where the surface speed is less than approximately 1.5 m/s. Mixed-film lubrication is rarely used because it is difficult to quantify the actual value of the coefficient of friction (note the steep gradient in for this zone). The design of boundary-lubricated bearings is outlined in Section 5.3 and that of full-film hydrodynamic bearings is outlined in Section 5.4.

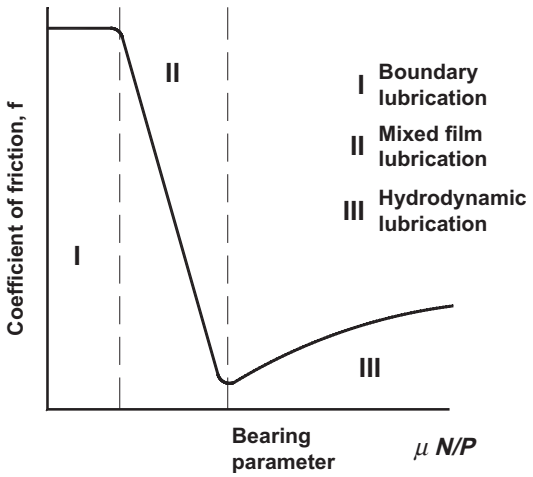


Figure 5.6
Schematic representation of the variation of bearing performance with lubrication.

5.2.1 Lubricants

As can be seen from Figure 5.6, bearing performance is dependent on the type of lubrication occurring and the viscosity of the lubricant. The viscosity is a measure of a fluid's resistance to shear. Lubricants can be solid, liquid, or gaseous, although the most commonly known are oils and greases. The principal classes of liquid lubricants are mineral oils and synthetic oils. Their viscosity is highly dependent on temperature, as illustrated in Figure 5.7. They are typically solid at −35 °C, thin as paraffin at 100 °C, and burn above 240 °C. Many additives are used to affect their performance. For example, EP (extreme pressure) additives add fatty acids and other compounds to the oil, which attack the metal surfaces to form "contaminant" layers, which protect the surfaces and reduce friction even when the oil film is squeezed out by high contact loads. Greases are oils mixed with soaps to form a thicker lubricant that can be retained on surfaces.

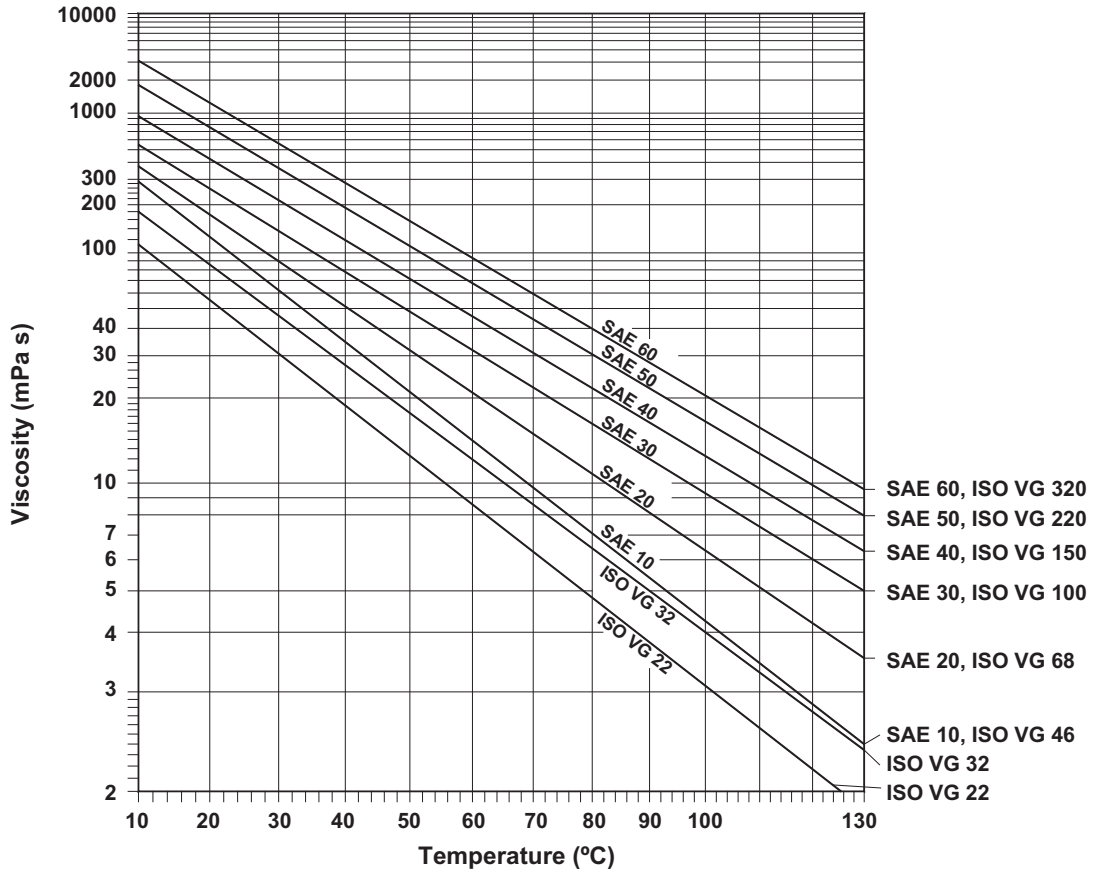


Figure 5.7
Variation of absolute viscosity with temperature for various lubricants.

The viscosity variation with the temperature of oils has been standardized and oils are available with a class number, for example SAE 10, SAE 20, SAE 30, SAE 40, SAE 5W, SAE 10W, etc. The origin of this identification system developed by the Society of Automotive Engineers was to classify oils for general-purpose use and winter use, the "W" signifying the latter. The lower the numerical value, the thinner or less viscous the oil. Multigrade oil, e.g., SAE 10W/40, is formulated to meet the viscosity requirements of two oils giving some of the benefits of the constituent parts. An equivalent identification system is also available from the International Organization for Standardization (ISO 3448).

5.3 Design of Boundary-Lubricated Bearings

As described in Section 5.2, the journal and bearing surfaces in a boundary-lubricated bearing are in direct contact in places. These bearings are typically used for very low-speed applications such as bushes and linkages where their simplicity and compact nature are advantageous. Examples include household lawnmower wheels, garden hand tools such as shears, ratchet wrenches and domestic and automotive door hinges incorporating the journal as a solid pin riding inside a cylindrical outer member. A further example, involving linear motion, is the slider in an ink jet printer.

General considerations in the design of a boundary-lubricated bearing are as follows:

- The coefficient of friction (both static and dynamic),
- The load capacity,
- The relative velocity between the stationary and moving components,
- The operating temperature,
- Wear limitations, and
- The production capability.

A useful measure in the design of boundary-lubricated bearings is the PV factor (load capacity $\times$ peripheral speed), which indicates the ability of the bearing material to accommodate the frictional energy generated in the bearing. At the limiting PV value, the temperature will be unstable and failure will occur rapidly. A practical value for PV is half the limiting PV value. Values for PV for various bearing materials are given in Table 5.2. The preliminary design of a boundary-lubricated bearing essentially consists of setting the bearing proportions, i.e. its length and its diameter, and selecting the bearing material such that an acceptable PV value is obtained. This approach is set out as a step-by-step procedure below.

1. Determine the speed of rotation of the bearing and the load to be supported.
2. Set the bearing proportions. Common practice is to set the length-to-diameter ratio between 0.5 and 1.5. If the diameter D is known as an initial trial, set L equal to D.
3. Calculate the load capacity, $P = W/LD$, Eqn (5.2).

Table 5.2: Characteristics of some rubbing materials.

Material	Maximum load capacity P (MN/m^2)	Limiting PV value (MN/m s)	Maximum operating temperature (°C)	Coefficient of friction	Coefficient of expansion ($\times 10^{-6}$/°C)	Comments	Typical application
Carbon/graphite	1.4—2	0.11	350—500	0.1—0.25 dry	2.5—5.0	For continuous dry operation	Food and textile machinery
Carbon/graphite with metal	3.4	0.145	130—350	0.1—0.35 dry	4.2—5		
Graphite impregnated metal	70	0.28—0.35	350—600	0.1—0.15 dry	12—13		
Graphite/ thermosetting resin	2	0.35	250	0.13—0.5 dry	3.5—5	Suitable for sea water operation	
Reinforced thermosetting plastics	35	0.35	200	0.1—0.4 dry	25—80		Roll-neck bearings
Thermoplastic material without filler	10	0.035	100	0.1—0.45 dry	100		Bushes and thrust washers
Thermoplastic with filler or metal backed	10—14	0.035—0.11	100	0.15—0.4 dry	80—100		Bushes and thrust washers
Thermoplastic material with filler bonded to metal back	140	0.35	105	0.2—0.35 dry	27	With initial lubrication only	For conditions of intermittent operation or boundary lubrication e.g. ball joints, suspension, steering
Filled PTFE	7	Up to 0.35	250	0.05—0.35 dry	60—80	Glass, mica, bronze, graphite	For dry operations where low friction and wear required
PTFE with filler, bonded to steel backing	140	Up to 1.75	280	0.05—0.3 dry	20	Sintered bronze bonded to steel backing impregnated with PTFE/lead	Aircraft controls, linkages, gearbox, clutch, conveyors, bridges
Woven PTFE reinforced and bonded to metal backing	420	Up to 1.6	250	0.03—0.3	—	Reinforcement may be interwoven glass fiber or rayon	Aircraft and engine controls, linkages, engine mountings, bridge bearings

Reproduced courtesy of Neale (1995).

4. Determine the maximum tangential speed of the journal.
5. Calculate the PV factor.
6. Multiply the PV value obtained by a factor of safety of 2.
7. Interrogate manufacturer's data or Table 5.2 to identify an appropriate bearing material with a value for PV factor greater than that obtained in step (6).

Example 5.1

A bearing is to be designed to carry a radial load of 700 N for a shaft of diameter 25 mm running at a speed of 75 rpm (see Figure 5.8). Calculate the PV value, and by comparison with the available materials listed in Table 5.2 determine a suitable bearing material.

Solution

The primary data are $W = 700$ N, $D = 25$ mm, and $N = 75$ rpm.

Use $L/D = 1$ as an initial suggestion for the length to diameter ratio for the bearing.
$L = 25$ mm.

Calculating the load capacity, P

$$P = \frac{W}{LD} = \frac{700}{0.025 \times 0.025} = 1.12 \ \text{MN/m}^2$$

$$V = \omega r = \left(\frac{2\pi}{60}\right) N \frac{D}{2} = 0.1047 \times 75 \frac{0.025}{2} = 0.09816 \ \text{m/s}$$

$$\text{PV} = 0.11 \left(\text{MN/m}^2\right)(\text{m/s})$$

Multiplying this by a factor of 2 gives PV = 0.22 (MN/m s)

A material with a PV value greater than this, such as filled PTFE (limiting PV value up to 0.35), or PTFE with filler bonded to a steel backing (limiting PV value up to 1.75), would give an acceptable performance.

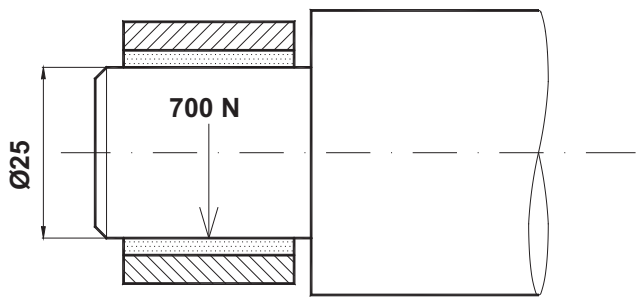


Figure 5.8
Boundary-lubricated bearing design example (dimensions in mm).

Example 5.2

The paper feed for a photocopier is controlled by two rollers that are sprung together with a force of approximately 20 N. The rollers each consist of a 20-mm-outer-diameter plastic cylinder pressed onto a 10-mm-diameter steel shaft (see Figure 5.9). The maximum feed rate for the copier is 30 pages per minute. Select bearings to support the rollers. After IMechE (1994).

Solution

The light load and low speed, together with the requirement that the bearings should be cheap, clean, and maintenance-free, suggest that dry rubbing bearings be considered.

The length of A4 paper is 297 mm.

The velocity of the roller surface is given by

$$V_{\text{roller}} = \frac{30 \times 0.297}{60} = 0.1485 \ \text{m/s}.$$

The angular velocity of the rollers is $0.1485/0.01 = 14.85$ rad/s.

The velocity of the bearing surface is therefore

$$V = 14.85 \times 0.005 = 0.07425 \ \text{m/s}.$$

An L/D ratio of 0.5 could be suitable giving a bearing length of 5 mm. The load capacity is

$$P = \frac{W}{LD} = \frac{20}{0.005 \times 0.01} = 0.4 \ \text{MPa}.$$

$$\text{PV} = 0.4 \times 0.07425 = 0.0297\left(\text{MN/m}^2\right)(\text{m/s}).$$

For this low value of PV, the cheapest bearing materials, namely thermoplastics, are adequate.

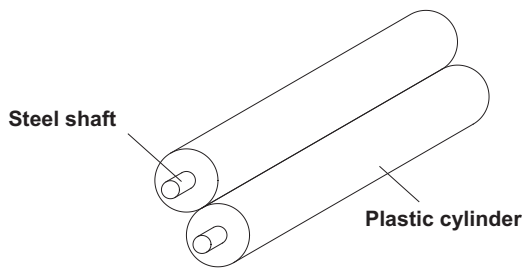


Figure 5.9
Copier rollers. *Reproduced from IMechE (1994).*

5.4 Design of Full-Film Hydrodynamic Bearings

In a full-film hydrodynamic bearing, the load on the bearing is supported on a continuous film of lubricant so that no contact between the bearing and the rotating journal occurs. The motion of the journal inside the bearing creates the necessary pressure to support the load, Figure 5.10. Hydrodynamic bearings are commonly found in internal combustion engines to support the crankshaft, and in turbocharger applications, Figure 5.11. Hydrodynamic bearings can consist of a full circumferential surface or a partial surface around the journal, Figure 5.12.

Bearing design can involve so many parameters that it is probably not possible to develop a step-by-step procedure capable of designing the best possible bearing. One route toward this goal would be to assign attribute points to each aspect of the design and undertake an optimization exercise using multiobjective design optimization or a similar scheme. This can, however, be time-consuming if the software was not already in a developed state and would not necessarily produce an optimum result due to inadequacies in modeling and incorrect assignment of attribute weightings. An alternative approach which is sensible as a starting point and outlined here is to develop one or a number of feasible designs and use judgment to select the best, or combine the best features of the proposed designs.

The design procedure for a journal bearing, recommended here as a starting point, includes the specification of the following:

- Journal radius, r (or a);
- Radial clearance, c;
- Axial length of the bearing surface, L;

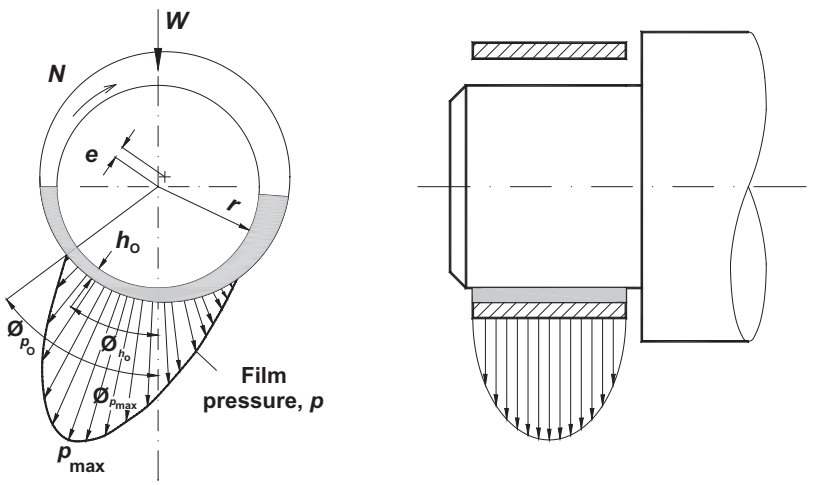


Figure 5.10

Motion of the journal generates pressure in the lubricant separating the two surfaces. Beyond h_o, the minimum film thickness, the pressure terms go negative and the film is ruptured.

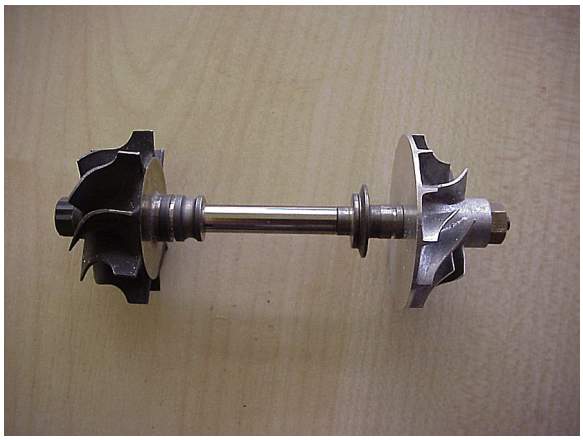


Figure 5.11
Turbocharger journal.

- Type of lubricant and its viscosity, μ;
- Journal speed, N;
- Load, W.

Values for the speed, the load, and possibly the journal radius are usually specified by the machine requirements and stress and deflection considerations. As such, journal bearing design consists of the determination of the radial clearance, the bearing length, and the lubricant viscosity. The design process for a journal bearing is usually iterative. Trial values for the clearance, the length, and the viscosity are chosen, various performance criteria are calculated, and the process is repeated until a satisfactory or optimized design is achieved. Criteria for optimization may be minimizing the frictional loss, minimizing the lubricant temperature rise, minimizing the lubricant supply, maximizing the load capability, and minimizing production costs.

The clearance between the journal and the bearing depends on the nominal diameter of the journal, the precision of the machine, surface roughness, and thermal expansion considerations. An overall guideline is for the radial clearance, c, to be in the range

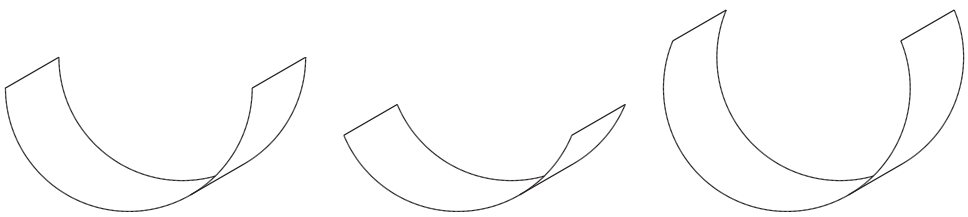


Figure 5.12
Partial surface journal bearings.

$0.001r < c < 0.002r$, where r is the nominal bearing radius ($0.001D < 2c < 0.002D$). Figure 5.13 shows values for the recommended diametral clearance ($2 \times c$) as a function of the journal diameter and rotational speed for steadily loaded bearings.

For a given combination of a, c, L, μ, N, and W, the performance of a journal bearing can be calculated. This requires determining the pressure distribution in the bearing, the minimum film thickness h_o, the location of the minimum film thickness $\theta_{p_{\max}}$ the coefficient of friction f, the lubricant flow Q, the maximum film pressure $p_{\max}$, and the temperature rise ΔT of the lubricant.

The pressure distribution in a journal bearing (see Figure 5.10) can be determined by solving the relevant form of the Navier–Stokes fluid flow equations, which in the reduced form for journal bearings is called the Reynolds equation, Eqn (5.3),

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 6V\frac{\partial h}{\partial x} + 6h\frac{\partial V}{\partial x} \tag{5.3}$$

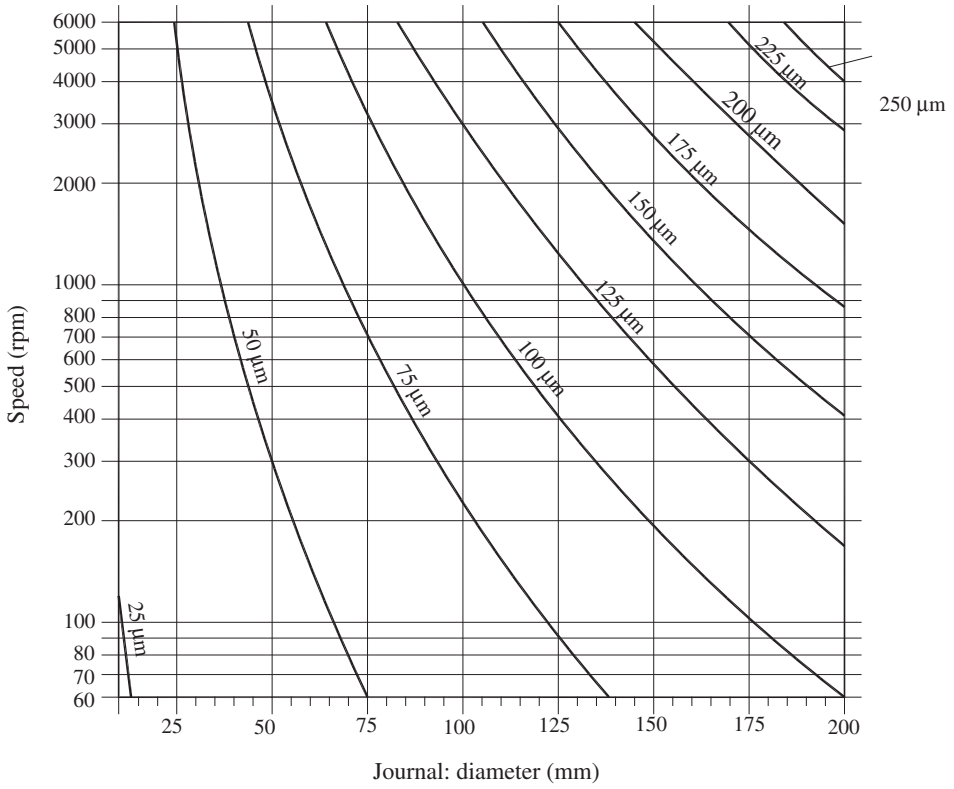


Figure 5.13
Minimum recommended values for the diametral clearance ($2 \times c$) for steadily loaded journal bearings. *Reproduced from Welsh (1983).*

where h = film thickness (m), μ = dynamic viscosity (Pa s), p = fluid pressure (Pa), V = journal velocity (m/s), and x,y,z = rectangular coordinates (m).

The Reynolds equation was first derived by Reynolds (1886) and further developed by Harrison (1913) to include the effects of compressibility. Here the derivation of the general Reynolds equation is given in Section 5.4.1 and is developed following the general outline given by Hamrock (1994) and Childs (2011). Some readers may prefer to omit the derivation and proceed directly to Section 5.4.2.

5.4.1 Reynolds Equation Derivation

The Navier–Stokes equations in a Cartesian coordinate system, for compressible flow assuming constant viscosity, are given by Eqns (5.4)–(5.6). Here x is taken as the coordinate in the direction of sliding, y is the coordinate in the direction of side leakage, and z is the coordinate across the lubricant film.

$$\rho\left(\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+\frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)+F_x \tag{5.4}$$

$$\rho\left(\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}\right) = -\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)+\frac{\mu}{3}\frac{\partial}{\partial y}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)+F_y \tag{5.5}$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)+\frac{\mu}{3}\frac{\partial}{\partial z}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)+F_z \tag{5.6}$$

If the density is assumed constant, then the dilation, Eqn (5.7), is zero.

$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}=0 \tag{5.7}$$

The Navier–Stokes equations for constant density and constant viscosity reduce to

$$\rho\left(\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+F_x \tag{5.8}$$

$$\rho\left(\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)+F_y \tag{5.9}$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)+F_z \tag{5.10}$$

For conditions known as slow viscous motion, where pressure and viscous terms predominate, simplifications are possible for the Navier–Stokes equations, making their solution more amenable to analytical and numerical techniques.

The Navier–Stokes equations can be nondimensionalized to enable a generalized solution (see Childs (2011)). Here the process is repeated to give dimensionless groups of specific relevance to journal bearings using the characteristic parameters given in Eqns (5.11)–(5.22).

$$x_* = \frac{x}{l_o} \tag{5.11}$$

where l_o is a characteristic length in the x direction.

$$y_* = \frac{y}{b_o} \tag{5.12}$$

where b_o is a characteristic length in the y direction.

$$z_* = \frac{z}{h_o} \tag{5.13}$$

where h_o is a characteristic length in the z direction.

$$t_* = \frac{t}{t_o} \tag{5.14}$$

where t_o is a characteristic time.

$$u_{x*} = \frac{u_x}{u_{x,o}} \tag{5.15}$$

where $u_{x,o}$ is a characteristic velocity in the x direction.

$$u_{y*} = \frac{u_y}{u_{y,o}} \tag{5.16}$$

where $u_{y,o}$ is a characteristic velocity in the y direction.

$$u_{z*} = \frac{u_z}{u_{z,o}} \tag{5.17}$$

where $u_{z,o}$ is a characteristic velocity in the z direction.

$$\rho_* = \frac{\rho}{\rho_o} \tag{5.18}$$

where ρ_o is a characteristic density.

$$\mu_* = \frac{\mu}{\mu_o} \tag{5.19}$$

where μ_o is a characteristic viscosity.

$$p_* = \frac{h_o^2 p}{\mu_o u_{x,o} l_o} \tag{5.20}$$

Substitution of the above dimensionless parameters, Eqns (5.11)−(5.20) in Eqn (5.4), gives

$$\begin{aligned}
&\frac{l_o}{u_{x,o}t_o}\frac{\partial u_{x*}}{\partial t_*} + u_{x*}\frac{\partial u_{x*}}{\partial x_*} + \frac{l_o u_{y,o}}{b_o u_{x,o}} u_{y*}\frac{\partial u_{x*}}{\partial y_*} + \frac{l_o u_{z,o}}{h_o u_{x,o}} u_{z*}\frac{\partial u_{x*}}{\partial z_*} \\
&\quad = \frac{l_o g}{u_{x,o}^2} - \frac{\mu_o}{\rho_o u_{x,o} l_o}\left(\frac{l_o}{h_o}\right)^2 \frac{1}{\rho_*}\frac{\partial p_*}{\partial x_*} - \frac{2}{3}\frac{\mu_o}{\rho_o u_{x,o} l_o}\frac{1}{\rho_*}\frac{\partial}{\partial x_*}\left[\mu_*\left(\frac{\partial u_{x*}}{\partial x_*} + \frac{u_{y,o} l_o}{u_{x,o} b_o}\frac{\partial u_{y*}}{\partial y_*} + \frac{u_{z,o} l_o}{u_{x,o} h_o}\frac{\partial u_{z*}}{\partial z_*}\right)\right] \\
&\qquad + \frac{2\mu_o}{\rho_o u_{x,o} l_o}\frac{1}{\rho_*}\frac{\partial}{\partial x_*}\left(\mu_*\frac{\partial u_{x*}}{\partial x_*}\right) + \frac{\mu_o}{\rho_o u_{x,o} l_o}\left(\frac{l_o}{b_o}\right)^2 \frac{1}{\rho_*}\frac{\partial}{\partial y_*}\left[\mu_*\left(\frac{\partial u_{x*}}{\partial y_*} + \frac{u_{y,o} b_o}{u_{x,o} l_o}\frac{\partial u_{y*}}{\partial x_*}\right)\right] \\
&\qquad + \frac{\mu_o}{\rho_o u_{x,o} l_o}\left(\frac{l_o}{h_o}\right)^2 \frac{1}{\rho_*}\frac{\partial}{\partial z_*}\left[\mu_*\left(\frac{\partial u_{x*}}{\partial z_*} + \frac{u_{z,o} h_o}{u_{x,o} l_o}\frac{\partial u_{z*}}{\partial x_*}\right)\right]
\end{aligned} \tag{5.21}$$

The relative importance of inertia and viscous forces can be determined by examining the value of the Reynolds number

$$\mathrm{Re} = \frac{\rho_o u_{x,o} l_o}{\mu_o} \tag{5.22}$$

The inverse of the Reynolds number occurs throughout Eqn (5.21).

In fluid film lubrication, because of the dominance of the viscous term $\partial^2 u_{x*}/\partial^2 z_*$, a modified form of the Reynolds number is used, defined for the x component of velocity by

$$\mathrm{Re}_x = \frac{\rho_o u_{x,o} h_o^2}{\mu_o l_o} \tag{5.23}$$

and similarly for the y and z directions

$$\mathrm{Re}_y = \frac{\rho_o u_{y,o} h_o^2}{\mu_o b_o} \tag{5.24}$$

$$\mathrm{Re}_z = \frac{\rho_o u_{z,o} h_o}{\mu_o} \tag{5.25}$$

The squeeze number is defined by

$$\sigma_s = \frac{\rho_o h_o^2}{\mu_o t_o} \tag{5.26}$$

Typically in hydrodynamically lubricated journal bearings, viscous forces are much greater than inertia forces and a typical Reynolds number, using Eqn (5.23), might be of the order of 1×10^{-4}.

Substitution of the Reynolds and squeeze numbers into Eqn (5.21) gives

$$\begin{aligned}
&\sigma_s \frac{\partial u_{x*}}{\partial t_*} + \mathrm{Re}_x u_{x*} \frac{\partial u_{x*}}{\partial x_*} + \mathrm{Re}_y u_{y*} \frac{\partial u_{x*}}{\partial y_*} + \mathrm{Re}_z u_{z*} \frac{\partial u_{x*}}{\partial z_*} \\
&\quad = \frac{l_o g}{u_{x,o}^2} \mathrm{Re}_x - \frac{1}{\rho_*} \frac{\partial p_*}{\partial x_*} + \frac{1}{\rho_*} \frac{\partial}{\partial z_*} \left(\mu_* \frac{\partial u_{x*}}{\partial z_*} \right) - \frac{2}{3} \left(\frac{h_o}{l_o} \right)^2 \frac{1}{\rho_*} \frac{\partial}{\partial x_*} \left[\mu_* \left(\frac{\partial u_{x*}}{\partial x_*} + \frac{u_{y,o} l_o}{u_{x,o} b_o} \frac{\partial u_{y*}}{\partial y_*} \right.\right. \\
&\qquad \left.\left. + \frac{u_{z,o} l_o}{u_{x,o} h_o} \frac{\partial u_{z*}}{\partial z_*} \right) \right] + \left(\frac{h_o}{b_o} \right)^2 \frac{1}{\rho_*} \frac{\partial}{\partial y_*} \left[\mu_* \left(\frac{\partial u_{x*}}{\partial y_*} + \frac{u_{y,o} b_o}{u_{x,o} l_o} \frac{\partial u_{y*}}{\partial x_*} \right) \right] + 2 \left(\frac{h_o}{l_o} \right)^2 \frac{1}{\rho_*} \frac{\partial}{\partial x_*} \left(\mu_* \frac{\partial u_{x*}}{\partial x_*} \right) \\
&\qquad + \frac{1}{\rho_*} \frac{\partial}{\partial z_*} \left(\mu_* \frac{u_{z,o} h_o}{u_{x,o} l_o} \frac{\partial u_{z*}}{\partial x_*} \right)
\end{aligned} \tag{5.27}$$

Examination of the order of magnitude of the terms in Eqn (5.27) provides an indication of the relative significance of the terms which need to be considered. The inertia terms, the gravity term, and $u_{z,o}/u_{x,o}$ are of order h_o/l_o. The pressure gradient term and the first viscous term are of order unity. The remaining viscous terms are of order $(h_o/l_o)^2$ or $(h_o/b_o)^2$ and therefore very small in comparison to the other terms. Neglecting terms of the order of $(h_o/l_o)^2$ and $(h_o/b_o)^2$ gives

$$\sigma_s \frac{\partial u_{x*}}{\partial t_*} + \mathrm{Re}_x u_{x*} \frac{\partial u_{x*}}{\partial x_*} + \mathrm{Re}_y u_{y*} \frac{\partial u_{x*}}{\partial y_*} + \mathrm{Re}_z u_{z*} \frac{\partial u_{x*}}{\partial z_*} = \frac{l_o g}{u_{x,o}^2} \mathrm{Re}_x - \frac{1}{\rho_*} \frac{\partial p_*}{\partial x_*} + \frac{1}{\rho_*} \frac{\partial}{\partial z_*} \left(\mu_* \frac{\partial u_{x*}}{\partial z_*} \right) \tag{5.28}$$

Similarly for the y and z components

$$\sigma_s \frac{\partial u_{y*}}{\partial t_*} + \mathrm{Re}_x u_{x*} \frac{\partial u_{y*}}{\partial x_*} + \mathrm{Re}_y u_{y*} \frac{\partial u_{y*}}{\partial y_*} + \mathrm{Re}_z u_{z*} \frac{\partial u_{y*}}{\partial z_*} = \frac{b_o g}{u_{y,o}^2} \mathrm{Re}_y - \frac{1}{\rho_*} \frac{\partial p_*}{\partial y_*} + \frac{1}{\rho_*} \frac{\partial}{\partial z_*} \left(\mu_* \frac{\partial u_{y*}}{\partial z_*} \right) \tag{5.29}$$

$$\frac{\partial p}{\partial z_*} = 0 \tag{5.30}$$

Examination of Eqns (5.28)–(5.30) shows that the pressure is a function of x_*, y_*, and t_*,

$$p = f(x_*, y_*, t_*) \tag{5.31}$$

The continuity equation can be expressed as

$$\sigma_s \frac{\partial p_*}{\partial t_*} + \mathrm{Re}_x \frac{\partial}{\partial x_*} (\rho_* u_{x*}) + \mathrm{Re}_y \frac{\partial}{\partial y_*} (\rho_* u_{y*}) + \mathrm{Re}_z \frac{\partial}{\partial z_*} (\rho_* u_{z*}) = 0 \tag{5.32}$$

Taylor vortices can be formed in a fluid-filled annulus with inner cylinder rotation as illustrated in Figures 5.14 and 5.15. Vortices involve significant acceleration and deceleration of the flow and therefore dissipation of energy and are normally undesirable in any bearing lubrication scheme. A check can readily be made to identify if Taylor vortices are likely to occur. If the Taylor number, Ta_m, based on the mean annulus radius, Eqn (5.33), is greater

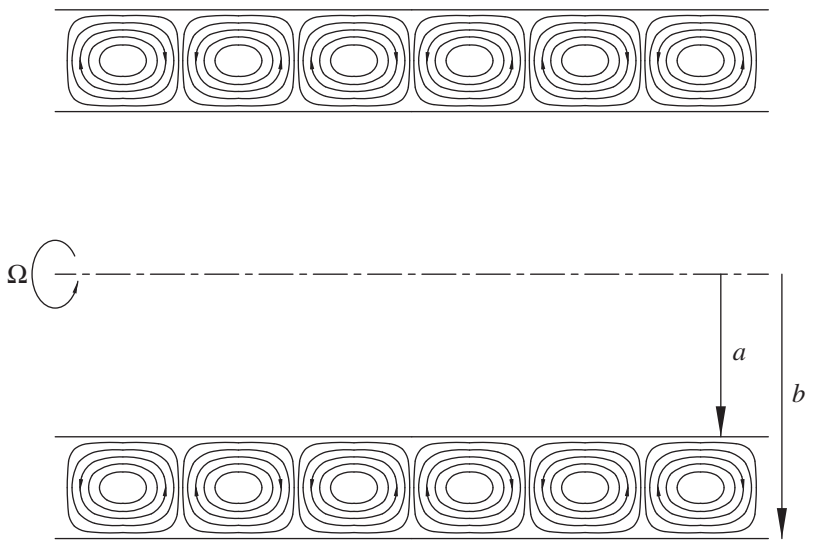


Figure 5.14
Taylor vortices.

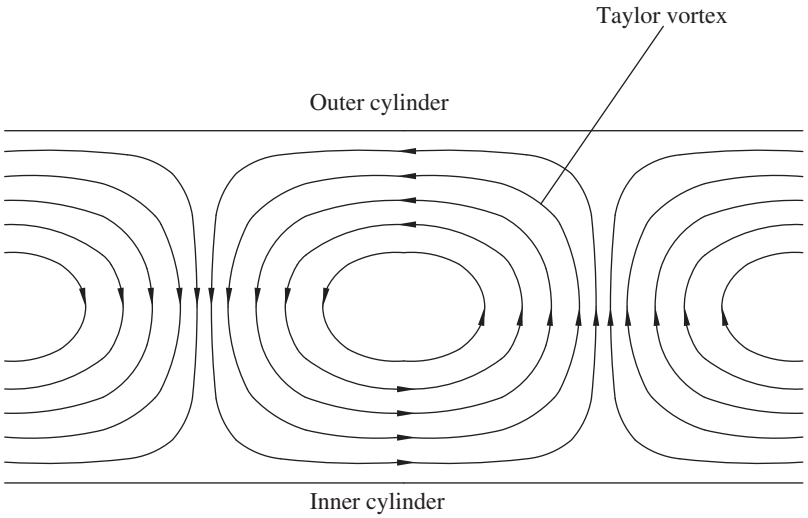


Figure 5.15
Taylor vortex cells.

than approximately 41.2, then Taylor vortices may form and laminar flow conditions no longer hold, invalidating the use of the equations developed in this section.

$$\mathrm{Ta}_m = \frac{\Omega r_m^{0.5}(b-a)^{1.5}}{\nu} \tag{5.33}$$

$$r_m = 0.5(b+a) \tag{5.34}$$

$$\mathrm{Ta}_{m,\mathrm{cr}} = \sqrt{1697} = 41.19 \tag{5.35}$$

The critical angular velocity for this case is given by

$$\Omega_{\mathrm{cr}} = \frac{41.19\nu}{r_m^{0.5}(b-a)^{1.5}} \tag{5.36}$$

For an annulus with a finite gap,

$$\Omega_{\mathrm{cr}} = \frac{41.19\nu F_g}{r_m^{0.5}(b-a)^{1.5}} \tag{5.37}$$

where F_g is a geometrical factor defined by

$$F_g = \frac{\pi^2}{41.19\sqrt{S}}\left(1 - \frac{b-a}{2r_m}\right)^{-1} \tag{5.38}$$

and S is given in an alternative form by

$$S = 0.0571\left(1 - 0.652\frac{(b-a)/r_m}{1-(b-a)/2r_m}\right) + 0.00056\left(1 - 0.652\frac{(b-a)/r_m}{1-(b-a)/2r_m}\right)^{-1} \tag{5.39}$$

The Froude number, Eqn (5.40), provides an indication of the relative importance of inertia and gravity forces. A typical Froude number for a journal bearing might be of the order of 10, providing an indication that gravity forces can be neglected in comparison with viscous forces.

$$\mathrm{Fr} = \frac{U_o}{\sqrt{gL}} \tag{5.40}$$

The importance of pressure relative to inertia can be judged by examining the Euler number, Eqn (5.41). For a typical journal bearing, the Euler number may be of the order of 100, giving an indication that the pressure term is much larger than the inertia term.

$$\mathrm{Eu} = \frac{p - p_o}{0.5\rho U_o^2} \tag{5.41}$$

If in addition to neglecting terms of order $(h_o/l_o)^2$ or $(h_o/b_o)^2$, terms of the order of h_o/l_o and h_o/b_o are neglected and only terms of the order of unity are considered, the Navier–Stokes equations reduce to

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z}\left(\mu\frac{\partial u_x}{\partial z}\right) \tag{5.42}$$

$$\frac{\partial p}{\partial y} = \frac{\partial}{\partial z}\left(\mu\frac{\partial u_y}{\partial z}\right) \tag{5.43}$$

Equation (5.31) shows that the pressure is only a function of x and y for steady-state conditions. Equations (5.42) and (5.43) can therefore be integrated directly to give general expressions for the velocity gradients as follows:

$$\frac{\partial u_x}{\partial z} = \frac{z}{\mu}\frac{\partial p}{\partial x} + \frac{A}{\mu} \tag{5.44}$$

$$\frac{\partial u_y}{\partial z} = \frac{z}{\mu}\frac{\partial p}{\partial y} + \frac{C}{\mu} \tag{5.45}$$

where A and C are constants of integration.

The temperature across the thin layer of lubricant in a journal bearing may vary significantly. As viscosity is highly dependent on temperature, this leads to increased complexity in

obtaining a solution to Eqns (5.44) and (5.45). It has been found acceptable, however, in many fluid film applications to model the viscosity of a fluid film using an average value for the viscosity across the film. With μ taken as the average value of viscosity across the fluid film, integration of Eqns (5.44) and (5.45) gives the velocity components as follows:

$$u_x = \frac{z^2}{2\mu}\frac{\partial p}{\partial x} + A\frac{z}{\mu} + B \tag{5.46}$$

$$u_y = \frac{z^2}{2\mu}\frac{\partial p}{\partial y} + C\frac{z}{\mu} + D \tag{5.47}$$

where B and D are constants of integration.

If the no-slip condition is assumed at the fluid-solid interface, then the boundary conditions are as follows and illustrated in Figure 5.16:

$$u_x = u_{x,a},\ u_y = u_{y,a} \quad \text{at } z = 0$$

$$u_x = u_{x,b},\ u_y = u_{y,b} \quad \text{at } z = h$$

Application of the boundary conditions to the equations for the velocity gradients and velocity components, Eqns (5.46) and (5.47), gives

$$\frac{\partial u_x}{\partial z} = \frac{2z - h}{2\mu}\frac{\partial p}{\partial x} - \frac{u_{x,a} - u_{x,b}}{h} \tag{5.48}$$

$$\frac{\partial u_y}{\partial z} = \frac{2z - h}{2\mu}\frac{\partial p}{\partial y} - \frac{u_{y,a} - u_{y,b}}{h} \tag{5.49}$$

$$u_x = \frac{z(z-h)}{2\mu}\frac{\partial p}{\partial x} + \frac{h - z}{h}u_{x,a} + \frac{z}{h}u_{x,b} \tag{5.50}$$

$$u_y = \frac{z(z-h)}{2\mu}\frac{\partial p}{\partial y} + \frac{h - z}{h}u_{y,a} + \frac{z}{h}u_{y,b} \tag{5.51}$$

The viscous shear stresses are defined by

$$\tau_{zx} = \mu\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) \tag{5.52}$$

$$\tau_{zy} = \mu\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) \tag{5.53}$$

The order of magnitude of $\partial u_z/\partial x$ and $\partial u_z/\partial y$ is much smaller than that of $\partial u_x/\partial z$ and $\partial u_y/\partial z$, so the viscous shear stresses can be approximated by

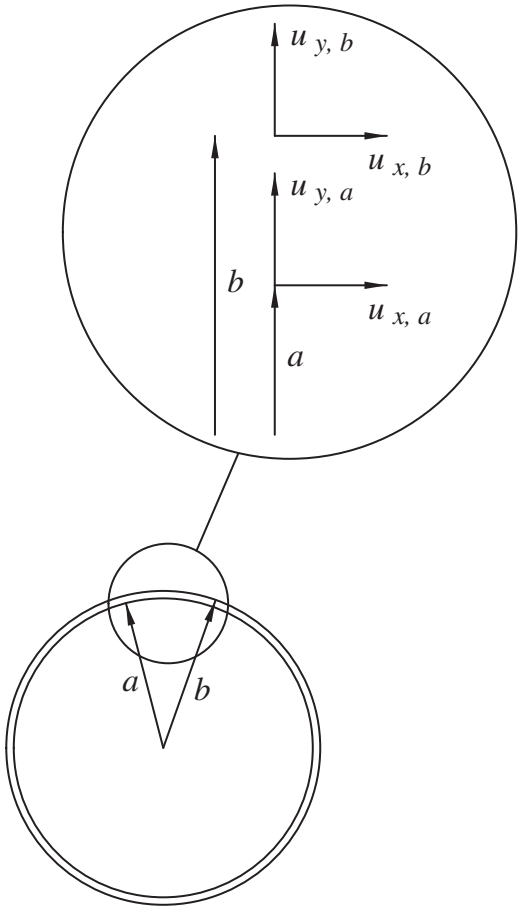


Figure 5.16
Journal bearing boundary conditions.

$$\tau_{zx} = \mu \frac{\partial u_x}{\partial z} \tag{5.54}$$

$$\tau_{zy} = \mu \frac{\partial u_y}{\partial z} \tag{5.55}$$

From Eqns (5.48) and (5.49), the viscous shear stresses acting on the solid surfaces can be expressed by

$$(\tau_{zx})_{z=o} = \left(\mu \frac{\partial u_x}{\partial z}\right)_{z=o} = -\frac{h}{2}\frac{\partial p}{\partial x} - \frac{\mu\left(u_{x,a} - u_{x,b}\right)}{h} \tag{5.56}$$

$$(-\tau_{zx})_{z=h} = -\left(\mu\frac{\partial u_x}{\partial z}\right)_{z=h} = -\frac{h}{2}\frac{\partial p}{\partial x} + \frac{\mu(u_{x,a} - u_{x,b})}{h} \tag{5.57}$$

$$(\tau_{zy})_{z=o} = \left(\mu\frac{\partial u_y}{\partial z}\right)_{z=o} = -\frac{h}{2}\frac{\partial p}{\partial y} - \frac{\mu(u_{y,a} - u_{y,b})}{h} \tag{5.58}$$

$$(-\tau_{zy})_{z=h} = -\left(\mu\frac{\partial u_y}{\partial z}\right)_{z=h} = -\frac{h}{2}\frac{\partial p}{\partial y} + \frac{\mu(u_{y,a} - u_{y,b})}{h} \tag{5.59}$$

The negative signs for the viscous shear stresses, in Eqns (5.56)−(5.59), indicate that the stress acts in a direction opposite to the motion.

The volumetric flow rates per unit width in the x and y directions are defined by

$$q_x = \int_o^h u_x \mathrm{d}z \tag{5.60}$$

$$q_y = \int_o^h u_y \mathrm{d}z \tag{5.61}$$

Substituting for the velocity components using Eqns (5.50) and (5.51) in Eqns (5.60) and (5.61) gives

$$q_x = -\frac{h^3}{12\mu}\frac{\partial p}{\partial x} + \frac{u_{x,a} + u_{x,b}}{2}h \tag{5.62}$$

$$q_y = -\frac{h^3}{12\mu}\frac{\partial p}{\partial y} + \frac{u_{y,a} + u_{y,b}}{2}h \tag{5.63}$$

The Reynolds equation is formed by substituting the expressions for the volumetric flow rate into the continuity equation.

Integrating the continuity equation gives

$$\int_o^h \left[\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z)\right]\mathrm{d}z = 0 \tag{5.64}$$

The integral

$$\int_o^h \frac{\partial}{\partial x}[f(x,y,z)]\mathrm{d}z = -f(x,y,h)\frac{\partial h}{\partial x} + \frac{\partial}{\partial x}\left[\int_o^h f(x,y,z)\mathrm{d}z\right] \tag{5.65}$$

If the density is assumed to be the mean density of the fluid across the film, then the u_x term in Eqn (5.64) is

$$\int_o^h \frac{\partial}{\partial x}(\rho u_x)\mathrm{d}z = -(\rho u_x)_{z=h}\frac{\partial h}{\partial x} + \frac{\partial}{\partial x}\left(\int_o^h \rho u_x \mathrm{d}z\right) = -\rho u_{x,b}\frac{\partial h}{\partial x} + \frac{\partial}{\partial x}\left(\int_o^h \rho u_x \mathrm{d}z\right) \tag{5.66}$$

Similarly for the u_y term in Eqn (5.64),

$$\int_o^h \frac{\partial}{\partial y}\left(\rho u_y\right)\mathrm{d}z = -\rho u_{y,b}\frac{\partial h}{\partial y} + \frac{\partial}{\partial y}\left(\int_o^h \rho u_y \mathrm{d}z\right) \tag{5.67}$$

The u_z term can be integrated directly giving

$$\int_o^h \frac{\partial}{\partial z}(\rho u_z)\mathrm{d}z = \rho\left(u_{z,b} - u_{z,a}\right) \tag{5.68}$$

The integral form of the continuity equation, Eqn (5.64), upon substitution of Eqns (5.66) and (5.68), can be stated as

$$h\frac{\partial \rho}{\partial t} - \rho u_{x,b}\frac{\partial h}{\partial x} + \frac{\partial}{\partial x}\left(\rho\int_o^h u_x \mathrm{d}z\right) - \rho u_{y,b}\frac{\partial h}{\partial y} + \frac{\partial}{\partial y}\left(\rho\int_o^h u_y \mathrm{d}z\right) + \rho\left(u_{z,b} - u_{z,a}\right) = 0 \tag{5.69}$$

The integrals in Eqn (5.69) represent the volumetric flow rates per unit width. Substitution of the values for these from Eqns (5.62) and (5.63) gives the general Reynolds equation.

$$\begin{aligned}\frac{\partial}{\partial x}\left(-\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial x}\left[\frac{\rho h\left(u_{x,a} + u_{x,b}\right)}{2}\right] + \frac{\partial}{\partial y}\left[\frac{\rho h\left(u_{y,a} + u_{y,b}\right)}{2}\right] \\ + \rho\left(u_{z,b} - u_{z,a}\right) - \rho u_{x,b}\frac{\partial h}{\partial x} - \rho u_{y,b}\frac{\partial h}{\partial y} + h\frac{\partial p}{\partial t} = 0\end{aligned} \tag{5.70}$$

The first two terms of Eqn (5.70) are the Poiseuille terms and they describe the net flow rates due to pressure gradients within the lubricated area. The third and fourth terms are the Couette terms and they describe the net entrained flow rates due to surface velocities. The fifth to seventh terms are due to a squeezing motion. The eighth term describes the net flow rate due to local expansion as a result of density changes with time. Eqn (5.70) is repeated below with the identification of the various terms emphasized.

$$\underbrace{\frac{\partial}{\partial x}\left(-\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(-\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial y}\right)}_{\text{Poiseuille}}+\underbrace{\frac{\partial}{\partial x}\left[\frac{\rho h\left(u_{x,a}+u_{x,b}\right)}{2}\right]+\frac{\partial}{\partial y}\left[\frac{\rho h\left(u_{y,a}+u_{y,b}\right)}{2}\right]}_{\text{Couette}}$$

$$+\underbrace{\rho\left(u_{z,b}-u_{z,a}\right)-\rho u_{x,b}\frac{\partial h}{\partial x}-\rho u_{y,b}\frac{\partial h}{\partial y}}_{\text{Squeeze}}+\underbrace{h\frac{\partial p}{\partial t}}_{\text{Net flow due to local expansion}}=0$$

For tangential-only motion, where

$$u_{z,b}=u_{x,b}\frac{\partial h}{\partial x}+u_{y,b}\frac{\partial h}{\partial y} \tag{5.71}$$

and $u_{z,a}=0$, Eqn (5.70) reduces to

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\mu}\frac{\partial p}{\partial y}\right)=12\overline{u}_x\frac{\partial(\rho h)}{\partial x}+12\overline{u}_y\frac{\partial(\rho h)}{\partial y} \tag{5.72}$$

where

$$\overline{u}_x=\frac{u_{x,a}+u_{x,b}}{2}=\text{constant} \tag{5.73}$$

$$\overline{u}_y=\frac{u_{y,a}+u_{y,b}}{2}=\text{constant} \tag{5.74}$$

For hydrodynamic lubrication, the fluid properties do not vary significantly through the bearing and can be considered constant. In addition, for hydrodynamic lubrication, the motion is pure sliding and $\overline{u}_y=0$.

The Reynolds equation can therefore be simplified to

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right)=12\overline{u}_x\mu_o\frac{\partial h}{\partial x} \tag{5.75}$$

In some lubrication applications, side leakage can be neglected and Eqn (5.72) can be restated as

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\mu}\frac{\partial p}{\partial x}\right)=12\overline{u}_x\frac{\partial(\rho h)}{\partial x} \tag{5.76}$$

Equation (5.76) can be integrated giving

$$\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x}=\frac{12\overline{u}_x}{h^2}+\frac{A}{\rho h^3} \tag{5.77}$$

Boundary conditions $dp/dx = 0$ when $x = x_m$, $\rho = \rho_m$, $h = h_m$ give $A = -12\overline{u}_x\rho_m h_m$. The subscript m refers to the condition for which $dp/dx = 0$ such as the point of maximum pressure. Substituting for A in Eqn (5.77) gives

$$\frac{dp}{dx} = 12\overline{u}_x\mu\frac{\rho h - \rho_m h_m}{\rho h^3} \tag{5.78}$$

If the density can be considered constant, then Eqn (5.78) becomes

$$\frac{dp}{dx} = 12\overline{u}_x\mu\frac{h - h_m}{h^3} \tag{5.79}$$

For the case of a gas-lubricated bearing, the density, using the ideal gas law, is given by

$$\rho = \frac{p}{RT} \tag{5.80}$$

From Eqn (5.72), and taking the viscosity as constant,

$$\frac{\partial}{\partial x}\left(\rho h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho h^3\frac{\partial p}{\partial y}\right) = 12\overline{u}_x\mu_o\frac{\partial(\rho h)}{\partial x} \tag{5.81}$$

Expressing Eqn (5.72) in cylindrical coordinates

$$\frac{\partial}{\partial r}\left(\frac{r\rho h^3}{\mu}\frac{\partial p}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial\phi}\left(\frac{\rho h^3}{\mu}\frac{\partial p}{\partial\phi}\right) = 12\left[\overline{u}_r\frac{\partial(\rho r h)}{\partial r} + \overline{u}_\phi\frac{\partial(\rho h)}{\partial\phi}\right] \tag{5.82}$$

where

$$\overline{u}_r = \frac{u_{r,a} + u_{r,b}}{2} \tag{5.83}$$

$$\overline{u}_\phi = \frac{u_{\phi,a} + u_{\phi,b}}{2} \tag{5.84}$$

If the viscosity and density can be assumed to be constant, then Eqn (5.82) becomes

$$\frac{\partial}{\partial r}\left(rh^3\frac{\partial p}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial\phi}\left(h^3\frac{\partial p}{\partial\phi}\right) = 12\mu_o\left[\overline{u}_r\frac{\partial(rh)}{\partial r} + \overline{u}_\phi\frac{\partial h}{\partial\phi}\right] \tag{5.85}$$

Equation (5.85) applies to a thrust bearing where the fluid film is in the z direction and the bearing dimensions are in the r, ϕ directions.

For turbulent flow, the Reynolds equation is

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu k_x}\frac{\partial\overline{p}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{h^3}{\mu k_y}\frac{\partial\overline{p}}{\partial y}\right) = \frac{\overline{u}_x}{2}\frac{\partial h}{\partial x} \tag{5.86}$$

where (from Constantinescu, 1962)

$$k_x = 12 + 0.026\left(\frac{\rho\Omega rh}{\mu}\right)^{0.102} \tag{5.87}$$

$$k_y = 12 + 0.0198\left(\frac{\rho\Omega rh}{\mu}\right)^{0.091} \tag{5.88}$$

For an infinitely wide journal bearing, the pressure in the axial direction can be assumed to be constant. This approach is valid for a length-to-diameter ratio of $L/D > 2$. The integrated form of the Reynolds equation, Eqn (5.79), upon substitution for $\overline{u}_x = (u_a + u_b)/2 = \Omega a/2$, as $u_b = 0$ for a stationary bearing surface, gives

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{6\mu_o a\Omega(h - h_m)}{h^3} \tag{5.89}$$

$$\mathrm{d}x = a\mathrm{d}\phi \tag{5.90}$$

$$\frac{\mathrm{d}p}{\mathrm{d}\phi} = \frac{6\mu_o a^2\Omega(h - h_m)}{h^3} \tag{5.91}$$

$$\cos\alpha = \frac{1}{b}[h + a + e\cos(\pi - \phi)] \tag{5.92}$$

$$h = b\cos\alpha - a + e\cos\phi \tag{5.93}$$

Hence, using trigonometric relationships,

$$h = b\left[1 - \left(\frac{e}{b}\right)^2 \sin^2\phi\right]^{0.5} - a + e\cos\phi \tag{5.94}$$

Expanding the series

$$\left[1 - \left(\frac{e}{b}\right)^2 \sin^2\phi\right]^{0.5} = 1 - \frac{1}{2}\left(\frac{e}{b}\right)^2 \sin^2\ \phi - \frac{1}{8}\left(\frac{e}{b}\right)^4 \sin^4\phi - \ldots \tag{5.95}$$

Hence

$$h = b\left[1 - \frac{1}{2}\left(\frac{e}{b}\right)^2 \sin^2\phi - \frac{1}{8}\left(\frac{e}{b}\right)^4 \sin^4\phi - \ldots\right] - a + e\cos\phi \tag{5.96}$$

$$b - a = c \tag{5.97}$$

$$h = c + e\left[\cos\phi - \frac{1}{2}\left(\frac{e}{b}\right)\sin^2\phi - \frac{1}{8}\left(\frac{e}{b}\right)^3 \sin^4\phi - \ldots\right] \tag{5.98}$$

The ratio *e/b* is typically of the order of 1×10^{-3}. As a result, terms in Eqn (5.98) with this ratio can readily be neglected, giving

$$h = c + e\cos\phi = c\left(1 + \frac{e}{c}\cos\phi\right) = c(1 + \varepsilon\cos\phi) \tag{5.99}$$

where ε is the eccentricity ratio defined by

$$\varepsilon = \frac{e}{c} \tag{5.100}$$

Substitution of Eqn (5.99) in Eqn (5.91) gives

$$\frac{\mathrm{d}p}{\mathrm{d}\phi} = 6\mu\Omega\left(\frac{a}{c}\right)^2\left[\frac{1}{(1+\varepsilon\cos\phi)^2} - \frac{h_m}{c(1+\varepsilon\cos\phi)^3}\right] \tag{5.101}$$

Integration of Eqn (5.101) gives the following expression for the pressure distribution:

$$p = 6\mu\Omega\left(\frac{a}{c}\right)^2\int\left[\frac{1}{(1+\varepsilon\cos\phi)^2} - \frac{h_m}{c(1+\varepsilon\cos\phi)^3}\right]\mathrm{d}\phi + A \tag{5.102}$$

where A is a constant of integration.

For a journal bearing,

$$x = a\phi \tag{5.103}$$

and

$$\overline{u} = \frac{u_{x,a}}{2} = \frac{a\Omega}{2} \tag{5.104}$$

From Eqn (5.75),

$$\frac{\partial}{\partial\phi}\left(h^3\frac{\partial p}{\partial\phi}\right) + a^2\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right) = 6\mu_o\Omega a^2\frac{\partial h}{\partial\phi} \tag{5.105}$$

Substituting for the film thickness gives

$$\frac{\partial}{\partial\phi}\left(h^3\frac{\partial p}{\partial\phi}\right) + a^2h^3\frac{\partial^2 p}{\partial y^2} = -6\mu_o\Omega a^2 e\sin\phi \tag{5.106}$$

The Reynolds equation can be solved by approximate mathematical methods or numerically. Once the pressure distribution has been established, the journal performance can be determined in terms of the bearing load capacity, frictional losses, lubricant flow requirements, and lubricant temperature rise.

5.4.2 Design Charts for Full-Film Hydrodynamic Bearings

If the designer wishes to avoid the direct solution of Eqn (5.3), use can be made of a series of design charts. These were originally produced by Raimondi and Boyd (1958a–c), who used an iterative technique to solve the Reynolds equations. These charts give the film thickness, coefficient of friction, lubricant flow, lubricant side flow ratio, minimum film thickness location, maximum pressure ratio, maximum pressure ratio position, and film termination angle versus the Sommerfield number (see Figures 5.17–5.24).

The Sommerfield number, which is also known as the bearing characteristic number, is defined in Eqn (5.107). It is used as it encapsulates all the parameters usually defined by the designer. Great care needs to be taken with units in the design of journal bearings. Many design charts have been produced using English units (psi, reyn, Btu, etc.). As long as a consistent set of units is maintained, use of the charts will yield sensible results. In particular, note the use of revolutions per second in the definition of speed in the Sommerfield number.

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N_s}{P} \tag{5.107}$$

where S is the bearing characteristic number, r is the journal radius (m), c is the radial clearance (m), μ is the absolute viscosity (Pa s), N_s is the journal speed (revolutions per second, rps), $P = W/LD$ is the load per unit of projected bearing area (N/m^2), W is the load on the bearing (N), D is the journal diameter (m), and L is the journal bearing length (m).

Consider the journal shown in Figure 5.25. As the journal rotates, it will pump lubricant in a clockwise direction. The lubricant is pumped into a wedge shaped space and the journal is forced over to the opposite side. The angular position where the lubricant film is at its minimum thickness h_o is called the attitude angle, ϕ. The center of the journal is displaced from the center of the bearing by a distance e called the eccentricity. The ratio of the eccentricity e to the radial clearance c is called the eccentricity ratio ($\varepsilon = e/c$). The relationship between the film thickness, the clearance, the eccentricity, and the eccentricity ratio are defined in Eqns (5.108)–(5.110).

$$h_o = c - e \tag{5.108}$$

$$\varepsilon = \frac{e}{c} \tag{5.109}$$

$$\frac{h_o}{c} = 1 - \varepsilon \tag{5.110}$$

One of the assumptions made in the analysis of Raimondi and Boyd is that the viscosity of the lubricant is constant as it passes through the bearing. However, work is done on the lubricant in the bearing and the temperature of the lubricant leaving the bearing zone will be higher than the entrance value. Figure 5.7 shows the variation of viscosity with temperature for some

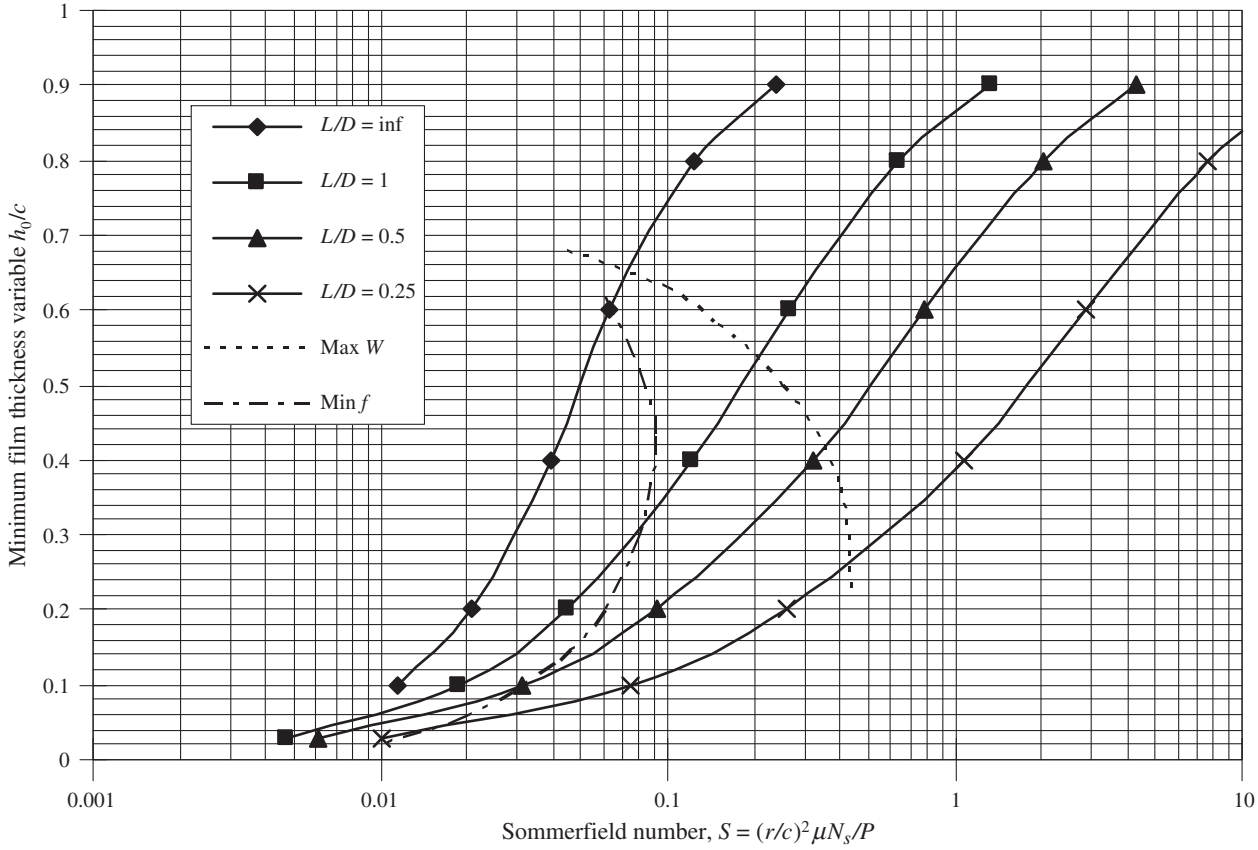


Figure 5.17

Chart for the minimum film thickness variable (h_o/c) versus the Sommerfield number. *Data from Raimondi and Boyd (1958c).*

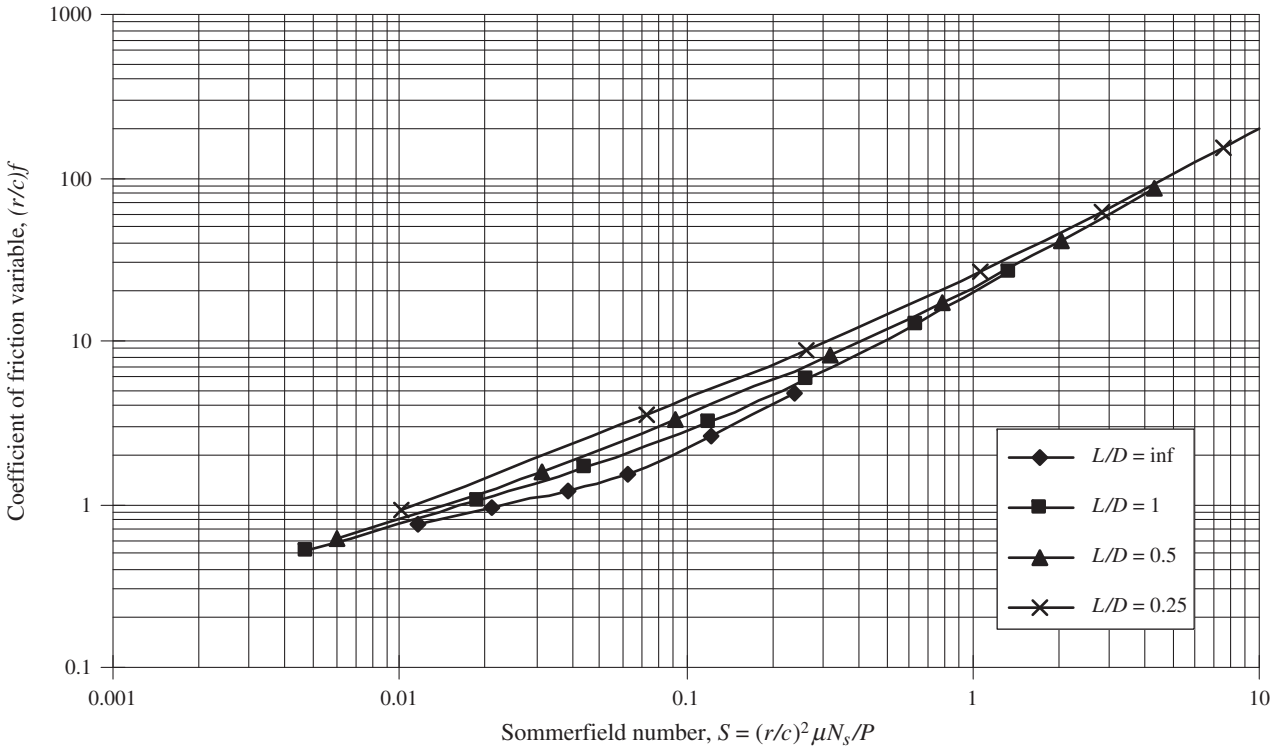


Figure 5.18
Chart for determining the coefficient of friction variable, $(r/c)f$. *Data from Raimondi and Boyd (1958c).*

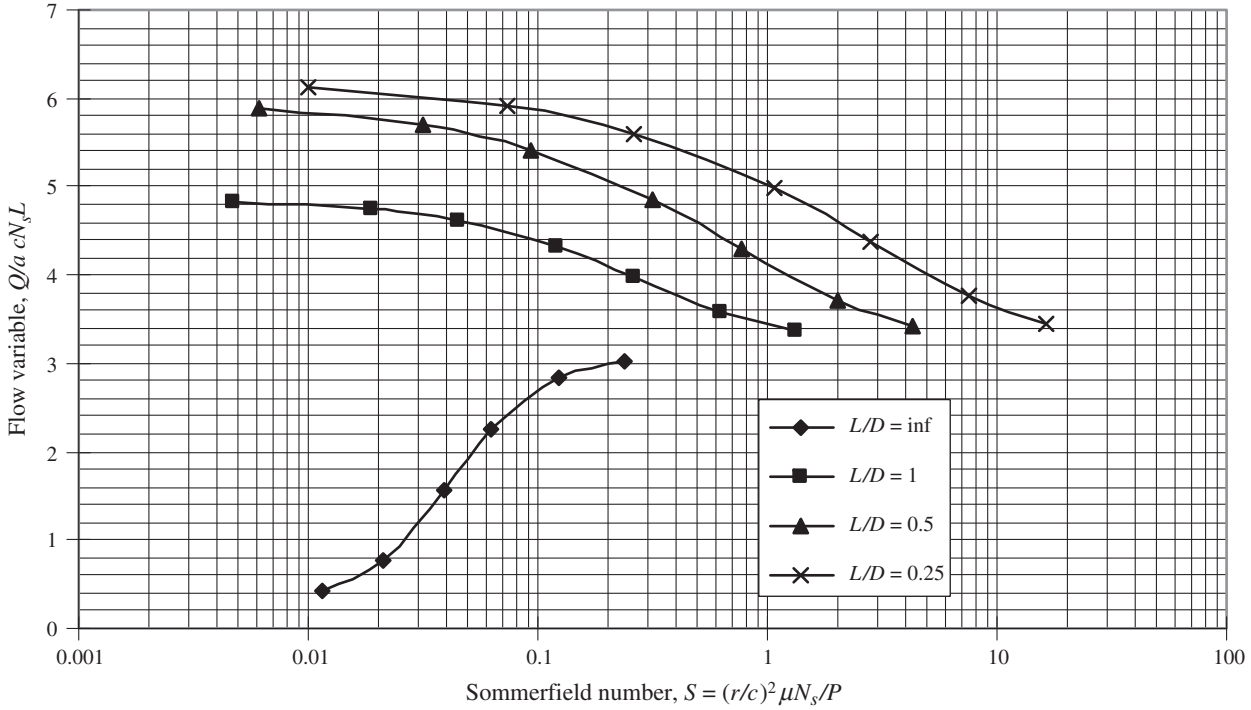


Figure 5.19
Chart for determining the flow variable, $Q/(acN_sL)$. *Data from Raimondi and Boyd (1958c).*

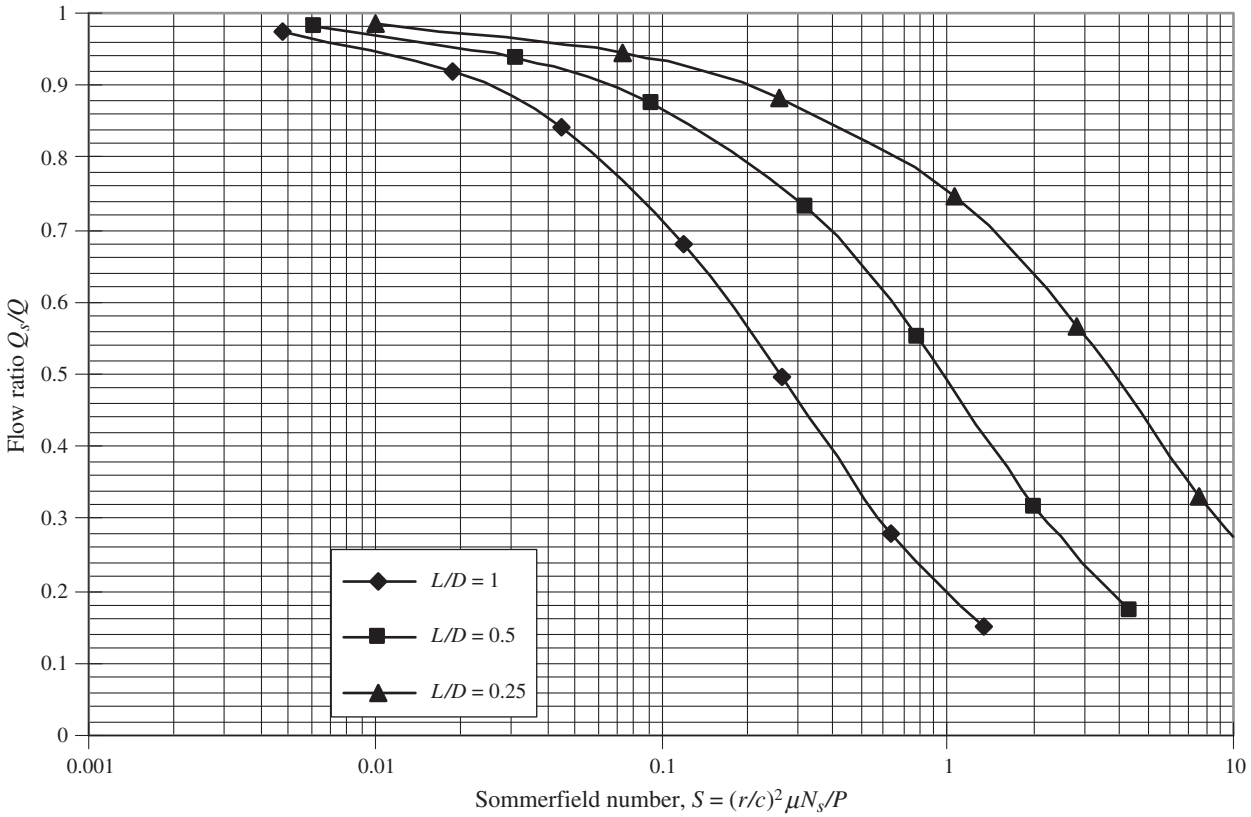


Figure 5.20
Chart for determining the ratio of side flow, Q_s, to total flow, Q. *Data from Raimondi and Boyd (1958c).*

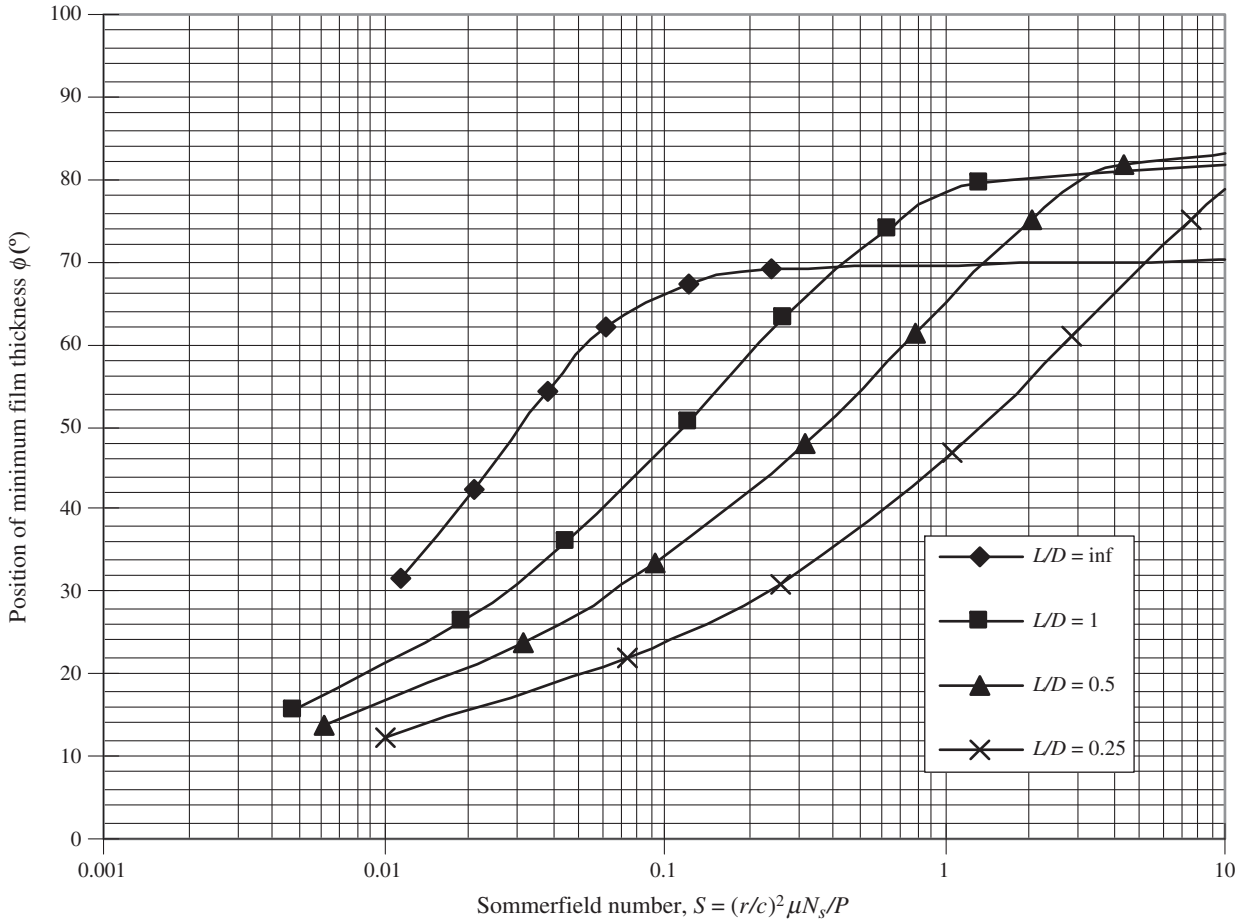


Figure 5.21
Chart for determining the position of the minimum film thickness ϕ. *Data from Raimondi and Boyd (1958c).*

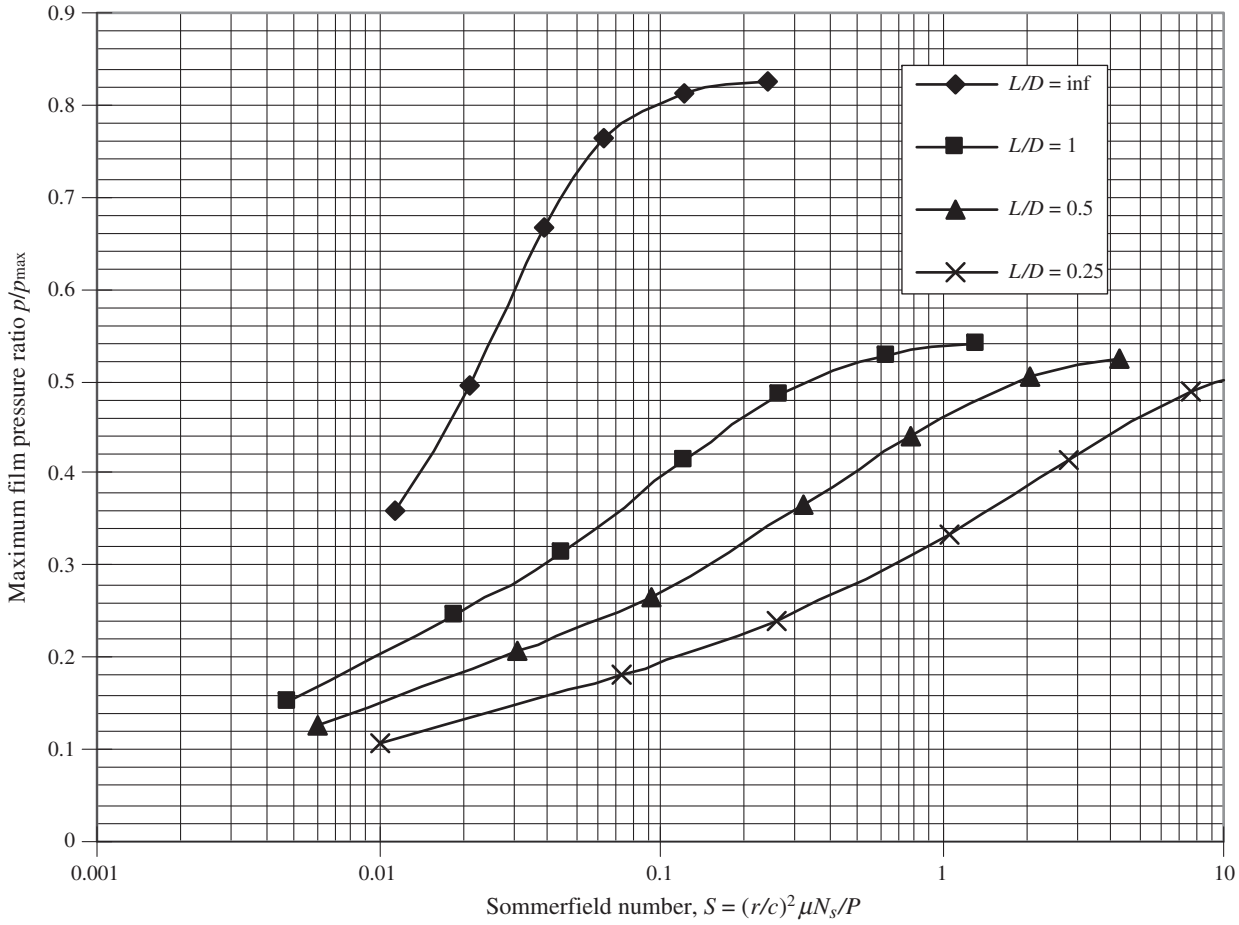


Figure 5.22
Chart for determining the maximum film pressure ratio, p/p_{max}. *Data from Raimondi and Boyd (1958c).*

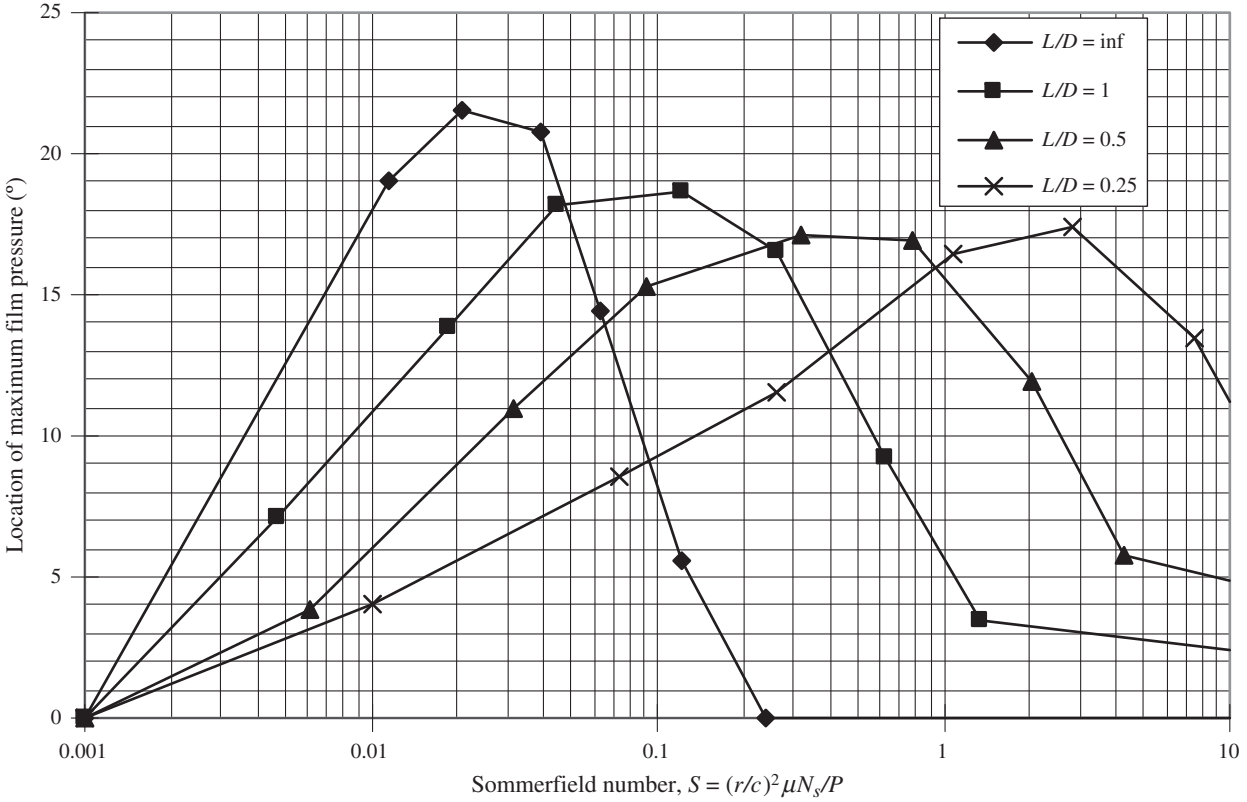


Figure 5.23
Chart for determining the position of maximum pressure, $\phi_{p_{max}}$. *Data from Raimondi and Boyd (1958c).*

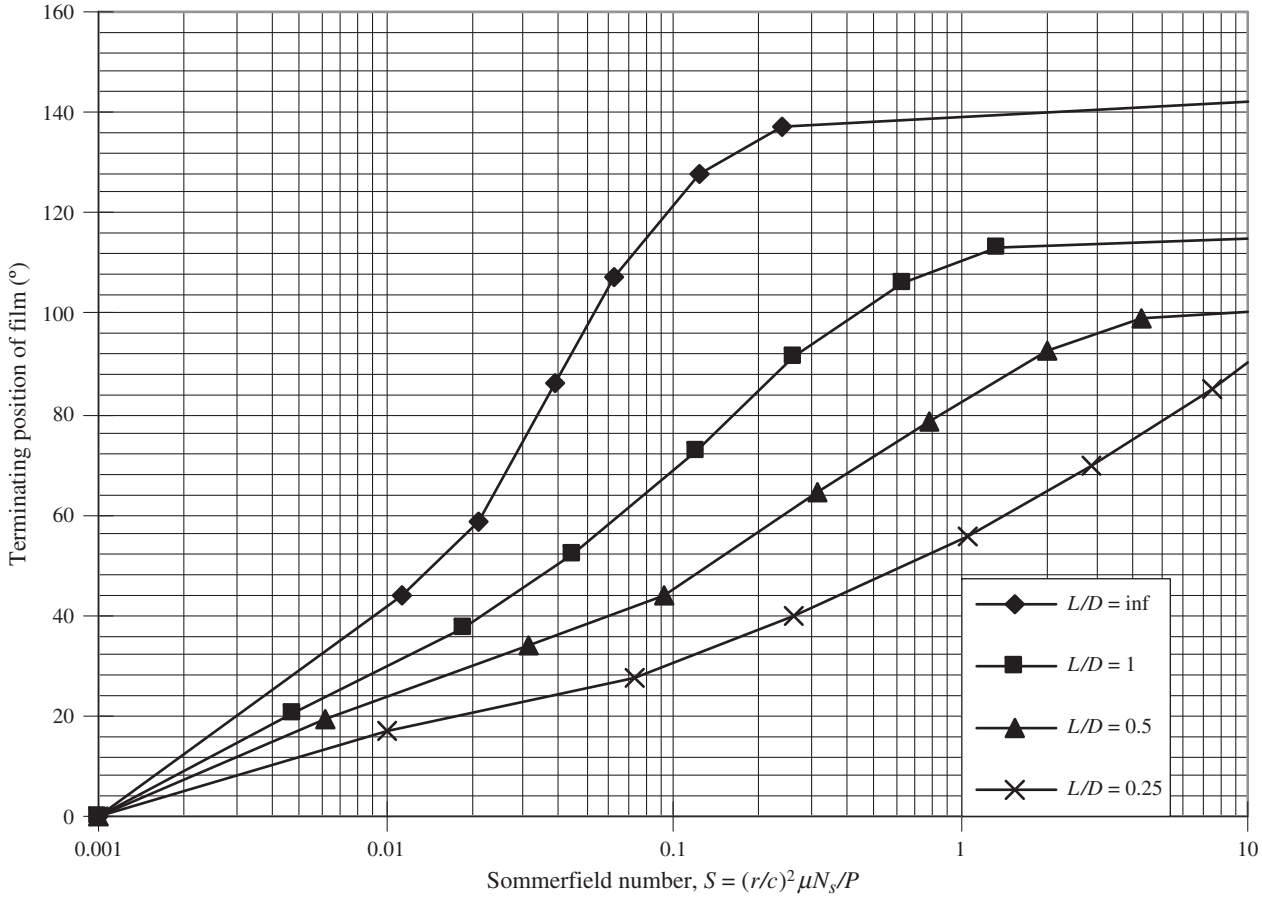


Figure 5.24
Chart for determining the film termination angle, ϕ_{p_o}. *Data from Raimondi and Boyd (1958c).*

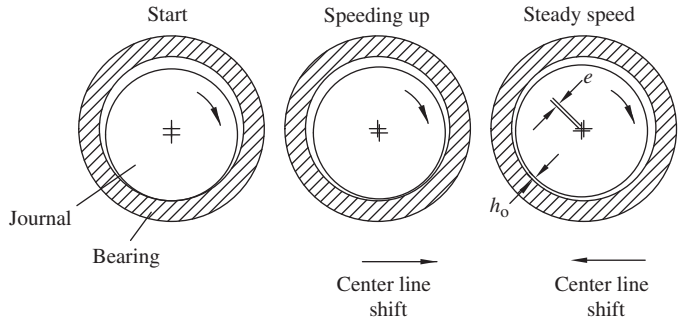


Figure 5.25
Full-film hydrodynamic bearing motion from start-up.

of the SAE and ISO defined lubricants and it can be seen that the value of the viscosity for a particular lubricant is highly dependent on the temperature. Some of the lubricant entrained into the bearing film emerges as side flow which carries away some of the heat. The remainder flows through the load-bearing zone and carries away the remainder of the heat generated. The temperature used for determining the viscosity can be taken as the average of the inlet and exit lubricant temperatures as given by Eqn (5.111).

$$T_{\text{av}} = T_1 + \frac{T_2 - T_1}{2} = T_1 + \frac{\Delta T}{2} \tag{5.111}$$

where T_1 is the temperature of the lubricant supply and T_2 is the temperature of the lubricant leaving the bearing. Generally, petroleum lubrication oils should be limited to a maximum temperature of approximately 70 °C in order to prevent excessive oxidation.

One of the parameters that needs to be determined is the bearing lubricant exit temperature T_2. This is a trial and error or iterative process. A value of the temperature rise, ΔT, is guessed; the viscosity for a standard oil, corresponding to this value, is determined; and the analysis performed. If the temperature rise calculated by the analysis does not correspond closely to the guessed value, the process should be repeated using an updated value for the temperature rise until the two match. If the temperature rise is unacceptable it may be necessary to try a different lubricant or modify the bearing configuration.

Given the length, the journal radius, the radial clearance, the lubricant type, and its supply temperature, the steps for determining the various bearing operating parameters are listed below.

1. Determine the speed of the journal and the load to be supported.
2. If L and D are not already determined, set the proportions of the bearing so that the load capacity, $P = W/LD$, is somewhere between 0.34 MN/m^2 for light machinery and 13.4 MN/m^2 for heavy machinery.
3. Determine a value for the radial clearance of the bearing using the data presented in Figure 5.13.

4. If not already specified, select a lubricant. Lubricant oil selection is a function of speed or compatibility with other lubricant requirements. Generally as the design speed rises, oils with a lower viscosity should be selected.
5. Estimate a value for the temperature rise ΔT across the bearing. The value taken for the initial estimate is relatively unimportant. As a guide, a value of $\Delta T = 10\ ^\circ\text{C}$ is generally a good starting guess. This value can be increased for high-speed bearings and for low bearing clearances.
6. Determine the average lubricant temperature $T_{\text{av}} = T_1 + \Delta T/2$ and find the corresponding value for the viscosity for the chosen lubricant.
7. Calculate the Sommerfield number, $S = (r/c)^2 \mu N_s/P$, and the length-to-diameter ratio.
8. Use the charts (Figures 5.18–5.20) to determine values for the coefficient of friction variable, the total lubricant flow variable, and the ratio of the side flow to the total lubricant flow with the values for the Sommerfield number and the L/D ratio.
9. Calculate the temperature rise of the lubricant through the bearing using

$$\Delta T = \frac{8.30 \times 10^{-6} P}{1 - \frac{1}{2}(Q_s/Q)} \times \frac{(r/c)f}{Q/rcN_sL} \tag{5.112}$$

10. If this calculated value does not match the estimated value for ΔT to within say 1 °C, repeat the procedure from step (6) using the updated value of the temperature rise to determine the average lubricant temperature.
11. Check that the values for the Sommerfield number and the length-to-diameter ratio give a design that is in the optimal operating region for minimal friction and maximum load capability on the chart for the minimum film thickness variable (Figure 5.17). If the operating Sommerfield number and L/D ratio combination do not fall within this zone, then it is likely that the bearing design can be improved by altering the values for c, L, D, the lubricant type, and the operating temperature as appropriate.
12. If the value for the temperature rise across the bearing has converged, values for the total lubricant flow rate, Q, the side flow rate Q_s, and the coefficient of friction f can be calculated. The journal bearing must be supplied with the value of the total lubricant calculated in order for it to perform as predicted by the charts.
13. The charts given in Figures 5.17–5.24 can be used to determine values for the maximum film thickness, the maximum film pressure ratio, the location of the maximum film pressure, and its terminating angular location as required.
14. The torque required to overcome friction in the bearing can be calculated by

$$\text{Torque} = fWr \tag{5.113}$$

15. The power lost in the bearing is given by

$$\text{Power} = \omega \times \text{Torque} = 2\pi N_s \times \text{Torque} \tag{5.114}$$

where ω is the angular velocity (rad/s).

16. Specify the surface roughness for the journal and bearing surfaces. A ground journal with an arithmetic average surface roughness of 0.4—0.8 μm is recommended for bearings of good quality. For high-precision equipment, both surfaces can be lapped or polished to give a surface roughness of 0.2—0.4 μm. The specified roughness should be less than the minimum film thickness.
17. Design the recirculation and sealing system for the bearing lubricant.

For values of the length to diameter (L/D) ratio other than those shown in the charts given in Figures 5.17—5.24, values for the various parameters can be found by interpolation using Eqn (5.115) (see Raimondi and Boyd (1958b)).

$$y = \frac{1}{(L/D)^3}\left[-\frac{1}{8}\left(1-\frac{L}{D}\right)\left(1-2\frac{L}{D}\right)\left(1-4\frac{L}{D}\right)y_\infty + \frac{1}{3}\left(1-2\frac{L}{D}\right)\left(1-4\frac{L}{D}\right)y_1 - \frac{1}{4}\left(1-\frac{L}{D}\right)\left(1-4\frac{L}{D}\right)y_{1/2} + \frac{1}{24}\left(1-\frac{L}{D}\right)\left(1-2\frac{L}{D}\right)y_{1/4}\right] \tag{5.115}$$

Here y is the desired variable and y_∞, y_1, $y_{1/2}$, and $y_{1/4}$ are the variable at $L/D = \infty$, 1, 1/2, and 1/4, respectively.

Example 5.3

A full journal bearing has a nominal diameter of 50.0 mm and a bearing length of 25.0 mm (see Figure 5.26). The bearing supports a load of 3000 N and the journal design speed is 3000 rpm. The radial clearance has been specified as 0.04 mm. An SAE 10 oil has been chosen and the lubricant supply temperature is 50 °C.

Find the temperature rise of the lubricant, the lubricant flow rate, the minimum film thickness, the torque required to overcome friction, and the heat generated in the bearing.

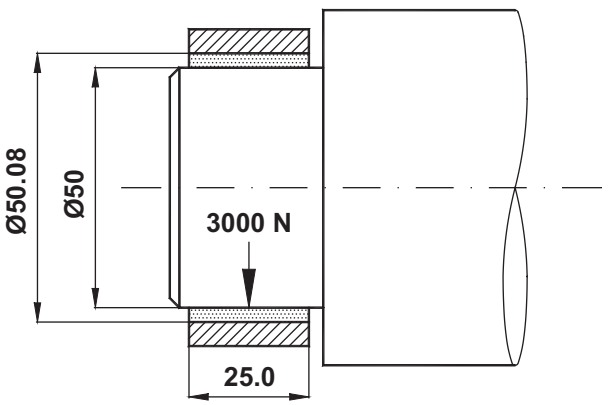


Figure 5.26
Bearing design example.

Solution

The primary data are $D = 50.0$ mm, $L = 25$ mm, $W = 3000$ N, $N = 3000$ rpm, $c = 0.04$ mm, SAE 10, and $T_1 = 50\ ^\circ$C.

Guess a value for the lubricant temperature rise ΔT across the bearing to be, say, $\Delta T = 20\ ^\circ$C.

$$T_{\text{av}} = T_1 + \frac{\Delta T}{2} = 50 + \frac{20}{2} = 60\ ^\circ\text{C}$$

From Figure 5.7 for SAE 10 at 60 °C, $\mu = 0.014$ Pa s.

The mean radius is given by

$$r_m = \frac{a+b}{2} = \frac{25 + 25.04}{2} = 25.02 \text{ mm}$$

$$b - a = 0.04 \text{ mm}$$

Assuming a density of 900 kg/m^3,

$$\nu = \mu/\rho = 0.014/900 = 1.55 \times 10^{-5}\ \text{m}^2/\text{s}$$

So the Taylor number using this approximate value for the kinematic viscosity is given by

$$\text{Ta}_m = \frac{\Omega r_m^{0.5}(b-a)^{1.5}}{\nu} = 0.81$$

As this is much less than the critical Taylor number, 41.19, laminar flow can be assumed.

$$N_s = 3000/60 = 50 \text{ rps}, \quad L/D = 25/50 = 0.5.$$

$$P = \frac{W}{LD} = \frac{3000}{0.025 \times 0.05} = 2.4 \times 10^6\ \text{N/m}^2$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N_s}{P} = \left(\frac{25 \times 10^{-3}}{0.04 \times 10^{-3}}\right)^2 \frac{0.014 \times 50}{2.4 \times 10^6} = 0.1139$$

from Figure 5.18 with $S = 0.1139$ and $L/D = 0.5$, $(r/c)f = 3.8$, from Figure 5.19 with $S = 0.1139$ and $L/D = 0.5$, $Q/(rcN_sL) = 5.34$, from Figure 5.20 with $S = 0.1139$ and $L/D = 0.5$, $Q_s/Q = 0.852$.

The value of the temperature rise of the lubricant can now be calculated using Eqn (5.112):

$$\Delta T = \frac{8.3 \times 10^{-6} P}{1 - 0.5\frac{Q_s}{Q}} \times \frac{\frac{r}{c}f}{\frac{Q}{rcN_sL}} = \frac{8.3 \times 10^{-6} \times 2.4 \times 10^6}{1 - (0.5 \times 0.852)} \times \frac{3.8}{5.34} = 24.70\ ^\circ\text{C}.$$

As the value calculated for the lubricant temperature rise is significantly different from the estimated value, it is necessary to repeat the above calculation but using the new improved estimate for determining the average lubricant temperature.

Using $\Delta T = 24.70\ ^\circ\text{C}$ to calculate T_{av} gives

$$T_{\text{av}} = 50 + \frac{24.70}{2} = 62.35\ ^\circ\text{C}$$

Repeating the procedure using the new value for T_{av} gives

$$\mu = 0.0136\ \text{Pa s},$$

$$S = 0.1107,$$

from Figure 5.18 with $S = 0.1107$ and $L/D = 0.5$, $(r/c)f = 3.7$, from Figure 5.19 with $S = 0.1107$ and $L/D = 0.5$, $Q/(rcN_sL) = 5.35$, from Figure 5.20 with $S = 0.1107$ and $L/D = 0.5$, $Q_s/Q = 0.856$,

$$\Delta T = \frac{8.3 \times 10^{-6} \times 2.4 \times 10^{6}}{1 - (0.5 \times 0.856)} \times \frac{3.7}{5.35} = 24.08\ ^\circ\text{C},$$

$$T_{\text{av}} = 50 + \frac{24.08}{2} = 62.04\ ^\circ\text{C}.$$

This value for T_{av} is close to the previous calculated value suggesting that the solution has converged. For $T_{\text{av}} = 62.04\ ^\circ\text{C}$, $\mu = 0.0136$ Pa s and $S = 0.1107$.

The other parameters can now be found.

$$Q = rcN_sL \times 5.35 = 25 \times 0.04 \times 50 \times 25 \times 5.35 = 6688\ \text{mm}^3/\text{s}.$$

From Figure 5.17, $h_o/c = 0.22$, $h_o = 0.0088$ mm.

$$f = 3.7 \times (c/r) = 3.7 \times (0.04/25) = 0.00592.$$

The torque is given by $\text{Torque} = fWr = 0.00592 \times 3000 \times 0.025 = 0.444$ N m.

The power dissipated in the bearing is given by $\text{Power} = 2\pi \times \text{Torque} \times N_s = 139.5$ W.

Example 5.4

A full journal bearing is required for a 30-mm-diameter shaft, rotating at 1200 rpm supporting a load of 1500 N (see Figure 5.27). From previous experience, the lubrication system available is capable of delivering the lubricant to the bearing at 40 °C. Select an appropriate radial clearance, length, and lubricant for the bearing and determine the temperature rise of the lubricant, the total lubricant flow rate required, and the power absorbed in the bearing. Check that your chosen design operates within the optimum zone indicated by the dotted lines on the minimum film thickness ratio chart (Figure 5.17).

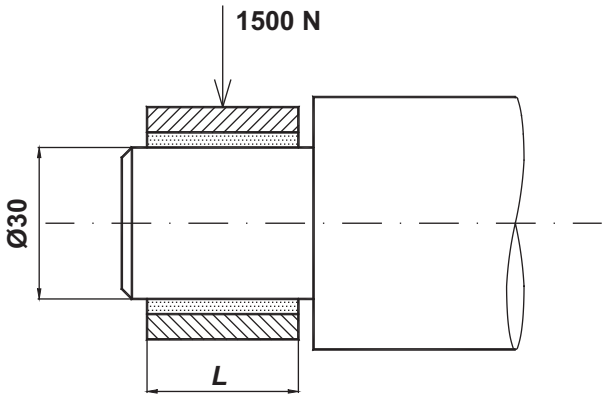


Figure 5.27
Bearing design example.

Solution

The primary design information is $D = 30.0$ mm, $W = 1500$ N, $N = 1200$ rpm, SAE 10, $T_1 = 40\ ^\circ$C.

From Figure 5.13 for a speed of 1200 rpm and a nominal journal diameter of 30 mm, a suitable value for the diametral clearance is 0.05 mm. The radial clearance is therefore 0.025 mm. The next step is to set the length of the bearing. Typical values for L/D ratios are between 0.5 and 1.5. Here a value for the ratio is arbitrarily set as $L/D = 1$, i.e., $L = 30$ mm.

Given the speed of the bearing, 1200 rpm, as an initial proposal an SAE 10 lubricant is selected.

The procedure for determining the average lubricant temperature is the same as for the previous example. As a first estimate, the temperature rise of the lubricant is taken as 10 °C and the results of the iterative procedure are given in Table 5.3.

$$P = \frac{W}{LD} = \frac{1500}{0.03 \times 0.03} = 1.667 \times 10^6\ \text{N/m}^2.$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N_s}{P} = \left(\frac{15 \times 10^{-3}}{0.025 \times 10^{-3}}\right)^2 \frac{\mu 20}{1.667 \times 10^6} = 4.319\ \mu.$$

$$T_{\text{av}} = T_1 + \frac{\Delta T}{2} = 40 + \frac{10}{2} = 45\ ^\circ\text{C}.$$

The mean radius is given by

$$r_m = \frac{a+b}{2} = \frac{15 + 15.025}{2} = 15.01\ \text{mm}$$

$$b - a = 0.025\ \text{mm}$$

Table 5.3: Tabular data for Example 5.4.

Temperature rise, ΔT (°C)	10	15.90	13.76	13.76
Average lubricant temperature, T_{av} (°C)	45	47.95	46.88	46.88
Average lubricant viscosity, μ (Pa s)	0.028	0.023	0.024	Converged
Sommerfield number, S	0.1209	0.09934	0.1037	
Coefficient of friction variable, $(r/c)f$	3.3	2.8	2.8	
Flow variable, Q/rcN_sL	4.35	4.4	4.4	
Side flow to total flow ratio, Q_s/Q	0.68	0.72	0.72	

An estimate for the kinematic viscosity can be found, taking the average temperature of the oil as 45 °C and assuming a density of 900 kg/m^3. At 45 °C, the viscosity for SAE 10 is approximately 0.028 Pa s

$$\nu = \mu/\rho = 0.028/900 = 3.11 \times 10^{-5}\,\mathrm{m}^2/\mathrm{s}$$

So the Taylor number using this approximate value for the kinematic viscosity is given by

$$\mathrm{Ta}_m = \frac{\Omega r_m^{0.5}(b-a)^{1.5}}{\nu} = 0.062$$

As this is much less than the critical Taylor number, 41.19, laminar flow can be assumed.

The total lubricant flow rate required is given by

$$Q = rcN_sL \times 4.4 = 15 \times 0.025 \times 20 \times 30 \times 4.4 = 990\ \mathrm{mm}^3/\mathrm{s}.$$

With $S = 0.1037$ and $L/D = 1$, the design selected is within the optimum operating zone for minimum friction and optimum load capacity indicated in Figure 5.17.

The friction factor, $f = 2.8 \times (c/r) = 2.8 \times 0.025/15 = 0.004\ 667$.

Torque $= fWr = 0.004667 \times 1500 \times 0.015 = 0.105$ N m.

Power $= 2\pi N_s \times$ Torque $= 13.19$ W.

Example 5.5

A full journal bearing has a nominal diameter of 25.0 mm and bearing length 25.0 mm. The bearing supports a load of 1450 N and the journal design speed is 2500 rpm. Two radial

clearances are proposed: (1) 0.025 mm, (2) 0.015 mm. An SAE 10 oil has been chosen and the lubricant supply temperature is 50 °C. Using the standard Raimondi and Boyd design charts, find the temperature rise of the lubricant, the lubricant flow rate, the torque required to overcome friction, and the heat generated in the bearing for both of the proposed clearances. Determine which clearance, if any, should be selected for the bearing.

Solution

$D = 25$ mm, $L = 25$ mm, $W = 1450$ N, $N = 2500$ rpm, $N_s = 2500/60$ rps $= 41.67$ rps, SAE 10, $T_1 = 50$ °C. $L/D = 1$.

$$P = \frac{W}{LD} = \frac{1450}{0.025 \times 0.025} = 2.32 \times 10^6 \text{ N/m}^2.$$

1. $c = 0.025$ mm.

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N_s}{P} = \left(\frac{12.5 \times 10^{-3}}{0.025 \times 10^{-3}}\right)^2 \frac{\mu 41.67}{2.32 \times 10^6} = 4.49\ \mu.$$

The mean radius is given by

$$r_m = \frac{a+b}{2} = \frac{12.5 + 12.525}{2} = 12.51 \text{ mm}$$

$$b - a = 0.025 \text{ mm}$$

An estimate for the kinematic viscosity can be found, taking the average temperature of the oil as 55 °C and assuming a density of 900 kg/m^3. At 55 °C, the viscosity for SAE 10 is approximately 0.017 Pa s

$$\nu = \mu/\rho = 0.017/900 = 1.89 \times 10^{-5} \text{ m}^2/\text{s}$$

So the Taylor number using this approximate value for the kinematic viscosity is given by

$$\text{Ta}_m = \frac{\Omega r_m^{0.5} (b-a)^{1.5}}{\nu} = 0.19$$

As this is much less than the critical Taylor number, 41.19, laminar flow can be assumed. Reducing the clearance results in a smaller Taylor number so laminar flow can also be assumed for the smaller clearance.

The solution for the iterative procedure using the Raymondi and Boyd charts (Figures 5.18−5.20) to find the average temperature rise of the lubricant across the bearing is given in tabular form below. The first estimate for the temperature rise is taken as $\Delta T = 10$ °C.

Temperature rise, ΔT (°C)	10	16.13	15.36
Average lubricant temperature, T_{av} (°C)	55	58.07	57.68
Average lubricant viscosity, μ (Pa s)	0.017	0.015	Converged
Sommerfield number, S	0.07633	0.06735	
Coefficient of friction variable, $(r/c)f$	2.3	2.2	
Flow variable, Q/rcN_sL	4.45	4.52	
Side flow to total flow ratio, Q_s/Q	0.766	0.78	

$\Delta T \approx 15.4$ °C.
$Q = rcN_sL \times 4.52 = 12.5 \times 0.025 \times 41.67 \times 25 \times 4.52 = 1471$ mm^3/s.
$f = 2.2 \times (c/r) = 2.2 \times 0.025/12.5 = 0.0044$.
$T = fWr = 0.0044 \times 1450 \times 0.0125 = 0.07975$ N m.
Heat generated $= 2\pi N_s \times$ Torque $= 20.88$ W.

With $S = 0.06735$ and $L/D = 1$, the design selected is outside the optimum operating zone for minimum friction and optimum load capacity indicated in Figure 5.17.

2. $c = 0.015$ mm.

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N_s}{P} = \left(\frac{12.5 \times 10^{-3}}{0.015 \times 10^{-3}}\right)^2 \frac{\mu 41.67}{2.32 \times 10^6} = 12.47\,\mu.$$

Assuming $\Delta T = 10$ °C.

Temperature rise, ΔT (°C)	10	27.01	25.17	25.85
Average lubricant temperature, T_{av} (°C)	55	63.51	62.58	62.93
Average lubricant viscosity, μ (Pa s)	0.017	0.0125	0.013	Converged
Sommerfield number, S	0.2120	0.1559	0.1621	
Coefficient of friction variable, $(r/c)f$	4.8	3.8	3.9	
Flow variable, Q/rcN_sL	4.08	4.22	4.21	
Side flow to total flow ratio, Q_s/Q	0.55	0.622	0.62	

$\Delta T \approx 25.9$ °C.
$Q = rcN_sL \times 4.21 = 12.5 \times 0.015 \times 41.67 \times 25 \times 4.21 = 822.3$ mm^3/s.
$f = 3.9 \times (c/r) = 3.9 \times 0.015/12.5 = 0.00468$.
$T = fWr = 0.00468 \times 1450 \times 0.0125 = 0.08{,}483$ N m.
Heat generated $= 2\pi N_s \times$ Torque $= 22.21$ W.

With $S = 0.1621$ and $L/D = 1$ the design selected is inside the optimum operating zone for minimum friction and optimum load capacity indicated in Figure 5.17.

Therefore this clearance ($c = 0.015$ mm) should be selected.

Example 5.6

A full-film hydrodynamic journal bearing is required for a shaft supporting a radial load of 7500 N while rotating at 800 rpm. The nominal diameter of the shaft is 80 mm. Select a suitable lubricant and appropriate values for the radial clearance and bearing length. Determine the overall temperature rise in the bearing and the flow rate of lubricant required. Establish that the selected design lies within the optimum zone for minimum friction and maximum load on the minimum film thickness ratio chart. The lubricant can be supplied at an inlet temperature of 50 °C using an available cooling system.

Solution

From Figure 5.13 for $N = 800$ rpm and $D = 80$ mm, $2c = 80$ μm.

Hence $c = 40$ μm.

$$13.4 > P > 0.34 \text{ MPa}$$

$$P = \frac{W}{LD} = \frac{7500}{0.08L}$$

Try $L/D = 0.5$

$$P = \frac{W}{LD} = \frac{7500}{0.08 \times 0.04} = 2.34 \text{ MPa}$$

Try SAE 20. As $N_s \Downarrow$ SAE number $\Uparrow$

The mean radius is given by

$$r_m = \frac{a+b}{2} = \frac{40 + 40.04}{2} = 40.02 \text{ mm}$$

$$b - a = 0.04 \text{ mm}$$

An estimate for the kinematic viscosity can be found, taking the average temperature of the oil as 65 °C and assuming a density of 900 kg/m^3. At 55 °C, the viscosity for SAE 20 is approximately 0.017 Pa s

$$\nu = \mu/\rho = 0.017/900 = 1.89 \times 10^{-5} \text{ m}^2/\text{s}$$

So the Taylor number using this approximate value for the kinematic viscosity is given by

$$\text{Ta}_m = \frac{\Omega r_m^{0.5} (b-a)^{1.5}}{\nu} = 0.22$$

As this is much less than the critical Taylor number, 41.19, laminar flow can be assumed.

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N_s}{P} = \left(\frac{40 \times 10^{-3}}{0.04 \times 10^{-3}}\right)^2 \frac{\mu 13.33}{2.34 \times 10^6} = 5.69\,\mu$$

Temperature rise, ΔT (°C)	30	21.6	23.2
Average lubricant temperature, T_{av} (°C)	65	60.8	61.66
Average lubricant viscosity, μ (Pa s)	0.017	0.02	0.02
Sommerfield number, S	0.0967	0.114	Converged
Coefficient of friction variable, $(r/c)f$	3.4	3.7	
Flow variable, Q/rcN_sL	5.4	5.35	
Side flow to total flow ratio, Q_s/Q	0.866	0.84	

With $S = 0.114$ and $L/D = 0.5$, the design is well within the optimum zone for minimum friction and maximum load.

$$Q = rcN_sL \times 5.35 = 40 \times 0.04 \times 13.33 \times 40 \times 5.35 = 4565\ \text{mm}^3/\text{s}$$

Example 5.7

A design proposal for a full-film hydrodynamic bearing comprises a bearing of nominal diameter 30 mm, length 15 mm using ISO VG 22 lubricant supplied at 50 °C. The bearing load is 1280 N and the rotational speed is 6000 rpm. The radial clearance for the bearing is 0.022 mm. Use the viscosity chart given in Figure 5.7 and the Raimondi and Boyd design charts given in Figures 5.17–5.20 to determine the lubricant flow rate, lubricant temperature rise, and minimum film thickness. The initial guess for the temperature rise across the bearing can be taken as 30 °C.

If in place of ISO VG 22 oil, SAE 10 oil was used in the bearing, determine the resulting temperature rise of the lubricant.

Comment on the suitability of the proposed design and justify the selection of either ISO VG 22 or SAE 10 as a lubricant.

Solution

Initial data: $L = 15$ mm, $D = 30$ mm, $L/D = 0.5$

$N = 6000$ rpm, $N_s = 100$ rps

$c = 0.022$ mm

ISO VG 22

$T_1 = 60$ °C

$\Delta T = 30\ ^\circ\text{C}$

$W = 1280\ \text{N}$

$$T_{\text{av}} = 50 + \frac{30}{2} = 65\ ^\circ\text{C}$$

The mean radius is given by

$$r_m = \frac{a+b}{2} = \frac{15 + 15.022}{2} = 15.01\ \text{mm}$$

$$b - a = 0.022\ \text{mm}$$

An estimate for the kinematic viscosity can be found, taking the average temperature of the oil as 65 °C and assuming a density of 900 kg/m^3. At 55 °C, the viscosity for ISO VG 22 is approximately 0.0074 Pa s

$$\nu = \mu/\rho = 0.0074/900 = 8.22 \times 10^{-6}\ \text{m}^2/\text{s}$$

So the Taylor number using this approximate value for the kinematic viscosity is given by

$$\text{Ta}_m = \frac{\Omega r_m^{0.5}(b-a)^{1.5}}{\nu} = 0.97$$

As this is much less than the critical Taylor number, 41.19, laminar flow can be assumed.

$$P = \frac{W}{LD} = \frac{1280}{0.015 \times 0.03} = 2.844 \times 10^6\ \text{Pa}$$

Temperature rise, ΔT (°C)	30	30.8
Average lubricant temperature, T_{av} (°C)	65	65.4
Average lubricant viscosity, μ (Pa s)	0.0074	Converged
Sommerfield number, S	0.12	
Coefficient of friction variable, $(r/c)f$	4	
Flow variable, Q/rcN_sL	5.3	
Side flow to total flow ratio, Q_s/Q	0.842	

$\Delta T = 31\ ^\circ\text{C}$.
$Q = rcN_sL \times 5.3 = 15 \times 0.022 \times 100 \times 15 \times 5.3 = 2624\ \text{mm}^3/\text{s}$.
$h_o/c = 0.24$.
$h_o = 0.24 \times 0.022\ \text{mm} = 5.28 \times 10^{-3}\ \text{mm}$.
$h_o = 5.3\ \mu\text{m}$.

From the graph for h_o/c versus the Sommerfield number, the design is in the optimum zone for maximum load and minimum friction.

For SAE 10:

Temperature rise, ΔT (°C)	30	43.2	36.9	38.6
Average lubricant temperature, T_{av} (°C)	65	71.6	68.5	69.3
Average lubricant viscosity, μ (Pa s)	0.0116	0.0092	0.0102	0.01
Sommerfield number, S	0.19	0.15	0.167	Converged
Coefficient of friction variable, $(r/c)f$	5.6	4.8	5	
Flow variable, Q/rcN_sL	5.1	5.2	5.15	
Side flow to total flow ratio, Q_s/Q	0.8	0.82	0.812	

$\Delta T \approx 38.6$ °C.

With $S = 0.167$ and $L/D = 0.5$, the design also falls within the optimum zone for maximum load and minimum friction.

Use ISO VG 22 as the lubricant temperature rise is lower unless other components or considerations dictate the use of SAE 10.

5.4.3 Alternative Method for the Design of Full-Film Hydrodynamic Bearings

An alternative method to using the design charts of Raimondi and Boyd is to use an approximate method developed by Reason and Narang (1982). They defined an approximation for the pressure variation in a journal bearing given by

$$p = \frac{\frac{3\mu V}{rc^2}\left[\frac{L^2}{4} - z^2\right]\left\{\frac{\varepsilon \sin\theta}{(1+\varepsilon\cos\theta)^3}\right\}}{1 + \frac{2+\varepsilon^2}{2r^2}\left[\frac{L^2}{4} - z^2\right]\left\{\frac{1}{(1+\varepsilon\cos\theta)(2+\varepsilon\cos\theta)}\right\}} \tag{5.116}$$

where μ = viscosity (Pa s), V = surface velocity (m/s), r = journal radius (m), c = radial clearance (m), L = bearing length (m), z = axial coordinate (m), ε = eccentricity variable, and θ = circumferential coordinate.

This equation was integrated using an approximate method and the solution given in terms of two integrals I_s and I_c, the sine and cosine load integrals, respectively. Both I_s and I_c are functions of the L/D ratio and the eccentricity ratio ε. The values of these integrals are given in Table 5.4 along with the corresponding Sommerfield number calculated using Eqn (5.117).

$$S = \frac{1}{6\pi\sqrt{I_c^2 + I_s^2}}. \tag{5.117}$$

Table 5.4: Values of the sine and cosine integrals, I_s and I_c, and the Sommerfield number S, as functions of the L/D ratio and the eccentricity ratio ε.

ε	$L/D = 0.25$	$L/D = 0.5$	$L/D = 0.75$	$L/D = 1.0$	$L/D = 1.5$	$L/D = 2$	$L/D = \infty$
0.10	$I_s = 0.0032$	0.0120	0.0244	0.0380	0.0636	0.0839	0.1570
	$I_c = -0.0004$	−0.0014	−0.0028	−0.0041	−0.0063	−0.0076	−0.0100
	$S = 16.4506$	4.3912	2.1601	1.3880	0.8301	0.6297	0.3372
0.20	$I_s = 0.0067$	0.0251	0.0505	0.0783	0.1300	0.1705	0.3143
	$I_c = -0.0017$	−0.0062	−0.0118	−0.0174	−0.0259	−0.0312	−0.0408
	$S = 7.6750$	2.0519	1.0230	0.6614	0.4002	0.3061	0.1674
0.30	$I_s = 0.0109$	0.0404	0.0804	0.1236	0.2023	0.2628	0.4727
	$I_c = -0.0043$	−0.0153	−0.0289	−0.0419	−0.0615	−0.0733	−0.0946
	$S = 4.5276$	1.2280	0.6209	0.4065	0.2509	0.1944	0.1100
0.40	$I_s = 0.0164$	0.0597	0.1172	0.1776	0.2847	0.3649	0.6347
	$I_c = -0.0089$	−0.0312	−0.0579	−0.0825	−0.1183	−0.1391	−0.1763
	$S = 2.8432$	0.7876	0.4058	0.2709	0.1721	0.1359	0.0850
0.50	$I_s = 0.0241$	0.0862	0.1656	0.2462	0.3835	0.4831	0.8061
	$I_c = -0.0174$	−0.0591	−0.1065	−0.1484	−0.2065	−0.2391	−0.2962
	$S = 1.7848$	0.5076	0.2694	0.1845	0.1218	0.0984	0.0618
0.60	$I_s = 0.0363$	0.1259	0.2345	0.3396	0.5102	0.6291	0.9983
	$I_c = -0.0338$	−0.1105	−0.1917	−0.2590	−0.3474	−0.3949	−0.4766
	$S = 1.0696$	0.3167	0.1752	0.1242	0.0859	0.0714	0.0480
0.70	$I_s = 0.0582$	0.1927	0.3430	0.4793	0.6878	0.8266	1.2366
	$I_c = -0.0703$	−0.2161	−0.3549	−0.4612	−0.5916	−0.6586	−0.7717
	$S = 0.5813$	0.1832	0.1075	0.0798	0.0585	0.0502	0.0364
0.80	$I_s = 0.1071$	0.3264	0.5425	0.7220	0.9771	1.1380	1.5866
	$I_c = -0.1732$	−0.4797	−0.7283	−0.8997	−1.0941	−1.1891	−1.3467
	$S = 0.2605$	0.0914	0.0584	0.0460	0.0362	0.0322	0.0255
0.90	$I_s = 0.2761$	0.7079	1.0499	1.3002	1.6235	1.8137	2.3083
	$I_c = -0.6644$	−1.4990	−2.0172	−2.3269	−2.6461	−2.7932	−3.0339
	$S = 0.0737$	0.0320	0.0233	0.0199	0.0171	0.0159	0.0139
0.95	$I_s = 0.6429$	1.3712	1.8467	2.1632	2.5455	2.7600	3.2913
	$I_c = -2.1625$	−3.9787	−4.8773	−5.3621	−5.8315	−6.0396	−6.3776
	$S = 0.0235$	0.0126	0.0102	0.0092	0.0083	0.0080	0.0074
0.99	$I_s = 3.3140$	4.9224	5.6905	6.1373	6.6295	6.8881	8.7210
	$I_c = -22.0703$	−28.5960	−30.8608	−31.9219	−32.8642	−33.2602	−33.5520
	$S = 0.0024$	0.0018	0.0017	0.0016	0.0016	0.0016	0.0015

Reason and Narang (1982).

Knowing values for the integrals I_s and I_c enables the bearing parameters to be calculated as shown below.

- Given: W, N_s, L, D, c, the type of lubricant, and T_1.
- Guess a value for the temperature rise of the lubricant ΔT and calculate T_{av}. Determine the viscosity at T_{av} from Figure 5.7.
- Calculate the bearing loading pressure, $P = \frac{W}{LD}$.
- Calculate the Sommerfield number, $S = \left(\frac{r}{c}\right)^2 \frac{\mu N_s}{P}$.
- Knowing the length-to-diameter ratio and the Sommerfield number, search the correct L/D column in Table 5.4 for a matching Sommerfield number. A matching Sommerfield

number will give the eccentricity ratio reading across the table to the left. Interpolation may be necessary for values of S between the values listed in the table. Once the correct eccentricity has been found, determine the values for the sine and cosine integrals I_s and I_c to correspond with the value of the Sommerfield number.

Recall that for a linear function, $y = f(x)$, the value for y between the bounding points (x_1, y_1) and (x_2, y_2) for a given value of x can be found by linear interpolation:

$$y = \left(\frac{x - x_1}{x_2 - x_1}\right)(y_2 - y_1) + y_1 \tag{5.118}$$

- Calculate the minimum film thickness, $h_o = c(1 - \varepsilon)$.
- Calculate the attitude angle ϕ, (°)

$$\phi = \tan^{-1}\left(-\frac{I_s}{I_c}\right). \tag{5.119}$$

- Calculate the flow variable $Q/(rcN_sL)$, where Q is the lubricant supply.

$$\frac{Q}{rcN_sL} = \pi(1 + \varepsilon) - \pi\varepsilon\eta_1\left[1 - \eta_1\left(\frac{D}{L}\right)^2 \frac{1}{\sqrt{\left(\frac{D}{L}\right)^2\frac{\eta_1}{2} + \frac{1}{4}}} \tanh^{-1}\left(\frac{1}{2\sqrt{\left(\frac{D}{L}\right)^2\frac{\eta_1}{2} + \frac{1}{4}}}\right)\right] \tag{5.120}$$

where

$$\eta_1 = \frac{(1 + \varepsilon)(2 + \varepsilon)}{(2 + \varepsilon^2)} \tag{5.121}$$

- Calculate the flow through the minimum film thickness Q_π.

$$\frac{Q_\pi}{rcN_sL} = \pi(1 - \varepsilon) + \pi\varepsilon\eta_2\left[1 - \eta_2\left(\frac{D}{L}\right)^2 \frac{1}{\sqrt{\left(\frac{D}{L}\right)^2\frac{\eta_2}{2} + \frac{1}{4}}} \tanh^{-1}\left(\frac{1}{2\sqrt{\left(\frac{D}{L}\right)^2\frac{\eta_2}{2} + \frac{1}{4}}}\right)\right] \tag{5.122}$$

where

$$\eta_2 = \frac{(1 - \varepsilon)(2 - \varepsilon)}{(2 + \varepsilon^2)} \tag{5.123}$$

Note that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right) \tag{5.124}$$

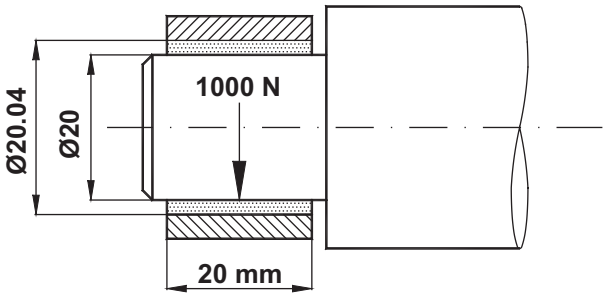


Figure 5.28
Bearing design example.

- The flow ratio can be calculated using Eqn (5.125).

$$\frac{Q_s}{Q} = 1 - \frac{Q_\pi}{Q} \tag{5.125}$$

- Calculate the friction variable $(r/c)f$

$$f\left(\frac{r}{c}\right) = 6\pi S\left\{\frac{\pi}{3\sqrt{(1-\varepsilon^2)}} + \frac{\varepsilon}{2}I_s\right\} \tag{5.126}$$

- Calculate the temperature rise,

$$\Delta T = \frac{8.30 \times 10^{-6}P}{1 - \frac{1}{2}(Q_s/Q)} \times \frac{(r/c)f}{Q/rcN_sL}$$

Although these equations may appear complex, the advantage of the method developed by Reason and Narang is that it is suitable for conventional programming or utilization within a spreadsheet.

Example 5.8

A full journal bearing has a nominal diameter of 20.0 mm and a bearing length of 20.0 mm (see Figure 5.28). The bearing supports a load of 1000 N, and the journal design speed is 6000 rpm. The radial clearance has been specified as 0.02 mm. An SAE 20 oil has been chosen to be compatible with requirements elsewhere in the machine and the lubricant supply temperature is 60 °C. Using the approximate method of Reason and Narang, find the temperature rise of the lubricant, the total lubricant flow rate, the minimum film thickness, the torque required to overcome friction, and the heat generated in the bearing.

Solution

The primary design data are $D = 20$ mm, $L = 20$ mm, $W = 1000$ N, $N = 6000$ rpm, $c = 0.02$ mm, SAE 20, and $T_1 = 60\ ^\circ$C.

Guess $\Delta T = 15\ ^\circ$C. Hence:

$$T_{\text{av}} = T_1 + \frac{\Delta T}{2} = 60 + \frac{15}{2} = 67.5\ ^\circ\text{C}.$$

For SAE 20, at 67.5 °C, from Figure 5.7, $\mu = 0.016$ Pa s.

The mean radius is given by

$$r_m = \frac{a+b}{2} = \frac{20 + 20.02}{2} = 20.01 \text{ mm}$$

$$b - a = 0.02 \text{ mm}$$

Assuming a density of 900 kg/m^3.

$$\nu = \mu/\rho = 0.016/900 = 1.78 \times 10^{-5}\text{m}^2/\text{s}$$

So the Taylor number using this approximate value for the kinematic viscosity is given by

$$\text{Ta}_m = \frac{\Omega r_m^{0.5}(b-a)^{1.5}}{\nu} = 0.45$$

As this is much less than the critical Taylor number, 41.19, laminar flow can be assumed.

$$P = \frac{W}{LD} = \frac{1000}{0.02 \times 0.02} = 2.5 \text{ MPa}.$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N_s}{P} = \left(\frac{0.01}{0.00002}\right)^2 \frac{0.016 \times 100}{2.5 \times 10^6} = 0.16.$$

From Table 5.4 for $L/D = 1$, the nearest bounding values of the Sommerfield number to $S = 0.16$ are for $\varepsilon = 0.5$ ($S = 0.1845$) and $\varepsilon = 0.6$ ($S = 0.1242$).

Interpolating to find the values of ε, I_s, and I_c, which correspond to $S = 0.16$:

$$\varepsilon = \frac{0.16 - 0.1845}{0.1242 - 0.1845}(0.6 - 0.5) + 0.5 = 0.5406,$$

$$I_s = \frac{0.16 - 0.1845}{0.1242 - 0.1845}(0.3396 - 0.2462) + 0.2462 = 0.2841,$$

$$I_c = \frac{0.16 - 0.1845}{0.1242 - 0.1845}(-0.2590 - -0.1484) + -0.1484 = -0.1933.$$

From Eqn (5.120) with $\eta_1 = 1.708$,

$$\frac{Q}{rcN_sL} = 4.84 - 2.901\left[1 - 1.708 \times 0.9517 \tanh^{-1}(0.4759)\right] = 4.379.$$

From Eqn (5.122) with $\eta_2 = 0.2925$,

$$\frac{Q_\pi}{rcN_sL} = 1.443 + 0.4968\left[1 - 0.2925 \times 1.589 \tanh^{-1}(0.7943)\right] = 1.690.$$

From Eqn (5.125), $Q_s/Q = 0.6141$.

From Eqn (5.126), $f\left(\frac{r}{c}\right) = 3.016(1.245 + 0.07679) = 3.987$.

$$\Delta T = \left(\frac{8.3 \times 10^{-6} P}{1 - 0.5(Q_s/Q)}\right) \frac{(r/c)f}{Q/rcN_sL} = 27.26\ ^\circ\text{C}.$$

Repeating the procedure with this value for ΔT gives:

$T_{\text{av}} = 73.6\ ^\circ$C, hence $\mu = 0.013$ Pa s,
$S = 0.13$, hence $\varepsilon = 0.5904$,
$Q/(rcN_sL) = 4.492$,
$Q_\pi/(rcN_sL) = 1.531$,
$(r/c)f = 3.418$.

Using these values gives $\Delta T = 23.56\ ^\circ$C.

Repeating the procedure with this value for ΔT gives:

$T_{\text{av}} = 71.78\ ^\circ$C, hence $\mu = 0.014$ Pa s,
$S = 0.14$, hence $\varepsilon = 0.5738$,
$Q/(rcN_sL) = 4.454$,
$Q_\pi/(rcN_sL) = 1.585$,
$(r/c)f = 3.613$.

Using these values gives $\Delta T = 24.83\ ^\circ$C. Hence $T_{\text{av}} = 72.4\ ^\circ$C. This is close enough to the previous value, indicating that the correct value for T_{av} has been determined.

The values for the total lubricant supply, minimum film thickness, coefficient of friction, torque, and the heat generated in the bearing can now be determined:

$Q = 1782\ \text{mm}^3/\text{s}$,
$h_o = c(1 - \varepsilon) = 0.009$ mm,
$f = 0.007226$,
Torque $= 0.072$ N m,
Power $= 45.4$ W.

Example 5.9

A full journal bearing has a nominal diameter of 80.0 mm and a bearing length 60.0 mm. The bearing supports a load of 7500 N, and the journal design speed is 800 rpm. The radial clearance has been specified as 0.05 mm. An SAE 30 oil has been chosen and the lubricant supply temperature is 50 °C. Using the approximate method of Reason and Narang for determining the design and performance of full-film hydrodynamic bearings, find the temperature rise of the lubricant, the lubricant flow rate, the torque required to overcome friction, the heat generated in the bearing, and the minimum film thickness.

Solution

$L/D = 0.75$, $P = 1.562$ MN/m^2.

The mean radius is given by

$$r_m = \frac{a+b}{2} = \frac{40 + 40.05}{2} = 40.03 \text{ mm}$$

$$b - a = 0.05 \text{ mm}$$

An estimate for the kinematic viscosity can be found, taking the average temperature of the oil as 60 °C and assuming a density of 900 kg/m^3. At 55 °C, the viscosity for SAE 30 is approximately 0.032 Pa s

$$\nu = \mu/\rho = 0.032/900 = 3.56 \times 10^{-5} \text{ m}^2/\text{s}$$

So the Taylor number using this approximate value for the kinematic viscosity is given by

$$\text{Ta}_m = \frac{\Omega r_m^{0.5}(b-a)^{1.5}}{\nu} = 0.17$$

As this is much less than the critical Taylor number, 41.19, laminar flow can be assumed.

Temperature rise, ΔT (°C)	20	19.16
Average lubricant temperature, T_{av} (°C)	60	59.58
Average lubricant viscosity, μ (Pa s)	0.032	Converged
Sommerfeld number, S	0.1748	
ε	0.6006	
l_s	0.2352	
l_c	−0.1928	
Flow variable, Q/rcN_sL	4.715	
Q_π/rcN_sL	1.440	
Side flow to total flow ratio, Q_s/Q	0.6946	
Coefficient of friction variable, $(r/c)f$	4.547	

$\Delta T \approx 19.2\ ^{\circ}\text{C}$.
$h_o = 0.05 \times 10^{-3}(1 - 0.6006) = 19.97\ \mu\text{m} \approx 20.0\ \mu\text{m}$.
$Q = 4.715 \times 40 \times 0.05 \times (800/60) \times 60 = 7544\ \text{mm}^3/\text{s}$.
Torque $= fWr = 4.547(0.05/40) \times 7500 \times 0.04 = 1.705$ N m.
Heat generated $= 2\pi N_s \times$ Torque $= 142.8$ W.

5.5 Conclusions

Bearings are used to support a load while allowing relative motion between two elements of a machine. Typical applications are shafts where the shaft rotates relative to a housing, but bearings can also be used to facilitate linear relative motion for, say, machine slides. This chapter has introduced the wide range of bearing types and their selection and the design of boundary-lubricated and full-film hydrodynamic journal bearings. Boundary-lubricated bearings represent an important class of bearings where the combination of low speed and load make it possible for surface-to-surface contact to occur without significant wear, and this type of bearing is widely used in plant machinery and product design. In full-film hydrodynamic bearings, rotation results in high pressures in the lubricant, which act to separate the journal from the bearing thus enabling prolonged high-speed operation.

References

Books and Papers

Brandlein, J., Eschmann, P., Hasbargen, L., Weigand, K., 1999. Ball and Roller Bearings. Theory, Design and Application. John Wiley & Sons.

Childs, P.R.N., 2011. Rotating Flow. Elsevier.

Constantinescu, V.N., 1962. Analysis of bearings operating in turbulent regime. Trans. ASME, J. Basic Eng. 84, 139–151.

Hamrock, B.J., 1994. Fundamentals of Fluid Film Lubrication. McGraw Hill.

Harris, T.A., 2001. Rolling Bearing Analysis, fourth ed. John Wiley & Sons.

Harrison, W.J., 1913. The hydrodynamical theory of lubrication with special reference to air as a lubricant. Trans. Cambridge Philos. Soc. xxii (1912–1925), 6–54.

Hindhede, U., Zimmerman, J.R., Hopkins, R.B., Erisman, R.J., Hull, W.C., Lang, J.D., 1983. Machine Design Fundamentals. A Practical Approach. Wiley.

Institution of Mechanical Engineers, 1994. Tribological Design Data. Part 1: Bearings. I.Mech.E.

Neale, M.J. (Ed.), 1995. The Tribology Handbook. Butterworth Heinemann.

Raimondi, A.A., Boyd, J., 1958a. A solution for the finite journal bearing and its application to analysis and design: I. ASLE Transactions 1, 159–174.

Raimondi, A.A., Boyd, J., 1958b. A solution for the finite journal bearing and its application to analysis and design: II. ASLE Transactions 1, 175–193.

Raimondi, A.A., Boyd, J., 1958c. A solution for the finite journal bearing and its application to analysis and design: III. ASLE Transactions 1, 194–209.

Reason, B.R., Narang, I.P., 1982. Rapid design and performance evaluation of steady state journal bearings - a technique amenable to programmable hand calculators. ASLE Transactions 25 (4), 429–444.

Reynolds, O., 1886. On the theory of lubrication and its application to Mr Beauchamp Tower's experiments including an experimental determination of the viscosity of olive oil. Philos. Trans. Royal Soc. 177, 157–234.

Welsh, R.J., 1983. Plain Bearing Design Handbook. Butterworth.

Further Reading

Harnoy, A., 2002. Bearing Design in Machinery. Marcel Dekker.

Standards

American National Standards Institute, 1990. Load Ratings and Fatigue Life for Ball Bearings. ANSI/AFBMA Standard 9−1990. ANSI.
American National Standards Institute, 1990. Load Ratings and Fatigue Life for Roller Bearings. ANSI/AFBMA Standard 11−1990. ANSI.
American National Standards Institute, 1995. Shaft and Housing Fits for Metric Radial Ball and Roller Bearings (Except Tapered Roller Bearings) Conforming to Basic Boundary Plans. ANSI/ABMA Standard 7−1995. ANSI.
British Standards Institution, 1979. BS 3134-1:1979. Metric Tapered Roller Bearings. Specification for Dimensions and Tolerances of Single Row Bearings.
British Standards Institution, 1982. BS 292: Part 1: 1982. Rolling Bearings: Ball Bearings, Cylindrical and Spherical Roller Bearings. Specification for Dimensions of Ball Bearings, Cylindrical and Spherical Roller Bearings (Metric Series).
British Standards Institution, 1984. BS 6560:1984. Glossary of Terms for Rolling Bearings.
British Standards Institution, 1991. BS5512:1991. Method of Calculating Dynamic Load Rating and Rating Life of Rolling Bearings.
British Standards Institution, 1992. BS 3134-4:1992, ISO 10317:1992. Metric tapered roller bearings. Specification for designation system.
British Standards Institution, 1992. BS 4480: Part 1: 1992. Plain Bearings Metric Series. Sintered Bushes. Dimensions and Tolerances.
British Standards Institution, 1995. BS ISO 12128:1995. Plain Bearings. Lubrication Holes, Grooves and Pockets. Dimensions, Types, Designation and Their Application to Bearing Bushes.
British Standards Institution, 1998. BS ISO 15:1998. Rolling Bearings. Radial Bearings. Boundary Dimensions, General Plan.
British Standards Institution, 1999. BS ISO 113:1999. Rolling Bearings. Plummer Block Housings. Boundary Dimensions.
British Standards Institution, 1999. BS ISO 3547-1:1999. Plain Bearings. Wrapped Bushes. Dimensions.
British Standards Institution, 1999. BS ISO 3547-3:1999. Plain Bearings. Wrapped Bushes. Lubrication Holes, Lubrication Grooves and Lubrication Indentations.
British Standards Institution, 1999. BS ISO/TS 16799:1999. Rolling Bearings. Dynamic Load Ratings and Rating Life. Discontinuities in the Calculating of Basic Dynamic Load Ratings.
British Standards Institution, 2001. BS ISO 12128:2001. Plain Bearings. Lubrication Holes, Grooves and Pockets. Dimensions, Types, Designation and Their Application to Bearing Bushes.
British Standards Institution, 2001. BS ISO 2982-2:2001. Rolling Bearings. Accessories. Locknuts and Locking Devices. Dimensions.
British Standards Institution, 2001. BS ISO 3290:2001. Rolling Bearings. Balls. Dimensions and Tolerances.
British Standards Institution, 2002. BS ISO 104:2002. Rolling Bearings. Thrust Bearings. Boundary Dimensions, General Plan.
British Standards Institution, 2002. BS ISO 492:2002. Rolling Bearings. Radial Bearings. Tolerances.
Organization for Standardization, 1992. ISO 3448 Industrial Liquid Lubricants—ISO Viscosity Classification.
International Organization for Standardization, 2001. ISO 14728−1. Rolling Bearings. Linear Motion Rolling Bearings. Part 1: Dynamic Load Ratings and Rating Life.
International Organization for Standardization, 2001. ISO 14728−2. Rolling Bearings. Linear Motion Rolling Bearings. Part 2: Static Load Ratings.
International Organization for Standardization, 2001. ISO 15243. Rolling Bearings. Damages and Failures. Terms, Characteristics and Causes.
Society of Automotive Engineers, 1989. SAE J300. SAE Standard Engine Oil Viscosity Classification.

Web Sites

At the time of going to press the world-wide-web contained useful information relating to this chapter at the following sites.

www.abc-bearing.com
www.arb-bearings.com
www.bocabearings.com
www.championbearings.com
www.cooperbearings.com
www.fag.de
www.ggbearings.com
www.hb-bearings.com
www.ina.com
www.jskbearings.com
www.mrcbearings.com
www.nationalbearings.com
www.newindo.com
www.nsk-rhp.com
www.ntn.ca
www.railko.co.uk
www.rbcbearings.com
www.revolve.com
www.rosebearings.com
www.skf.com
www.snfa-bearings.co.uk
www.spn.com.tw
www.sun-sunshine.com
www.thordonbearings.com
www.timken.com
www.torrington.com
www.toyobearings.com

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated.

a	radius (m)
b	outer radius (m)
b_o	characteristic length (m)
c	radial clearance (m)
D	journal diameter (m)
e	eccentricity
Eu	Euler number (dimensionless)
F	coefficient of friction
F_g	geometrical factor
Fr	Froude number (dimensionless)
F_x	body force component in the x direction (N/m^3)

F_y	body force component in the y direction (N/m^3)
F_z	body force component in the z direction (N/m^3)
g	acceleration due to gravity (m/s^2)
h	film thickness (m)
h_o	minimum film thickness (m)
I_c	cosine load integral
I_s	sine load integral
L	characteristic length, length (m), bearing length (m)
l_o	characteristic length (m)
N	speed (normally in rpm)
N_s	journal speed (revolutions per second)
p	static pressure (Pa)
$p_{\max}$	maximum annulus static pressure (N/m^2)
P	load capacity (N/m^2), load (N)
q_x	volumetric flow rate per unit width in the x direction (m^2/s)
q_y	volumetric flow rate per unit width in the y direction (m^2/s)
Q	lubricant flow rate (m^3/s)
Q_π	flow through the minimum film thickness (m^3/s)
Q_s	side flow rate (m^3/s)
r	radius (m)
r_m	mean radius (m)
R	specific gas constant (J/kg K)
Re	Reynolds number (dimensionless)
$\mathbf{Re}_x$	Reynolds number based on x component of velocity (dimensionless)
$\mathbf{Re}_y$	Reynolds number based on y component of velocity (dimensionless)
$\mathbf{Re}_z$	Reynolds number based on z component of velocity (dimensionless)
S	geometrical factor, bearing characteristic number
SAE	Society of Automotive Engineers
t	time (s)
t_o	characteristic time scale (s)
t_*	dimensionless time ratio (dimensionless)
T	temperature (°C, K)
T_{av}	average temperature (°C)
T_1	temperature of the lubricant supply (°C)
T_2	temperature of the lubricant leaving the bearing (°C)
Ta	Taylor number (dimensionless)
$\mathbf{Ta}_m$	Taylor number based on mean radius (dimensionless)
$\mathbf{Ta}_{m,\mathrm{cr}}$	critical Taylor number based on mean annulus radius (dimensionless)
u_r	velocity component in the r direction (m/s)
u_x	velocity component in the x direction (m/s)
$u_{x,o}$	characteristic velocity component in the x direction (m/s)

u_y	velocity component in the y direction (m/s)
$u_{y,o}$	characteristic velocity component in the y direction (m/s)
u_z	velocity component in the z direction (m/s)
$u_{z,o}$	characteristic velocity component in the z direction (m/s)
$u\phi$	velocity component in the tangential direction (m/s)
$\overline{u}$	average velocity (m/s)
U_o	characteristic velocity (m/s), reference velocity (m/s)
V	journal velocity (m/s); velocity (m/s)
W	applied load (N)
x	distance along the x axis (m)
x_*	dimensionless location (dimensionless)
y_*	dimensionless location (dimensionless)
z	distance along the z axis (m)
z_*	dimensionless axial location (dimensionless)
α	angle (° or rad)
ε	eccentricity ratio
$\phi_{p_{\max}}$	angular position of maximum pressure (°)
ϕ_{h_o}	angular position of minimum film thickness (°)
ϕ_{p_o}	film termination angle (°)
ϕ	position of the minimum film thickness (°), attitude angle (°), azimuth angle (rad)
ϕ'	relative azimuth angle (rad)
ϕ_*	nondimensional circumferential location (dimensionless)
μ	viscosity (Pa s)
μ_o	reference viscosity (Pa s)
μ_*	dimensionless viscosity (dimensionless)
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
ρ_*	dimensionless density (dimensionless)
ρ_o	reference density (kg/m^3)
σ_s	squeeze number (dimensionless)
τ_{zx}	viscous shear stress acting in the x direction, on a plane normal to the z direction (N/m^2)
τ_{zy}	viscous shear stress acting in the y direction, on a plane normal to the z direction (N/m^2)
ω	angular velocity (rad/s)
Ω	angular velocity magnitude (rad/s)

CHAPTER 6

Rolling Element Bearings

Chapter Outline

6.1 Introduction 201
6.2 Bearing Life and Selection 203
6.2.1 Modified Life Equation 207
6.3 Bearing Installation 245
6.4 Radial Location 248
6.5 Conclusions 251
Reference 251
Further Reading 251
Nomenclature 252

Abstract

The purpose of a bearing is to support a load, typically applied to a shaft, while allowing relative motion between two elements of a machine. The selection of bearing type was introduced in Chapter 5, along with the design of boundary lubricated and hydrodynamically lubricated journal bearings. In this chapter, rolling element bearings are considered with a specific focus on the selection of stock items from original equipment manufacturers.

6.1 Introduction

The term rolling contact bearings encompasses a wide variety of bearings that use spherical balls or some type of roller between the stationary and moving elements, as illustrated in Figure 6.1. The most common type of bearing supports a rotating shaft resisting a combination of radial and axial (or thrust) loads. Some bearings are designed to carry only radial or only thrust loads.

The selection of the type of bearing used for a given application can be aided by the comparison charts, an example of which is given in Table 5.1. Several bearing manufacturers produce excellent catalogs (e.g. NSK, SKF, FAG, and INA) including design guides. The reader is commended to gain access to this information, which is available in hard copy (just telephone and ask the manufacturer for a brochure) and via internet sources. The comparative ratings shown in Table 6.1 can be justified for, say, radial load carrying capacity; roller bearings are better than ball bearings because of their shape and the area over which the load

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00006-X
Copyright © 2014 Elsevier Ltd. All rights reserved.

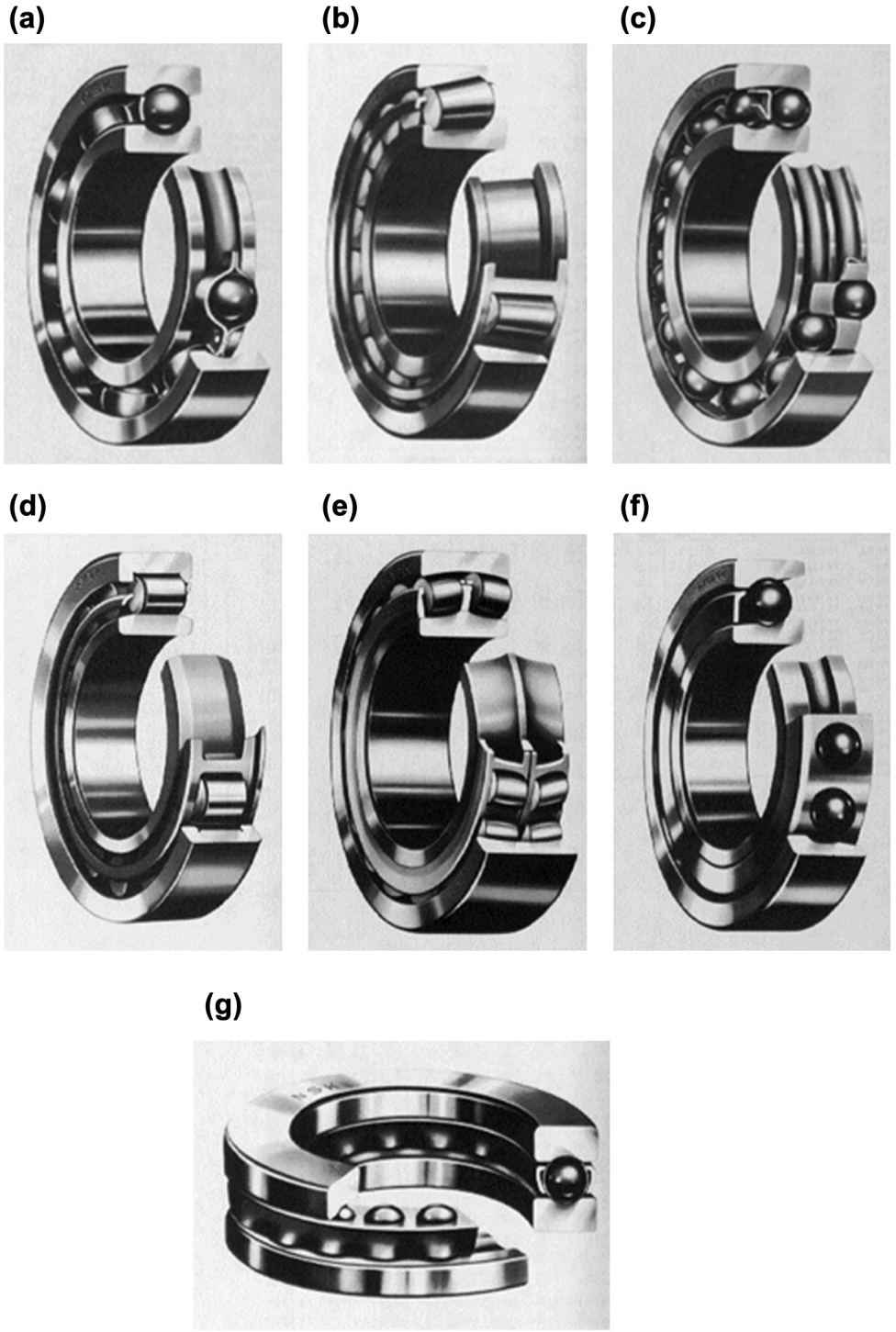


Figure 6.1
A selection of the various types of rolling element bearings. (a) Deep groove ball bearing. (b) Taper roller bearing. (c) Self-aligning ball bearing. (d) Cylindrical roller bearing. (e) Spherical roller bearing. (f) Angular contact ball bearing. (g) Thrust ball bearing. *Photographs courtesy of NSK RHP Bearings.*

Table 6.1: Merits of different rolling contact bearings.

Bearing type	Radial load capacity	Axial or thrust load capacity	Misalignment capability
Single row	Good	Fair	Fair
Double row deep groove ball	Excellent	Good	Fair
Angular contact	Good	Excellent	Poor
Cylindrical roller	Excellent	Poor	Fair
Needle roller	Excellent	Poor	Poor
Spherical roller	Excellent	Fair/good	Excellent
Tapered roller	Excellent	Excellent	Poor

is spread. Thrust load capacity varies dramatically with design. The grooves in the races of the deep groove ball bearing permit the transfer of moderate thrust in combination with radial load. The angular contact bearing (Figure 6.1(f)) is better than the single row ball bearing, because the races are higher on one side providing a more favorable load path. Cylindrical and needle bearings should not generally be subjected to any thrust load.

6.2 Bearing Life and Selection

The load on a rolling contact bearing is exerted on a very small area as illustrated in Figure 6.2. The resulting contact stresses are very high and of the order of 2000 MPa. Despite very strong steels (e.g. BS 970 534 A99, AISI 52100) all bearings have a finite life and will eventually fail

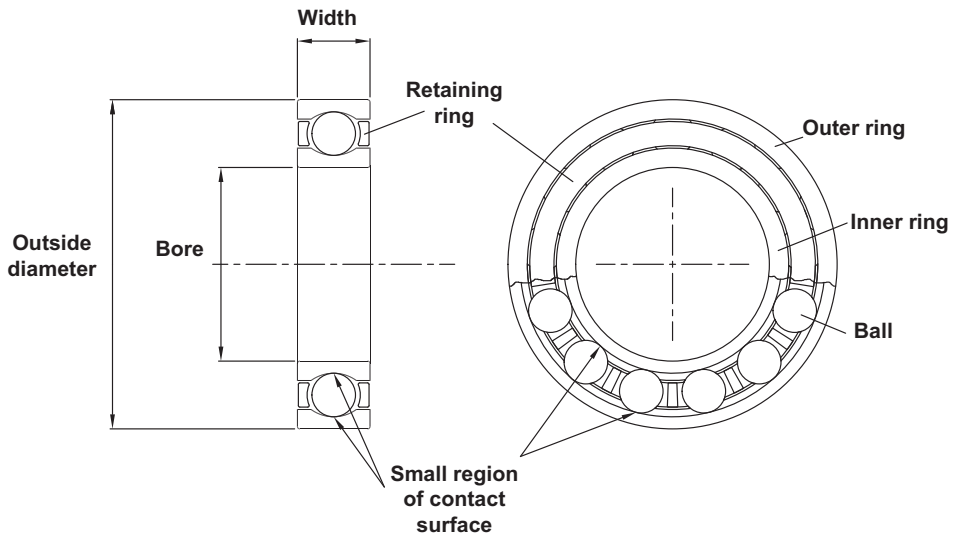


Figure 6.2
Contact area for a ball bearing.

due to fatigue. For two groups of apparently identical bearings tested under loads P_1 and P_2, the respective lives L_1 and L_2 are related by the life equation given in Eqn (6.1).

$$\frac{L_1}{L_2} = \left(\frac{P_2}{P_1}\right)^k \tag{6.1}$$

where

$L_1 =$ life at load P_1 (number of revolutions),
$L_2 =$ life at load P_2 (number of revolutions),
$P_1 =$ load (N),
$P_2 =$ load (N),

and

$k = 3$ for ball bearings,
$k = 3.33$ for cylindrical roller bearings.

The dimensions for rolling element bearings have been standardized and can be purchased as stock items from specialist manufacturers and suppliers. The selection of a bearing from a manufacturer's catalog involves consideration of the bearing load carrying capacity and the bearing geometry. For a given bearing, the load carrying capacity is given in terms of the basic dynamic load rating and the basic static load rating. The various commonly used definitions for rolling element bearing life specification are outlined and illustrated by the example below.

The basic dynamic load rating, C, is the constant radial load which a bearing can endure for 1×10^6 revolutions without evidence of the development of fatigue in any of the bearing components.

The life of a ball bearing, L, is the number of revolutions (or hours at some constant speed), which the bearing runs before the development of fatigue in any of the bearing components.

Fatigue occurs over a large number of cycles of loading. For a bearing, this would mean a large number of revolutions. Fatigue is a statistical phenomenon with considerable spread of the actual life of a group of bearings of a given design. The rated life is the standard means of reporting the results of many tests of bearings. It represents the life that 90% of the bearings would achieve successfully at a rated load. The rated life is referred to as the L_{10} life at the rated load. The rated life, L_{10}, of a group of apparently identical bearings is defined as the number of revolutions (or hours at some constant speed) that 90% of the group of bearings will complete before the first evidence of fatigue develops.

If in Eqn (6.1), $P_2 = C$ and the corresponding life $L_2 = 1 \times 10^6$, then the life of a bearing L, with basic dynamic load rating C with a load P, is given by

$$L = \left(\frac{C}{P}\right)^k \text{ million revolutions} \tag{6.2}$$

where

L = life (millions of revolutions),
C = basic dynamic load rating (N), and
P = load (N).

When selecting a particular bearing from a manufacturer's catalog, it is useful to know the required basic dynamic load rating C for a given load P and life L, which is given by

$$C = P\left(\frac{L}{10^6}\right)^{(1/k)} \tag{6.3}$$

Example 6.1

A straight cylindrical roller bearing operates with a load of 7.5 kN. The required life is 8760 h at 1000 rpm. What load rating should be used for selection from the catalog?

Solution

Using Eqn (6.3),

$$C = P_d\left(L_d/10^6\right)^{1/k} = 7500\left(\frac{8760 \times 1000 \times 60}{10^6}\right)^{1/3.33} = 49.2 \text{ kN}$$

Example 6.2

A catalog lists the basic dynamic load rating for a ball bearing to be 33,800 N for a rated life of 1 million revolutions. What would be the expected L_{10} life of the bearing if it were subjected to 15,000 N and determine the life in hours that this corresponds to if the speed of rotation is 2000 rpm? Comment on the value obtained and its suitability for a machine.

Solution

$C_{\text{cat}} = 33{,}800$ N, $P_d = 15{,}000$ N, $L_{\text{cat}} = 10^6$ (L_{10} life at load C), $k = 3$ (ball). Using Eqn (6.2), the basic rating life is given by

$$L = 10^6\left(\frac{33{,}800}{15{,}000}\right)^3 = 11.44 \times 10^6 \text{ revolutions } (= L_{10} \text{ life at } 15{,}000 \text{ N}).$$

If the rotational speed is 2000 rpm, $L = 11.44 \times 10^6/(2000 \times 60) = 95$ h operation. This is not very long and illustrates the need to use a bearing with a high basic dynamic load rating.

Equations (6.1)–(6.3) have been traditionally used to model the life of a bearing. However, the life of practical rolling element bearings in service depends on a series of factors that include lubrication, contamination, misalignment, installation, and environmental conditions, as well as the quality of manufacture and design. ISO 281:1990/Amd 2:2000 provides a modified life equation to be used as a supplement to the basic rating life. The modified life

equation is presented in Section 6.2.1. The modified life equation can result in predicted lives that are an order of magnitude greater than the standard life equation, Eqn (6.1) or Eqn (6.3), depending of course on the various parameters for the application concerned.

The basic static load rating, C_o, is the load the bearing can withstand without any permanent deformation of any component. If this load is exceeded, it is likely the bearing races will be indented by the rolling elements (called brinelling). Subsequently, the operation of the bearing would be noisy and the impact loads on the indented area would produce rapid wear and progressive failure of the bearing would ensue. Unintentional damage to machine bearings can occur during transportation. If a rotor is unsecured it can vibrate and, although not rotating, the loads can be transferred through the small areas of contact between the bearing elements and raceways. If these loads are above the basic static load rating, brinelling can take place.

Loads on bearings often vary with time and may not be entirely radial. An example is the thrust bearing illustrated in Figure 6.3 from the Rolls-Royce Trent series of engines, see Figure 6.4. This bearing has to take a substantial thrust load, as well as a radial load. In addition, and somewhat unusually for a bearing, both inner and outer raceways rotate. This is because Rolls-Royce plc use a three-spool technology for their engines in order to operate the turbine and compressor stages at optimum speeds. The combined effects of radial and thrust loads can be accommodated in the life equation by an equivalent load.

The equivalent load, P, is defined as the constant radial load which, if applied to a bearing, would give the same life as that which the bearing would attain under the actual conditions of

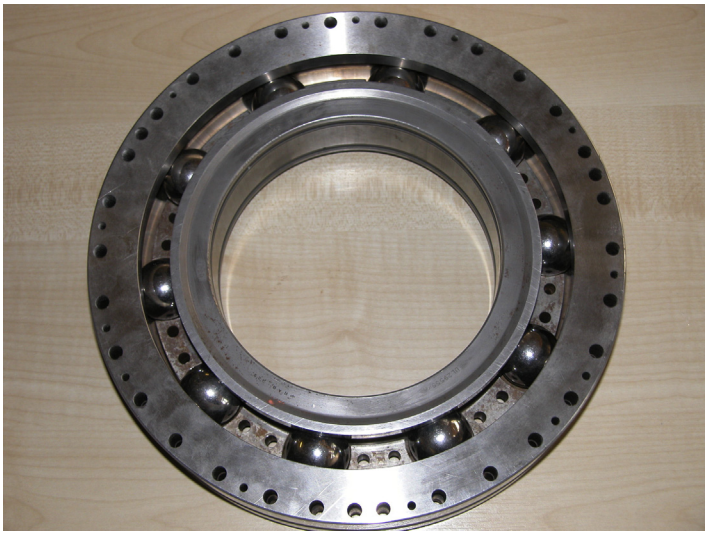


Figure 6.3
Thrust bearing from the Rolls-Royce Trent series of engines.

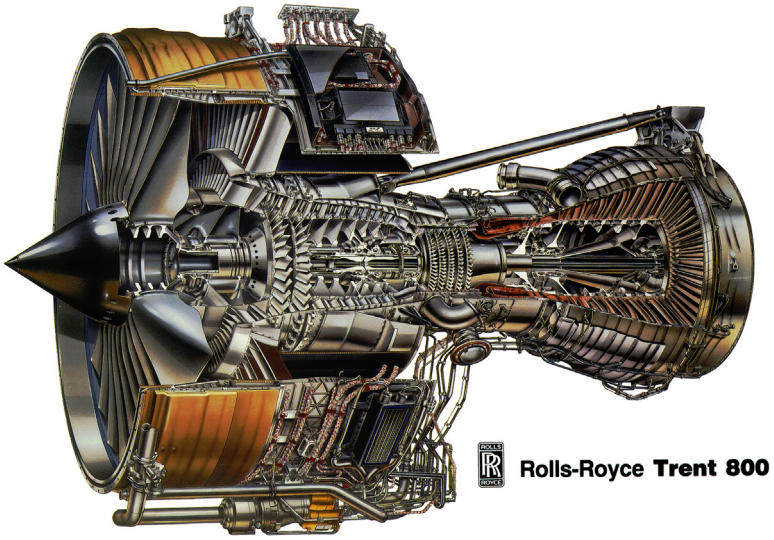


Figure 6.4
The Trent 800 engine illustrating three spools, each rotating at a different speed.
Figure courtesy of Rolls-Royce plc.

load and rotation. When both radial and thrust loads are exerted on a bearing, the equivalent load is thereby the constant radial load that would produce the same rated life for the bearing as the combined loading.

Normally

$$P = VXR + YT \tag{6.4}$$

where

P = equivalent load (N),
V = 1.2 if the mounting rotates is recommended, V = 1.0 if shaft rotates,
X = radial factor (given in bearing catalogs, see Table 6.2 for example data),
R = applied radial load (N),
Y = thrust factor (given in bearing catalogs, see Table 6.2 for example data), and
T = applied thrust load (N).

6.2.1 Modified Life Equation

The basic rating life of a rolling element bearing for a specific application can deviate significantly from the actual service life. Influencing factors include lubrication, contamination, misalignment, installation, and environmental conditions. ISO 281:1990/Amd 2:2000 gives a provision for a modification to the basic rating life accounting for lubrication, contamination conditions, and the fatigue limit of the material. The standard also makes provisions for a specific manufacturer to recommend how to calculate the life

Table 6.2: Values for the radial and thrust factors for determining the equivalent load for deep groove ball single bearings and bearing pairs arranged in tandem.

	Normal clearance			C3 clearance			C4 clearance		
F_a/C_o	*e*	*X*	*Y*	*E*	*X*	*Y*	*e*	*X*	*Y*
0.025	0.22	0.56	2	0.31	0.46	1.75	0.4	0.44	1.42
0.04	0.24	0.56	1.8	0.33	0.46	1.62	0.42	0.44	1.36
0.07	0.27	0.56	1.6	0.36	0.46	1.46	0.44	0.44	1.27
0.13	0.31	0.56	1.4	0.41	0.46	1.3	0.48	0.44	1.16
0.25	0.37	0.56	1.2	0.46	0.46	1.14	0.53	0.44	1.05
0.5	0.44	0.56	1	0.54	0.46	1	0.56	0.44	1

Source: After SKF.

modification factor. In the case of SKF, for example, the factor a_{SKF} is used, and the equation for SKF rating life is given by

$$L_{nm} = a_1 a_{SKF} \left(\frac{C}{P}\right)^k \tag{6.5}$$

where

L_{nm} = rating life (at $100 - n\%$ reliability) in millions of revolutions,
a_1 = the life adjustment factor for reliability, and
a_{SKF} = SKF life modification factor.

Values of a_1 are given in Table 6.3. Values for the life modification factor can be found from tables provided by the specific manufacture in catalogs, online or using an online tool where the user inputs data for the fatigue load limit ratio, the lubrication condition, and the contamination level. The approach requires calculation of the kinematic viscosity ratio, Eqn (6.6), of the actual operating kinematic viscosity of the lubricant to the rated kinematic viscosity depending on the mean diameter and the rotational speed.

$$\kappa = \frac{\nu}{\nu_1} \tag{6.6}$$

Table 6.3: Modified life equation factors.

Reliability %	Failure probability *n*%	SKF rating life L_{nm}	a_1
90	10	L_{10m}	1
95	5	L_{5m}	0.62
96	4	L_{4m}	0.53
97	3	L_{3m}	0.44
98	2	L_{2m}	0.33
99	1	L_{1m}	0.21

Table 6.4: Typical values for the contamination factor.

Condition	$d_m < 100$ mm	$d_m \geq 100$ mm
Extreme cleanliness	1	1
High cleanliness	0.8 to 0.6	0.9 to 0.8
Normal cleanliness	0.6 to 0.5	0.8 to 0.6
Slight contamination	0.5 to 0.3	0.6 to 0.4
Typical contamination	0.3 to 0.1	0.4 to 0.2
Severe contamination	0.1 to 0	0.1 to 0
Very severe contamination	0	0

where

$\kappa =$ viscosity ratio,
$\nu =$ operating kinematic viscosity of the lubricant (m^2/s), and
$\nu_1 =$ rated kinematic viscosity depending on the bearing mean diameter and rotational speed (m^2/s).

A guideline for the contamination factor is given in Table 6.4. Tables for a_{SKF} are provided as a function of the viscosity ratio, κ, and the product of the contamination factor and the fatigue load limit and load ratio, $\eta_c P_u/P$.

Tables 6.5–6.7 give an overview of the information typically available in bearing manufacturers' catalogs.

Example 6.3

A bearing is required to carry a radial load of 2.8 kN and provide the axial location for a shaft of 30 mm diameter rotating at 1500 rpm. An L_{10} life of 10,000 h is required.

1. Select and specify an appropriate bearing using the standard bearing life equation (Eqn (6.3)).
2. Determine the rating life using the modified life equation assuming that the rated oil kinematic viscosity is 14 mm^2/s and the actual kinematic viscosity at operating conditions is 40 mm^2/s, normal cleanliness with a contamination factor of 0.6, and an SKF life modification factor of 14. The desired reliability is 90%.

Solution

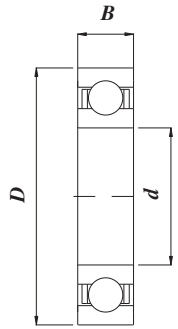
1. Axial shaft location is required, so a deep groove ball bearing, which provides axial location capability in both directions, would be suitable. The total number of revolutions in life is

$$10{,}000 \times 1500 \times 60 = 900 \text{ million.}$$

So $L = 900$. The load is purely radial so $P = 2800$ N. The required dynamic loading is given by

$$C = PL^{1/3} = 2800 \times 900^{1/3} = 27{,}033 \text{ N.}$$

Table 6.5: Single row deep groove ball bearings. Note that this table presents a selection only of the bearings available.



d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit, P_u (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
3	10	4	0.54	0.18	0.007	130,000	80,000	0.002	623
4	9	3.5	0.54	0.18	0.007	140,000	70,000	0.001	628/4-2Z
4	9	4	0.54	0.18	0.007	140,000	70,000	0.001	638/4-2Z
4	9	2.5	0.54	0.18	0.007	140,000	85,000	0.001	618/4
4	11	4	0.715	0.232	0.01	130,000	80,000	0.002	619/4
4	12	4	0.806	0.28	0.012	120,000	75,000	0.002	604
4	13	5	0.936	0.29	0.012	110,000	67,000	0.003	624
4	16	5	1.11	0.38	0.016	95,000	60,000	0.005	634
5	11	5	0.637	0.255	0.011	120,000	60,000	0.002	638/5-2Z
5	11	4	0.637	0.255	0.011	120,000	60,000	0.001	628/5-2Z
5	11	3	0.637	0.255	0.011	120,000	75,000	0.001	618/5
5	16	5	1.14	0.38	0.016	95,000	60,000	0.005	625*
7	19	6	2.34	0.95	0.04	85,000	53,000	0.007	607-Z*
7	22	7	3.45	1.37	0.057	70,000	45,000	0.013	627*
8	16	5	1.33	0.57	0.024	—	26,000	0.004	628/8-2RS1
8	16	4	1.33	0.57	0.024	90,000	56,000	0.003	618/8
8	16	6	1.33	0.57	0.024	90,000	45,000	0.004	638/8-2Z
8	19	6	1.9	0.735	0.031	80,000	50,000	0.007	619/8

8	19	6	2.34	0.95	0.04	85,000	43,000	0.007	607/8-2Z*
8	22	7	3.45	1.37	0.057	75,000	48,000	0.012	608*
8	22	11	3.45	1.37	0.057	—	22,000	0.016	630/8-2RS1
8	24	8	3.9	1.66	0.071	63,000	40,000	0.017	628*
8	28	9	4.62	1.96	0.083	60,000	30,000	0.03	638-2RZ
9	17	5	1.43	0.64	0.027	85,000	43,000	0.004	628/9-2Z
9	17	4	1.43	0.64	0.027	85,000	53,000	0.003	618/9
9	20	6	2.08	0.865	0.036	80,000	48,000	0.008	619/9
9	24	7	3.9	1.66	0.071	70,000	43,000	0.014	609*
9	26	8	4.75	1.96	0.083	60,000	38,000	0.02	629*
9.525	22.225	7.144	3.32	1.4	0.06	60,000	43,000	0.012	EEB 3-2Z
9.525	22.225	5.556	3.32	1.4	0.06	60,000	43,000	0.009	EE 3 TN9
10	19	5	1.38	0.585	0.025	80,000	48,000	0.005	61800
10	22	6	2.08	0.85	0.036	75,000	45,000	0.01	61900
10	26	8	4.75	1.96	0.083	67,000	40,000	0.019	6000*
10	26	12	4.62	1.96	0.083	—	19,000	0.025	63000-2RS1
10	30	9	5.4	2.36	0.1	56,000	34,000	0.032	6200*
10	30	14	5.07	2.36	0.1	—	17,000	0.04	62200-2RS1
10	35	17	8.06	3.4	0.143	—	15,000	0.06	62300-2RS1
10	35	11	8.52	3.4	0.143	50,000	32,000	0.053	6300*
12	21	5	1.43	0.67	0.028	70,000	43,000	0.006	61801
12	21	5	1.43	0.67	0.028	70,000	36,000	0.006	61801-2Z
12	24	6	2.25	0.98	0.043	67,000	40,000	0.011	61901
12	28	8	5.4	2.36	0.1	60,000	38,000	0.022	6001*
12	28	12	5.07	2.36	0.1	—	17,000	0.029	63001-2RS1
12	30	8	5.07	2.36	0.1	56,000	28,000	0.023	16101-2Z
12	32	10	7.28	3.1	0.132	50,000	32,000	0.037	6201*
12	32	14	6.89	3.1	0.132	—	15,000	0.045	62201-2RS1
12	37	17	9.75	4.15	0.176	—	14,000	0.07	62301-2RS1
12.7	28.575	6.35	5.4	2.36	0.1	60,000	38,000	0.023	R 8
12.7	28.575	7.938	5.4	2.36	0.1	56,000	28,000	0.024	R 8-2Z
12.7	33.337	9.525	6.89	3.1	0.132	45,000	32,000	0.037	RLS 4
15	24	5	1.56	0.8	0.034	60,000	38,000	0.007	61802
15	28	7	4.36	2.24	0.095	56,000	34,000	0.016	61902
15	32	8	5.85	2.85	0.12	50,000	32,000	0.025	16002*
15	32	9	5.85	2.85	0.12	—	14,000	0.03	6002-2RSH*

Continued

Table 6.5: Single row deep groove ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit, P_u (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
15	35	11	8.06	3.75	0.16	43,000	22,000	0.045	6202-2Z*
15	42	17	11.4	5.4	0.228	—	12,000	0.11	62302-2RS1
15	42	13	11.9	5.4	0.228	38,000	24,000	0.082	6302*
15.875	39.688	11.113	9.56	4.75	0.2	34,000	24,000	0.065	RLS 5
15.875	46.038	15.875	13.5	6.55	0.275	30,000	22,000	0.12	RMS 5
17	26	5	1.68	0.93	0.039	56,000	34,000	0.008	61803
17	35	10	6.37	3.25	0.137	45,000	28,000	0.039	6003*
17	35	8	6.37	3.25	0.137	45,000	28,000	0.032	16003*
17	35	14	6.05	3.25	0.137	—	13,000	0.052	63003-2RS1
17	40	16	9.56	4.75	0.2	—	12,000	0.083	62203-2RS1
17	40	9	9.56	4.75	0.2	38,000	24,000	0.048	98203
17	40	12	9.95	4.75	0.2	38,000	24,000	0.065	6203*
17	47	14	14.3	6.55	0.275	34,000	22,000	0.12	6303*
17	47	19	13.5	6.55	0.275	—	11,000	0.15	62303-2RS1
17	62	17	22.9	10.8	0.455	28,000	18,000	0.27	6403
19.05	47.625	14.287	12.7	6.55	0.28	30,000	20,000	0.11	RLS 6
19.05	50.8	17.462	15.9	7.8	0.335	28,000	19,000	0.14	RMS 6
20	32	7	4.03	2.32	0.104	45,000	28,000	0.018	61804
20	37	9	6.37	3.65	0.156	43,000	26,000	0.038	61904
20	42	8	7.28	4.05	0.173	38,000	24,000	0.05	16004*
20	42	12	9.95	5	0.212	38,000	24,000	0.069	6004*
20	42	9	7.93	4.5	0.19	38,000	24,000	0.051	98204 Y
20	42	16	9.36	5	0.212	—	11,000	0.086	63004-2RS1
20	47	18	12.7	6.55	0.28	—	10,000	0.13	62204-2RS1
20	47	14	13.5	6.55	0.28	32,000	20,000	0.11	6204*
20	52	21	15.9	7.8	0.335	—	9500	0.2	62304-2RS1
20	52	15	16.8	7.8	0.335	30,000	19,000	0.14	6304*
20	72	19	30.7	15	0.64	24,000	15,000	0.4	6404
22	50	14	14	7.65	0.325	30,000	19,000	0.12	62/22
22	56	16	18.6	9.3	0.39	28,000	18,000	0.18	63/22
22.225	47.625	9.525	11.7	6.3	0.265	30,000	20,000	0.073	EE 8 TN9
22.225	50.8	14.287	14	7.65	0.325	26,000	19,000	0.12	RLS 7

22.225	57.15	17.462	18.6	9.3	0.39	26,000	18,000	0.18	RMS 7
25	37	7	4.36	2.6	0.125	38,000	24,000	0.022	61805
25	42	9	7.02	4.3	0.193	36,000	22,000	0.045	61905
25	47	12	11.9	6.55	0.275	32,000	20,000	0.08	6005*
25	47	8	8.06	4.75	0.212	32,000	20,000	0.06	16005*
25	52	15	14.8	7.8	0.335	28,000	14,000	0.13	6205-2RSL*
25	62	17	23.4	11.6	0.49	24,000	16,000	0.23	6305*
25	62	24	22.5	11.6	0.49	—	7500	0.32	62305-2RS1
25	80	21	35.8	19.3	0.815	20,000	13,000	0.53	6405
25.4	57.15	15.875	17.8	9.65	0.405	24,000	17,000	0.17	RLS 8
25.4	57.15	15.875	17.8	9.65	0.405	24,000	12,000	0.17	RLS 8-2Z
25.4	63.5	19.05	22.5	11.6	0.49	22,000	16,000	0.23	RMS 8
28	58	16	16.8	9.5	0.405	26,000	16,000	0.18	62/28
28	68	18	25.1	13.7	0.585	22,000	14,000	0.29	63/28
28.575	63.5	15.875	19.5	11.2	0.475	22,000	15,000	0.2	RLS 9
28.575	71.438	20.637	28.1	16	0.67	19,000	13,000	0.35	RMS 9
30	42	7	4.49	2.9	0.146	32,000	20,000	0.027	61806
30	55	9	11.9	7.35	0.31	28,000	17,000	0.085	16006*
30	55	13	13.8	8.3	0.355	28,000	17,000	0.12	6006*
30	55	13	13.8	8.3	0.355	—	8000	0.12	6006-2RS1*
30	55	19	13.3	8.3	0.355	—	8000	0.16	63006-2RS1
30	62	20	19.5	11.2	0.475	—	7500	0.24	62206-2RS1
30	62	16	20.3	11.2	0.475	24,000	15,000	0.2	6206*
30	72	27	28.1	16	0.67	—	6300	0.48	62306-2RS1
30	72	19	29.6	16	0.67	20,000	13,000	0.35	6306*
30	90	23	43.6	23.6	1	18,000	11,000	0.74	6406
31.75	69.85	17.462	22.5	13.2	0.55	20,000	14,000	0.3	RLS 10
31.75	79.375	22.225	33.2	19	0.815	17,000	12,000	0.5	RMS 10
34.925	76.2	17.462	27	15.3	0.655	18,000	13,000	0.35	RLS 11
34.925	88.9	22.225	41	24	1.02	15,000	11,000	0.63	RMS 11
35	47	7	4.75	3.2	0.166	28,000	18,000	0.03	61807
35	55	10	9.56	6.8	0.29	26,000	16,000	0.08	61907
35	62	14	16.8	10.2	0.44	24,000	15,000	0.16	6007*
35	62	9	13	8.15	0.375	24,000	15,000	0.11	16007*
35	62	20	15.9	10.2	0.44	—	7000	0.21	63007-2RS1
35	72	23	25.5	15.3	0.655	—	6300	0.37	62207-2RS1

Continued

Table 6.5: Single row deep groove ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit, P_u (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
35	72	17	27	15.3	0.655	20,000	13,000	0.29	6207*
35	80	31	33.2	19	0.815	—	6000	0.66	62307-2RS1
35	80	21	35.1	19	0.815	19,000	12,000	0.46	6307*
35	100	25	55.3	31	1.29	16,000	10,000	0.95	6407
38.1	82.55	19.05	30.7	19	0.8	16,000	11,000	0.45	RLS 12
38.1	95.25	23.813	41	24	1.02	15,000	11,000	0.79	RMS 12
40	52	7	4.94	3.45	0.186	26,000	16,000	0.034	61808
40	62	12	13.8	10	0.425	24,000	12,000	0.12	61908-2RZ
40	68	9	13.8	9.15	0.44	22,000	14,000	0.13	16008*
40	68	15	17.8	11.6	0.49	22,000	14,000	0.19	6008*
40	68	21	16.8	11.6	0.49	—	6300	0.26	63008-2RS1
40	80	23	30.7	19	0.8	—	5600	0.44	62208-2RS1
40	80	18	32.5	19	0.8	18,000	11,000	0.37	6208*
40	80	18	32.5	19	0.8	18,000	9000	0.37	6208-2Z*
40	90	33	41	24	1.02	—	5000	0.89	62308-2RS1
40	90	23	42.3	24	1.02	17,000	11,000	0.63	6308*
40	90	23	42.3	24	1.02	17,000	11,000	0.63	6308-Z*
40	110	27	63.7	36.5	1.53	14,000	9000	1.25	6408
41.275	88.9	19.05	33.2	21.6	0.915	15,000	11,000	0.51	RLS 13
41.275	101.6	23.813	52.7	31.5	1.34	14,000	9500	0.91	RMS 13
44.45	95.25	20.637	35.1	23.2	0.98	14,000	10,000	0.66	RLS 14
44.45	107.95	26.988	61.8	38	1.6	12,000	8500	1.15	RMS 14
45	58	7	6.63	6.1	0.26	22,000	14,000	0.04	61809
45	68	12	14	10.8	0.465	20,000	13,000	0.14	61909
45	75	16	22.1	14.6	0.64	20,000	12,000	0.25	6009*
45	75	10	16.5	10.8	0.52	20,000	12,000	0.17	16009*
45	75	23	20.8	14.6	0.64	—	5600	0.34	63009-2RS1
45	85	19	35.1	21.6	0.915	17,000	11,000	0.41	6209*
45	100	36	52.7	31.5	1.34	—	4500	1.15	62309-2RS1
45	100	25	55.3	31.5	1.34	15,000	9500	0.83	6309*
45	120	29	76.1	45	1.9	13,000	8500	1.55	6409
50	65	7	6.76	6.8	0.285	20,000	13,000	0.052	61810

50	72	12	14.6	11.8	0.5	19,000	12,000	0.14	61910
50	80	10	16.8	11.4	0.56	18,000	11,000	0.18	16010*
50	80	16	22.9	16	0.71	18,000	11,000	0.26	6010*
50	80	16	22.9	16	0.71	18,000	11,000	0.26	6010-Z*
50	90	20	37.1	23.2	0.98	15,000	10,000	0.46	6210*
50	90	20	37.1	23.2	0.98	15,000	8000	0.46	6210-2RZ*
50	110	27	65	38	1.6	13,000	8500	1.05	6310*
50	130	31	87.1	52	2.2	12,000	7500	1.9	6410
50.8	101.6	20.637	35.1	23.2	0.98	14,000	10,000	0.7	RLS 16
55	72	9	9.04	8.8	0.375	19,000	12,000	0.083	61811
55	80	13	16.5	14	0.6	17,000	11,000	0.19	61911
55	90	18	29.6	21.2	0.9	16,000	10,000	0.39	6011*
55	100	21	46.2	29	1.25	14,000	9000	0.61	6211*
55	120	29	74.1	45	1.9	12,000	8000	1.35	6311
55	140	33	99.5	62	2.6	11,000	7000	2.3	6411
57.15	114.3	22.25	52.7	36	1.53	11,000	8000	0.96	RLS 18
57.15	127	31.75	81.9	52	2.2	10,000	7000	1.7	RMS 18
60	78	10	11.9	11.4	0.49	17,000	11,000	0.11	61812
60	85	13	16.5	14.3	0.6	16,000	10,000	0.2	61912
60	95	11	20.8	15	0.735	15,000	9500	0.28	16012*
60	95	18	30.7	23.2	0.98	15,000	9500	0.42	6012*
60	110	22	55.3	36	1.53	13,000	8000	0.78	6212*
60	130	31	85.2	52	2.2	11,000	7000	1.7	6312*
63.5	127	23.813	60.5	45	1.9	10,000	7000	1.25	RLS 20
63.5	139.7	31.75	92.3	60	2.5	9500	6700	2.05	RMS 20
65	85	10	12.4	12.7	0.54	16,000	10,000	0.13	61813
65	100	18	31.9	25	1.06	14,000	9000	0.44	6013*
65	100	11	22.5	16.6	0.83	14,000	9000	0.3	16013*
65	120	23	58.5	40.5	1.73	12,000	7500	0.99	6213*
65	140	33	97.5	60	2.5	10,000	6700	2.1	6313*
65	160	37	119	78	3.15	9500	6000	3.3	6413
69.85	158.75	34.925	104	68	2.75	9000	6300	3	RMS 22
70	90	10	12.4	13.2	0.56	15,000	9000	0.14	61814
70	100	16	23.8	21.2	0.9	14,000	7000	0.35	61914-2RZ
70	110	13	29.1	25	1.06	13,000	8000	0.43	16014*
70	110	20	39.7	31	1.32	13,000	8000	0.6	6014*

Continued

Table 6.5: Single row deep groove ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit, P_u (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
70	125	24	63.7	45	1.9	11,000	7000	1.05	6214*
70	150	35	111	68	2.75	9500	6300	2.5	6314*
70	180	42	143	104	3.9	8500	5300	4.85	6414
75	95	10	12.7	14.3	0.61	14,000	8500	0.15	61815
75	105	16	24.2	19.3	0.965	13,000	8000	0.37	61915
75	110	12	28.6	27	1.14	13,000	8000	0.38	16115
75	115	20	41.6	33.5	1.43	12,000	7500	0.64	6015*
75	130	25	68.9	49	2.04	10,000	6700	1.2	6215*
75	160	37	119	76.5	3	9000	5600	3	6315*
75	190	45	153	114	4.15	8000	5000	6.8	6415
76.2	146.05	26.988	70.2	55	2.2	8500	6000	1.4	RLS 24
80	100	10	13	15	0.64	13,000	6300	0.15	61816-2RZ
80	110	16	25.1	20.4	1.02	12,000	7500	0.4	61916
80	125	22	49.4	40	1.66	11,000	7000	0.85	6016*
80	125	14	35.1	31.5	1.32	11,000	7000	0.6	16016*
80	140	26	72.8	55	2.2	9500	6000	1.4	6216*
80	170	39	130	86.5	3.25	8500	5300	3.6	6316*
80	200	48	163	125	4.5	7500	4800	8	6416
85	110	13	19.5	20.8	0.88	12,000	7500	0.27	61817
85	130	14	35.8	33.5	1.37	11,000	6700	0.63	16017*
85	130	22	52	43	1.76	11,000	6700	0.89	6017*
85	150	28	87.1	64	2.5	9000	5600	1.8	6217*
85	180	41	140	96.5	3.55	8000	5000	4.25	6317*
85	210	52	174	137	4.75	7000	4500	9.5	6417
88.9	165.1	28.575	95.6	73.5	2.8	7500	5300	2.85	RLS 28
88.9	206.375	44.45	153	117	4.15	6300	4500	6.5	RMS 28
90	115	13	19.5	22	0.915	11,000	7000	0.28	61818
90	125	18	33.2	31.5	1.23	11,000	6700	0.59	61918
90	140	24	60.5	50	1.96	10,000	6300	1.15	6018*
90	140	16	43.6	39	1.56	10,000	6300	0.85	16018*
90	160	30	101	73.5	2.8	8500	5300	2.15	6218*
90	190	43	151	108	3.8	7500	4800	4.9	6318*

90	225	54	186	150	5	6700	4300	11.5	6418
95	120	13	19.9	22.8	0.93	11,000	6700	0.3	61819
95	130	18	33.8	33.5	1.43	10,000	6300	0.61	61919
95	145	16	44.8	41.5	1.63	9500	6000	0.89	16019*
95	145	24	63.7	54	2.08	9500	6000	1.2	6019*
95	170	32	114	81.5	3	8000	5000	2.6	6219*
95	200	45	159	118	4.15	7000	4500	5.65	6319*
100	125	13	19.9	24	0.95	10,000	6300	0.31	61820
100	140	20	42.3	41	1.63	9500	6000	0.83	61920
100	150	24	63.7	54	2.04	9500	5600	1.25	6020*
100	150	16	46.2	44	1.73	9500	5600	0.91	16020*
100	180	34	127	93	3.35	7500	4800	3.15	6220*
100	215	47	174	140	4.75	6700	4300	7	6320
101.6	215.9	44.45	174	140	4.75	6000	4300	6.55	RMS 32
105	130	13	20.8	19.6	1	10,000	6300	0.32	61821
105	145	20	44.2	44	1.7	9500	5600	0.87	61921
105	160	18	54	51	1.86	8500	5300	1.2	16021*
105	160	26	76.1	65.5	2.4	8500	5300	1.6	6021*
105	190	36	140	104	3.65	7000	4500	3.7	6221*
105	225	49	182	153	5.1	6300	4000	8.25	6321
110	140	16	28.1	26	1.25	9500	5600	0.6	61822
110	150	20	43.6	45	1.66	9000	5600	0.9	61922
110	170	28	85.2	73.5	2.4	8000	5000	1.95	6022*
110	170	19	60.2	57	2.04	8000	5000	1.45	16022*
110	200	38	151	118	4	6700	4300	4.35	6222*
110	240	50	203	180	5.7	6000	3800	9.55	6322
114.3	238.125	50.8	203	180	5.7	5300	3800	8.9	RMS 36
120	150	16	29.1	28	1.29	8500	5300	0.65	61824
120	165	22	55.3	57	2.04	8000	5000	1.2	61924
120	180	19	63.7	64	2.2	7500	4800	1.6	16024*
120	180	28	88.4	80	2.75	7500	4800	2.05	6024*
120	215	40	146	118	3.9	6300	4000	5.15	6224
120	260	55	208	186	5.7	5600	3400	12.5	6324
130	165	18	37.7	43	1.6	8000	4800	0.93	61826
130	180	24	65	67	2.28	7500	4500	1.85	61926
130	200	33	112	100	3.35	7000	4300	3.15	6026*

Continued

Table 6.5: Single row deep groove ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit, P_u (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
130	200	22	83.2	81.5	2.7	7000	4300	2.35	16026*
130	230	40	156	132	4.15	5600	3600	5.8	6226
130	280	58	229	216	6.3	5000	4500	17.5	6326 M
140	175	18	39	46.5	1.66	7500	4500	0.99	61828
140	190	24	66.3	72	2.36	7000	5600	1.7	61928 MA
140	210	22	80.6	86.5	2.8	6700	4000	2.5	16028
141	210	33	111	108	3.45	6700	4000	3.35	6028
140	250	42	165	150	4.55	5300	3400	7.45	6228
140	300	62	251	245	7.1	4800	4300	22	6328 M
150	190	20	48.8	61	1.96	6700	4300	1.4	61830
150	210	28	88.4	93	2.9	6300	5300	3.05	61
140	300	62	251	245	7.1	4800	4300	22	6328 M
150	190	20	48.8	61	1.96	6700	4300	1.4	61830
150	210	28	88.4	93	2.9	6300	5300	3.05	61930 MA
150	225	35	125	125	3.9	6000	3800	4.8	6030
150	225	24	92.2	98	3.05	6000	3800	3.15	16030
150	270	45	174	166	4.9	5000	3200	9.4	6230
150	320	65	276	285	7.8	4300	4000	26	6330 M
160	200	20	49.4	64	2	6300	4000	1.45	61832
160	220	28	92.3	98	3.05	6000	5000	3.25	61932 MA
160	240	25	99.5	108	3.25	5600	3600	3.7	16032
160	240	38	143	143	4.3	5600	3600	5.9	6032
160	290	48	186	186	5.3	4500	3000	14.5	6232
160	340	68	276	285	7.65	4000	3800	29	6332 M
170	215	22	61.8	78	2.4	6000	3600	1.9	61834
170	230	28	93.6	106	3.15	5600	4800	3.4	61934 MA
170	260	42	168	173	5	5300	4300	7.9	6034 M
170	260	28	119	129	3.75	5300	3200	5	16034
170	310	52	212	224	6.1	4300	3800	17.5	6234 M
170	360	72	312	340	8.8	3800	3400	34.5	6334 M
180	225	22	62.4	81.5	2.45	5600	3400	2	61836
180	250	33	119	134	3.9	5300	4300	5.05	61936 MA

180	280	31	138	146	4.15	4800	4000	6.6	16036
180	280	46	190	200	5.6	4800	4000	10.5	6036 M
180	320	52	229	240	6.4	4000	3600	18.5	6236 M
180	380	75	351	405	10.4	3600	3200	42.5	6336 M
190	240	24	76.1	98	2.8	5300	3200	2.6	61838
190	260	33	117	134	3.8	5000	4300	5.25	61938 MA
190	290	46	195	216	5.85	4800	3800	11	6038 M
190	290	31	148	166	4.55	4800	3000	7.9	16038
190	340	55	255	280	7.35	3800	3400	23	6238 M
190	400	78	371	430	10.8	3400	3000	49	6338 M
200	250	24	76.1	102	2.9	5000	3200	2.7	61840
200	280	38	148	166	4.55	4800	3800	7.4	61940 MA
200	310	34	168	190	5.1	4300	2800	8.85	16040
200	310	51	216	245	6.4	4300	3600	14	6040 M
200	360	58	270	310	7.8	3600	3200	28	6240 M
220	270	24	78	110	3	4500	2800	3	61844
220	300	38	151	180	4.75	4300	3600	8	61944 MA
220	340	56	247	290	7.35	4000	3200	18.5	6044 M
220	340	37	174	204	5.2	4000	2400	11.5	16044
220	400	65	296	365	8.8	3200	3000	37	6244 M
220	460	88	410	520	12	3000	2600	72.5	6344 M
240	300	28	108	150	3.8	4000	2600	4.5	61848
240	320	38	159	200	5.1	4000	3200	8.6	61948 MA
240	360	37	203	255	6.3	2800	2200	14	16048 MA
240	360	56	255	315	7.8	3600	3000	19.5	6048 M
240	440	72	358	465	10.8	3000	2600	51	6248 M
240	500	95	442	585	12.9	2600	2400	92.5	6348 M
260	320	28	111	163	4	3800	2400	4.8	61852
260	360	46	212	270	6.55	3600	3000	14.5	61952 MA
260	400	65	291	375	8.8	3200	2800	29.5	6052 M
260	400	44	238	310	7.2	2600	2000	22.5	16052 MA
260	480	80	390	530	11.8	2600	2400	65.5	6252 M
260	540	102	507	710	15	2400	2200	115	6352 M
280	420	44	242	335	7.5	2400	1900	23.5	16056 MA
280	420	65	302	405	9.3	3000	2600	31	6056 M
280	500	80	423	600	12.9	2600	2200	71	6256 M

Continued

Table 6.5: Single row deep groove ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit, P_u (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
280	580	108	572	850	17.3	2200	2000	140	6356 M
300	380	38	172	245	5.6	3200	2600	10.5	61860 MA
300	420	56	270	375	8.3	3000	2400	24.5	61960 MA
300	460	74	358	500	10.8	2800	2400	44	6060 M
300	460	50	286	405	8.8	2800	2400	32	16060 MA
300	540	85	462	670	13.7	2400	2000	88.5	6260 M
320	400	38	172	255	5.7	3000	2400	11	61864 MA
320	440	56	276	400	8.65	2800	2400	25.5	61964 MA
320	440	37	216	310	6.7	2800	2400	16	60964 MA
320	480	50	281	405	8.65	2600	2200	34	16064 MA
320	480	74	371	540	11.4	2600	2200	46	6064 M
320	580	92	527	800	16	2200	1900	110	6264 MA
330	460	56	281	425	9	2600	2200	30	306728
340	420	38	178	275	6	2800	2400	11.5	61868 MA
340	460	56	281	425	9	2600	2200	26.5	61968 MA
340	480	60	291	430	9	2600	2200	36	306890
340	520	82	423	640	13.2	2400	2200	62	6068 M
340	520	57	345	520	10.6	2400	2000	45	16068 MA
340	620	92	559	900	17.3	2000	1800	110	6268 MA
350	500	70	319	475	9.8	2400	2000	46	306674
360	440	25	121	212	4.5	2600	2200	6.5	60872 MA
360	440	38	182	285	6.1	2600	2200	12	61872 MA
360	480	56	291	450	9.15	2600	2200	28	61972 MA
360	540	82	462	735	15	2400	1900	64.5	6072 M
360	540	57	351	550	11	1800	1400	49	16072 MA
380	480	46	242	390	8	2400	2000	20	61876 MA
380	480	46	242	390	8	2400	1500	18.5	61876 TN9
380	520	65	338	540	10.8	2400	1900	40	61976 MA
380	560	57	377	620	12.2	2200	1800	51	16076 MA
380	560	82	462	750	14.6	2200	1800	67.5	6076 M
400	500	31	163	280	5.6	2400	1900	15.5	60880 MA
400	500	46	247	405	8.15	2400	1900	20.5	61880 MA

400	540	65	345	570	11.2	2200	1800	41.5	61980 MA
400	540	44	265	440	8.5	2200	1800	27.5	60980 M
400	590	74	436	710	13.4	2000	1700	70	306614
400	600	90	520	865	16.3	2000	1700	87.5	6080 M
400	720	130	663	1160	20.8	1800	500	235	BB1B 362809
420	520	46	251	425	8.3	2200	1800	21.5	61884 MA
420	560	65	351	600	11.4	2200	1800	43	61984 MA
420	620	90	507	880	16.3	2000	1600	91.5	6084 M
440	540	31	159	290	5.6	2200	1800	16.5	60888 MA
440	540	46	255	440	8.5	2200	1800	22.5	61888 MA
440	599	80	410	720	13.2	2000	1600	66	BB1B 363471
440	600	74	410	720	13.2	2000	1600	60.5	61988 MA
440	600	50	312	550	10.2	2000	1600	40	60988 MB
440	650	94	553	965	17.6	1900	1500	105	6088 M
460	580	56	319	570	10.6	2000	1600	35	61892 MA
460	620	74	423	750	13.7	1900	1600	62.5	61992 MA
460	680	100	582	1060	19	1800	1500	120	6092 MB
480	600	56	325	600	10.8	1900	1600	36.5	61896 MA
480	650	78	449	815	14.6	1800	1500	74	61996 MA
480	700	100	618	1140	20	1700	1400	125	6096 MB
500	620	37	229	440	7.8	1800	1500	20	608/500 MA
500	670	78	462	865	15	1700	1400	77	619/500 MA
500	689	100	475	865	15	1700	480	110	BB1B 363472
500	720	100	605	1140	19.6	1600	1300	135	60/500 N1MAS
530	650	56	332	655	11.2	1700	1400	39.5	618/530 MA
530	710	57	423	815	13.7	1600	1300	61	609/530 MA
530	710	82	488	930	15.6	1600	1300	90.5	619/530 MA
530	760	100	585	1120	18.6	1500	1300	150	360476 A
530	780	112	650	1270	20.8	1500	1200	185	60/530 N1MAS
540	625	40	225	425	7.5	1700	1400	14	BB1B 362692
560	680	56	345	695	11.8	1600	1300	42	618/560 MA
560	680	37	229	465	8	1600	1300	30.5	608/560 MA
560	750	85	494	980	16.3	1500	1200	105	619/560 MA
560	820	115	663	1470	22	1400	1200	210	60/560 N1MAS
600	730	42	265	550	9	1500	1200	40	608/600 MA
600	730	60	364	765	12.5	1500	1200	52	618/600 MA

Continued

Table 6.5: Single row deep groove ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

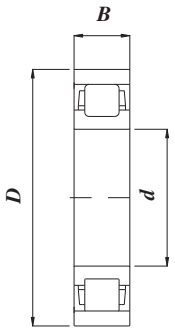
d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit, P_u (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
600	800	90	585	1220	19.6	1400	1100	125	619/600 MA
600	870	118	728	1500	23.6	1300	1100	230	60/600 MA
630	780	48	364	765	12.2	1400	1100	41	608/630 MA
630	780	69	442	965	15.3	1400	1100	73	618/630 MA
630	850	100	624	1340	21.2	1300	1100	160	619/630 N1MA
630	850	71	488	1060	16.6	1300	1100	110	609/630 MB
630	920	128	819	1760	27	1200	1000	285	60/630 N1MBS
650	920	118	780	1730	26	1200	1000	250	306708 D
670	820	69	442	1000	15.6	1300	1100	83.5	618/670 MA
670	820	69	442	1000	15.6	1300	800	77.5	618/670 TN
670	900	103	676	1500	22.4	1200	1000	185	619/670 MA
670	900	73	553	1220	18.6	1200	1000	145	609/670 MA
670	980	136	904	2040	30	1100	900	345	60/670 N1MAS
710	870	74	475	1100	16.6	1200	1000	93.5	618/710 MA
710	950	106	663	1500	22	1100	900	220	619/710 MA
710	950	78	559	1290	19	1100	900	150	609/710 MA
710	1000	140	832	1900	27.5	1100	900	335	306704 C
730	940	100	650	1500	22	1100	900	175	361840
750	920	78	527	1250	18.3	1100	900	110	618/750 MA
750	1000	112	761	1800	25.5	1000	850	255	619/750 MA
750	1090	150	995	2360	33.5	950	800	485	60/750 MA
760	1080	150	923	2200	31	950	800	430	306474 D
800	980	57	403	1000	14.3	1000	850	100	608/800 MA
800	1060	115	832	2040	28.5	950	800	275	619/800 MA
800	1080	115	819	2040	28.5	950	750	320	361844
800	1150	155	1010	2550	34.5	900	750	535	60/800 N1MAS
850	1030	57	397	1020	14	950	750	75	608/850 MB
850	1030	82	559	1430	19.6	950	750	140	618/850 MA
850	1120	118	832	2160	29	850	750	310	619/850 MA
850	1220	165	1040	2700	35.5	800	670	630	306493 AA
900	1090	85	618	1600	21.6	850	700	160	618/900 MA
900	1180	122	852	2280	30	800	670	350	619/900 MB

900	1280	170	1140	3100	39	750	630	720	60/900 MB
950	1150	90	637	1730	22.4	800	670	190	618/950 MA
950	1250	132	1010	2800	36	750	630	390	619/950 MB
950	1360	180	1170	3250	40.5	700	560	860	60/950 MB
970	1125	75	488	1340	17.6	800	670	125	BB1B 363297
1000	1220	71	553	1560	20	750	600	175	608/1000 MB
1000	1220	100	637	1800	22.8	750	600	245	618/1000 MA
1000	1320	140	1010	2800	35.5	700	560	515	619/1000 MB
1000	1420	185	1350	3900	47.5	630	530	930	60/1000 MB
1030	1250	100	605	1760	22	700	600	250	BB1B 630533
1060	1280	100	728	2120	26.5	670	560	260	618/1060 MA
1060	1400	150	1010	3000	36	630	530	620	619/1060 MB
1060	1500	195	1350	3900	46.5	600	500	1080	60/1060 MB
1120	1360	106	741	2200	26.5	630	530	315	618/1120 MA
1120	1460	150	1040	3100	36.5	600	480	650	619/1120 MB
1120	1580	200	1460	4400	50	530	450	1250	60/1120 MB
1180	1420	106	761	2360	27.5	560	480	330	618/1180 MB
1180	1540	160	1140	3600	41.5	220	280	775	619/1180 MB
1250	1500	112	852	2750	31.5	530	430	385	618/1250 MB
1320	1600	122	956	3150	35.5	480	400	500	618/1320 MA
1320	1720	128	1210	4050	44	450	360	830	609/1320 MB
1400	1700	132	1120	4000	42.5	430	360	615	618/1400 MA
1400	1820	185	1590	5500	57	400	340	1250	619/1400 MB
1500	1820	140	1210	4400	46.5	380	240	690	618/1500 TN
1500	1950	195	1720	6100	62	380	320	1500	619/1500 MB
1600	1950	155	1270	4800	48	320	280	965	618/1600 MB
1600	2060	200	1860	6950	69.5	300	260	1650	619/1600 MB
1700	2060	160	1270	4900	48	300	240	1100	618/1700 MB
1700	2180	212	1990	7650	73.5	280	220	1950	619/1700 MB
2000	2200	75	936	4500	41.5	220	190	290	BB1B 363270
2390	2690	120	1300	6200	52	120	100	975	BB1-8001

* SKF Explorer bearing.

Source: Table after SKF.

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.



d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
15	35	11	12.5	10.2	1.22	22,000	26,000	0.047	NU 202 ECP
15	35	11	12.5	10.2	1.22	22,000	34,000	0.047	NU 202 ECPHA
15	35	11	12.5	10.2	1.22	22,000	26,000	0.049	NJ 202 ECP
17	40	12	17.2	14.3	1.73	19,000	22,000	0.07	NJ 203 ECP
17	40	16	23.8	21.6	2.65	19,000	22,000	0.095	NJ 2203 ECP
17	47	14	24.6	20.4	2.55	15,000	20,000	0.12	N 303 ECP
20	47	18	29.7	27.5	3.45	16,000	19,000	0.14	NJ 2204 ECP
20	47	14	25.1	22	2.75	16,000	19,000	0.11	NJ 204 ECP
20	47	14	25.1	22	2.75	16,000	19,000	0.11	NU 204 ECP
20	52	15	35.5	26	3.25	15,000	18,000	0.15	N 304 ECP*
20	52	21	47.5	38	4.8	15,000	18,000	0.21	NU 2304 ECP*
25	47	12	14.2	13.2	1.4	18,000	18,000	0.084	NU 1005
25	52	15	28.6	27	3.35	14,000	16,000	0.13	N 205 ECP
25	52	18	34.1	34	4.25	14,000	16,000	0.17	NUP 2205 ECP
25	52	15	28.6	27	3.35	14,000	16,000	0.14	NU 205 ECJ
25	52	18	34.1	34	4.25	14,000	16,000	0.18	NJ 2205 ECP
25	52	18	34.1	34	4.25	14,000	26,000	0.17	NU 2205 ECML
25	62	17	46.5	36.5	4.55	12,000	22,000	0.29	NJ 305 ECML*
25	62	24	64	55	6.95	12,000	15,000	0.42	NJ 2305 ECML*
25	62	17	46.5	36.5	4.55	12,000	15,000	0.28	NU 305 ECP*
25	62	24	64	55	6.95	12,000	15,000	0.39	NJ 2305 ECP*
30	55	13	17.9	17.3	1.86	15,000	15,000	0.12	NU 1006
30	62	16	44	36.5	4.55	13,000	22,000	0.22	NUP 206 ECML*
30	62	20	55	49	6.1	13,000	22,000	0.29	NU 2206 ECML*

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
30	62	16	44	36.5	4.55	13,000	14,000	0.22	NUP 206 ECP*
30	62	16	44	36.5	4.55	13,000	22,000	0.23	NU 206 ECML*
30	72	19	58.5	48	6.2	11,000	12,000	0.44	NU 306 ECM*
30	72	19	58.5	48	6.2	11,000	12,000	0.4	NU 306 ECP*
30	72	19	58.5	48	6.2	11,000	12,000	0.41	NJ 306 ECP*
30	72	27	83	75	9.65	11,000	19,000	0.58	NJ 2306 ECML*
35	62	14	35.8	38	4.55	12,000	20,000	0.16	NU 1007 ECMP
35	72	17	56	48	6.1	11,000	18,000	0.36	NUP 207 ECML*
35	72	23	69.5	63	8.15	11,000	12,000	0.4	N 2207 ECM*
35	80	21	75	63	8.15	9500	11,000	0.51	NUP 307 ECP*
35	80	31	106	98	12.7	9500	11,000	0.75	NUP 2307 ECP*
35	80	21	75	63	8.15	9500	11,000	0.55	N 307 ECM*
35	100	25	76.5	69.5	9	8000	9500	1.05	N 407
40	68	15	25.1	26	3	12,000	18,000	0.22	NU 1008 ML
40	80	23	81.5	75	9.65	9500	11,000	0.54	NU 2208 ECP*
40	80	18	62	53	6.7	9500	11,000	0.42	NU 208 ECJ*
40	80	18	62	53	6.7	9500	11,000	0.42	NU 208 ECP*
40	90	23	93	78	10.2	8000	15,000	0.81	NJ 308 ECML*
40	90	33	129	120	15.3	8000	9500	0.96	NJ 2308 ECJ*
40	110	27	96.8	90	11.8	7000	8500	1.35	NJ 408
40	110	27	96.8	90	11.8	7000	11,000	1.45	NJ 408 MA
45	75	16	44.6	52	6.3	9500	11,000	0.26	NU 1009 ECP
45	75	19	52.8	64	7.8	9000	11,000	0.34	NJ 2009 ECP
45	85	23	69.5	64	8.15	9000	9500	0.46	NUB 209 ECJ
45	85	19	69.5	64	8.15	9000	9500	0.48	NU 209 ECJ*
45	100	36	160	153	20	7500	13,000	1.45	NJ 2309 ECML*
45	100	25	112	100	12.9	7500	8500	0.95	NUP 309 ECP*
45	100	25	112	100	12.9	7500	8500	0.88	N 309 ECP*
45	100	25	112	100	12.9	7500	13,000	0.95	NUP 309 ECML*
45	100	25	112	100	12.9	7500	8500	1	NU 309 ECNP*
45	120	29	106	102	13.4	6700	7500	1.65	NU 409
50	80	16	30.8	34.5	4	10,000	15,000	0.31	NU 1010 ML
50	90	23	90	88	11.4	8500	9000	0.59	NUP 2210 ECNP*
50	90	20	73.5	69.5	8.8	8500	14,000	0.51	NUP 210 ECML*

Continued

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
50	90	20	73.5	69.5	8.8	8500	9000	0.49	NU 210 ECKP*
50	90	20	73.5	69.5	8.8	8500	14,000	0.57	NU 210 ECML*
50	90	20	73.5	69.5	8.8	8500	14,000	0.5	NJ 210 ECML*
50	110	27	127	112	15	6700	8000	1.15	NJ 310 ECP*
50	110	40	186	186	24.5	6700	12,000	2.05	NJ 2310 ECML*
50	130	31	130	127	16.6	6000	7000	2	NU 410
55	90	18	57.2	69.5	6.3	8000	8500	0.4	NU 1011 ECP
55	100	25	96.5	95	12.2	7500	8000	0.68	NUB 211 ECJ
55	100	21	96.5	95	12.2	7500	8000	0.72	NU 211 ECM*
55	120	43	232	232	15.3	6000	11,000	2.48	NJ 2311 ECML*
55	120	29	156	143	18.6	6000	11,000	1.58	NJ 311 ECML*
60	95	18	37.4	44	5.3	8000	13,000	0.48	NU 1012 ML
60	110	28	146	153	20	6700	7500	1.1	NU 2212 ECM*
60	110	22	108	102	13.4	6700	7500	0.81	NU 212 ECP*
60	110	22	108	102	13.4	6700	11,000	0.86	NUP 212 ECML*
60	130	46	260	265	34.5	5600	10,000	3.2	NJ 2312 ECML*
60	130	31	173	160	20.8	5600	10,000	2	NJ 312 ECML*
60	130	46	260	265	34.5	5600	6700	2.85	NUP 2312 ECP*
60	130	31	173	160	20.8	5600	6700	1.95	NU 312 ECM*
60	150	35	168	173	22	5000	6000	3.1	NJ 412
65	100	26	105	146	18.3	6300	7500	0.7	NJ 3013 ECP
65	100	18	38	46.5	5.5	7500	12,000	0.51	NU 1013 ML
65	120	23	122	118	15.6	6300	10,000	1.1	NUP 213 ECML*
65	120	31	170	180	24	6300	6700	1.5	NUP 2213 ECP*
65	140	48	285	290	38	5300	9500	3.6	NJ 2313 ECML*
65	160	37	183	190	24	4800	5600	3.65	NJ 413
70	110	20	56.1	67.2	8	7000	11,000	0.8	NU 1014 ML
70	125	31	180	193	25.5	6000	10,000	1.7	NJ 2214 ECML*
70	125	24	137	137	18	6000	6300	1.15	NU 214 ECP*
70	125	24	137	137	18	6000	6300	1.25	NJ 214 ECM*
70	150	35	236	228	29	4800	5600	2.85	NUP 314 ECM*

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
70	150	51	315	325	41.5	4800	5600	4.15	NUP 2314 ECP*
75	115	20	58.3	71	8.5	6700	10,000	0.74	NU 1015 ML
75	130	31	150	156	20.4	5600	6000	1.35	NUB 215 ECJ
75	130	25	150	156	20.4	5600	6000	1.35	NU 215 ECM*
75	160	37	280	265	33.5	4500	5300	3.45	NUP 315 ECP*
75	160	55	380	400	50	4500	5300	5.1	NUP 2315 ECP*
75	190	45	264	280	34	4000	4800	6.9	NJ 415
80	125	22	99	127	16.3	5600	9500	1.1	NJ 1016 ECML
80	140	33	212	245	31	5300	5600	2.1	NUP 2216 ECJ*
80	140	26	160	166	21.2	5300	8500	1.65	NUP 216 ECML*
80	170	58	415	440	55	4300	7500	6.6	NJ 2316 ECML*
80	170	39	300	290	36	4300	7500	4.7	NJ 316 ECML*
80	200	48	303	320	39	3800	4500	7.45	NJ 416
85	130	22	68.2	86.5	10.5	6000	6000	1.05	NU 1017 M
85	150	36	190	200	24.5	4800	5300	2.05	NUB 217 ECJ
85	150	28	190	200	24.5	4800	5300	2.05	NU 217 ECM*
85	180	41	340	335	41.5	4000	4800	4.9	NUP 317 ECP*
85	180	60	455	490	60	4000	4800	7	NUP 2317 ECP*
90	140	24	80.9	104	12.7	5600	8500	1.35	NU 1018 ML
90	160	40	280	315	39	4500	5000	3.15	NU 2218 ECP*
90	160	30	208	220	27	4500	5000	2.35	NU 218 ECJ*
90	190	64	500	540	65.5	3800	6700	8.8	NJ 2318 ECML*
90	190	43	365	360	43	3800	6700	6	NJ 318 ECML*
90	190	53	365	360	43	3800	4500	5.55	NJ 318 ECJ*
90	225	54	380	415	48	3400	4000	10.5	NU 418
95	145	24	84.2	110	13.2	5300	8000	1.4	NU 1019 ML
95	170	32	255	265	32.5	4300	4800	2.85	NU 219 ECJ*
95	170	43	325	375	45.5	4300	4800	3.85	NU 2219 ECM*
95	200	45	390	390	46.5	3600	4300	6.25	NUP 319 ECP*
95	200	67	530	585	69.5	3600	4300	9.75	NUP 2319 ECP*
100	150	24	85.8	114	13.7	5000	5000	1.45	NU 1020 M
100	180	46	285	305	36.5	4000	4500	3.75	NUB 220 ECJ
100	180	34	285	305	36.5	4000	4500	3.45	NU 220 ECJ*
100	215	73	670	735	85	3200	6000	13.2	

Continued

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
									NJ 2320 ECML*
100	215	47	450	440	51	3200	6000	9	NJ 320 ECML*
100	250	58	429	475	53	3000	3600	14	NU 420
105	160	26	101	137	16	4800	4800	1.85	NU 1021 M
105	190	36	300	315	36.5	3800	6300	4	NUP 221 ECML*
105	225	49	500	500	57	3200	5600	8.9	NU 321 ECML*
105	260	60	501	570	64	2800	3400	19	NU 421 M
110	170	28	128	166	19.3	4500	4500	2.3	NU 1022 M
110	200	53	335	365	42.5	3600	4000	5.2	NUB 222 ECJ
110	200	38	335	365	42.5	3600	4000	4.8	NU 222 ECP*
110	240	50	530	540	61	3000	3400	11.2	NUP 322 ECP*
110	240	80	780	900	102	3000	4500	18.5	NU 2322 ECKMA*
110	280	65	532	585	64	2600	3200	20	NU 422
120	180	28	134	183	20.8	4000	6300	2.5	NJ 1024 ML
120	215	58	520	630	72	3400	3600	9	NUP 2224 ECJ*
120	215	40	390	430	49	3400	5600	6	NUP 224 ECML*
120	260	55	610	620	69.5	2800	5000	15.1	NJ 324 ECML*
120	260	86	915	1040	116	2800	4300	24.3	NJ 2324 ECMA*
120	260	86	915	1040	116	2800	3200	24	NU 2324 ECM*
120	310	72	644	735	78	2400	2800	28	NU 424
127	254	50.8	484	585	65.5	2800	3200	12	CRM 40 AMB
130	200	33	165	224	25	3800	3800	3.8	NJ 1026 M
130	200	42	297	440	51	3200	4800	4.6	NU 2026 ECMA
130	230	40	415	455	51	3200	5300	7	NUP 226 ECML*
130	230	64	610	735	83	3200	5300	12.2	NUP 2226 ECML*
130	280	58	720	750	81.5	2400	4500	19.4	NJ 326 ECML*
130	280	93	1060	1250	137	2400	3800	30.5	NJ 2326 ECPA*
140	210	33	179	255	28	3600	5300	4.05	NU 1028 ML
140	250	42	450	510	57	2800	3200	8.6	NU 228 ECJ*
140	250	68	655	830	93	2800	4800	15.6	NUP 2228 ECML*
140	300	62	780	830	88	2400	4300	23	NJ 328 ECML*

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
140	300	102	1200	1430	150	2400	3600	37.5	NUP 2328 ECMA*
150	225	35	198	290	31.5	3200	5000	4.85	NU 1030 ML
150	270	45	510	600	64	2600	4500	11	NUP 230 ECML*
150	270	73	735	930	100	2600	2800	18.5	NU 2230 ECM*
150	320	65	900	965	100	2200	3400	27.7	NJ 330 ECM*
150	320	108	1370	1630	166	2200	3400	46	NJ 2330 ECMA*
160	240	48	418	670	72	2600	4000	7.9	NU 2032 ECMA
160	240	38	229	325	35.5	3000	3000	5.95	NU 1032 M
160	290	80	930	1200	129	2400	3400	25	NUP 2232 ECMA*
160	290	48	585	680	72	2400	4000	14.5	NUP 232 ECML*
160	340	68	1000	1080	112	2000	2400	32.2	NJ 332 ECM*
160	340	114	1250	1730	176	2000	3200	53.5	NJ 2332 ECMA
160	340	68	1000	1080	112	2000	3200	32.5	NUP 332 ECMA*
170	260	54	473	735	41.5	2400	3600	11	NU 2034 ECMA
170	260	42	275	400	41.5	2800	4300	8.15	NJ 1034 ML
170	310	52	695	815	85	2400	3200	20	NUP 234 ECMA*
170	310	86	1060	1430	140	2400	3200	30	NU 2234 ECMA*
170	360	120	1450	2040	204	1700	3000	63	NJ 2334 ECMA
170	360	72	952	1180	116	1700	2200	38.5	N 334 ECM
180	280	46	336	475	51	2600	2600	10.5	NU 1036 M
180	320	52	720	850	88	2200	3200	21	NUP 236 ECMA*
180	320	86	1100	1430	146	2200	2400	31.5	NU 2236 ECM*
180	380	75	1020	1290	125	1600	2200	45	NU 336 ECM
180	380	126	1610	2240	216	1600	2800	71.5	NU 2336 ECMA
180	440	95	1250	1600	150	1600	2400	80	NUP 436 MA
190	260	33	251	400	40.5	2600	4000	5.5	NJ 1938 ECML
190	290	46	347	500	53	2600	3800	11	NJ 1038 ML
190	340	55	800	965	98	2000	2200	24	NU 238 ECM*

Continued

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
190	340	92	1220	1600	160	2000	3000	39	NU 2238 ECM*
190	400	132	1830	2550	236	1500	2600	82.5	NU 2338 ECMA
190	400	78	1140	1500	143	1500	2000	50	NU 338 ECM
200	250	30	183	345	32.5	2800	3600	3.55	NJ 2840 ECMA
200	280	38	341	540	55	2600	3800	7.4	NJ 1940 ECMP
200	310	51	380	570	58.5	2400	2400	14.5	NU 1040 M
200	360	58	850	1020	100	1900	2200	28.5	NU 240 ECM*
200	360	128	1540	2450	236	1600	2400	59	NU 3240 ECM
200	420	80	1230	1630	150	1400	2400	57.5	NU 340 ECMA
200	420	138	1980	2800	255	1400	2400	96.5	NU 2340 ECMA
220	300	38	336	560	55	2400	3000	8.3	NJ 1944 ECMA
220	300	48	457	830	83	2400	3600	10	NU 2944 M2P
220	340	56	495	735	73.5	2200	3200	19	NJ 1044 MP
220	400	65	1060	1290	125	1600	1900	38	NU 244 ECM*
220	400	108	1570	2280	212	1600	2400	62.5	NU 2244 ECMA
220	460	145	2380	3450	310	1300	2200	120	NU 2344 ECMA
220	460	88	1210	1630	150	1500	1700	72.5	NU 344 M
240	300	28	201	365	32	2400	3600	4.5	NUZ 1848 ECMA
240	360	56	523	800	78	2000	2600	20	NU 1048 MA
240	440	72	952	1370	129	1600	2200	52.5	NJ 248 MA
240	440	120	1450	2360	224	1500	2200	85	NJ 2248 MA
240	500	155	2600	3650	320	1200	2000	155	NU 2348 ECMA
240	500	95	1450	2000	180	1300	1600	94.5	NU 348 M
260	400	65	627	965	96.5	1800	1800	29.5	NU 1052 M
260	400	104	1420	2320	232	1500	2400	49.5	NU 3052 ECMA
260	440	144	2090	3450	310	1300	2200	98	NU 3152 ECMA
260	480	80	1170	1700	156	1400	2000	72	NUP 252 MA
260	480	130	1790	3000	275	1300	2000	112	NJ 2252 MA
260	540	165	3190	4500	380	1100	1800	190	NU 2352 ECMA
260	540	102	1940	2700	236	1100	1800	125	NU 352 ECMA
280	420	82	1230	2160	204	1400	2200	40	NU 2056 ECMA
280	420	65	660	1060	102	1700	2200	31.5	NU 1056 MA

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
280	460	146	2290	3900	335	1200	2000	105	NU 3156 EMA/HB1
280	500	80	1140	1700	153	1400	1900	71.5	NU 256 MA
280	500	165.1	2920	5000	430	1200	1500	160	BC1B 322312
280	500	130	2200	3250	285	1200	1900	115	NU 2256 ECMA
280	580	175	2700	4300	365	1000	1700	230	NU 2356 MA
300	380	48	473	980	88	1700	1700	14	NU 2860 ECM
300	460	95	1510	2750	255	1300	2000	59.5	NU 2060 ECMA
300	460	74	858	1370	129	1500	2000	46.5	NU 1060 MA
300	540	140	2090	3450	300	1200	1800	145	NU 2260 MA
300	540	85	1420	2120	183	1300	1400	89.5	NU 260 M
300	620	185	4020	5850	480	950	1600	270	NU 2360 ECMA
320	400	48	495	1060	93	1600	2400	14.5	NU 2864 ECMA
320	400	38	369	710	60	1600	2400	11	NU 1864 ECMP
320	440	72	765	1500	140	1400	1400	34.5	314756
320	440	56	693	1200	110	1500	2000	27	NU 1964 ECMA
320	480	121	1830	3200	285	1200	1900	75	NU 3064 ECM
320	480	95	1450	265	245	1200	1900	62.5	NU 2064 ECMA
320	480	74	880	1430	132	1400	1400	49	NJ 1064 MA
320	540	176	3140	5400	450	1000	1700	175	NU 3164 ECMA
320	580	92	1610	2450	204	1200	1600	115	NU 264 MA
320	580	150	3190	5000	415	1000	1600	180	NU 2264 ECMA
340	420	48	512	1120	98	1500	2000	15	NJ 2868 ECMA
340	420	35	212	400	33.5	1500	2000	9.6	316197
340	460	72	809	1660	146	1400	1400	36	NU 2968 M
340	460	56	682	1200	108	1400	1900	27.5	NU 1968 ECMA
340	520	82	1080	1760	156	1300	1700	68	NJ 1068 MA
340	580	190	3190	5700	475	950	1600	210	NU 3168 ECMA
340	620	165	2640	4500	365	1000	1500	220	NU 2268 MA
350	480	85	1060	2160	196	1200	1300	48	612129 A
360	540	106	1940	3600	320	1100	1600	88.5	NU 2072 ECMA
360	540	82	1100	1830	163	1300	1600	67.5	NU 1072 MA

Continued

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
360	600	192	3410	5700	415	900	1500	225	NU 3172 ECMA
360	650	170	2920	4900	400	950	1400	250	NU 2272 MA
360	750	224	5010	8150	630	850	1300	510	NU 2372 ECMA
380	480	46	539	1060	86.5	1300	1300	23	NU 1876 ECM
380	480	40	270	520	37.5	1300	1300	15.5	316010 A
380	560	135	2380	4750	415	1000	1800	115	NU 3076 ECMP
380	560	106	1980	3750	335	1000	1600	92.5	NU 2076 ECMA
380	560	82	1140	1930	170	1200	1600	71	NU 1076 MA
380	680	175	3960	6400	510	850	1300	275	NU 2276 ECMA
400	500	46	572	1180	100	1300	1900	21.8	NJ 1880 MP
400	540	82	1400	2850	255	1200	1600	57	NU 2980 ECMA
400	540	65	952	1730	153	1200	1600	41	NU 1980 ECMA
400	600	118	2200	4750	380	950	1400	120	NU 2080 ECMA
400	600	148	2810	5500	440	950	1400	175	NU 3080 MA6
400	600	90	1380	2320	204	1100	1500	92.5	NU 1080 MA
420	520	75	935	2240	196	1100	1600	33	NJ 3884 MA
420	520	46	550	1120	95	1200	1600	20.5	NJ 1884 MA6
420	520	46	550	1120	95	1200	1600	20	NU 1884 MA
420	560	82	1210	2550	228	1100	1500	59	NU 2984 MA
420	560	65	968	1800	156	1100	1500	48	NJ 1984 ECMA
420	620	150	2920	5400	430	900	1400	160	NU 3084 ECMA
420	620	118	2460	4750	380	900	1400	125	NU 2084 ECMA
420	620	90	1420	2450	212	1100	1400	96	NU 1084 MA
420	700	224	4950	9000	695	750	1300	380	NU 3184 ECMA/HB1
440	540	60	809	1900	153	1100	1500	34	NU 2888 ECMA
440	540	40	292	610	41.5	1100	1500	18.5	316011
440	600	74	1060	2000	17	1100	1400	64	NU 1988 MA
440	600	95	1720	3600	310	1100	1400	84	NJ 2988 ECM/HB1
440	600	118	1980	4250	355	900	1100	105	NUP 3988 ECM
440	650	122	2550	4900	390	8500	1300	145	

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
									NU 2088 ECMA
440	650	94	1510	2650	212	1000	1300	105	NU 1088 MA
440	720	226	5120	9650	735	700	1200	395	NU 3188 ECMA/HB1
460	580	56	825	1700	140	1100	1400	37	N 1892 M2P/ HB3
460	580	72	1080	2400	193	1100	1400	48	NJ 2892 ECMA
460	620	95	1720	3600	310	1000	1300	89	NJ 2992 ECMA
460	680	163	3470	6400	500	800	1200	210	NU 3092 ECMA
460	680	128	2810	5400	415	800	1200	165	NU 2092 ECMA
460	680	100	1650	2850	224	950	1200	115	NU 1092 MA
460	760	240	5280	9650	735	670	1100	455	NU 3192 ECMA/HB1
460	830	212	5120	8650	655	700	1100	530	NU 2292 MA
460	830	165	4180	6800	510	750	1100	415	NU 1292 MA
480	600	56	765	1630	120	1000	1300	39	NU 1896 MA
480	600	72	1100	2450	200	1000	1300	47.5	NJ 2896 ECMA
480	700	128	2860	5600	430	750	1200	170	NU 2096 ECMA
480	700	100	1680	3000	232	900	1200	130	NU 1096 MA
480	790	248	5940	10,800	800	630	1100	500	NU 3196 ECMA/HB1
500	620	72	1170	2700	204	950	1300	50	NU 28/500 ECMA
500	620	45	396	830	55	1000	1000	29	316198
500	620	56	825	1730	129	950	950	39	NF 18/500 ECM
500	670	128	2330	5200	440	800	1200	130	N 39/500 ECMB/W33
500	670	78	1210	2360	193	900	1200	79	NU 19/500 MA
500	720	128	2920	5850	430	750	1100	180	NU 20/500 ECMA
500	720	167	4020	8000	610	750	1100	225	NU 30/500 ECFR
500	720	100	1720	3100	236	900	1100	135	NU 10/500 MA
500	830	264	6440	12,000	880	600	1000	595	NU 31/500 ECMA/HB1

Continued

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
500	920	185	5280	8500	620	670	950	585	NU 12/500 MA
530	650	45	418	900	62	2200	3400	30	315835 A
530	650	72	1170	2750	220	900	1200	52	NJ 28/530 ECMA
530	710	82	1570	3050	240	850	1100	94	NJ 19/530 ECM/HB1
530	710	106	2380	5000	415	850	1100	120	NU 29/530 ECMA
530	780	145	3740	7350	550	670	1000	255	NU 20/530 ECMA
530	780	112	2290	4050	305	800	1000	190	NU 10/530 MA
530	870	272	7480	14,600	1040	560	950	660	NU 31/530 ECMA/HB1
560	680	56	809	1830	134	850	1100	44.5	NJ 18/560 ECMA6/ 343016
560	680	45	429	950	62	2000	3200	32	BC1B 320938/ HB3
560	750	85	1650	3250	260	800	1000	110	NU 19/560 ECMA
560	750	112	2460	5400	425	800	1000	145	NU 29/560 ECMA
560	820	150	3800	7650	560	630	1000	290	NU 20/560 ECMA
560	820	115	2330	4250	310	750	1000	210	NU 10/560 MA
560	1030	206	7210	11,200	780	560	800	805	NU 12/560 MA
600	730	60	897	2040	146	800	1000	50.5	NU 18/600 ECMA/HB1
600	730	52	468	1020	65.5	1800	2400	41	315836
600	730	78	1320	3450	260	750	1000	70	NU 28/600 ECMA
600	800	118	2920	6550	475	750	950	165	NU 29/600 ECMA/HB1
600	800	90	1900	3800	275	750	950	130	NU 19/600 ECMA/HA1
600	870	118	2750	5100	365	700	900	245	NU 10/600 MA
600	870	200	5390	11,000	780	600	900	415	NU 30/600 ECMA/HA1
600	870	155	4180	8000	570	600	900	325	NU 20/600 ECMA

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
630	780	88	1570	3800	285	750	950	98	NU 28/630 MA
630	780	56	605	1290	85	1700	2600	55	315933
630	780	88	1570	3900	305	750	950	100	N 28/630 MB
630	780	112	2200	5700	450	670	670	120	N 38/630 MB
630	780	69	1100	2500	183	800	1000	75	NJ 18/630 ECMA/HB1
630	850	128	3300	7200	510	700	900	230	NJ 29/630 ECMAS/HB1
630	850	100	2240	4400	315	700	900	165	NU 19/630 ECMA
630	920	128	3410	6200	430	630	850	285	NU 10/630 ECMA/HA2
630	920	212	6440	14,300	1000	530	850	490	NU 30/630/ 342771 A
630	920	170	4730	9500	670	560	850	400	NU 20/630 ECMA
630	1150	230	8580	13,700	915	450	700	1100	NU 12/630 ECMA
640	790	56	605	1290	83	1700	2200	56	315837
670	820	69	1230	2800	204	670	900	84.5	NJ 18/670 MA6/343016
670	820	56	616	1370	83	1600	2200	60	316012
670	900	103	2330	4750	325	630	850	195	NU 19/670 ECMA
670	980	136	3740	6800	465	600	800	350	NU 10/670 ECMA
670	980	180	5390	11,000	750	500	800	480	NU 20/670 ECMA
670	980	230	6600	14,000	950	500	800	600	NU 30/670 MA/342771
710	870	74	1450	3350	236	630	850	93.5	NU 18/710 ECMA/HB1
710	870	95	1940	5000	365	630	850	130	N 28/710 ECMB
710	950	140	3740	8300	570	600	800	295	NU 29/710 ECMA/HA1
710	1030	140	4680	8500	570	560	750	415	NU 10/710 ECN2MA
710	1030	185	5940	12,000	815	480	700	540	NU 20/710 ECMA
720	880	62	704	1560	91.5	1400	1900	74.5	315799

Continued

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
750	920	78	1470	3450	228	600	800	110	NU 18/750 MA
750	1000	112	2700	5600	365	560	700	255	319166 A
750	1090	150	4730	8800	585	430	670	490	NU 10/750 ECN2MA
750	1090	195	7040	14,600	980	430	670	635	NU 20/750 ECMA
800	980	82	1720	4150	280	530	700	145	NJ 18/800 ECMA
800	1150	200	7040	14,600	950	400	630	715	NU 20/800 ECMA
800	1150	155	5500	10,600	695	480	630	560	NU 10/800 ECMA/HB1
820	990	72	858	1960	112	1200	1600	100	315800 B
820	990	72	858	1960	112	1200	1600	100	315800
850	1030	106	2120	6000	405	500	670	195	N 28/850 MB
850	1030	74	935	2120	122	1200	1600	115	316200
850	1030	106	2120	6000	405	500	670	190	NU 28/850 MA
850	1120	118	3190	6950	440	480	600	330	NU 19/850 ECMA/HA1
850	1120	155	4680	11,200	720	480	600	430	N 29/850 ECMB6
900	1090	112	2700	7200	490	450	600	240	N 28/900 MB
900	1090	85	1120	2550	156	1100	1500	145	319161
900	1090	112	2700	7200	490	450	600	235	NU 28/900 MA/343017
900	1180	165	5830	14,000	880	430	560	560	NU 29/900 ECMA/HB1
900	1180	122	4130	8800	560	430	560	380	NU 19/900 ECMA
950	1150	90	1340	3100	190	1000	1300	170	315869 A
950	1250	175	5830	14,000	880	400	530	745	NU 29/950 ECMA
1000	1220	100	2640	6550	400	400	530	265	NU 18/1000 MA/HB1
1000	1220	128	3690	10,000	620	400	530	350	NF 28/1000 ECMP/HA1
1000	1320	185	7040	17,300	1080	360	480	700	NU 29/1000 ECFR
1030	1250	100	1510	3450	216	320	380	225	BC1B 319579
1060	1280	128	3580	10,400	640	300	360	360	N 28/1060 MB
1060	1400	195	7210	17,300	1060	280	340	870	NU 29/1060 ECMA/HB1

Table 6.6: Single row cylindrical roller bearings. Note that this table presents a selection only of the bearings available.—cont'd

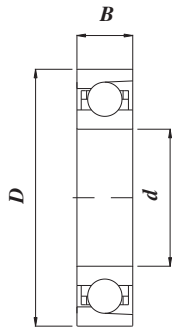
d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
1060	1400	250	9130	24,000	1460	260	320	1070	NU 39/1060 ECKMA/HA1
1060	1500	325	13,000	32,500	1950	220	280	1900	N 30/1060/ 241663
1120	1360	104	1650	3800	224	220	280	285	316201
1120	1580	345	15,700	39,000	2320	190	240	2150	NU 30/1120 K/VA901
1180	1420	106	3030	7800	455	260	320	350	NJ 18/1180 ECMA
1180	1540	206	8970	21,600	1290	190	240	1090	NUP 29/1180 ECMA/HA1
1180	1540	272	11,200	29,000	1730	200	260	1400	N 39/1180 MB
1180	1540	206	8970	21,600	1290	190	240	1050	NU 29/1180 ECMA/HA1
1250	1500	106	1720	4150	240	220	280	330	315913
1250	1750	290	12,800	30,500	1760	170	200	2320	N 20/1250 MB
1320	1600	122	3800	10,000	570	260	340	530	NU 18/1320 MAS/343016
1320	1720	230	11,400	30,500	1760	240	320	1500	NU 29/1320 ECFR
1320	1720	300	13,200	34,000	1960	220	320	1900	N 39/1320 MB
1320	1850	400	21,600	55,000	3100	190	240	3550	N 30/1320/ VE901
1400	1700	175	6600	18,300	1040	220	280	860	N 28/1400 ECMP
1500	1820	140	3300	8000	440	200	260	665	319301
1700	2060	160	3690	9150	490	160	200	925	319286
1700	2060	160	7210	19,300	1040	160	200	1150	NU 18/1700 ECMA/HB1

* SKF Explorer bearing.
Source: Table after SKF.

Reference to the deep groove bearing chart (Table 6.5) shows a suitable bearing could be:
ISO designation 6306*,
bore diameter 30 mm, outer diameter 72 mm,
width 19 mm,
$C = 29{,}600$ N, $C_o = 16{,}000$ N,
Reference speed 20,000 rpm,
Limiting speed 13,000 rpm.

2. Normally, it would be necessary to determine the viscosity ratio, κ, and $\eta_c P_u/P$ in order to determine a_{SKF}. In this case, the SKF life modification factor has been given. Nevertheless,

Table 6.7: Single row angular contact ball bearings. Note that this table presents a selection only of the bearings available.



d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
10	30	9	7.02	3.35	0.14	30,000	30,000	0.03	7200 BECBP
10	30	9	7.02	3.35	0.14	30,000	30,000	0.03	7200 BEP
12	32	10	7.61	3.8	0.16	26,000	26,000	0.036	7201 BECBP
12	32	10	7.61	3.8	0.16	26,000	26,000	0.036	7201 BEGAP
12	32	10	7.61	3.8	0.16	26,000	26,000	0.036	7201 BEP
12	37	12	10.6	5	0.208	24,000	24,000	0.063	7301 BEP
15	35	11	9.5	5.1	0.216	26,000	26,000	0.045	7202 BECBP*
15	35	11	9.5	5.1	0.216	26,000	26,000	0.045	7202 BEGAP*
15	35	11	9.5	5.1	0.216	26,000	26,000	0.045	7202 BEGBP*
15	35	11	8.84	4.8	0.204	24,000	24,000	0.045	7202 BEP
15	42	13	13	6.7	0.28	20,000	20,000	0.081	7302 BECBP
15	42	13	13	6.7	0.28	20,000	20,000	0.081	7302 BEP
17	40	12	11	5.85	0.25	22,000	22,000	0.07	7203 BECBM*
17	40	12	10.4	5.5	0.236	20,000	20,000	0.064	7203 BEP
17	47	14	15.9	8.3	0.355	19,000	19,000	0.11	7303 BECBP
17	47	14	15.9	8.3	0.355	19,000	19,000	0.11	7303 BEGAP
17	47	14	15.9	8.3	0.355	19,000	19,000	0.11	7303 BEP
20	47	14	13.3	7.65	0.325	18,000	19,000	0.11	7204 BECBM
20	47	14	14.3	8.15	0.345	19,000	19,000	0.11	7204 BECBP*
20	52	15	19	10	0.425	18,000	18,000	0.15	7304 BECBM*
20	52	15	17.4	9.5	0.4	16,000	16,000	0.14	7304 BEP

Table 6.7: Single row angular contact ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
25	52	15	15.6	10	0.43	17,000	17,000	0.14	7205 BECBM*
25	52	15	15.6	10.2	0.43	15,000	15,000	0.13	7205 BECBY
25	62	17	26.5	15.3	0.655	15,000	15,000	0.24	7305 BECBM*
25	62	17	24.2	14	0.6	14,000	14,000	0.23	7305 BEP
25	80	21	39.7	23.6	1	11,000	11,000	0.61	7405 BCBM
30	62	16	24	15.6	0.655	14,000	14,000	0.19	7206 BECAP*
30	72	19	35.5	21.2	0.9	13,000	13,000	0.37	7306 BECBM*
30	72	19	34.5	21.2	0.9	12,000	12,000	0.37	7306 BECBY
30	72	19	32.5	19.3	0.815	12,000	12,000	0.33	7306 BEP
30	90	23	47.5	29	1.22	10,000	10,000	0.85	7406 BM
35	72	17	31	20.8	0.88	12,000	12,000	0.3	7207 BECBM*
35	80	21	41.5	26.5	1.14	11,000	11,000	0.49	7307 BECBM*
35	80	21	39	24.5	1.04	10,000	10,000	0.45	7307 BEP
35	100	25	60.5	38	1.6	8500	9000	1.1	7407 BCBM
40	80	18	36.4	26	1.1	10,000	10,000	0.38	7208 BECBJ
40	80	18	34.5	24	1.02	10,000	10,000	0.39	7208 BEM
40	90	23	50	32.5	1.37	10,000	10,000	0.61	7308 BECAP*
40	110	27	70.2	45	1.9	8000	8000	1.4	7408 BCBM
45	85	19	37.7	28	1.2	9000	9000	0.43	7209 BECBJ
45	85	19	38	28.5	1.22	10,000	10,000	0.44	7209 BECBM*
45	100	25	61	40.5	1.73	9000	9000	0.82	7309 BECAP*
45	100	25	60.5	41.5	1.73	8000	8000	0.86	7309 BECBJ
45	120	29	85.2	55	2.36	7000	7500	1.8	7409 BCBM
47.625	114.3	26.988	83.2	58.5	2.28	8000	8000	1.25	AMS 15 ABP
50	90	20	39	30.5	1.29	8500	8500	0.47	7210 BECBJ
50	90	20	40	31	1.32	9000	9000	0.51	7210 BECBM*
50	90	20	39	30.5	1.29	8500	8500	0.47	7210 BEY
50	110	27	74.1	51	2.2	7500	7500	1.13	7310 BECBJ
50	110	27	75	51	2.16	8000	8000	1.16	7310 BEGAM*
50	130	31	95.6	64	2.7	6300	6700	2.25	7410 BM
50.8	114.3	26.988	83.2	58.5	2.28	8000	8000	1.2	AMS 16 ABP

Continued

Table 6.7: Single row angular contact ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
55	100	21	48.8	38	1.63	7500	7500	0.62	7211 BECBJ
55	100	21	49	40	1.66	8000	8000	0.66	7211 BECBM*
55	120	29	85.2	60	2.55	6700	6700	1.48	7311 BECBJ
55	120	29	85.2	60	2.55	6700	6700	1.48	7311 BECBY
55	140	33	111	76.5	3.25	6000	6300	2.75	7411 BCBM
55	140	33	111	76.5	3.25	6000	6300	2.75	7411 BM
57.15	127	31.75	111	80	3.35	6700	6700	1.6	AMS 18 ABP
60	110	22	57.2	45.5	1.93	7000	7000	0.83	7212 BECBJ
60	110	22	61	50	2.12	7500	7500	0.85	7212 BECBM*
60	130	31	104	76.5	3.2	6700	6700	1.71	7312 BECAP*
60	150	35	119	86.5	3.55	5600	5600	3.3	7412 BCBM
63.5	139.7	31.75	124	91.5	3.8	6300	6300	2	AMS 20 ABP
65	120	23	66.3	54	2.28	6300	6300	1	7213 BECBJ
65	120	23	69.5	57	2.45	6700	6700	1.1	7213 BECBM
65	140	33	108	80	3.35	5600	5600	2.35	7313 BECBF
65	160	37	130	96.5	3.8	5000	5300	3.85	7413 BGAM
70	125	24	71.5	60	2.5	6000	6000	1.1	7214 BECBJ
70	125	24	75	64	2.7	6300	6300	1.1	7214 BECBP*
70	150	35	127	98	3.9	5600	5600	2.55	7314 BECAP*
70	180	42	159	127	4.8	4500	4800	5	7414 BCBM
75	130	25	73.5	65.5	2.7	6300	6300	1.29	7215 BECBM
75	160	37	132	104	4.15	5300	5300	3.26	7315 BECBM*
75	190	45	168	140	5.1	4300	4500	6.85	7415 BCBM
80	140	26	85	75	3.05	5600	5600	1.59	7216 BECBM*
80	170	39	143	118	4.5	5000	5000	4.03	7316 BECBM*
80	170	39	143	118	4.5	4500	4500	3.7	7316 BEGAY
80	200	48	178	153	5.5	4000	4300	8	7416 CBM
80	200	48	178	153	5.5	4000	4300	8	7416 M
85	150	28	95.6	83	3.25	5000	5000	1.9	7217 BECBJ
85	150	28	102	90	3.55	5300	5300	1.83	7217 BECBP*
85	180	41	156	132	4.9	4800	4800	4.74	7317 BECBM*
85	180	41	153	132	4.9	4300	4300	4.59	7317 BECBY

Table 6.7: Single row angular contact ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
85	210	52	190	166	5.85	3800	4000	10.3	7417 BACBMC
90	160	30	108	96.5	3.65	4500	4500	2.34	7218 BECBJ
90	160	30	108	96.5	3.65	4500	4800	2.41	7218 BECBM
90	190	43	165	146	5.2	4000	4000	5.22	7318 BECBJ
90	190	43	166	146	5.3	4500	4500	5.53	7318 BECBM*
90	225	54	216	200	6.7	3600	3800	11.5	7418 CBM
90	225	54	216	200	6.7	3600	3800	11.5	7418 GAM
95	170	32	129	118	4.4	4800	4800	2.95	7219 BECBM*
95	200	45	180	163	5.7	4300	4300	6.41	7319 BECBM*
95	200	45	168	150	5.2	3800	3800	6.41	7319 BEGAF
95	250	55	251	245	7.8	3400	3400	13.5	7419 M
100	180	34	135	122	4.4	4000	4300	3.61	7220 BECBM
100	215	47	216	208	6.95	4000	4000	8	7320 BECBM*
100	215	47	203	190	6.4	3600	3600	8	7320 BEGAF
100	265	60	276	275	8.65	3200	3200	15.5	7420 CBM
105	190	36	148	137	4.8	3800	4000	4.18	7221 BECBM
105	225	49	203	193	6.4	3400	3600	9.12	7321 BECBM
110	200	38	153	143	4.9	3600	3800	4.95	7222 BECBM
110	240	50	225	224	7.2	3200	3400	10.7	7322 BECBM
120	180	28	87.1	93	3.2	3800	4000	2.4	7024 BGM
120	215	40	165	163	5.3	3400	3600	5.89	7224 BM
120	260	55	238	250	7.65	3000	3200	13.8	7324 BGBM
130	230	40	186	193	6.1	3200	3400	6.76	7226 BCBM
130	280	58	276	305	9	2800	2800	17.1	7326 BCBM
140	210	33	114	129	4.15	3200	3400	3.85	7028 BGM
140	250	42	199	212	6.4	2800	3000	8.83	7228 BCBM
140	300	62	302	345	9.8	2600	2600	21.3	7328 BCBM
150	225	35	133	146	4.55	3000	3200	4.7	7030 BGM
150	270	45	216	240	6.95	2600	2800	10.8	7230 BCBM
150	320	65	332	390	10.8	2400	2400	25	7330 BCBM
160	290	48	255	300	8.5	2400	2600	13.6	7232 BCBM

Continued

Table 6.7: Single row angular contact ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
160	340	68	358	440	11.8	2200	2400	30.3	7332 BCBM
170	310	52	281	345	9.5	2400	2400	16.7	7234 BCBM
170	360	72	390	490	12.7	2000	2200	34.6	7334 BCBM
180	280	46	195	240	6.7	2400	2600	10	7036 BGM
180	320	52	291	375	10	2200	2400	17.6	7236 BCAM
180	320	52	291	375	10	2200	2400	17.6	7236 BCBM
180	380	75	410	540	13.7	2000	2000	40	7336 BCBM
190	340	55	307	405	10.4	2000	2200	21.9	7238 BCBM
190	400	78	442	600	14.6	1900	1900	48.3	7338 BCBM
200	360	58	325	430	11	1800	2000	25	7240 BCBM
200	420	80	462	655	15.6	1800	1800	52.8	7340 BCBM
220	340	56	255	355	9	2000	2000	18	7044 BGM
220	400	65	390	560	13.4	1800	1800	35.2	7244 BCBM
220	460	88	494	720	16.6	1600	1700	70	7344 BCBM
240	360	56	260	375	9.15	1800	1900	19	7048 BGM
240	440	72	364	540	12.5	1600	1700	49	7248 BCBM
240	500	95	559	865	19	1400	1500	88.5	7348 BCBM
260	480	80	507	780	17.3	1400	1500	66	7252 BM
280	420	65	338	540	12.2	1500	1600	30	7056 BGM
280	500	80	520	850	18.3	1400	1400	69.5	7256 BM
300	460	74	423	695	15.3	1500	1600	42.5	7060 AGM
320	440	56	351	585	12.9	1500	1600	25.5	71964 AC
320	480	74	390	670	14	1300	1400	44.5	7064 BGM
335	450	56	281	480	10.4	1400	1400	25.5	466952
340	460	56	338	585	12.5	1500	1500	24	71968 ACMB
340	520	82	449	815	16.6	1200	1300	61.5	7068 BGM
340	620	92	702	1340	26	1100	1100	125	7268 BGM
360	440	38	234	425	9	1500	1500	12	71872 ACM/P5
360	480	56	351	630	13.2	1400	1500	28.5	71972 ACMB
360	540	82	520	950	19	1300	1300	62.5	7072 AGM
380	480	31	190	345	8	1300	1400	13.5	70876 AMB
380	560	82	507	950	18.6	1200	1300	65.5	7076 AM
400	540	65	423	780	15.6	1200	1300	42	71980 AM
400	600	90	605	1180	22.4	950	1200	89.5	307238
400	720	103	728	1500	26.5	900	950	190	7280 BM
410	560	70	423	830	16	1100	1100	49.5	468431
420	560	65	423	800	16	1200	1200	44.5	71984 AM
420	620	90	605	1180	22	1100	1100	95	7084 AM
440	650	94	650	1320	24	1000	1100	100	7088 AM
460	580	37	265	560	10.4	1100	1100	24.5	70892 AM

Table 6.7: Single row angular contact ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
460	620	74	507	1040	19	100	1100	58	71992 ACM
460	680	100	689	1460	25.5	950	1000	120	7092 AM
465	635	76	468	980	17.6	900	950	70.5	307352
480	700	100	702	1530	26.5	950	1000	125	7096 AM
500	620	37	276	620	11	1000	1000	27	708/500 AMB
500	670	78	553	1220	21.2	950	1000	78	719/500 AGMB
500	720	100	715	1600	27.5	900	950	130	70/500 AM
530	650	56	390	900	15.6	950	1000	39.5	718/530 AMB
530	710	82	618	1340	23.6	850	900	92	719/530 ACM
530	760	100	702	1600	27.5	750	800	150	307368 B
530	780	112	832	1900	31.5	800	850	175	70/530 AM
540	630	45	260	600	10.6	850	900	21.5	BA1B 311585
560	750	85	592	1290	22.8	800	850	105	719/560 AMB
600	730	42	338	735	14	800	850	38.5	708/600 AGMB
600	800	90	715	1730	27.5	750	800	125	719/600 ACM
600	870	118	884	2160	34	700	750	230	70/600 AGMB
630	700	22	135	440	7.2	750	750	11.5	BA1B 311712
630	920	128	956	2450	37.5	560	700	270	70/630 AMB
670	820	69	553	1290	22.4	700	750	77	718/670 ACMB
670	980	136	1170	3100	45.5	560	600	340	70/670 AMB
710	870	74	605	1630	24.5	630	670	93.5	718/710 ACMB
710	1030	140	1190	3250	46.5	560	600	370	70/710 AMB
750	920	78	650	1800	26.5	600	630	110	718/750 ACMB
750	1090	150	1300	3650	52	530	560	445	70/750 AMB
762	889	63.5	449	1270	18.6	560	600	58	BA1B 311576
800	1130	120	1080	3200	44	450	480	395	

Continued

Table 6.7: Single row angular contact ball bearings. Note that this table presents a selection only of the bearings available.—cont'd

d (mm)	*D* (mm)	*B* (mm)	*C* (kN)	C_o (kN)	Fatigue load limit (kN)	Reference speed (rpm)	Limiting speed (rpm)	Mass (kg)	Designation
									BA1B 311745
800	1150	155	1330	3800	52	480	500	500	70/800 AMB
850	1030	82	689	1860	29	500	530	140	718/850 AMB
850	1220	165	1530	4650	61	450	480	595	70/850 AMB
900	1030	63	416	1270	17.3	450	480	90	BA1B 307788
900	1280	170	1560	4900	63	400	430	665	70/900 AMB
950	1360	180	1630	5200	65.5	380	400	805	70/950 AMB
1000	1220	100	923	2750	39	400	430	245	718/1000 AMB
1000	1320	103	832	2450	34.5	340	360	370	307101 C
1000	1420	185	1630	5400	67	360	380	890	70/1000 AMB
1060	1500	195	1680	5700	71	320	340	1050	70/1060 AMB
1120	1360	106	1060	3750	45	340	360	320	718/1120 AMB
1120	1580	200	1720	5850	71	300	320	1150	70/1120 AMB
1180	1660	212	1740	6200	73.5	280	280	1350	70/1180 AMB
1250	1500	80	806	2700	36	280	300	295	708/1250 AMB
1250	1750	218	1780	6550	76.5	240	260	1600	70/1250 AMB
1700	1900	80	975	4550	45.5	160	170	310	BA1B 307756

* SKF Explorer bearing.
Source: Table after SKF.

the steps are included below. The viscosity ratio is given by $\kappa = 40/14 = 2.86$. The contamination factor is that for high cleanliness, $\eta_c = 0.6$.

$$\eta_c \frac{P_u}{P} = 0.6 \frac{0.67}{2.8} = 0.6 \times 0.239 = 0.14$$

From manufacturer's tables, using κ and $\eta_c P_u/P$, $a_{SKF} = 14$.

Hence from Eqn (6.5), for a desired reliability of 90%,

$$L_{nm} = a_1 a_{SKF} \left(\frac{C}{P}\right)^p = 1 \times 14 \times \left(\frac{29{,}600}{2800}\right)^3 = 16{,}540 \text{ million revolutions}$$

This high life suggests either a very extended operation or an opportunity for an alternative design that does not require this quality of bearing installation.

6.3 Bearing Installation

The practical use of rolling element bearings requires consideration of their installation, as well as correct selection. Bearing installation considerations include the bearing combination, the mounting of the bearings, and the provision of lubrication.

A typical application of rolling element bearings is the support of a rotating shaft. If the operating temperature of the machine varies, the shaft length can grow relative to the casing or mounting arrangement. An idea of the magnitude of the axial shaft growth can be estimated by

$$\Delta L = L_o \alpha \Delta T \tag{6.7}$$

where

ΔL is the change in length (m),
L_o is the original length (m),
α is the coefficient of linear thermal expansion (per °C), and
ΔT is the temperature rise (°C).

For a gas turbine engine the difference in temperature between the casing and the shaft can be 50 °C. If the original length of the steel shaft was 1.0 m the growth of the shaft would be

$$\Delta L = L_o \alpha \Delta T = 1.0 \times 11 \times 10^{-6} \times 50 = 5.5 \times 10^{-4} \text{ m} = 0.55 \text{ mm}.$$

This is a considerable axial movement within a machine and must be allowed for if significant loadings and resultant stresses and possible contact between stationary and rotating components are to be avoided. A typical solution to this kind of situation is to allow for a limited axial movement on one bearing as illustrated in Figure 6.5. Here, one bearing has the location of its inner and outer races "fixed" relative to the shaft and housing by means of shoulders and locking rings. The location of the other bearing is fixed only for the inner race. If the shaft expands, the axial movement can be accommodated by limited sliding motion of the outer race of the right-hand bearing within the housing bore. For this kind of arrangement, the bearings are referred to as fixed and floating bearings. Similar movement is also possible for the arrangement shown in Figure 6.6. Here the right-hand bearing is a cylindrical roller bearing, and the axial location of the roller is not fixed and can move or float axially to the limited extents of the race to take up any axial movement or expansion of the shaft.

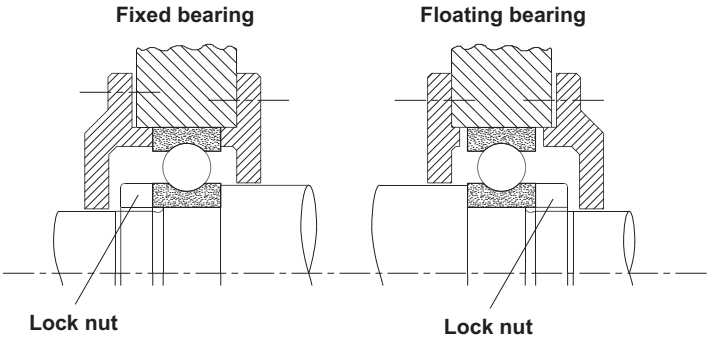


Figure 6.5
Basic bearing mounting using two deep groove ball bearings for a rotating horizontal shaft for moderate radial and axial loading.

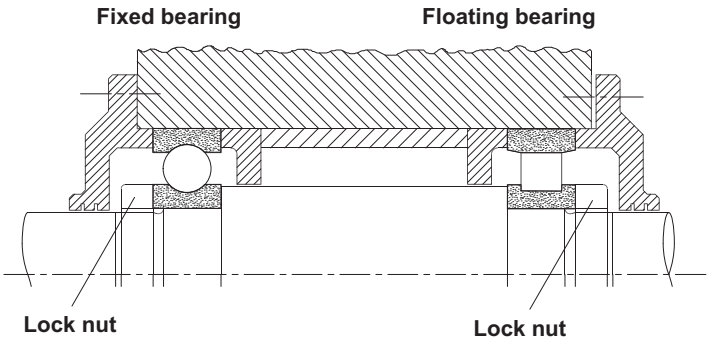


Figure 6.6
Basic bearing mounting using a deep groove and a cylindrical roller bearing for moderate radial loads at the "locating deep groove bearing" and high radial load capacity at the cylindrical roller bearing.

Correct lubrication is essential in order to ensure that the calculated life of the bearing is achieved. Too much lubrication can result in increased levels of viscous dissipation of energy in the bearings and resultant overheating of the lubricant and the bearing. Too little lubrication can result in excessive wear and early failure. The form of lubrication depends on what is required by the bearing application. Grease lubrication is normally the easiest and requires injection of a specific quantity of grease into the bearing and some method of dirt and dust exclusion. Bearings can be purchased which are grease filled and sealed for life. Oil lubrication can be supplied by means of partial submersion, as shown in Figure 6.7, a recirculation circuit as shown in Figure 6.8 or oil spot lubrication. A recirculation system typically consists of a cooler, an oil pump, a filter, oil jets, scavenge or collector holes, and a reservoir. Oil spot lubrication involves the application of very small quantities of lubricant

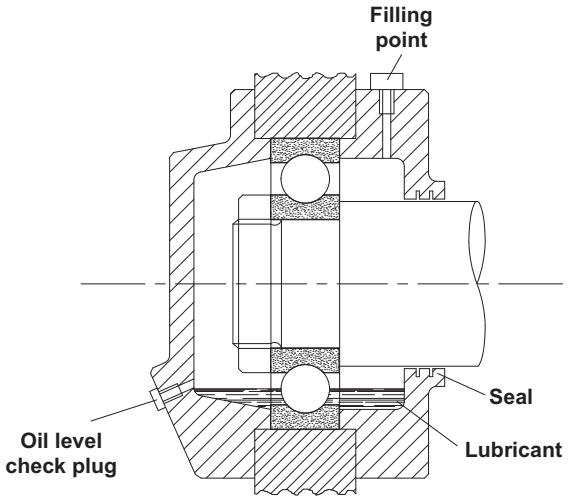


Figure 6.7
Partial submersion lubrication arrangement.

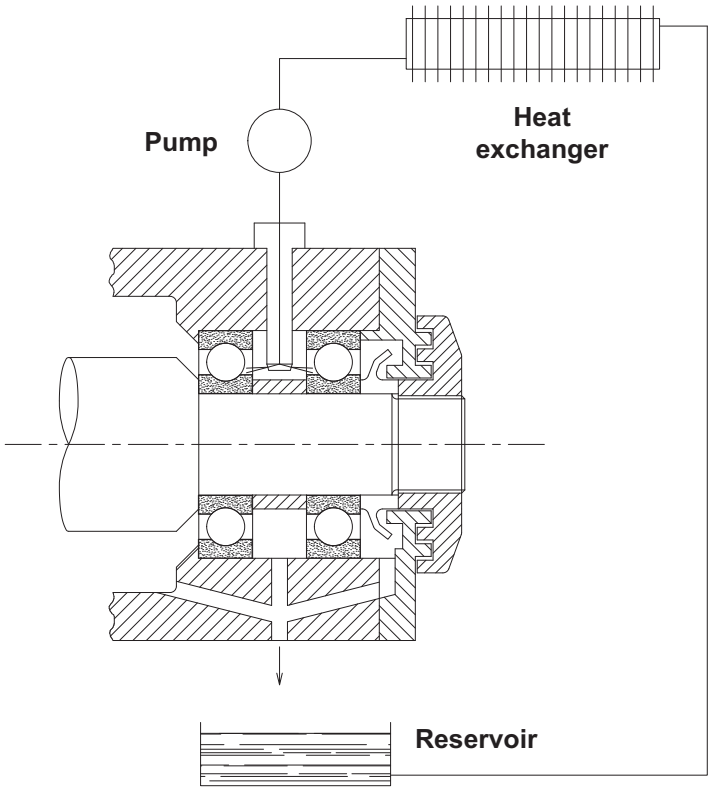


Figure 6.8
Recirculation lubrication system.

directly to the bearing by means of compressed air. Once again the bearing manufacturer's handbooks are a good source of information for lubricant supply rates and systems.

6.4 Radial Location

The load capacity for a bearing, as defined by test procedures, within catalogs and by the basic dynamic rating, is based upon full support of the raceways at the inner and outer diameters, in the case of a radial bearing and the rings and washers in the case of, for instance, a thrust or taper roller bearing. The mating bearing seats for a bearing must be manufactured with adequate accuracy to provide the necessary support and prevent them from turning within their seats under load. The surfaces should be uninterrupted without grooves or other such features. Generally, an interference fit is required for one ring and a loose fit is required for the other. Table 6.8 provides an indication of the type of fit for a given application and Tables 6.9 and 6.10 provide recommended fits for the case of radial bearings with a solid steel shaft and housings respectively.

If a bearing is mounted on a hollow cylindrical shaft, it is generally necessary to use a slightly higher interference fit than the equivalent sized solid shaft in order to provide a similar surface pressure between the shaft and inner raceway bearing seat. The fit is not normally appreciably affected until the ratio $c_i = d_i/d \geq 0.5$. For values above this a correction factor, normally available from bearing manufacturers, can be applied to the mean probable interference based on a solid shaft application to provide the requisite interference for the hollow shaft application.

Table 6.8: Recommended type of fit depending on operating conditions for radial bearings.

Operating conditions	Load condition	Example application	Recommended fits
Rotating inner ring Stationary outer ring Constant load direction	Rotating load on inner ring Stationary load on outer ring	Belt drives	Interference fit for the inner ring Loose fit for the outer ring
Stationary inner ring Rotating outer ring Constant load direction	Stationary load on inner ring Rotating load on outer ring	Conveyer idlers Wheel hub bearings	Loose fit for the inner ring Interference fit for the outer ring
Rotating inner ring Stationary outer ring Load rotates with the inner ring	Stationary load on inner ring Rotating load on outer ring	Vibrating applications Motors	Interference fit for the outer ring Loose fit for the inner ring
Stationary inner ring Rotating outer ring Load rotates with the outer ring	Rotating load on inner ring Stationary load on outer ring	Gyratory crusher Merry go round	Interference fit for the inner ring Loose fit for the outer ring

Table 6.9: Recommended radial rolling element bearing fits for solid cylindrical steel shafts.

Load conditions		Examples	Shaft diameter (mm)			Tolerance of shaft	Remarks
			Ball brgs	Cylindrical roller brgs, Tapered roller brgs	Spherical roller brgs		
Rotating outer ring load	Easy axial displacement of inner ring on shaft desirable	Wheels on stationary axles		All shaft diameters		g6	Use g5 and h5 where accuracy is required. In case of large bearings, f6 can be used to allow easy axial movement.
	Easy axial displacement of inner ring on shaft unnecessary	Tension pulleys Rope sheaves				h6	
Rotating inner ring load or direction of load indeterminate	Light loads or variable loads (<0.06C)	Electrical home appliances, Pumps, Blowers,	<18	—	—	js5	
			18 to 100	<40	—	js6(j6)	
		Transport vehicles, Precision machinery, Machine tools	100 to 200	40 to 140	—	k6	
			—	140 to 200	—	m6	
	Normal loads (0.06–0.13C)	General bearing applications,	<18	—	—	js5 or js6 (j5 or j6)	k6 and m6 can be used for single row tapered roller bearings and single row angular contact ball bearings instead of k5 and m5.
		Medium and large motors,	18 to 100	<40	<40	k5 or k6	
		Turbines, Pumps,	100 to 140	40 to 100	40 to 65	m5 or m6	
		Engine main bearings, Gears,	140 to 200	100 to 140	65 to 100	m6	
		Woodworking machines	200 to 280	140 to 200	100 to 140	n6	
			—	200 to 400	140 to 280	p6	
			—	—	280 to 500	r6	
			—	—	Over 500	r7	
	Heavy loads or shock loads (>0.13C)	Railway axleboxes,	—	50 to 140	50 to 100	n6	More than CN bearing internal clearance is necessary.
		Industrial vehicles,	—	140 to 200	100 to 140	p6	
		Traction motors,	—	Over 200	140 to 200	r6	
		Construction equipment, Crushers	—	—	200 to 500	r7	
Axial loads only				All shaft diameters		js6 (j6)	—

Source: After NSK.

Table 6.10: Recommended radial rolling element bearing fits with housings. This table is applicable to cast iron and steel housings. For housings made of light alloys, the interference should be tighter than those in this table.

Load conditions			Examples	Tolerances for housing bores	Axial displacement of outer ring	Remarks
Solid housings	Rotating outer ring load	Heavy loads on bearing in thin-walled housing or heavy shock loads	Automotive wheel hubs (roller bearings) Crane travelling wheels	P7	Impossible	—
		Normal or heavy loads	Automotive wheel hubs (ball bearings) Vibrating screens	N7		
		Light or variable loads	Conveyor rollers Rope sheaves Tension pulleys	M7		
	Direction of load indeterminate	Heavy shock loads	Traction motors	M7		
		Normal or heavy loads	Pumps Crankshaft main bearings Medium and large motors	K7	Generally impossible	If axial displacement of the outer ring is not required.
Solid or split housings		Normal or light loads		JS7 (J7)	Possible	Axial displacement of outer ring is necessary.
	Rotating inner ring load	Loads of all kinds	General bearing applications Railway axleboxes	H7	Easily possible	—
		Normal or light loads	Plummer blocks	H8		
		High temperature rise of inner ring through shaft	Paper dryers	G7		
Solid housing		Accurate running desirable under normal or light loads	Grinding spindle Rear ball bearings High speed centrifugal compressor free bearings	JS6 (J6)	Possible	—
	Direction of load indeterminate		Grinding spindle Front ball bearings High speed centrifugal compressor fixed bearings	K6	Generally impossible	For heavy loads, interference fit tighter than K is used. When high accuracy is required, very strict tolerances should be used for fitting.
	Rotating inner ring load	Accurate running and high rigidity desirable under variable loads	Cylindrical roller Bearings for machine Tool main spindle	M6 or N6	Impossible	
		Minimum noise is required	Electrical home appliances	H6	Easily possible	—

Source: After NSK.

6.5 Conclusions

Bearings are used to support a load while allowing relative motion between two elements of a machine. Typical applications are shafts where the shaft rotates relative to a housing, but bearings can also be used to facilitate linear relative motion for, say, machine slides. This chapter has introduced the selection and specification of rolling element bearings. Rolling element bearings are produced by specialist manufacturers, and many types and sizes are available as stock items. For further information on bearings, the reader is recommended to view specific manufacturer's literature and the texts by Harris (2001) and Brandlein et al. (1999).

Reference

Books and Papers

Brandlein, J., Eschmann, P., Hasbargen, L., Weigand, K., 1999. Ball and Roller Bearings. Theory, Design and Application. John Wiley & Sons.
Harris, T.A., 2001. Rolling Bearing Analysis, fourth ed. John Wiley & Sons.

Further Reading

Harnoy, A., 2002. Bearing Design in Machinery. Marcel Dekker.
Hindhede, U., Zimmerman, J.R., Hopkins, R.B., Erisman, R.J., Hull, W.C., Lang, J.D., 1983. Machine Design Fundamentals. A Practical Approach. Wiley.

Standards

ANSI/ABMA Standard 11-1990(R1999). Load ratings and fatigue life for roller bearings.
ANSI/ABMA Standard 7-1995(R2001). Shaft and housing fits for metric radial ball and roller bearings (except tapered roller bearings) conforming to basic boundary plans.
ANSI/ABMA Standard 9-1990(R2000). Load ratings and fatigue life for ball bearings.
BS 4231:1992, ISO 3448:1992. Classification for viscosity grades of industrial liquid lubricants.
BS ISO 10317:2008. Rolling bearings. Tapered roller bearings. Designation system.
BS ISO 104:2002. Rolling bearings. Thrust bearings. Boundary dimensions, general plan.
BS ISO 113:2010. Rolling bearings. Plummer block housings. Boundary dimensions.
BS ISO 14728−1:2004. Rolling bearings. Linear motion rolling bearings. Dynamic load ratings and rating life.
BS ISO 14728−2:2004. Rolling bearings. Linear motion rolling bearings. Static load ratings.
BS ISO 15:2011. Rolling bearings. Radial bearings. Boundary dimensions, general plan.
BS ISO 15243:2004. Rolling bearings. Damage and failures. Terms, characteristics and causes.
BS ISO 281:2007. Rolling bearings. Dynamic load ratings and rating life.
BS ISO 2982−2:2001. Rolling bearings. Accessories. Locknuts and locking devices. Dimensions.
BS ISO 3290−1:2008. Rolling bearings. Balls. Steel balls.
BS ISO 355:2007+A1:2012. Rolling bearings. Tapered roller bearings. Boundary dimensions and series designations.
BS ISO 492:2002. Rolling bearings. Radial bearings. Tolerances.
BS ISO 5593:1997. Rolling bearings. Vocabulary.
BS ISO 76:2006. Rolling bearings. Static load ratings.
PD ISO/TR 1281−2:2008. Rolling bearings. Explanatory notes on ISO 281. Modified rating life calculation, based on a systems approach to fatigue stresses.
Society of Automotive Engineers. SAE J300. Engine oil viscosity classification. 2013.

Web Sites

At the time of going to press the world-wide-web contained useful information relating to this chapter at the following sites.

www.abcbearings.com/
www.arb-bearings.com/
www.bocabearings.com/
www.championballbearings.com/
www.cooperbearings.com/
www.fag.de/
www.hb-bearings.com/
www.ina.com/
www.nationalbearings.com/
www.newindo.com/
www.nsk.com/
www.ntnamericas.com/en/
www.peerbearing.com/
www.rbcbearings.com/
www.skf.com/
www.thordonbearings.com/
www.timken.com/
www.toyobearings.com/

Nomenclature

The following symbols are used in this chapter. Generally, preferred SI units have been stated:

a_1	life adjustment factor for reliability
a_{SKF}	SKF life modification factor
B	width (m)
C	basic dynamic load rating (N)
C_o	basic static load rating (N)
d	bore diameter (m)
d_m	mean diameter (m)
D	outer diameter (m)
ISO	International Organization for Standardization
k	constant
L	life (number of revolutions)
L_1	life at load P_1 (number of revolutions)
L_{10}	rated life at the rated load (number of revolutions)
L_2	life at load P_2 (number of revolutions)
L_{cat}	catalog rating (N)
L_d	design life (number of revolutions)
L_{nm}	rating life (at $100 - n\%$ reliability) in millions of revolutions

L_o original length (m)
P load (N)
P_1 load (N)
P_2 load (N)
P_d design load (N)
P_u fatigue load limit (N)
R applied radial load (N)
SAE Society of Automotive Engineers
T thrust load (N)
V rotating factor
X radial factor
Y thrust factor
α coefficient of expansion (per K)
ΔL change in length (m)
ΔT temperature rise (^oC)
η_c contamination factor
κ viscosity ratio
ν operating kinematic viscosity of the lubricant (m^2/s)
ν_1 rated kinematic viscosity depending on the bearing mean diameter and rotational speed (m^2/s).

CHAPTER 7

Shafts

Chapter Outline

7.1 Introduction to Shaft Design 255
7.2 Shaft–Hub Connection 260
7.3 Shaft–Shaft Connection Couplings 263
7.4 Critical Speeds and Shaft Deflection 265
7.4.1 Macaulay's Method for Calculating the Deflection of Beams 268
7.4.2 Castigliano's Theorem for Calculating Shaft Deflections 279
7.5 ASME Design Code for Transmission Shafting 282
7.6 Detailed Design Case Study 302
7.7 Conclusions 312
References 313
Further Reading 313
Nomenclature 314

Abstract

The objective of this chapter is to introduce the concepts and principles of shaft design. Specific attention is given to the arrangement of machine elements and features on a shaft, the connection of shafts, determining the deflection of shafts and critical speeds, as well as specifying shaft dimensions for strength and fluctuating load integrity. An overall shaft design procedure is presented including consideration of bearing and component mounting and shaft dynamics using the ASME design code for transmission shafting.

7.1 Introduction to Shaft Design

The term shaft usually refers to a component of circular cross-section that rotates and transmits power from a driving device, such as a motor or engine, through a machine. Shafts can carry gears, pulleys, and sprockets to transmit rotary motion and power via mating gears, belts, and chains. Alternatively, a shaft may simply connect to another shaft via a coupling. A shaft can be stationary and support a rotating member, such as the short shafts that support the nondriven wheels of automobiles often referred to as spindles. Some common shaft arrangements are illustrated in Figure 7.1.

Shaft design considerations include:

1. Size and spacing of components (as on a general assembly drawing), tolerances,
2. Material selection, material treatments,

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00007-1

Copyright © 2014 Elsevier Ltd. All rights reserved.

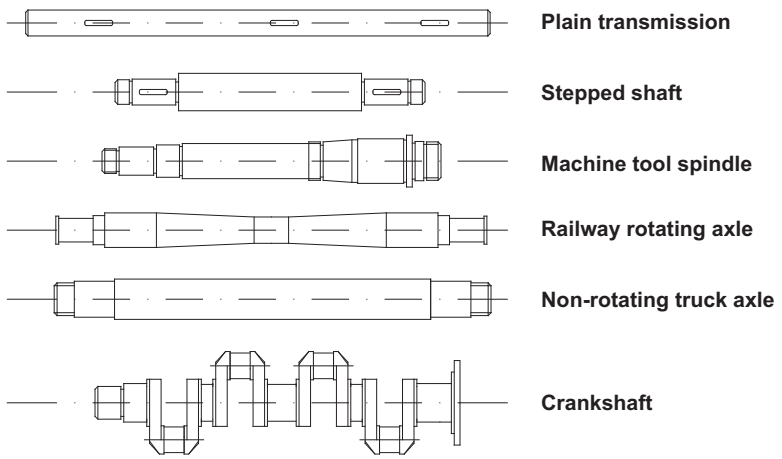


Figure 7.1
Typical shaft arrangements. *Adapted from Reshetov (1978).*

3. Deflection and rigidity,
   a. Bending deflection,
   b. Torsional deflection,
   c. Slope at bearings,
   d. Shear deflection,
4. Stress and strength,
   a. Static strength,
   b. Fatigue,
   c. Reliability,
5. Frequency response,
6. Manufacturing constraints.

Shafts typically consist of a series of stepped diameters accommodating bearing mounts and providing shoulders for locating devices such as gears, sprockets, and pulleys to butt up against and keys often used to prevent rotation, relative to the shaft, of these "added" components. A typical arrangement for a transmission shaft supporting a gear and pulley wheel illustrating the use of constant diameter sections and shoulders is shown in Figure 7.2.

Shafts must be designed so that deflections are within acceptable levels. Too much deflection can, for example, degrade gear performance and cause noise and vibration. The maximum allowable deflection of a shaft is usually determined by limitations set on the critical speed, minimum deflections required for gear operation, and bearing requirements. In general, deflections should not cause mating gear teeth to separate more than about 0.13 mm and the slope of the gear axes should not exceed about 0.03°. The deflection of the journal section of a shaft across a plain bearing should be small in comparison with the oil film thickness. Torsional and lateral deflection both contribute to lower critical speed. In addition,

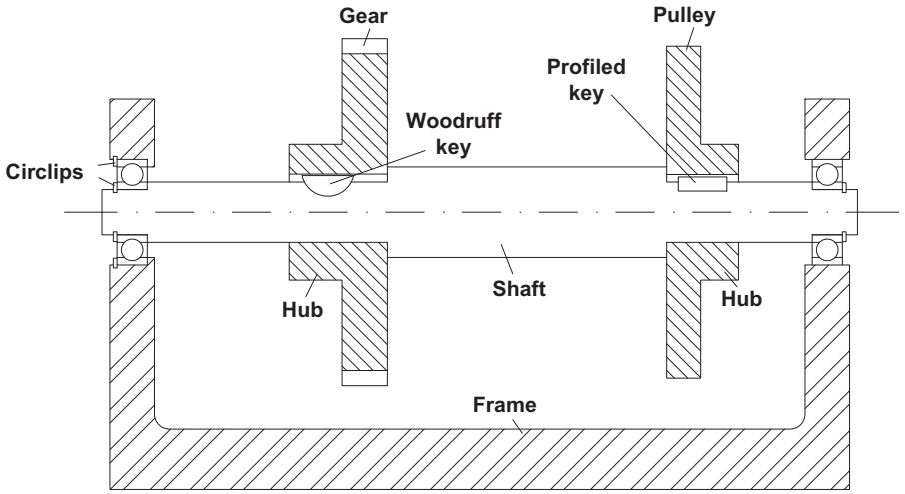


Figure 7.2
Typical shaft arrangement incorporating constant diameter sections and shoulders for locating added components.

shaft angular deflection at rolling element bearings should not exceed 0.04°, with the exception being self-aligning rolling element bearings.

Shafts can be subjected to a variety of combinations of axial, bending, and torsional loads (see Figure 7.3) which may fluctuate or vary with time. Typically, a rotating shaft transmitting power is subjected to a constant torque together with a completely reversed bending load, producing a mean torsional stress and an alternating bending stress respectively.

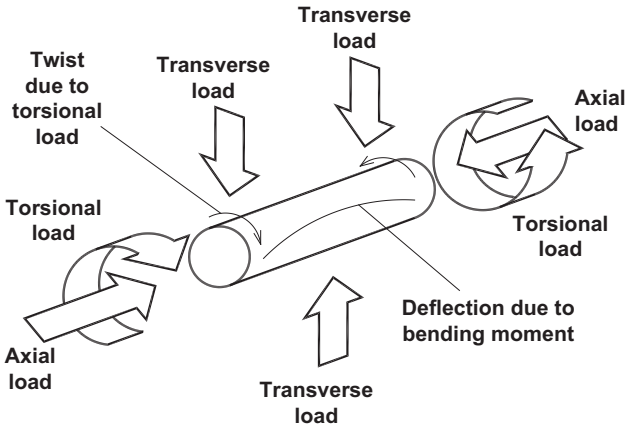


Figure 7.3
Typical shaft loading and deflection. *Adapted from Beswarick (1994a).*

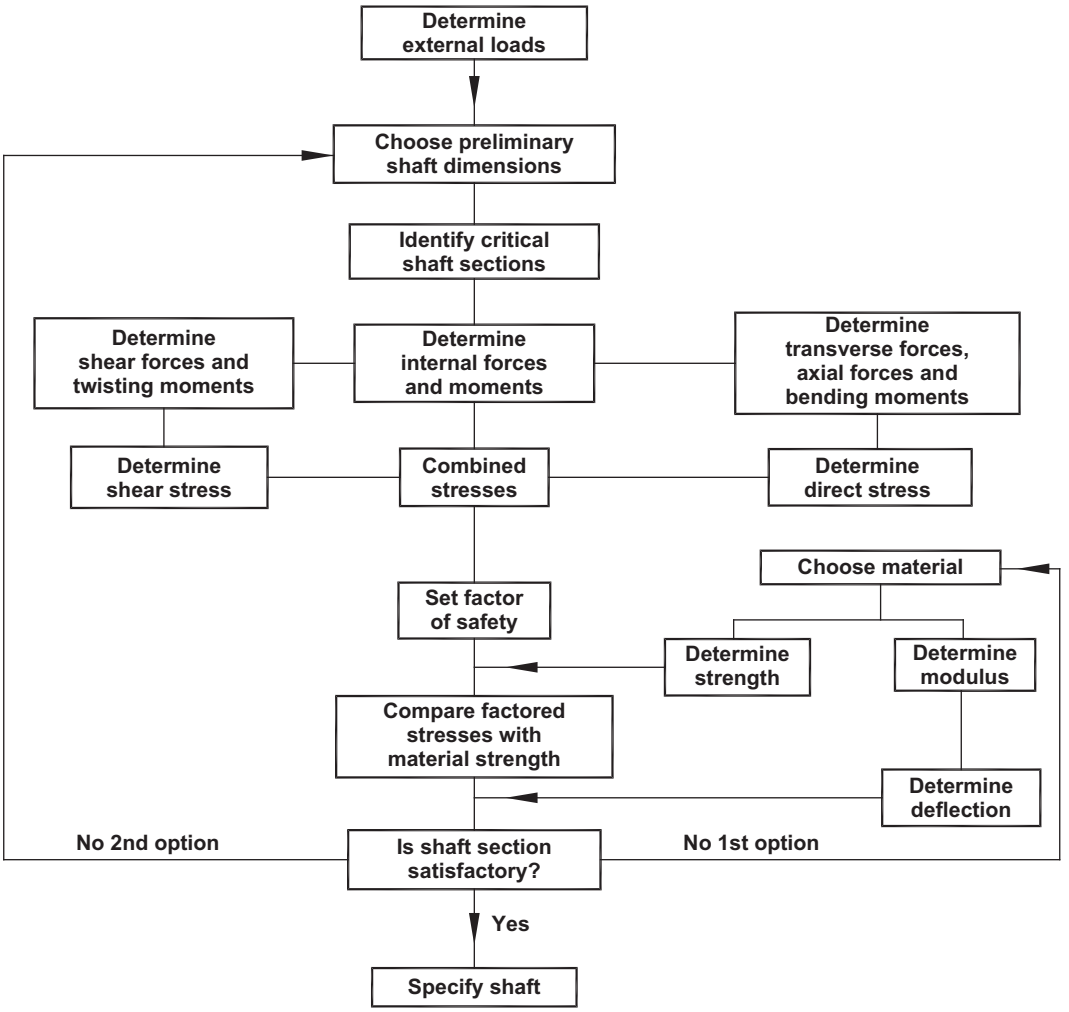


Figure 7.4
Design procedure flow chart for shaft strength and rigidity. *Adapted from Beswarick (1994a).*

Shafts should be designed to avoid operation at, or near, critical speeds. This is usually achieved by the provision of sufficient lateral rigidity so that the lowest critical speed is significantly above the range of operation. If torsional fluctuations are present (e.g. engine crankshafts, cam-shafts, compressors) the torsional natural frequencies of the shaft must be significantly different to the torsional input frequency. This can be achieved by providing sufficient torsional stiffness so that the shaft's lowest natural frequency is much higher than the highest torsional input frequency.

Rotating shafts must generally be supported by bearings. For simplicity of manufacture, it is desirable to use just two sets of bearings. If more bearings are required, precise alignment of

the bearings is necessary. Provision for thrust load capability and axial location of the shaft is normally supplied by just one thrust bearing taking thrust in each direction. It is important that the structural members supporting the shaft bearings are sufficiently strong and rigid.

The following list outlines a shaft design procedure for a shaft experiencing constant loading. The flow charts given in Figures 7.4 and 7.5 can be used to guide and facilitate design for shaft strength and rigidity and fluctuating load capability. Pyrhonen (2008) provides an overview of shaft design with specific reference to electrical machine design.

1. Determine the shaft rotational speed.
2. Determine the power or torque to be transmitted by the shaft.
3. Determine the dimensions of the power transmitting devices and other components mounted on the shaft and specify locations for each device.
4. Specify the locations of the bearings to support the shaft.
5. Propose a general form or scheme for the shaft geometry considering how each component will be located axially and how power transmission will take place.

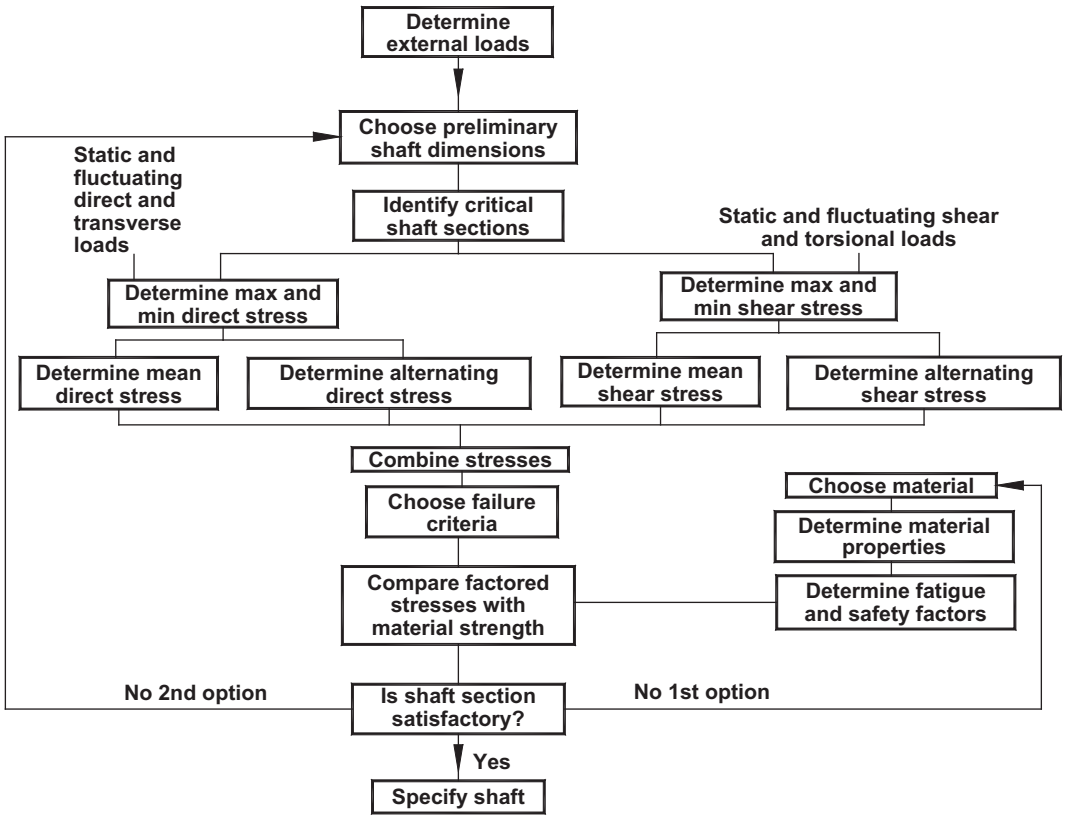


Figure 7.5
Design procedure flow chart for a shaft with fluctuating loading. *Adapted from Beswarick (1994b).*

6. Determine the magnitude of the torques throughout the shaft.
7. Determine the forces exerted on the shaft.
8. Produce shearing force and bending moment diagrams so that the distribution of bending moments in the shaft can be determined.
9. Select a material for the shaft and specify any heat treatments, etc.
10. Determine an appropriate design stress taking into account the type of loading (whether smooth, shock, repeated, reversed).
11. Analyze all the critical points on the shaft and determine the minimum acceptable diameter at each point to ensure safe design.
12. Determine the deflections of the shaft at critical locations and estimate the critical frequencies.
13. Specify the final dimensions of the shaft. This is best achieved using a detailed manufacturing drawing to a recognized standard (see, for example, the Manual of British Standards in Engineering and Drawing Design), and the drawing should include all the information required to ensure the desired quality. Typically, this will include material specifications, dimensions, and tolerances (bilateral, runout, data, etc. (see Chapter 19)), surface finishes, material treatments, and inspection procedures.

The following general principles should be observed in shaft design.

- Keep shafts as short as possible with the bearings close to applied loads. This will reduce shaft deflection and bending moments and increase critical speeds.
- If possible, locate stress raisers away from highly stressed regions of the shaft. Use generous fillet radii and smooth surface finishes and consider using local surface strengthening processes such as shot peening and cold rolling.
- If weight is critical use hollow shafts.

An overview of shaft−hub connection methods is given in Section 7.2, shaft to shaft connection methods in Section 7.3, and the determination of critical speeds in Section 7.4. In Section 7.5, the ASME equation for the design of transmission shafts is introduced.

7.2 Shaft−Hub Connection

Power transmitting components such as gears, pulleys, and sprockets need to be mounted on shafts securely and located axially with respect to mating components. In addition, a method of transmitting torque between the shaft and the component must be supplied. The portion of the component in contact with the shaft is called the hub and can be attached to, or driven by, the shaft by keys, pins, setscrews, press and shrink fits, splines, and taper bushes. Table 7.1 identifies the merits of various connection methods. Alternatively, the component can be formed as an integral part of a shaft. An example of this would be the cam on an automotive cam-shaft.

Table 7.1: Merits of various shaft–hub connections.

	Pin	Grub screw	Clamp	Press fit	Shrink fit	Spline	Key	Taper bush
High torque capacity	×	×	✓	×	✓	✓	✓	✓
Large axial loads	✓	×	✓	×	✓	×	×	✓
Axially compact	×	×	×	✓	✓	✓	✓	✓
Axial location provision	✓	✓	✓	✓	✓	×	×	✓
Easy hub replacement	×	✓	✓	×	×	✓	✓	✓
Fatigue	×	×	✓	✓	✓	×	×	✓
Accurate angular positioning	✓	×	×	×	×	✓	✓	(✓)
Easy position adjustment	×	✓	✓	×	×	×	×	✓

After Hurst (1994).

Figure 7.6 illustrates the practical implementation of several shaft–hub connection methods. Gears, for example, can be gripped axially between a shoulder on a shaft and a spacer with the torque transmitted through a key. Various configurations of keys exist including square, flat, and round keys as shown in Figure 7.7. The grooves in the shaft and hub, into which the key fits, are called keyways or keyseats. A simpler and less expensive method for transmitting light loads is to use pins and various pin types, is illustrated in Figure 7.8. An inexpensive method of providing axial location of hubs and bearings on shafts is to use circlips as shown

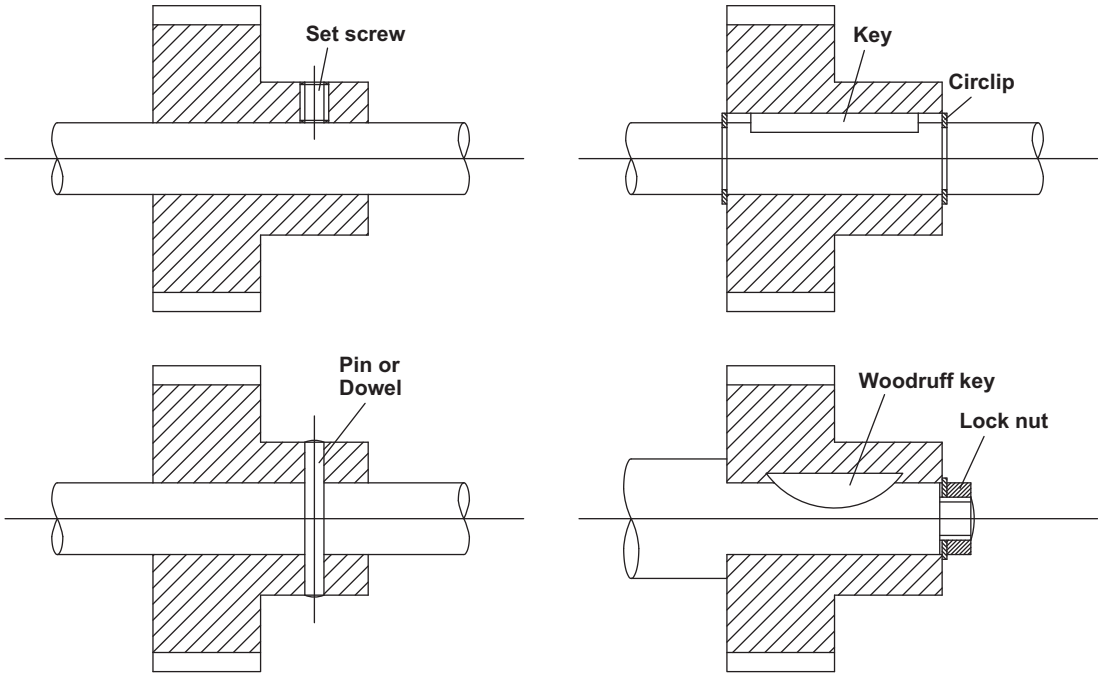


Figure 7.6
Alternative methods of shaft–hub connection.

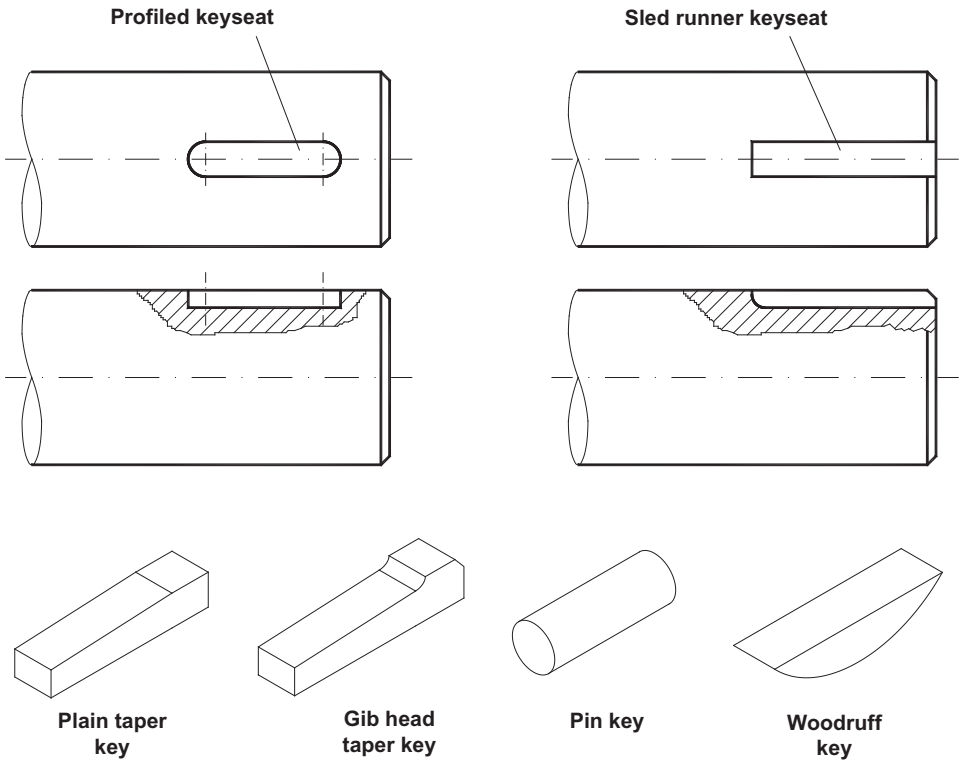


Figure 7.7
Keys for torque transmission and component location.

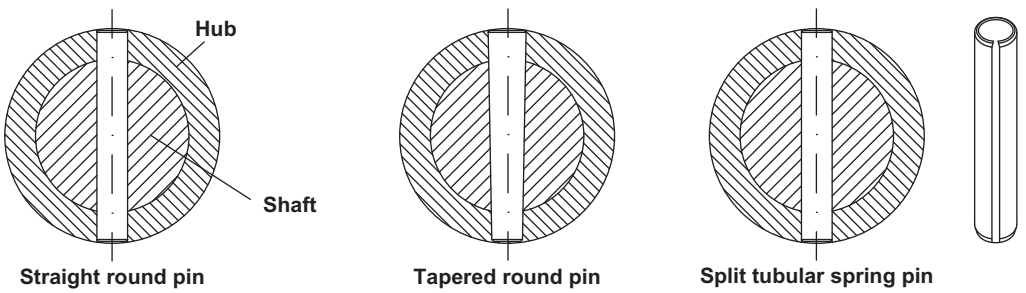


Figure 7.8
Pins for torque transmission and component location.

in Figures 7.2 and 7.9. One of the simplest shaft–hub attachments is to use an interference fit, where the hub bore is slightly smaller than the shaft diameter. Assembly is achieved by press fitting, or by thermal expansion of the outer ring by heating and thermal contraction of the inner ring by use of liquid nitrogen. The design of interference fits is covered in greater detail in Section 19.2.2. Mating splines, as shown in Figure 7.10, comprise teeth cut into both the

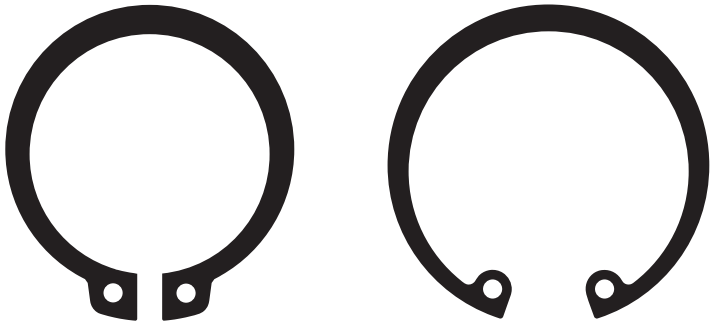


Figure 7.9
Snap rings or circlips.

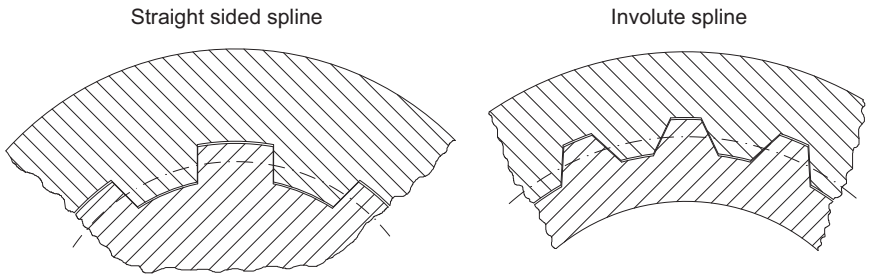


Figure 7.10
Splines.

shaft and the hub and provide one of the strongest methods of transmitting torque. Both splines and keys can be designed to allow axial sliding along the shaft.

7.3 Shaft–Shaft Connection Couplings

In order to transmit power from one shaft to another, a coupling or clutch can be used (for clutches see Chapter 13). There are two general types of coupling; rigid and flexible. Rigid couplings are designed to connect two shafts together so that no relative motion occurs between them (see Figure 7.11). Rigid couplings are suitable when precise alignment of two shafts is required. If significant radial or axial misalignment occurs, high stresses may result which can lead to early failure. Flexible couplings (see Figure 7.12) are designed to transmit torque while permitting some axial, radial, and angular misalignment. Many forms of flexible coupling are available (e.g. see manufacturers' catalogs such as Turboflex and Fenner). Each coupling is designed to transmit a given limiting torque. Generally, flexible couplings are able to tolerate up to $\pm 3^{\circ}$ of angular misalignment and up to 0.75 mm parallel misalignment depending on their design. If more misalignment is required, a universal joint can be used (see Figure 7.13). Couplings are considered in detail by Neale et al. (1998), Piotrowski (2006) and Sclater (2011).

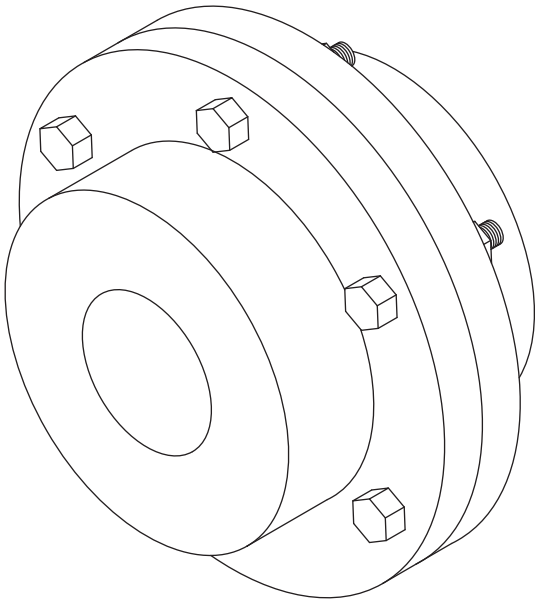


Figure 7.11
Rigid coupling.



Figure 7.12
Flexible couplings. *Photograph courtesy of Cross and Morse.*

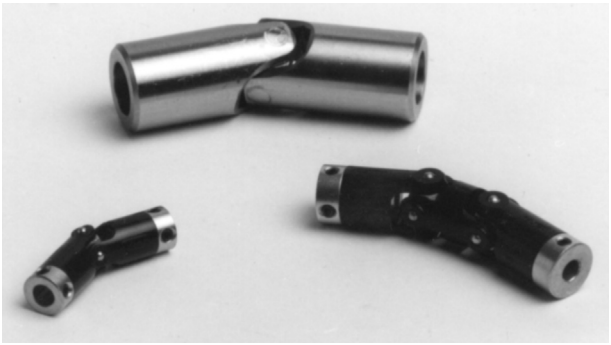


Figure 7.13
Universal joints.

7.4 Critical Speeds and Shaft Deflection

The center of mass of a rotating system (for example, a midmounted disk on a shaft supported by bearings at each end) will never coincide with the center of rotation due to manufacturing and operational constraints. As the shaft rotational speed is increased, the centrifugal force acting at the center of mass tends to bow the shaft. The more the shaft bows the greater the eccentricity and the greater the centrifugal force. Below the lowest critical speed of rotation, the centrifugal and elastic forces balance at a finite value of shaft deflection. The critical speed equilibrium theoretically requires infinite deflection of the center of mass, although realistically bearing damping, internal hysteresis, and windage cause equilibrium to occur at a finite displacement. This displacement can be large enough to break the shaft, damage bearings, and cause destructive machine vibration. Therefore, deflections need to be determined along the shaft and the consequences evaluated.

The critical speed of rotation is the same as the lateral frequency of vibration, which is induced when rotation is stopped and the center displaced laterally and suddenly released (i.e. the same as the frequency you would obtain if you "whacked" the stationary shaft with a hammer and monitored the frequency it vibrated at). For all shafts, except for the single concentrated mass shaft, critical speeds also occur at higher frequencies.

At the first critical speed, the shaft will bend into the simplest possible shape, and at the second critical speed, it will bend into the next simplest shape. For example, the shapes the shaft will bend into at the first two critical speeds (or modes) for an end-supported shaft with two masses are illustrated in Figure 7.14.

In certain circumstances, the fundamental frequency of a shaft system cannot be made higher than the shaft design speed. If the shaft can be accelerated rapidly through and beyond the first resonant critical frequency, before the vibrations have a chance to build up in amplitude, the system can be run at speeds higher than the natural frequency. This is the case with steam

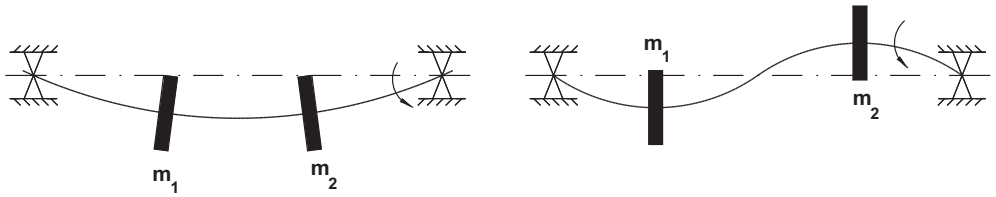


Figure 7.14
Shaft shapes for a simply supported shaft with two masses at the first and second critical speeds. Masses large in comparison with shaft mass.

and gas turbines, where the size of the turbomachinery and generators give low natural frequency but must be run at high speed due to efficiency considerations. As a general design principle, maintaining the operating speed of a shaft below half the shaft whirl critical frequency is normally acceptable.

A complete analysis of the natural frequencies of a shaft can be performed using a finite element analysis package, such as ANSYS, called a "nodal analysis." This can give a large number of natural frequencies in three dimensions from the fundamental upwards. This is the sensible and easiest approach for complex systems, but a quick estimate for a simplified system can be undertaken for design purposes as outlined in this section.

The critical speed of a shaft with a single mass attached can be approximated by

$$\omega_c = \sqrt{g/y} \tag{7.1}$$

where $\omega_c =$ critical angular velocity (rad/s), $y =$ static deflection at the location of the mass (m), and $g =$ acceleration due to gravity (m/s^2).

The first critical speed of a shaft carrying several concentrated masses is approximated by the Rayleigh–Ritz equation (see Eqn (7.2)). The dynamic deflections of a shaft are generally unknown. Rayleigh showed that an estimate of the deflection curve is suitable provided it represents the maximum deflection and the boundary conditions. The static deflection curve, due to the shaft's own weight and the weight of any attached components, gives a suitable estimate. Note that external loads are not considered in this analysis, only those due to gravitation. The resulting calculation gives a value higher than the actual natural frequency by a few percent.

$$\omega_c = \sqrt{\frac{g\sum_{i=1}^{n} W_i y_i}{\sum_{i=1}^{n} W_i y_i^2}} \tag{7.2}$$

where $\omega_c =$ critical angular velocity (rad/s), $W_i =$ mass or weight of node I (kg or N), and $y_i =$ static deflection of W_i (m).

Alternatively, the Dunkerley equation can be used to estimate the critical speed:

$$\frac{1}{\omega_c^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} + \ldots \tag{7.3}$$

where ω_1 is the critical speed if only mass number 1 is present, etc.

Both the Rayleigh–Ritz and the Dunkerley equations are approximations to the first natural frequency of vibration, which is assumed nearly equal to the critical speed. The Dunkerley equation tends to underestimate and the Rayleigh–Ritz equation tends to overestimate the critical frequency.

Often we need to know the second critical speed. For a two mass system, as illustrated in Figure 7.14, approximate values for the first two critical speeds can be found by solving the frequency equation given in Eqn (7.4).

$$\frac{1}{\omega^4} - (a_{11}m_1 + a_{22}m_2)\frac{1}{\omega^2} + (a_{11}a_{22} - a_{12}a_{21})m_1m_2 = 0 \tag{7.4}$$

where a_{11}, a_{12}, etc. are the influence coefficients. These correspond to the deflection of the shaft at the locations of the loads, as a result of 1 N loads. The first subscript refers to location of the deflection, the second to the location of the 1 N force. For example, the influence coefficients for a simply supported shaft with two loads, as illustrated in Figure 7.15, are listed next:

a_{11} is the deflection at the location of mass 1, which would be caused by a 1 N weight at the location of mass 1.

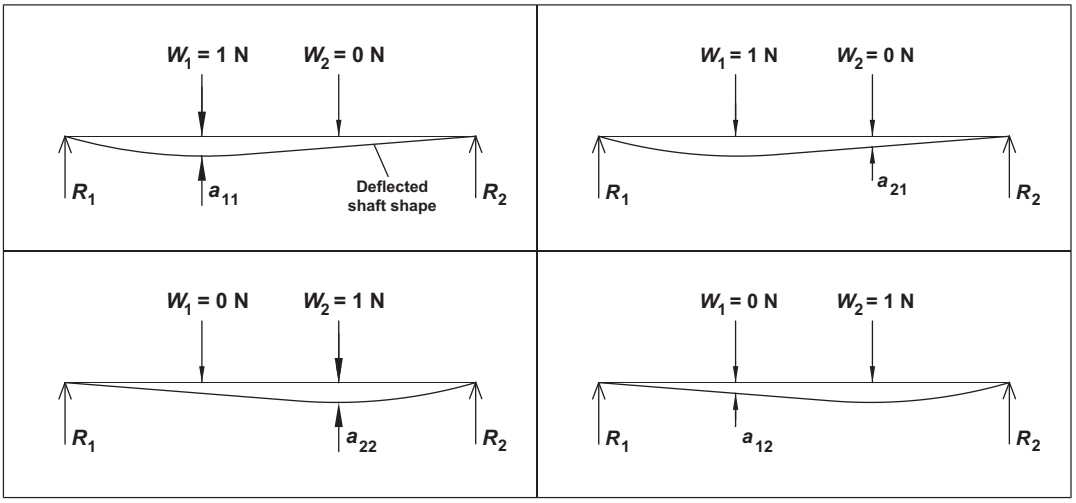


Figure 7.15
Example of influence coefficient definition for a simply supported shaft with two concentrated loads.

a_{21} is the deflection at the location of mass 2, which would be caused by a 1 N weight at the location of mass 1.
a_{22} is the deflection at the location of mass 2, which would be caused by a 1 N weight at the location of mass 2.
a_{12} is the deflection at the location of mass 1, which would be caused by a 1 N weight at the location of mass 2.

Note that values for a_{np} and a_{pn} are equal by the principle of reciprocity; for example, $a_{12} = a_{21}$.

For a multimass system, the frequency equation can be obtained by equating the following determinate to zero:

$$\begin{vmatrix} \left(a_{11}m_1 - \dfrac{1}{\omega^2}\right) & a_{12}m_2 & a_{13}m_3 & \cdots \\ a_{21}m_1 & \left(a_{22}m_2 - \dfrac{1}{\omega^2}\right) & a_{23}m_3 & \cdots \\ a_{31}m_1 & a_{32}m_2 & \left(a_{33}m_3 - \dfrac{1}{\omega^2}\right) & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix} = 0 \tag{7.5}$$

It should be noted that lateral vibration requires an external source of energy. For example, vibrations can be transferred from another part of a machine, and the shaft will vibrate in one or more lateral planes regardless of whether the shaft is rotating. Shaft whirl is a self-excited vibration caused by the shaft's rotation acting on an eccentric mass.

These analysis techniques for calculating the critical frequency require the determination of the shaft deflection. Section 7.4.1 introduces the Macaulay method, which is suitable for calculating the deflection of a constant diameter shaft and Section 7.4.2 introduces the strain energy method which is suitable for more complex shafts with stepped diameters.

7.4.1 Macaulay's Method for Calculating the Deflection of Beams

Macaulay's method can be used to determine the deflection of a constant cross-section shaft. The general rules for this method are given below. Having calculated the deflections of a shaft, this information can then be used to determine critical frequencies.

1. Take an origin at the left-hand side of the beam.
2. Express the bending moment at a suitable section XX in the beam to include the effect of all the loads.
3. Uniformly distributed loads must be made to extend to the right-hand end of the beam. Use negative loads to compensate.
4. Put in square brackets all functions of length other than those involving single powers of x.
5. Integrate as a whole any term in square brackets.

6. When evaluating the moment, slope, or deflection, neglect the square brackets terms when they become negative.
7. In the moment equation, express concentrated moments in the form $M_1[x-a]^0$ where M_1 is the concentrated moment and $x-a$ is its point of application relative to the section XX.

Macaulay's method for calculating the deflection of beams and shafts is useful in that it is relatively simple to use and easily programmed.

Example 7.1

As part of the preliminary design of a machine shaft, a check is performed to determine that the critical speed is significantly higher than the design speed of 7000 rpm. The components can be represented by three point masses, as shown in Figure 7.16. Assume the bearings are stiff and act as simple supports. The shaft diameter is 40 mm and the material is steel with a Young's modulus of 200×10^9 N/m^2.

Solution

Macaulay's method is used to determine the shaft deflections:

Resolving vertically: $R_1 + R_2 = W_1 + W_2 + W_3$.

Clockwise moments about O:

$$W_1L_1 + W_2(L_1 + L_2) - R_2(L_1 + L_2 + L_3) + W_3(L_1 + L_2 + L_3 + L_4) = 0.$$

Hence:

$$R_2 = \frac{W_1L_1 + W_2(L_1 + L_2) + W_3(L_1 + L_2 + L_3 + L_4)}{L_1 + L_2 + L_3}.$$

Calculating the moment at section XX:

$$M_{XX} = -R_1x + W_1[x - L_1] + W_2[x - (L_1 + L_2)] - R_2[x - (L_1 + L_2 + L_3)] \tag{7.6}$$

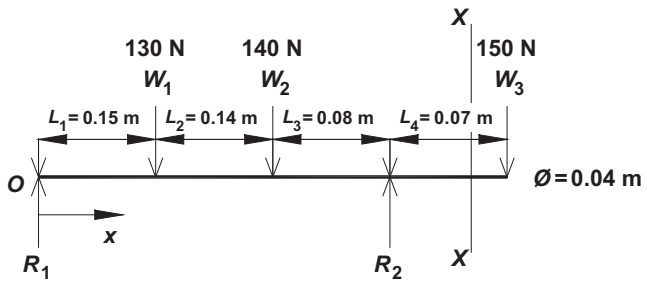


Figure 7.16
Machine shaft example.

The relationship for the deflection y of a beam subjected to a bending moment M is given by Eqn (7.7).

$$EI\frac{d^2y}{dx^2} = M \tag{7.7}$$

where E is the modulus of elasticity or Young's modulus (N/m^2), I is the second moment of area (m^4), y = deflection (m), x is the distance from the end of the beam to the location at which the deflection is to be determined (m), and M = moment (N m).

Equation (7.7) can be integrated once to find the slope dy/dx and twice to find the deflection y.

Integrating Eqn (7.6) to find the slope:

$$EI\frac{dy}{dx} = -R_1\frac{x^2}{2} + \frac{W_1}{2}[x - L_1]^2 + \frac{W_2}{2}[x - (L_1 + L_2)]^2 - \frac{R_2}{2}[x - (L_1 + L_2 + L_3)]^2 + A.$$

Integrating again to find the deflection equation gives

$$EIy = -R_1\frac{x^3}{6} + \frac{W_1}{6}[x - L_1]^3 + \frac{W_2}{6}[x - (L_1 + L_2)]^3 - \frac{R_2}{6}[x - (L_1 + L_2 + L_3)]^3 + Ax + B.$$

Note that Macaulay's method requires that terms within square brackets are to be ignored when the sign of the bracket goes negative.

It is now necessary to substitute boundary conditions to find the constants of integration. In the case of a shaft, the deflection at the bearings may be known, in which case these can be used as boundary conditions.

Assuming that the deflection at the bearing is zero then substituting $y = 0$ at $x = 0$ into the equation for deflection above gives $B = 0$.

$$\text{At } x = L_1 + L_2 + L_3, y = 0. \quad 0 = -\frac{R_1}{6}(L_1 + L_2 + L_3)^3 + \frac{W_1}{6}(L_2 + L_3)^3 + \frac{W_2}{6}(L_3)^3 + A(L_1 + L_2 + L_3).$$

Hence, $A = \dfrac{\frac{R_1}{6}(L_1 + L_2 + L_3)^3 - \frac{W_1}{6}(L_2 + L_3)^3 - \frac{W_2}{6}(L_3)^3}{L_1 + L_2 + L_3}$.

The second moment of area for a solid round shaft is given by

$$I = \frac{\pi d^4}{64}$$

so $I = \dfrac{\pi 0.04^4}{64} = 1.2566 \times 10^{-7}$ m^4

With $W_1 = 130$ N, $W_2 = 140$ N, $W_3 = 150$ N, $\phi = 0.04$ m, and $E = 200 \times 10^9$ N/m^2, substitution of these values into the above equations gives:

$R_1 = 79.19$ N,
$R_2 = 340.8$ N,
$A = 1.151$ N m^2.
At $x = 0.15$ m, $y = 5.097 \times 10^{-6}$ m.
At $x = 0.29$ m, $y = 2.839 \times 10^{-6}$ m.
At $x = 0.44$ m, $y = -1.199 \times 10^{-6}$ m.

Use absolute values for the displacement in the Rayleigh–Ritz equation:

$$\omega_c = \sqrt{\frac{9.81\left((130 \times 5.097 \times 10^{-6}) + (140 \times 2.839 \times 10^{-6}) + (150 \times |-1.199 \times 10^{-6}|)\right)}{\left(130 \times (5.097 \times 10^{-6})^2\right) + \left(140 \times (2.839 \times 10^{-6})^2\right) + \left(150 \times (|-1.199 \times 10^{-6}|)^2\right)}}$$

$$= \sqrt{\frac{9.81 \times 1.240 \times 10^{-3}}{4.722 \times 10^{-9}}} = 1605 \text{ rad/s}.$$

So the critical speed is $1605 \times 2\pi/60 = 15{,}530$ rpm.

Example 7.2

An air compressor consists of four compressor disks mounted on a steel shaft 50 mm apart, as shown in Figure 7.17. The shaft is simply supported at either end. Each compressor wheel mass is 18 kg. Calculate the deflection at each wheel using Macaulay's method then calculate, using the Rayleigh–Ritz equation, the first critical frequency of the shaft. The shaft outer diameter is 0.05 m, the inner diameter is 0.03 m.

Solution

$$-R_1 x + W_1[x - L_1] + W_2[x - (L_1 + L_2)] + W_3\,[x - (L_1 + L_2 + L_3)] + W_4[x - (L_1 + L_2 + L_3 + L_4)] = EI\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}.$$

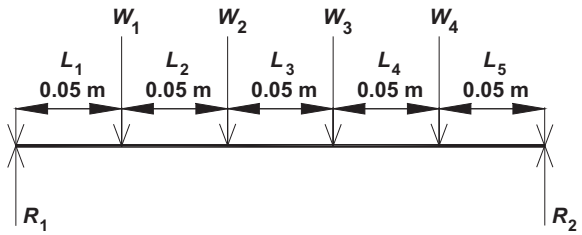


Figure 7.17
Air compressor shaft.

$$-\frac{R_1x^3}{6}+\frac{W_1}{6}[x-L_1]^3+\frac{W_2}{6}[x-(L_1+L_2)]^3+\frac{W_3}{6}[x-(L_1+L_2+L_3)]^3$$
$$+\frac{W_4}{6}[x-(L_1+L_2+L_3+L_4)]^3+Ax+B=EIy.$$

Boundary conditions:

$y=0$ at $x=0$,
$y=0$ at $x=L_1+L_2+L_3+L_4+L_5=L$.

Hence $B=0$.

$$A=\frac{1}{L}\left(\frac{R_1L^3}{6}-\frac{W_1}{6}(L-L_1)^3-\frac{W_2}{6}(L_3+L_4+L_5)^3-\frac{W_3}{6}(L_4+L_5)^3-\frac{W_4}{6}L_5^3\right).$$

$R_1+R_2=W_1+W_2+W_3+W_4$.

Moments about R_1:

$$W_1L_1+W_2(L_1+L_2)+W_3(L_1+L_2+L_3)+W_4(L_1+L_2+L_3+L_4)-R_2L=0.$$

Hence:

$A=2.20725$,
$R_2=353.16$ N,
$R_1=353.16$ N,
$I=2.67\times10^{-7}$ m^4, $E=200\times10^9$ N/m^2,
$y_{w1}=1.9286726\times10^{-6}$ m,
$y_{w2}=3.0996525\times10^{-6}$ m,
$y_{w3}=3.0996525\times10^{-6}$ m,
$y_{w4}=1.9286726\times10^{-6}$ m,
$\omega=1923.8$ rad/s $=18{,}371$ rpm.

Example 7.3

This example explores the influence on the critical frequency of adding an overhang load to a beam.

1. Determine the critical frequency for a 0.4 m long steel shaft running on bearings with a single point load, as illustrated in Figure 7.18. Assume that the bearings are rigid and act as simple supports. Ignore the self-weight of the shaft. Take Young's modulus of elasticity as 200×10^9 N/m^2.
2. Calculate the critical frequency if an overhang load of 150 N is added at a distance of 0.15 m from the right-hand bearing, as shown in Figure 7.19.

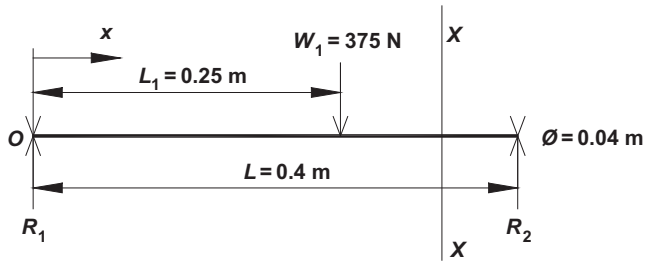


Figure 7.18
Simple shaft with single concentrated mass.

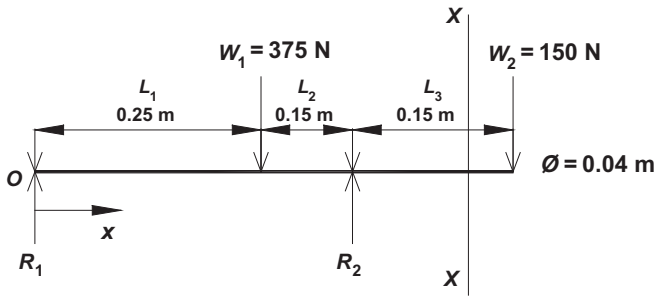


Figure 7.19
Simple shaft with a single concentrated mass between bearing and overhang mass.

Solution

1. Resolving vertically: $R_1 + R_2 = W_1$.
   Clockwise moments about O: $375 \times 0.25 - R_2 0.4 = 0$.
   Hence: $R_2 = 234.375$ N, $R_1 = 140.625$ N.

$$EI\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -R_1x + W_1[x - L_1].$$

$$EI\frac{\mathrm{d}y}{\mathrm{d}x} = -R_1\frac{x^2}{2} + \frac{W_1}{2}[x - L_1]^2 + A.$$

$$EIy = -R_1\frac{x^3}{6} + \frac{W_1}{6}[x - L_1]^3 + Ax + B.$$

Boundary conditions. Assuming that the deflection of the shaft is zero at the bearings, then substituting $y = 0$ and $x = 0$ into the above equation gives $B = 0$.
At $x = L$, $y = 0$. Hence

$$A = \frac{1}{L}\left(R_1\frac{L^2}{6} - \frac{W_1}{6}(L - L_1)^3\right) = \frac{1}{0.4}(1.5 - 0.2109) = 3.22266.$$

For a solid circular shaft $I = \dfrac{\pi d^4}{64} = \dfrac{\pi 0.04^4}{64} = 1.2566 \times 10^{-7}\ \text{m}^4$
$IE = 200 \times 10^9\ \text{N/m}^2$.

$$EIy = -140.625 \times \frac{0.25^3}{6} + (3.22266 \times 0.25).$$

$$y = 1.7485 \times 10^{-5}\ \text{m}.$$

Application of Macaulay's method to the geometry given in Figure 7.18 yields $y_1 = 1.74853 \times 10^{-5}$ m. So from Eqn (7.1):

$$\omega_c = \sqrt{\frac{g}{y_1}} = 749.03\ \text{rad/s} = 7152\ \text{rpm}.$$

2.

$$EI\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -R_1x + W_1[x - L_1] - R_2[x - (L_1 + L_2)]$$

$$EI\frac{\mathrm{d}y}{\mathrm{d}x} = -R_1\frac{x^2}{2} + \frac{W_1}{2}[x - L_1]^2 - \frac{R_2}{2}[x - (L_1 + L_2)]^2 + A$$

$$EIy = -R_1\frac{x^3}{6} + \frac{W_1}{6}[x - L_1]^3 - \frac{R_2}{6}[x - (L_1 + L_2)]^3 + Ax + B$$

Boundary conditions. Assuming negligible deflection at the bearings. At $x = 0$, $y = 0$. Hence $B = 0$.

At $x = L_1 + L_2$, $y = 0$. Hence $A = \dfrac{1}{L_1 + L_2}\left(\dfrac{R_1}{6}(L_1 + L_2)^3 - \dfrac{W_1}{6}L_2^3\right)$.

Resolving vertically: $R_1 + R_2 = W_1 + W_2$.

Moments about O: $W_1L_1 + W_2(L_1 + L_2 + L_3) - R_2(L_1 + L_2) = 0$.

Hence $R_2 = \dfrac{1}{L_1 + L_2}(W_1L_1 + W_2(\mathrm{L_1} + \mathrm{L_2} + \mathrm{L_3}))$, $R_1 = W_1 + W_2 - R_2$, $R_1 = 84.375$ N, $R_2 = 440.625$ N.

Substitution for A gives $A = 1.722656\ \text{N m}^2$.

Evaluating y at $x = L_1$ gives $y = 8.3929381 \times 10^{-6}$ m.

Evaluating y at $x = L_1 + L_2 + L_3$ gives $y = 1.8884145 \times 10^{-6}$ m.

Hence

$$\omega_c = \sqrt{\frac{9.81\left(3.147 \times 10^{-3} + 2.8326 \times 10^{-4}\right)}{2.64155 \times 10^{-8} + 5.34061 \times 10^{-10}}}$$

$$= \sqrt{\frac{9.81 \times 3.4306 \times 10^{-3}}{2.695 \times 10^{-8}}} = 1117.5\ \text{rad/s}.$$

$$\omega_c = 10{,}671\ \text{rpm}.$$

Comparison of the results from (1) and (2) shows that the addition of an overhang weight can have a significant effect in raising the critical frequency from 7150 rpm to 10,670 rpm in this case. This technique of adding overhang masses to increase the critical speed is sometimes used in plant machinery.

Example 7.4

1. Calculate the first two critical speeds for the loadings on a steel shaft, indicated in Figure 7.20. Assume the mass of the shaft can be ignored for the purpose of this calculation, and that the bearings can be considered to be stiff simple supports. The internal and external diameters of the shaft are 0.03 m and 0.05 m respectively. Take Young's modulus as 200 GN/m^2.
2. The design speed for the shaft is nominally 5000 rpm; what would be the implication of the bearing data given in Table 7.2 on the shaft operation assuming the bearings allow flexibility only in directions perpendicular to the shaft axis?

Solution

1. Applying Macaulay's method to find the influence coefficients: Firstly calculating the deflections at L_1 and L_2 with $W_1 = 1$ N and $W_2 = 0$ N to give a_{11} and a_{21}. Then, calculating the deflections at L_1 and L_2 with $W_1 = 0$ N and $W_2 = 1$ N to give a_{12} and a_{22}.

$$EIy = -R_1\frac{x^3}{6} + \frac{W_1}{6}[x - L_1]^3 + \frac{W_2}{6}[x - (L_1 + L_2)]^3 + Ax + B.$$

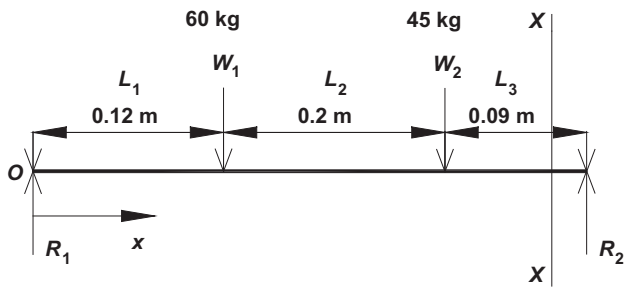


Figure 7.20
Simple shaft.

Table 7.2: Example bearing data for deflection versus load.

Load on bearing (N)	Deflection at bearing (mm)
50	0.003
100	0.009
200	0.010
300	0.0115
400	0.0126
500	0.0136
1000	0.0179
2000	0.0245

With $W_1 = 1$ N, $W_2 = 0$ N, $R_1 = 0.707317$ N, $R_2 = 0.292683$ N, $A = 9.90244 \times 10^{-3}$ N m^2 ($B = 0$).
Hence $a_{11} = 1.84355 \times 10^{-8}$ m, $a_{21} = 1.19688 \times 10^{-8}$ m.
With $W_1 = 0$ N, $W_2 = 1$ N, $R_1 = 0.2195122$ N, $R_2 = 0.7804878$ N, $A = 5.853659 \times 10^{-3}$ N m^2.
Hence $a_{22} = 1.26264 \times 10^{-8}$ m, $a_{12} = 1.19688 \times 10^{-8}$ m.
For this example: $a_{11} = 1.84355 \times 10^{-8}$ m, $a_{21} = 1.19688 \times 10^{-8}$ m, $a_{12} = 1.19688 \times 10^{-8}$ m, $a_{22} = 1.26264 \times 10^{-8}$ m.
Solving: $\dfrac{1}{\omega^4} - (a_{11}m_1 + a_{22}m_2)\dfrac{1}{\omega^2} + (a_{11}a_{22} - a_{12}a_{21})m_1m_2 = 0.$

By multiplying through by ω^4 and solving as a quadratic:

$$\omega^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

with
$a = (a_{11}a_{22} - a_{12}a_{21})m_1m_2,$
$b = -(a_{11}m_1 + a_{22}m_2),$
$c = 1,$
gives $\omega_{c1} = 812.5$ rad/s (7759 rpm) and $\omega_{c2} = 2503.4$ rad/s (23,905 rpm).

2. The deflection at the bearings should also be taken into account in determining the critical frequency of the shaft. The reaction at each of the bearings is approximately 500 N ($R_1 = 513.2$ N and $R_2 = 516.8$ N). The deflection at each bearing can be assumed roughly equal (so from Table 7.2 $y_{\text{bearing}} = 0.0136$ mm). This deflection can be simply added to the static deflection of the shaft due to bending by superposition. If the deflections were not equal, similar triangles could be used to calculate the deflection at the mass locations.

   The influence coefficients can also be used to determine the deflection under the actual loads applied. $W_1 = 60 \times 9.81 = 588.6$ N, $W_2 = 45 \times 9.81 = 441.45$ N:

$$y_{1\ \text{Load}} = W_1a_{11} + W_2a_{12} = 1.613 \times 10^{-5}\ \text{m},$$
$$y_{2\ \text{Load}} = W_1a_{21} + W_2a_{22} = 1.26 \times 10^{-5}\ \text{m},$$

$$y_{1\ \text{Total}} = 1.61 \times 10^{-5} + 1.36 \times 10^{-5} = 2.97 \times 10^{-5}\ \text{m},$$
$$y_{2\ \text{Total}} = 1.26 \times 10^{-5} + 1.36 \times 10^{-5} = 2.62 \times 10^{-5}\ \text{m}.$$

   These deflections give (using the Rayleigh–Ritz equation) a first critical frequency of

$$\omega_c = \sqrt{\frac{9.81 \times 2.96 \times 10^{-3}}{8.38 \times 10^{-8}}} = 588.6\ \text{rad/s} = 5620\ \text{rpm}.$$

   This is very close to the design speed. Different bearings or a stiffer shaft would be advisable.

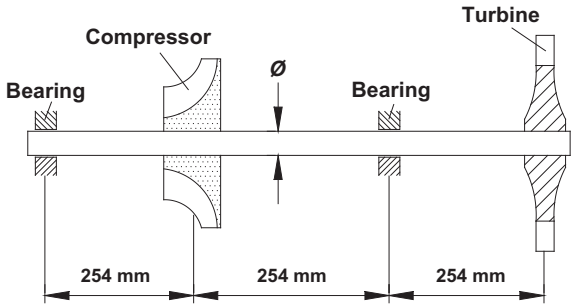


Figure 7.21
Turbine shaft.

Example 7.5

A 22.63 kg compressor impeller wheel is driven by a 13.56 kg turbine mounted on a common shaft (see Figure 7.21) manufactured from steel with Young's modulus $E = 207$ GN/m^2. The design speed is 10,000 rpm. Determine the shaft diameter so that the first critical speed is 12,000 rpm giving a safety margin of 2000 rpm. Use Rayleigh's equation and assume rigid bearings and a mass-less shaft (see Figure 7.22).

Solution

$W_1 = 9.81 \times 22.63 = 222$ N, $W_2 = 9.81 \times 13.56 = 133$ N.

Moments about the left-hand bearing:

$$222 \times 0.254 - R_R \times 0.508 + 133 \times 0.762 = 0$$

$R_R = 310.5$ N.
Resolving vertically $R_L + R_R = 222 + 133$.
$R_L = 44.5$ N.

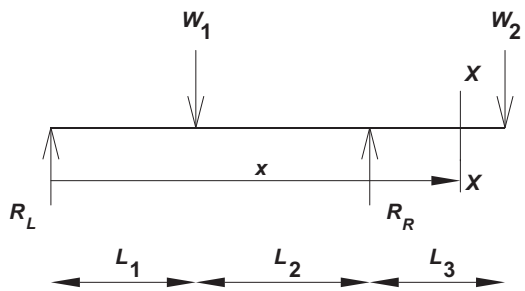


Figure 7.22
Principal loads.

Using Macaulay's method to determine the slope and deflection

$$M_{xx} = -R_L x + W_1[x - L_1] - R_R[x - (L_1 + L_2)]$$

$$EI\frac{dy}{dx} = -R_L\frac{x^2}{2} + \frac{W_1}{2}[x - L_1]^2 - \frac{R_R}{2}[x - (L_1 + L_2)]^2 + A$$

$$EIy = -R_L\frac{x^3}{6} + \frac{W_1}{6}[x - L_1]^3 - \frac{R_R}{6}[x - (L_1 + L_2)]^3 + Ax + B$$

Boundary conditions.

At $x = 0$, $y = 0$ (left-hand bearing), so from the above equation, $B = 0$.

At $x = L_1 + L_2$, $y = 0$ (right-hand bearing).

$$0 = -R_L\frac{(L_1 + L_2)^3}{6} + \frac{W_1}{6}L_2^3 + A(L_1 + L_2)$$

$$A = \frac{R_L\dfrac{(L_1 + L_2)^3}{6} - \dfrac{W_1}{6}L_2^3}{(L_1 + L_2)}$$

$$A = \frac{44.5(0.508)^3/6 - 222(0.254)^3/6}{0.508} = 0.7204$$

y_1 = deflection at location of load W_1.

y_2 = deflection at location of load W_2.

$$y_1 = \frac{1}{EI}\left(-\frac{44.5 \times 0.254^3}{6} + 0.7204 \times 0.254\right) = \frac{0.06144}{EI}$$

$$y_2 = \frac{1}{EI}\left(-\frac{44.5 \times 0.762^3}{6} + \frac{222}{6}(0.762 - 0.254)^3 - \frac{310.5}{6}(0.254)^3 + 0.7204 \times 0.762\right)$$
$$= \frac{1.270}{EI}$$

Let the first critical speed = 12,000 rpm = 12,000 × 2π/60 rad/s = 1257 rad/s

$$1257 = \sqrt{\frac{9.81\left(222 \times \dfrac{0.01644}{EI} + 133 \times \dfrac{1.270}{EI}\right)}{222\left(\dfrac{0.01644}{EI}\right)^2 + 133\left(\dfrac{1.270}{EI}\right)^2}} = \sqrt{EI \times 9.81 \times \frac{172.5}{214.6}} = \sqrt{EI \times 7.89}$$

$$1257^2 = EI \times 7.89 = \frac{\pi d^4}{64} \times 207 \times 10^9 \times 7.89$$

$$d = \left(1.971 \times 10^{-5}\right)^{0.25} = 0.06663 \text{ m}$$

$d \approx 67$ mm. Take $d = 70$ mm as the nearest standard size.

7.4.2 Castigliano's Theorem for Calculating Shaft Deflections

The strain energy, U, in a straight beam subjected to a bending moment M is

$$U = \int \frac{M^2 \mathrm{d}x}{2EI} \tag{7.8}$$

Castigliano's strain energy equation can be applied to problems involving shafts of nonconstant section to calculate deflections, and hence critical speeds.

Example 7.6

Determine the deflection of the stepped shaft, illustrated in Figure 7.23, under a single concentrated load.

Solution

The total strain energy due to bending is:

$$U = \int \frac{M^2 \mathrm{d}x}{2EI} = U_1 + U_2$$

where $U_1 =$ the energy from $x = 0$ to L_1, $U_2 =$ the energy from $x = L_1$ to L.

(In general, consider the strain energy for each section of a beam, i.e. between loads and between any change of shaft section.)

Resolving vertical loads: $R_1 + R_2 = W$.

Taking moments about the left-hand bearing

$$R_2 = \frac{WL_1}{L}.$$

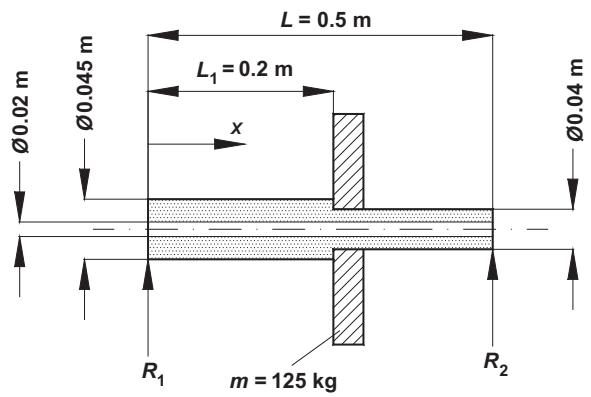


Figure 7.23
Stepped shaft with single load.

Hence,

$$R_1 = W\left(1 - \frac{L_1}{L}\right).$$

Splitting the beam between $x = 0$ and L_1 the moment can be expressed as $R_1 x$.

$$U_1 = \int_0^{L_1} \frac{(R_1 x)^2}{2EI_1} \mathrm{d}x = \left[\frac{(W(1 - (L_1/L)))^2 x^3}{6EI_1}\right]_0^{L_1} = \frac{(W(1 - (L_1/L)))^2 L_1^3}{6EI_1}.$$

Splitting the beam between $x = L_1$ and L the moment can be expressed as $R_2(L - x)$.

$$U_2 = \int_{L_1}^{L} \frac{(R_2(l - X))^2}{2EI_2} \mathrm{d}x = \left[\frac{(WL_1/L)^2\left(L^2 x - Lx^2 + x^3/3\right)}{2EI_2}\right]_{L_1}^{L}$$
$$= \frac{W^2 L_1^2}{2EI_2 L^2}\left(L^3 - L^3 + \frac{L^3}{3} - L^2 L_1 + LL_1^2 - \frac{L_1^3}{3}\right).$$

$$U = U_1 + U_2 = \frac{(W(1 - (L_1/L)))^2 L_1^3}{6EI_1} + \frac{W^2 L_1^2}{2EI_2 L^2}\left(L^3 - L^3 + \frac{L^3}{3} - L^2 L_1 + LL_1^2 - \frac{L_1^3}{3}\right).$$

Differentiating the above expression with respect to W gives the deflection at W:

$$\frac{\partial U}{\partial W} = \frac{L_1^3}{6EI_1}\left(2W - \frac{4WL_1}{L} + \frac{2WL_1^2}{L^2}\right) + \frac{2WL_1^2}{2EI_2 L^2}\left(\frac{L^3}{3} - L^2 L_1 + LL_1^2 - \frac{L_1^3}{3}\right). \tag{7.9}$$

$$I_1 = \frac{\pi\left(d_o^4 - d_i^4\right)}{64} = \frac{\pi\left(0.045^4 - 0.02^4\right)}{64} = 1.9343 \times 10^{-7}\ \mathrm{m}^4.$$

$$I_2 = \frac{\pi\left(0.04^4 - 0.02^4\right)}{64} = 1.1781 \times 10^{-7}\ \mathrm{m}^4.$$

Substitution of $W = 125 \times 9.81$ N, $L_1 = 0.2$ m, $L = 0.5$ m, $E = 200 \times 10^9$ N/m^2, I_1 and I_2 into Eqn (7.9) gives a deflection $\partial U/\partial W = 1.05372 \times 10^{-4}$ m and hence a first critical frequency using the approximation of Eqn (7.1) of

$$\sqrt{\frac{9.81}{1.05372 \times 10^{-4}}} = 305.121\ \mathrm{rad/s} = 2913\ \mathrm{rpm}.$$

Example 7.7

Using Castigliano's theorem for calculating deflections, determine the critical frequency of the steel shaft shown in Figure 7.24. Neglect the shaft mass.

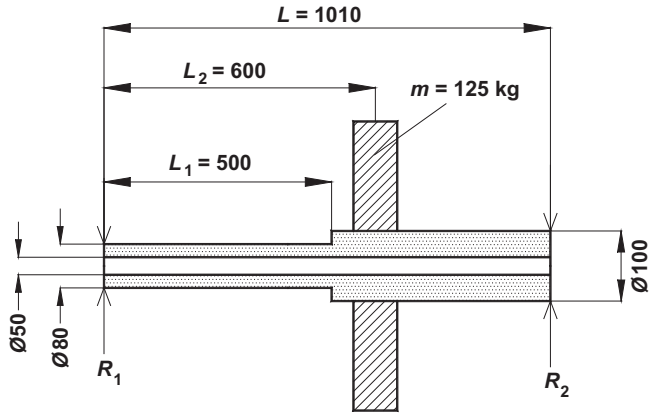


Figure 7.24
Simple stepped shaft and load (dimensions in mm).

Solution

Looking at the shaft from the left-hand side

Section I) $M = R_L x$,
Section II) $M = R_L x$,
Section III) $M = R_R\,(L - x)$.
$R_L + R_R = W$.
$WL_2 - R_R L = 0$.
$R_R = WL_2/L$.
$R_L = W\left(1 - \dfrac{L_2}{L}\right)$.

By definition,

$$U = \int \frac{M^2 \mathrm{d}x}{2EI}$$

$$U_1 = \int_0^{L_1} \frac{R_L^2 x^2}{2EI_1}\mathrm{d}x = \left[\frac{W^2\left(1 - \dfrac{L_2}{L}\right)^2 \dfrac{x^3}{3}}{2EI_1}\right]_0^{L_1} = \frac{W^2\left(1 - \dfrac{L_2}{L}\right)^2 \dfrac{L_1^3}{3}}{2EI_1}.$$

$$\frac{\partial U_1}{\partial W} = \frac{2W\left(1 - \dfrac{L_2}{L}\right)^2 \dfrac{L_1^3}{3}}{2EI_1}.$$

$$U_2 = \int_{L_1}^{L_2} \frac{R_L^2 x^2}{2EI_2}\mathrm{d}x = \left[\frac{W^2}{2EI_2}\left(1 - \frac{L_2}{L}\right)^2 \frac{x^3}{3}\right]_{L_1}^{L_2} = \frac{W^2}{2EI_2}\left(1 - \frac{L_2}{L}\right)^2 \left(\frac{L_2^3}{3} - \frac{L_1^3}{3}\right)$$

$$\frac{\partial U_2}{\partial W} = \frac{2W}{2EI_2}\left(1 - \frac{L_2}{L}\right)^2\left(\frac{L_2^3}{3} - \frac{L_1^3}{3}\right).$$

$I_3 = I_2$ (same section)

$$U_3 = \int \frac{W^2(L_2/L)^2}{2EI_3}\,\mathrm{d}x$$

$$= \left[\frac{W^2(L_2/L)^2\left(L^2x - x^2L + \frac{x^3}{3}\right)}{2EI_3}\right]_{L_2}^{L} = \frac{W^2}{2EI_3}\left(\frac{L_2}{L}\right)^2\left(L^3 - L^3 + \frac{L^3}{3} - L^2L_2 + L_2^2L - \frac{L_2^3}{3}\right)$$

$$\frac{\partial U_3}{\partial W} = \frac{W}{EI_3}\left(\frac{L_2}{L}\right)^2\left(\frac{L^3}{3} - L^2L_2 + L_2^2L - \frac{L_2^3}{3}\right).$$

$$\frac{\partial U_1}{\partial W} = 2.4708039 \times 10^{-5}$$

$$\frac{\partial U_2}{\partial W} = 6.6596749 \times 10^{-6}$$

$$\frac{\partial U_3}{\partial W} = 1.0801836 \times 10^{-5}$$

$$\frac{\partial U}{\partial W} = 4.2169551 \times 10^{-5}$$

$$\omega_c = 482.3 \text{ rad/s},$$
$$\omega_c = 4605.8 \text{ rpm}.$$

7.5 ASME Design Code for Transmission Shafting

According to Fuchs and Stephens (1980), between 50% and 90% of all mechanical failures are fatigue failures. This challenges the engineer to consider the possibility of fatigue failure at the design stage. Figure 7.25 shows the characteristic variation of fatigue strength for steel with the number of stress cycles. For low strength steels, a "leveling off" occurs in the graph between 10^6 and 10^7 cycles under noncorrosive conditions and, regardless of the number of stress cycles beyond this, the component will not fail due to fatigue. The value of stress corresponding to this leveling off is called the endurance stress, or the fatigue limit.

In order to design hollow or solid rotating shafts under combined cyclic bending and steady torsional loading for limited life, the ASME design code for the design of transmission shafting can be used. The ASME procedure ensures that the shaft is properly sized to provide

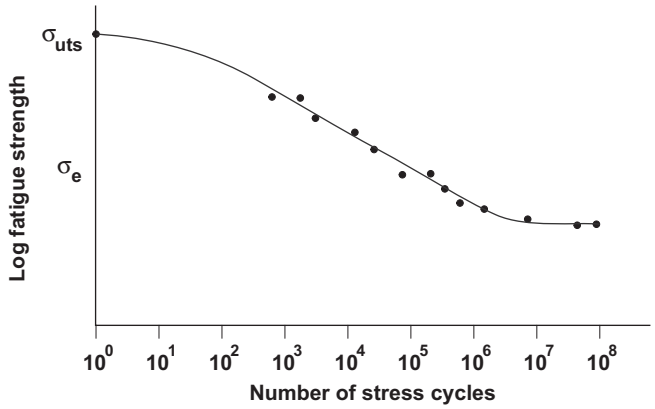


Figure 7.25
A typical strength cycle diagram for various steels.

adequate service life, but the designer must ensure that the shaft is stiff enough to limit deflections of power transfer elements, such as gears, and to minimize misalignment through seals and bearings. In addition, the shaft's stiffness must be such that it avoids unwanted vibrations through the running range.

The equation for determining the diameter for a solid shaft is given by

$$d = \left[\frac{32n_s}{\pi}\sqrt{\left(\frac{M}{\sigma_e}\right)^2 + \frac{3}{4}\left(\frac{T}{\sigma_y}\right)^2}\right]^{1/3} \tag{7.10}$$

where d = diameter (m), n_s = factor of safety, M = bending moment (N m), σ_e = endurance limit of the item (N/m^2), T = torque (N m), and σ_y = yield strength (N/m^2).

There is usually some uncertainty regarding what level a component will actually be loaded to, how strong a material is, and how accurate the modeling methods are. Factors of safety are frequently used to account for these uncertainties. The value of a factor of safety is usually based on experience of what has given acceptable performance in the past. The level is also a function of the consequences of component failure and the cost of providing an increased safety factor. As a guide, typical values for the factor of safety based on strength recommended by Vidosek (1957) are as follows:

- 1.25 to 1.5 for reliable materials under controlled conditions subjected to loads and stresses known with certainty,
- 1.5 to 2.0 for well-known materials under reasonably constant environmental conditions subjected to known loads and stresses,
- 2.0 to 2.5 for average materials subjected to known loads and stresses,
- 2.5 to 3.0 for less well-known materials under average conditions of load, stress, and environment,

- 3.0 to 4.0 for untried materials under average conditions of load, stress, and environment, and
- 3.0 to 4.0 for well-known materials under uncertain conditions of load, stress, and environment.

The endurance limit, σ_e, for a mechanical element can be estimated by Eqn (7.11) (after Marin, 1962). Here a series of modifying factors is applied to the endurance limit of a test specimen for various effects such as size, load, and temperature.

$$\sigma_e = k_a k_b k_c k_d k_e k_f k_g \sigma'_e \tag{7.11}$$

where k_a = surface factor, k_b = size factor, k_c = reliability factor, k_d = temperature factor, k_e = duty cycle factor, k_f = fatigue stress concentration factor, k_g = miscellaneous effects factor, and σ'_e = endurance limit of test specimen (N/m^2).

If the stress at the location under consideration is greater than σ_e, then the component will fail eventually due to fatigue (i.e. the component has a limited life).

Mischke (1987) has determined the following approximate relationships between the endurance limit of test specimens and the ultimate tensile strength of the material (for steels only):

$$\sigma'_e = 0.504\sigma_{\text{uts}} \quad \text{for} \quad \sigma_{\text{uts}} \leq 1400 \text{ MPa} \tag{7.12}$$

$$\sigma'_e = 700 \text{ MPa} \quad \text{for} \quad \sigma_{\text{uts}} \geq 1400 \text{ MPa} \tag{7.13}$$

The surface finish factor is given by Eqn (7.14).

$$k_a = a\sigma_{\text{uts}}^b \tag{7.14}$$

Values for a and b can be found in Table 7.3.

The size factor k_b can be calculated from Eqn (7.15) or (7.16) (Kuguel, 1961).

$$\text{For } d < 50 \text{ mm:} \quad k_b = \left(\frac{d}{7.62}\right)^{-0.1133} \tag{7.15}$$

$$\text{For } d > 50 \text{ mm:} \quad k_b = 1.85d^{-0.19} \tag{7.16}$$

The reliability factor k_c is given in Table 7.4 as a function of the nominal reliability desired.

Table 7.3: Surface finish factors (Noll and Lipson, 1946).

Surface finish	a (MPa)	b
Ground	1.58	−0.085
M/c or cold-drawn	4.51	−0.265
Hot rolled	57.7	−0.718
Forged	272.0	−0.995

Table 7.4: Reliability factors for use in the ASME transmission shaft equation.

Shaft nominal reliability	k_c
0.5	1.0
0.9	0.897
0.99	0.814
0.999	0.753

ANSI/ASME B106.1 M-1985.

For temperatures between −57 °C and 204 °C, the temperature factor, k_d, can be taken as 1.0. The ASME standard documents the values to use outside of this range.

The duty cycle factor k_e is used to account for cycle loading experienced by the shaft such as stops and starts, transient overloads, shock loading, etc. and requires prototype fatigue testing for its quantification. k_e is taken as 1.0 in the examples presented here.

The fatigue stress concentration factor k_f is used to account for stress concentration regions such as notches, holes, keyways, and shoulders. It is given by

$$k_f = \frac{1}{K_f} \tag{7.17}$$

where K_f is the component fatigue stress concentration factor which is given by

$$K_f = 1 + q(k_t - 1) \tag{7.18}$$

where q = notch sensitivity and k_t = geometric stress concentration factor.

Values for the notch sensitivity and typical geometric stress concentration factors are given in Figures 7.26–7.29 and Table 7.5.

The miscellaneous factor k_g is used to account for residual stresses, heat treatment, corrosion, surface coatings, vibration, environment, and unusual loadings. k_g is taken as 1.0 here.

In general, the following principles should be used when designing to avoid fatigue failures.

- Calculations should allow an appropriate safety factor, particularly where stress concentrations occur (e.g. keyways, notches, and change of section).
- Provide generous radii at changes of section and introduce stress relief grooves, etc.
- Choose materials if possible that have limiting fatigue stresses. For example, most steels.
- Provide for suitable forms of surface treatment. For example, shot-peening, work-hardening, and nitriding. Avoid treatments which introduce residual tensile stresses such as electroplating.
- Specify fine surface finishes.
- Avoid corrosive conditions.
- Stress relieving should be used where possible, particularly for welded structures.

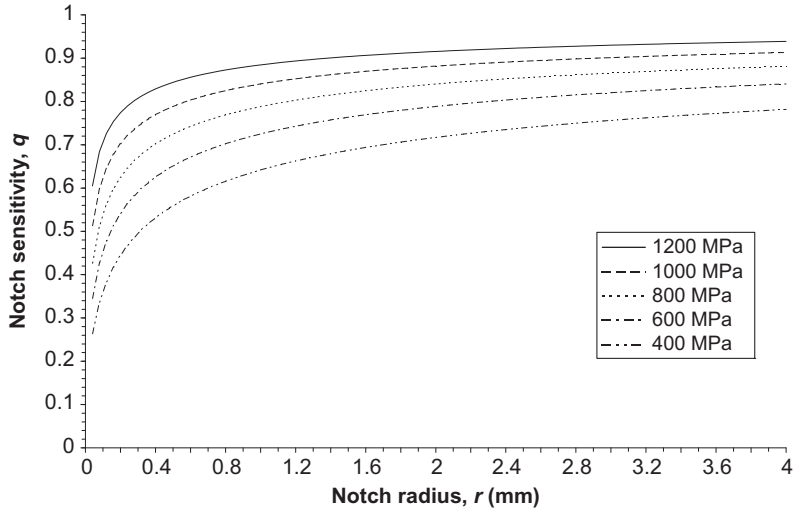


Figure 7.26
Notch sensitivity index versus notch radius for a range of steels subjected to reversed bending or reversed axial loads. *Data from Kuhn and Handrath (1952).*

Any appropriate material can be selected for a shaft. The requirements to resist high loads and high cycle fatigue often dictate consideration of steel. The coding system used for steels in the UK is given in British Standard 970. The standard uses a six digit designation; for example, 070M20.

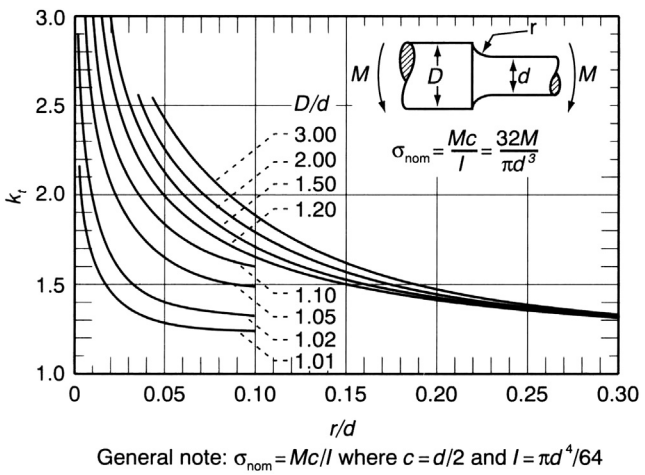


Figure 7.27
Stress concentration factors for a shaft with a fillet subjected to bending. *Reproduced from Peterson (1974).*

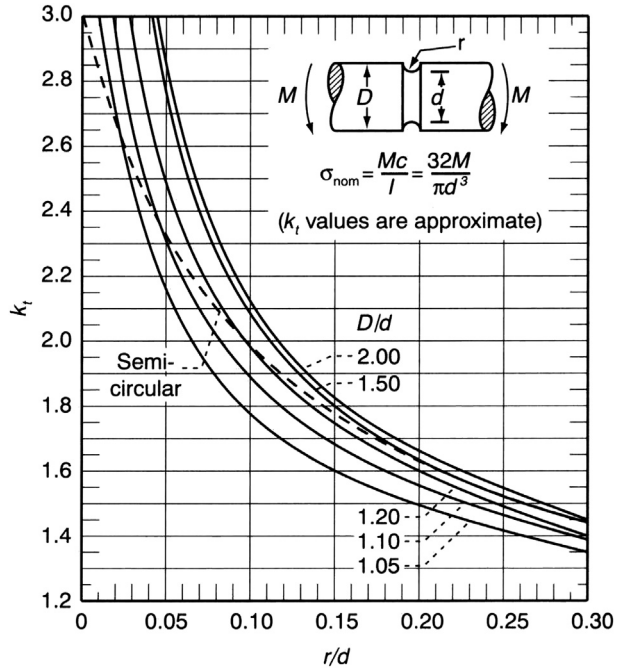


Figure 7.28

Stress concentration factors for a shaft with a groove subjected to bending. *Reproduced from Peterson (1974).*

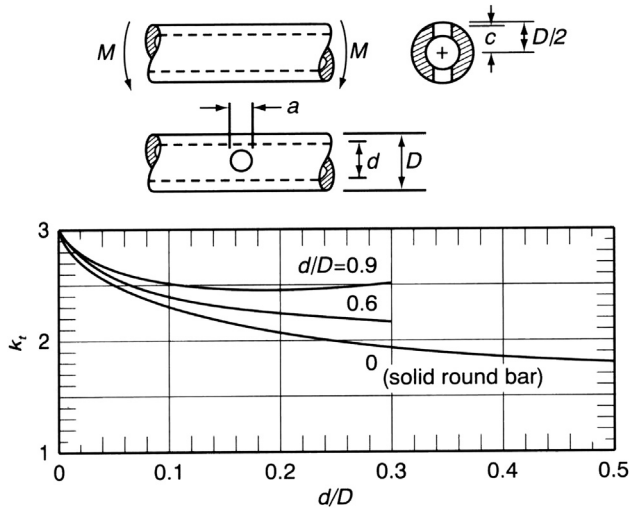


Figure 7.29

Stress concentration factors for a shaft with a transverse hole subjected to bending. *Reproduced from Peterson (1974).*

Table 7.5: Fatigue stress concentration factors, k_f, for keyways in steel shafts (Juvinall, 1967).

Steel	Profiled keyway bending stress	Sled runner keyway bending stress
Annealed <200 BHN	0.63	0.77
Quenched and drawn >200 BHN	0.5	0.63

The first three digits of the designation denote the family of steels to which the alloy belongs.

000−199 Carbon and carbon−manganese steels. The first three figures indicate one hundred times the mean manganese content.

200−240 Free cutting steels where the second and third digits represent one hundred times the minimum or mean sulfur content.

250 Silicon−manganese spring steels.

300−399 Stainless, heat resisting, and valve steels.

500−999 Alloy steels in groups of 10 or multiples in groups of 10 according to alloy type.

The fourth digit of the designation is a letter: either A, H, M, or S.

"A" denotes that the steel will be supplied to close limits of chemical composition,
"H" denotes a hardenability requirement for the material,
"M" denotes specified mechanical properties, and
"S" denotes a stainless steel.

The fifth and sixth digits of the designation represent one hundred times the mean carbon content of the steel. For example, 080A40 denotes a carbon−manganese steel supplied to close limits of chemical composition containing 0.7−0.9% manganese and 0.4% carbon.

Tables 7.6 and 7.7 list some steel types and typical material property values.

The choice of steel for a particular application can sometimes be a bewildering experience. Within the current British Standard alone for steels (BS970), there are several hundred steel specifications. In practice, relatively few steels are used for the majority of applications, and some of the popular specifications are listed below.

Steels can be divided into seven principal groupings.

1. Low carbon free cutting steels. These are the most popular types of steel for the production of turned components, where machinability and surface finish are important. Applications include automotive and general engineering. The principal specification is 230M07.

Table 7.6: Guide to the ultimate tensile and yield strengths for selected steels.

BS970 designation	σ_{uts} (MN/m^2)	σ_y (MN/m^2)	Old EN code
230M07	475		1A
080M15	450*	330*	32C
070M20	560*	440*	3B
210M15	490		32M
214M15	720		202
080M30	620*	480*	6
080M40	660*	530*	8
635M15	770_{min}		351
655M13	1000_{min}		36
665M17	770_{min}		34
605M36	700–850**	540**	16
709M40	700–1225**	540–955**	19
817M40	850–1000**	700**	24
410S21	700–850#	525#	56A
431S29	850–1000	680	57
403S17	420_{min}	280	
430S17	430_{min}	280	
304S15	480		58E
316S11	490		
303S31	510		58M
303S42	510		58AM

*Hot rolled and cold drawn.
**Hardened, tempered and cold drawn.
#Tempered.

Table 7.7: General physical properties.

Steel	ρ (kg/m^3)	E (GPa)	G (GPa)	ν	α (K^{-1})
080A15	7859	211.665	82.046	0.291	11.7×10^{-6}
080M40	7844	211.583	82.625	0.287	11.2×10^{-6}
708M40	7830	210.810	83.398	0.279	12.3×10^{-6}
303S–	7930	193.050	–	–	17.2×10^{-6}

2. Low carbon steels or mild steels. These are used for lightly stressed components, welding, bending, forming, and general engineering applications. Some of the popular specifications are 040A10, 045M10, 070M20, 080A15, and 080M15.
3. Carbon and carbon manganese case hardening steels. These steels are suitable for components that require a wear resisting surface and tough core. Specifications include 045A10, 045M10, 080M15, 210M15, and 214M15.
4. Medium carbon and carbon manganese steels. These offer greater strength than mild steels and respond to heat treatment. Tensile strengths in the range of 700–1000 MPa can be attained. Applications include gears, racks, pinions, shafts, rollers, bolts, and nuts. Specifications include 080M30, 080M40, 080A42, 080M50, 070M55, and 150M36.

5. Alloy case-hardening steels. These are used when a hard wear resisting surface is required but, because of the alloying elements, superior mechanical properties can be attained in comparison with carbon and carbon manganese case-hardening steels. Typical applications include gears, cams, rolled, and transmission components. Types include 635M15, 655M13, 665M17, 805M20, and 832M13.
6. Alloy direct hardening steels. These steels include alloying elements such as Ni, Cr, Mo, and V and are used for applications where high strength and shock resistance are important. Types include 605M36, 708M40, 709M40, 817M40, and 826M40.
7. Stainless steels. There are three types of stainless steels: martensitic, ferritic, and austenitic.
   Martensitic stainless steels can be hardened and tempered to give tensile strengths in the range from 550 to 1000 MN/m^2. Applications include fasteners, valves, shafts, spindles, cutlery, and surgical instruments. Specifications include 410S21, 420S29, 420S45, 431S29, 416S21, 416S41, 416S37, and 441S49.
   Ferritic stainless steels are common in strip and sheet form, and applications include domestic and automotive trim, catering equipment, and exhaust systems. They have good ductility and are easily formed. Specifications include 403S17 and 430S17.
   Austenitic stainless steels offer the highest resistance to corrosion, and applications are affiliated with the food, chemical, gas, and oil industries, as well as medical equipment and domestic appliances. Specifications include 302S31, 304S15, 316S11, 316S31, 320S31, 321S31, 303S31, 325S31, 303S42, and 326S36.

Example 7.8

Using the ASME equation for the design of transmission shafting determine a sensible minimum nominal diameter for the drive shaft, illustrated in Figure 7.30, consisting of a midmounted spur gear and overhung pulley wheel. The shaft is to be manufactured using 817M40 hot rolled alloy steel with $\sigma_{uts} = 1000$ MPa, $\sigma_y = 770$ MPa, and Brinell hardness approximately 220 BHN. The radius of the fillets at the gear and pulley shoulders is 3 mm. The power to be transmitted is 8 kW at 900 rpm. The pitch circle diameter of the 20° pressure angle spur gear is 192 mm and the pulley diameter is 250 mm. The masses of the gear and pulley are 8 kg and 10 kg respectively. The ratio of belt tensions should be taken as 2.5. Profiled keys are used to transmit torque through the gear and pulley. A shaft nominal reliability of 90% is desired. Assume the shaft is of constant diameter for the calculation.

Solution

In order to use the ASME design equation for transmission shafts, the maximum combination of torque and bending moment must be determined. A sensible approach is to determine the overall bending moment diagram for the shaft, as this information may be of use in other design calculations such as calculating the shaft deflection.

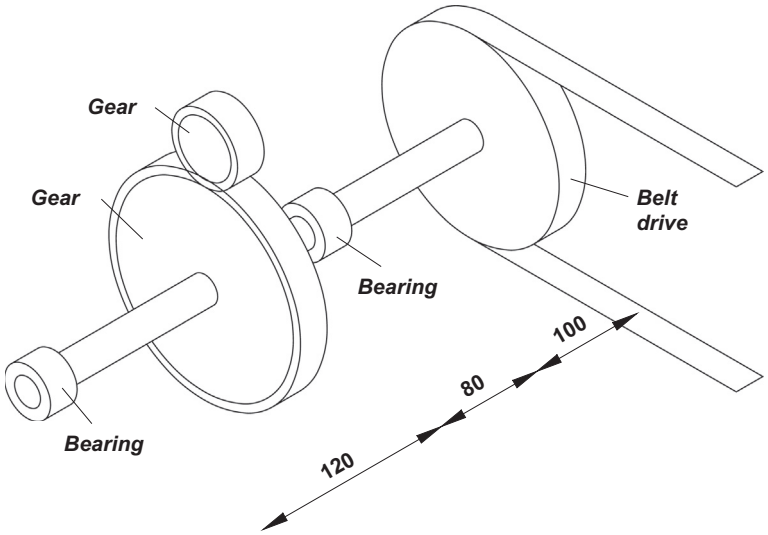


Figure 7.30
Transmission drive shaft.

The loading on the shaft can be resolved into both horizontal and vertical planes and must be considered in determining the resulting bending moments on the shaft. Figure 7.31 shows the combined loadings on the shaft from the gear forces, the gear's mass, the belt tensions, and the pulley's mass. The mass of the shaft itself has been ignored. The tension on a pulley belt is tighter on the "pulling" side than on the "slack" side and the relationship of these tensions is normally given as a ratio, which in this case is 2.5:1.

The power transmitted through the shaft is 8000 W.

The torque is given by: $\text{Torque} = \dfrac{\text{Power}}{\omega} = \dfrac{8000}{(2\pi/60) \times 900} = 84.9 \text{ N m}.$

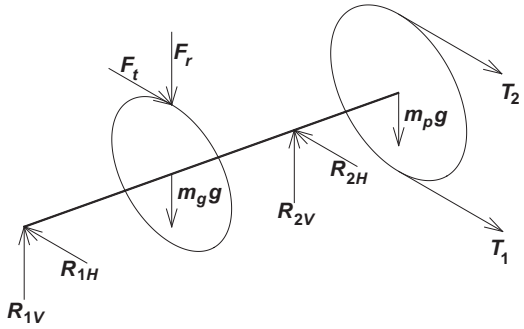


Figure 7.31
Shaft loading diagram.

Belts are introduced in more detail in Chapter 12. Here, only the tensions will be considered as necessary for the development of the solution.

The ratio of belt tensions is $T_1/T_2 = 2.5$, so $T_1 = 2.5T_2$.

The torque on the pulley in terms of the belt tensions is given by

$$\text{Torque} = T_1 r_p - T_2 r_p = 2.5\, T_2 r_p - T_2 r_p = 1.5 T_2 r_p.$$

The belt tensions are: $T_2 = 84.9/(1.5 \times 0.125) = 452.8$ N,

$$T_1 = 2.5T_2 = 1131.8 \text{ N}.$$

Forces on spur gears are considered in detail in Chapter 8. Here the relevant relationships will be stated as necessary and used in the development of the solution. A spur gear experiences both radial and tangential loading. The tangential load is given by $F_t = $ Torque/pitch circle radius.

$$F_t = \frac{\text{Torque}}{r_g} = \frac{84.9}{0.192/2} = 884.4 \text{ N}.$$

The radial load is given by $F_r = F_t \tan\phi$, where ϕ is the pressure angle.

$$F_r = F_t \tan\phi = 884.4 \times \tan 20 = 321.9 \text{ N}.$$

Note that when determining the horizontal and vertical bending moment diagrams both transmitted and gravitational loads should be included.

The vertical loads on the shaft are illustrated in Figure 7.32.

Moments about A:

$$\big(F_r + m_g g\big)L_1 - R_{2V}(L_1 + L_2) + m_p g(L_1 + L_2 + L_3) = 0.$$

$$R_{2V} = \frac{\big(F_r + m_g g\big)L_1 + m_p g(L_1 + L_2 + L_3)}{L_1 + L_2} = \frac{(321.9 + (8 \times 9.81))0.12 + (10 \times 9.81)0.3}{0.12 + 0.08}$$

$$= 387.4 \text{ N}.$$

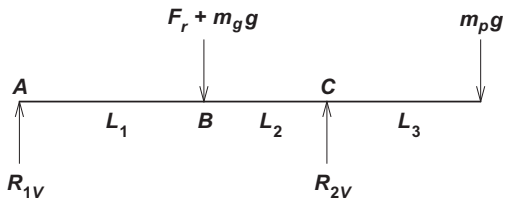


Figure 7.32
Vertical loading diagram.

Resolving vertical forces: $R_{1V} + R_{2V} = F_r + m_g g + m_p g$.

Hence, $R_{1V} = 321.9 + 78.5 + 98.1 - 387.4 = 111.1$ N.

The horizontal loads on the shaft are illustrated in Figure 7.33.

T = total tension $= T_1 + T_2 = 1131.8 + 452.8 = 1585$ N.

Moments about A: $F_t L_1 - R_{2H}(L_1 + L_2) + T(L_1 + L_2 + L_3) = 0$.

Hence,

$$R_{2H} = \frac{F_t L_1 + T(L_1 + L_2 + L_3)}{L_1 + L_2} = \frac{(884.4 \times 0.12) + (1585 \times 0.3)}{0.2} = 2908 \text{ N}.$$

Resolving horizontal forces: $R_{1H} + R_{2H} = F_t + T$.

Hence, $R_{1H} = 884.4 + 1585 - 2908 = -438.5$ N.

The bending moment diagrams can now be determined. You may wish to recall that the bending moment is the algebraic sum of the moments of the external forces to one side of the section about an axis through the section.

The vertical bending moments at B and C are given by (see Figure 7.34):

$M_{BV} = R_{1V} L_1 = 111.1 \times 0.12 = 13.3$ N m,
$M_{CV} = R_{1V}(L_1 + L_2) - ((F_r + m_g g)L_2) = 111.1(0.12 + 0.08) - (321.9 + 8 \times 9.81)$
$0.08 = -9.810$ N m.

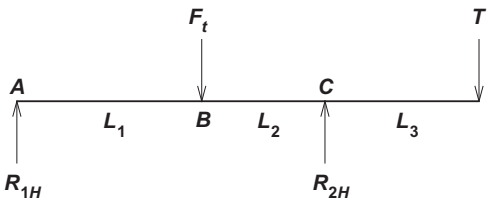


Figure 7.33
Horizontal loading diagram.

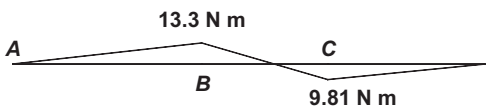


Figure 7.34
Vertical bending moment diagram.

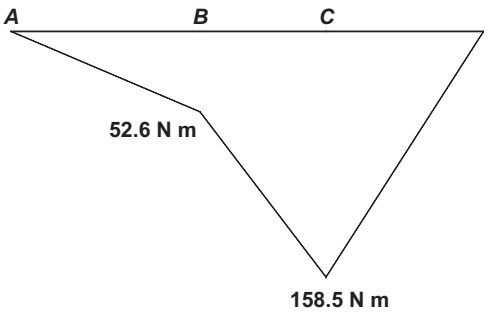


Figure 7.35
Horizontal bending moment diagram.

The horizontal bending moments at B and C are given by (see Figure 7.35):

$$M_{BH} = R_{1H}L_1 = -438.5 \times 0.12 = -52.62 \text{ N m},$$
$$M_{CH} = R_{1H}(L_1 + L_2) - (F_tL_2) = -438.5 \times 0.2 - 884.4 \times 0.08 = -158.5 \text{ N m}.$$

The resultant bending moment diagram can be determined by calculating the resultant bending moments at each point.

$$|M_B| = \sqrt{(13.3)^2 + (-52.62)^2} = 54.27 \text{ N m}.$$
$$|M_C| = \sqrt{(-9.81)^2 + (-158.5)^2} = 158.8 \text{ N m}.$$

From Figure 7.36 it can be seen that the maximum bending moment occurs at the right-hand bearing. The value of the resultant bending moment here should be used in the ASME design equation.

The next task is to determine the endurance limit of the shaft and the modifying factors. The shaft material is hot rolled steel, and the endurance limit of the test specimen, if unknown, can be estimated using $\sigma'_e = 0.504\sigma_{\text{uts}}$. The ultimate tensile strength for 817M40 is 1000 MPa and σ_y is 770 MPa. So, $\sigma'_e = 0.504 \times 1000 = 504$ MPa.

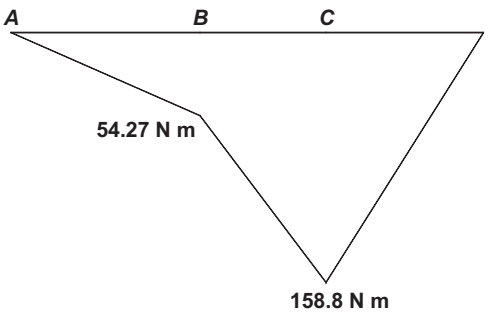


Figure 7.36
Resultant bending moment diagram.

The material is hot rolled so from Eqn (7.14) and Table 7.3, $k_a = a\sigma_{\text{uts}}^b = 57.7(1000)^{-0.718} = 0.405$.

Assuming that the diameter will be about 30 mm, the size factor can be estimated using $k_b = (d/7.62)^{-0.1133} = (30/7.62)^{-0.1133} = 0.856$. If the shaft is significantly different in size to 30 mm, then the calculation should be repeated until convergence between the assumed and the final calculated value for the shaft diameter is achieved.

The desired nominal reliability is 90%, so $k_c = 0.897$.

The operating temperature is not stated so a value of temperature factor of $k_d = 1$ is assumed. The duty cycle factor is assumed to be $k_e = 1$.

The fillet radius at the shoulders is 3 mm. The ratio of diameters $D/d = (3 + 3 + 30)/30 = 36/30 = 1.2$. $r/d = 3/30 = 0.1$. So from Figure 7.27 the geometric stress concentration factor $k_t = 1.65$.

From Figure 7.26 the notch sensitivity index for a 1000 MPa strength material with notch radius of 3 mm is $q = 0.9$.

$K_f = 1 + q(k_t - 1) = 1 + 0.9(1.65 - 1) = 1.59$.

$k_f = 1/K_f = 0.629$.

The miscellaneous factor is taken as $k_g = 1$.

The endurance limit can now be calculated from $\sigma_e = k_a k_b k_c k_d k_e k_f k_g \sigma'_e$.

$$\sigma_e = 0.405 \times 0.856 \times 0.897 \times 1 \times 1 \times 0.629 \times 1 \times 504 = 98.6 \text{ MPa}.$$

As a well known material has been selected, subject to known loads, the factor of safety can be taken as $n_s = 2$. The diameter can now be calculated from the ASME equation.

$$d = \left[\frac{32n_s}{\pi}\sqrt{\left(\frac{M}{\sigma_e}\right)^2 + \frac{3}{4}\left(\frac{T}{\sigma_y}\right)^2}\right]^{1/3} = \left[\frac{32 \times 2}{\pi}\sqrt{\left(\frac{158.8}{98.6 \times 10^6}\right)^2 + 0.75\left(\frac{84.9}{770 \times 10^6}\right)^2}\right]^{1/3}$$
$$= 0.032 \text{ m}$$

As this value is close to the assumed value used to evaluate the size and fatigue stress factors, further iteration is not necessary. For manufacturing convenience, it may be necessary to modify this diameter to the nearest standard size, as used within the company or taking into account materials readily available from suppliers, in this case 35 mm. Standard sizes can be found in texts such as the *Machinery's Handbook*. An advantage of using standard sizes is that standard stock bearings can be selected to fit.

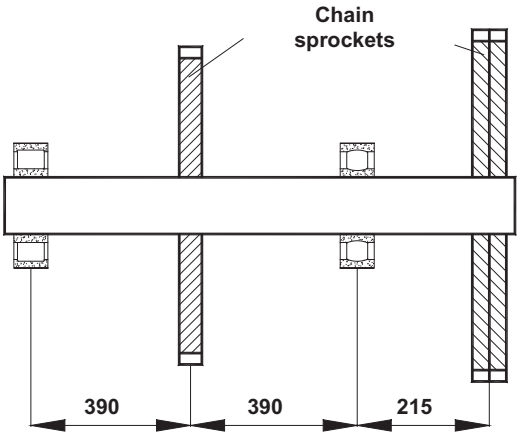


Figure 7.37
Chain conveyor drive shaft (dimensions in mm).

Example 7.9

Determine the diameter of the drive shaft for a chain conveyor, which has the loading parameters illustrated in Figure 7.37. A roller chain sprocket of 500 mm pitch diameter, weighing 90 kg will be midmounted between two bearings. A 400 mm, 125 kg roller chain sprocket will be mounted overhung. The drive shaft is to be manufactured using cold drawn 070M20 steel. Operating temperatures are not expected to exceed 65 °C and the operating environment is noncorrosive. The shaft is to be designed for nonlimited life of greater than 10^8 cycles with a 90% survival rate. The shaft will carry a steady driving torque of 1600 N m and rotate at 36 rpm. A sled runner keyway will be used for the overhung pulley and a profile keyway for the midmounted pulley. (Example adapted from ASME B106.1M − 1985.)

Solution

Figures 7.38 to 7.41 illustrate the principal loads and bending moment diagrams.

$$070M20 \;\; \sigma_{\text{uts}} = 560 \;\; \text{MPa}, \sigma_y = 440 \;\; \text{MPa}$$

Torque 1600 N m $= P \times d$.
$P_1 = 1600/0.25 = 6400$ N,
$P_2 = 1600/0.2 = 8000$ N.
Moments about R_1,
$6400 \times 0.39 - R_{2H} \times 0.78 + 8000 \times 0.995 = 0$.
$R_{2H} = 13{,}405.13$ N.

Resolving horizontal loads,

$$R_{1H} + R_{2H} = P_1 + P_2.$$

$$R_{1H} = 6400 + 8000 - 13{,}405.13 = 994.87 \text{ N}.$$

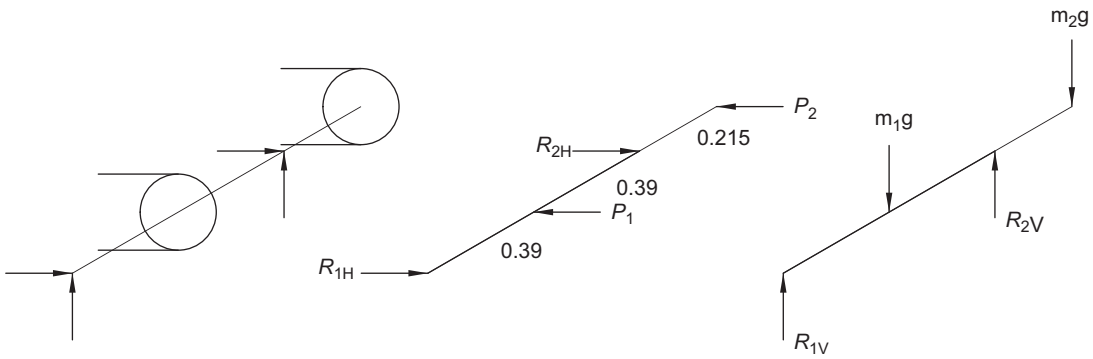


Figure 7.38
Principal loads.

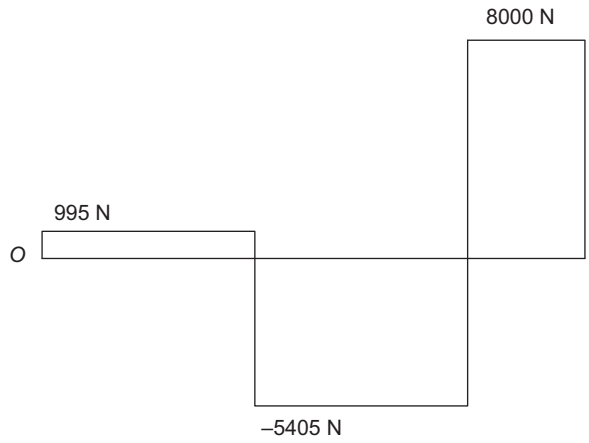


Figure 7.39
Horizontal shear force diagram.

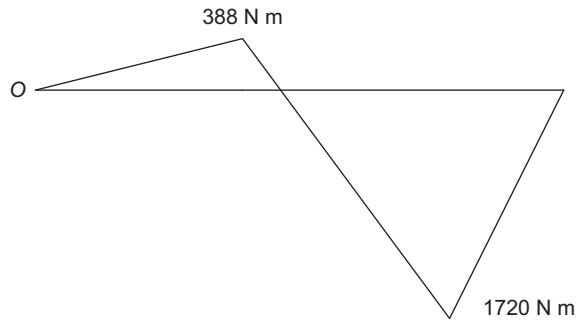


Figure 7.40
Horizontal bending moment diagram.

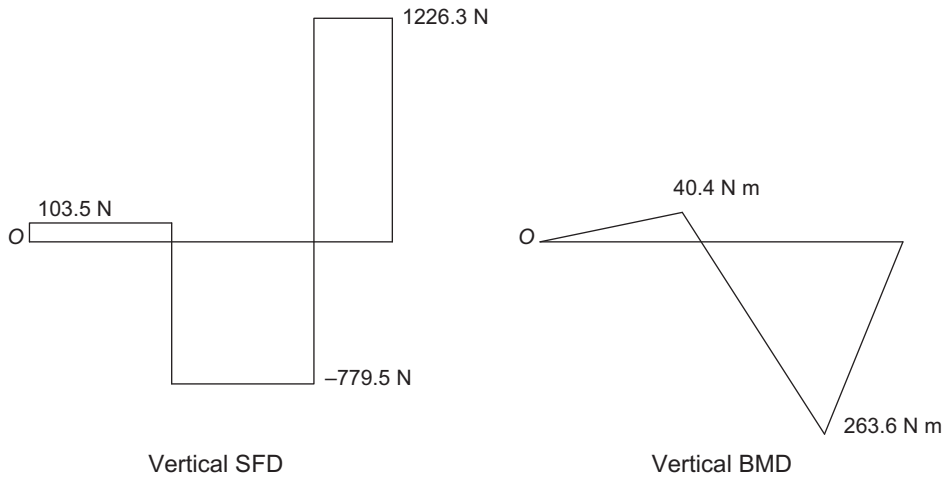


Figure 7.41
Vertical shear force diagram and bending moment diagram.

Horizontal bending moment diagram:

At pulley 1, $995 \times 0.39 = 388$ N m.
At pulley 2, $8000 \times 0.215 = 1720$ N m.

Vertical loads and bending moments:

$$R_{1V} + R_{2V} = 882.9 + 1226.3.$$

Moments about R_1

$$882.9 \times 0.39 - R_{2V} \times 0.78 + 1226.3 \times 0.995 = 0.$$

$R_{2V} = 2005.7$ N,
$R_{1V} = 103.45$ N.
Vertical moment at pulley 1: $103.45 \times 0.39 = 40.4$ N m.
Vertical moment at pulley 2: $1226.3 \times 0.215 = 263.6$ N m.

Resultant maximum bending moment,

$$M_{\max} = \left(263.6^2 + 1720^2\right)^{0.5} = 1740 \text{ N m}.$$

Factors for ASME equation are as follows:

$k_a = 4.51\sigma_{\text{uts}}^{-0.265} = 0.843$.
$k_b = 1.85\, d^{-0.19}$. Assume $d = 70$ mm, so $k_b = 0.83$.
$k_c = 0.9$.
$k_d = 1$.

$k_e = 1$.
$k_f = 0.5$ (The bending moment at the profiled keyway region is far greater. Also, in the absence of information about the hardness, assume worse case value.)
$k_g = 1$.
$\sigma'_e = 0.504\sigma_{\text{uts}} = 0.504 \times 560 = 282$ MPa.
$\sigma_e = 0.843 \times 0.83 \times 0.9 \times 1 \times 1 \times 0.5 \times 1 \times 282 = 88.87$ MPa.

Substituting values into the ASME equation for transmission shafts,

$$d = \left(\frac{32 \times 2}{\pi} \sqrt{\left(\frac{1740}{88.87 \times 10^6} \right)^2 + \frac{3}{4}\left(\frac{1600}{440 \times 10^6} \right)^2} \right)^{0.333} = 0.0739 \text{ m}.$$

$$d = 74 \text{ mm}$$

As a standard diameter, the shaft could be specified as 75 mm.

(Note if d assumed for k_b is significantly different from the result, repeat calculation until reasonable convergence.)

Example 7.10

A shaft is required for the chain drive illustrated in Figure 7.42. The power transmitted is 12 kW at 100 rpm. The masses of the sprockets are 5 kg and 16.3 kg with respective

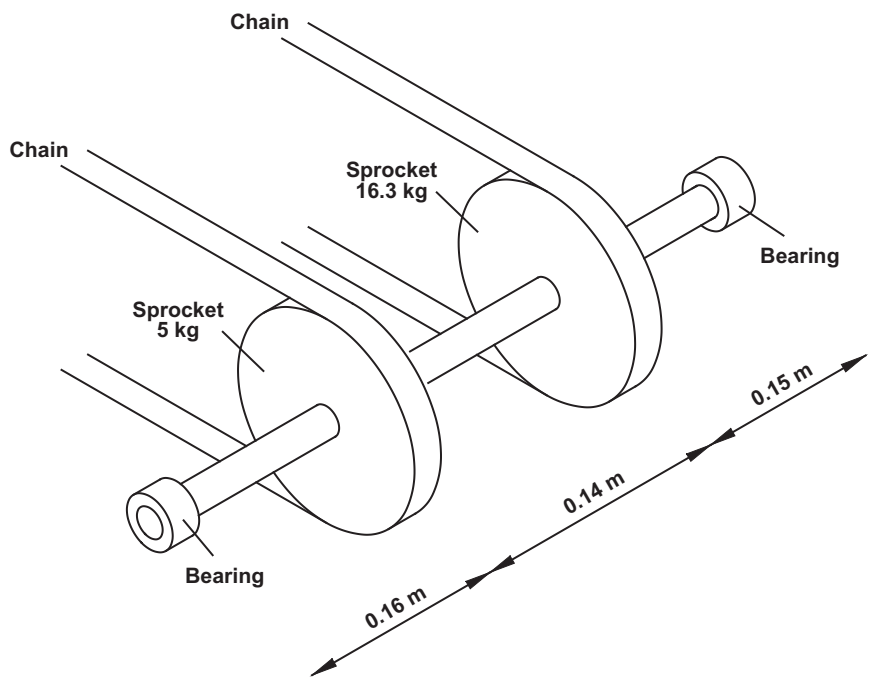


Figure 7.42
Chain drive.

diameters of 254.6 mm and 460.8 mm. The material selected for the shaft is quenched and drawn 817M40 with $\sigma_{uts} = 850$ MPa and $\sigma_y = 700$ MPa. The torque is transmitted between the shaft and sprockets by means of keys in profiled keyways. Determine the minimum diameter for the shaft based on the ASME equation for the design of transmission shafting if the desired reliability is 90%. Use an initial estimate for the shaft diameter of 60 mm.

Solution

Vertical loading (see Figure 7.43).

Moments about O (see Figure 7.44):

$$5 \times 9.81 \times 0.16 + 16.3 \times 9.81 \times 0.3 - R_{2V} \times 0.56 = 0.$$

$$R_{2V} = 124.0 \text{ N}.$$

Resolving vertically:

$$R_{1V} + R_{2V} = W_1 + W_2$$

$$R_{1V} = 49.05 + 159.9 - 124 = 84.91 \text{ N}$$

$$T = \frac{12{,}000}{100 \times (2\pi/60)} = 1146 \text{ N m}$$

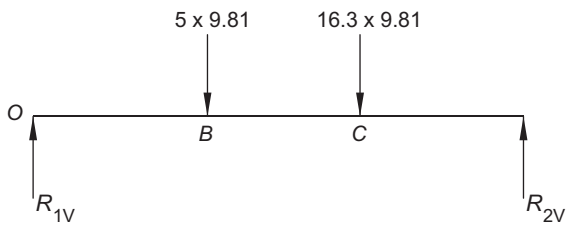


Figure 7.43
Vertical loading.

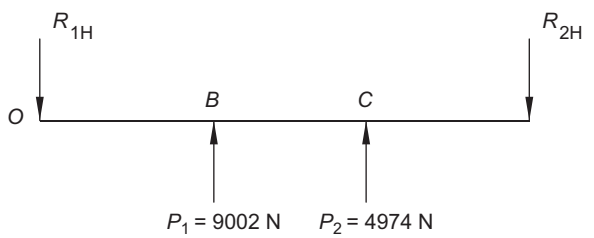


Figure 7.44
Horizontal loading.

$P_1 = T/r_1$

$P_2 = T/r_2$

$$P_1 = \frac{1146}{0.2546/2} = 9002 \text{ N}$$

$$P_2 = \frac{1146}{0.4608/2} = 4974 \text{ N}$$

Moments about O

$-9002 \times 0.16 - 4974 \times 0.3 + 0.45R_{2H} = 0$

$R_{2H} = 6517$ N

$R_{1H} + R_{2H} = 9002 + 4974$

$R_{1H} = 7459$ N

Vertical bending moments:

$M_{BV} = L_1 \times R_{1V} = 84.91 \times 0.16 = 13.59$ N m

$M_{CV} = R_{1V} \times 0.3 - 5 \times 9.81 \times 0.14 = 18.61$ N m

Horizontal bending moments:

$M_{BH} = R_{1H} \times L_1 = 7459 \times 0.16 = 1193$ N m

$M_{CH} = R_{1H} \times (L_1 + L_2) - 9002 \times 0.14 = 7459 \times 0.3 - 9002 \times 0.14 = 977.4$ N m

$|M_{\max}| = \sqrt{13.59^2 + 1193^2} = 1193$ N m

The vertical contribution was insignificant and could have been ignored.

$T_{\max} = 1146$ N m

$\sigma_e = k_a k_b k_c k_d k_e k_f k_g \sigma'_e$

$\sigma'_e = 0.504 \times 850 = 428.4$ MPa

$k_a = 4.51\sigma_{\text{uts}}^{-0.265} = 0.7549$

$k_b = 1.85\, d^{-0.19}$. Assume $d = 60$ mm, so $k_b = 0.8498$

$k_c = 0.897$

$k_d = 1.0$ (no special factors assumed)

$k_e = 1.0$ (no special factors assumed)

$k_f = 0.5$ (key)

$k_g = 1.0$ (no special factors assumed)

$\sigma_e = 0.7549 \times 0.8498 \times 0.897 \times 1 \times 1 \times 0.5 \times 1 \times 428.4 = 123.3$ MPa

Take $n_s = 2$. $n_s = 2$ is suitable for well-known loads and average materials.

$$d = \left(\frac{32 \times 2}{\pi} \sqrt{\left(\frac{1193}{123.3 \times 10^6} \right)^2 + \frac{3}{4} \left(\frac{1146}{700 \times 10^6} \right)^2} \right)^{0.333} = 0.058 \text{ m}$$

$d = 58$ mm is close enough to the guessed value of 60 mm, so it is not necessary to reevaluate k_b.

7.6 Detailed Design Case Study

A small and medium enterprise (SME) company has investigated the potential market for a small electric motor driven air compressor. The market analysis has confirmed the requirement for an 80 m^3/h, 7 bar air supply driven by a 5 kW electric motor. Preliminary design has already resulted in the selection of an electric motor, with a synchronous speed of 2850 rpm, and the piston and cylinder and associated components for a reciprocating compressor. The compressor design speed is 1500 rpm. A schematic for the compressor concept is given in Figure 7.45. The task here is to explore the detailed design of the shafts for this compressor if a geared transmission has been selected. Various alternatives might be possible for the transmission, based on the distance between the motor and the final drive and some of these areas illustrated in Figure 7.46, including a chain, belt, or gear drive as well as direct drive. Here it is assumed that the gear drive provides the most suitable arrangement and the detailed design for this option is explored.

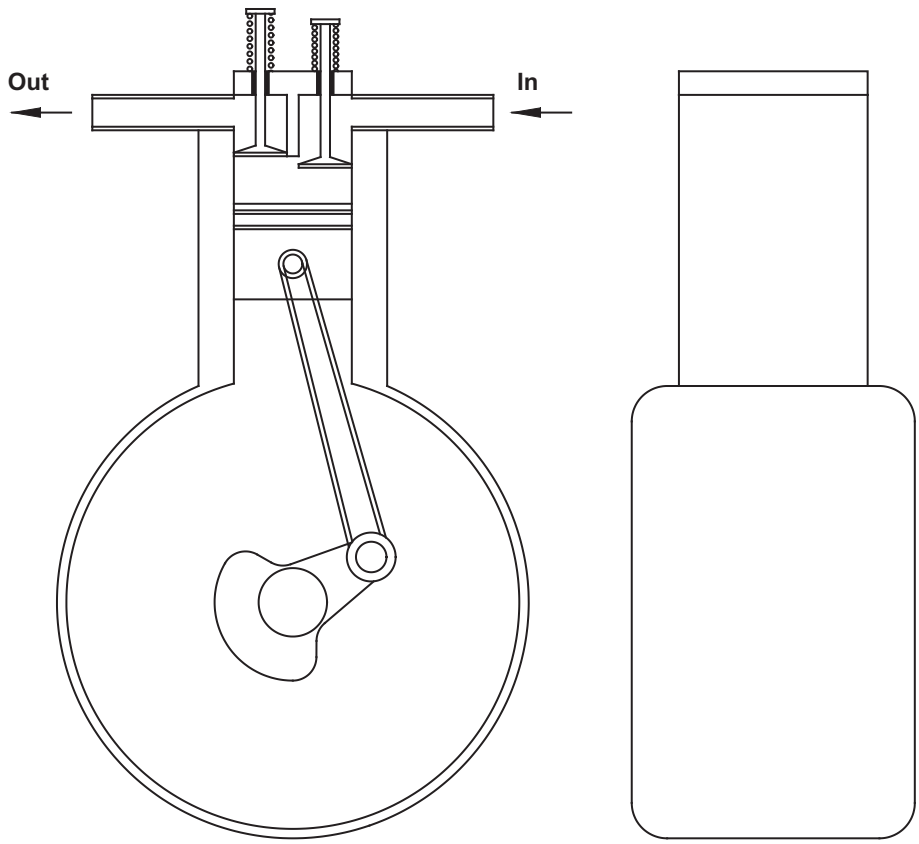


Figure 7.45
Schematic for a 7 bar, 80 m^3/h reciprocating compressor.

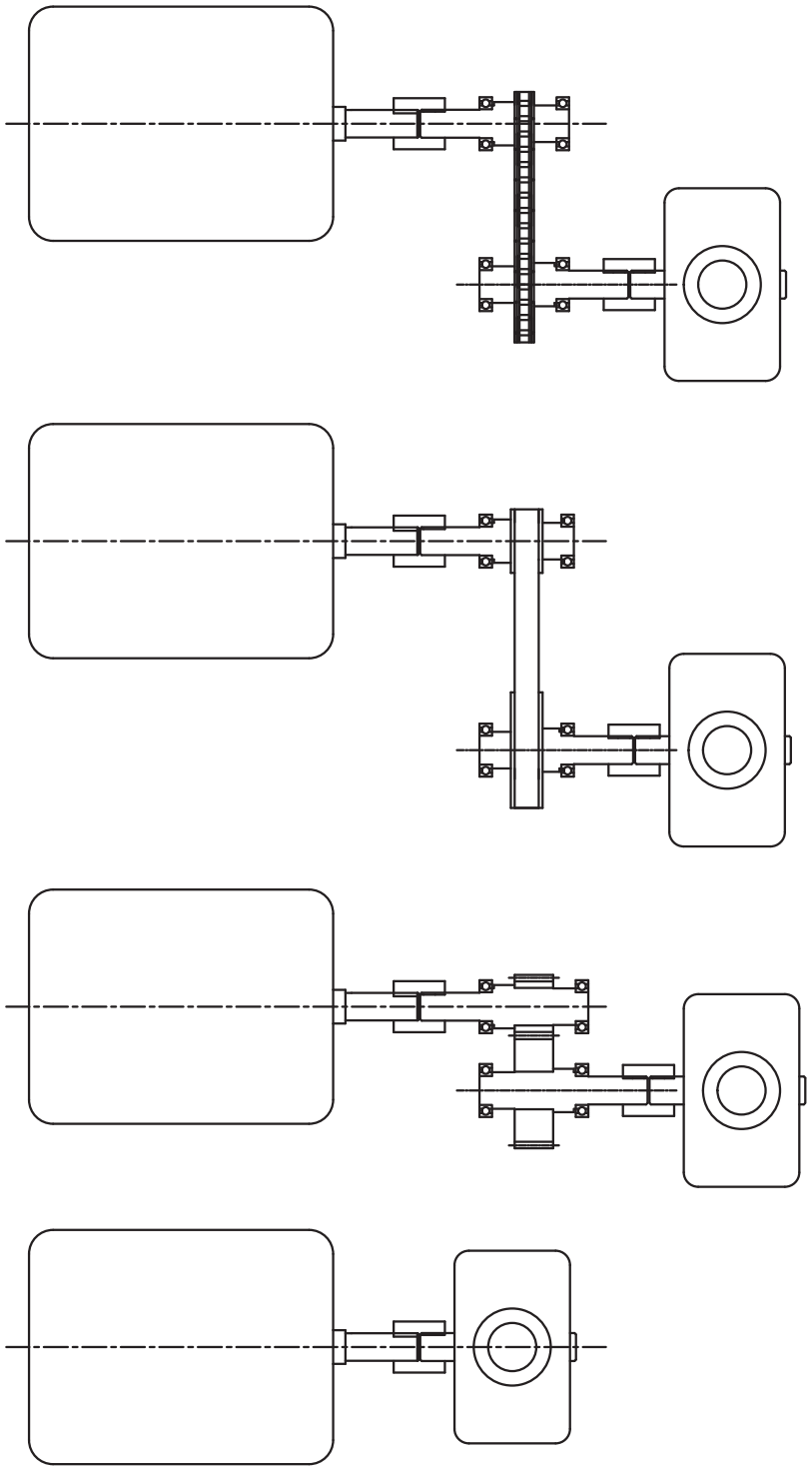


Figure 7.46
Conceptual arrangements for the transmission.

For a reduction ratio of 1.9, a single pair of gears is possible. Spur, helical, bevel, and worm gears are all possible. Helical gears are generally quieter and more compact than spur gears, but the thrust load they generate must be accommodated. Bevel gears provide drive through right angles, which is not a requirement here and worm gears are too inefficient. Spur gears appear to be the most appropriate selection for the design.

Given that this design must be capable of operating for 2 h a day, 200 days a year with a warrantee life of 18 months, detailed calculations based on the American Gear Manufacturers Association (AGMA) standards can be made for the gears. The size of the gears is unknown at this stage. The results of calculations for a range of modules and face widths are given in Table 7.8 (see Childs, 2004).

Examination of Table 7.8 shows that all the proposals have high factors of safety for bending. The highest threat of failure, for all the proposals, is due to wear on the pinions. The proposal with the highest factor of safety for contact stress on the pinions and acceptable values for the other parameters is $m = 3$. This design proposal has, therefore, been selected for the transmission.

The next step in the design is to size the shaft. This design experiences bending moments and torque so the ASME design code for transmission shafting is appropriate. Figure 7.47 gives an approximate arrangement for the shaft illustrating proposed dimensions, shoulders, steps, and keyways. In order to determine the shaft diameter using Eqn (7.10), it is necessary to know the torque, bending moment and endurance limit for the material.

The transmitted load from the gears is 558 N.

For a spur gear the radial load is given by

$$W_r = W_t \tan \phi = 558 \tan 20 = 203 \text{ N}$$

Table 7.8: AGMA bending and contact safety factors for a variety of modules and face widths.

Module	$m = 2$	$m = 2$	$m = 2.5$	$m = 3$
N_P	20	20	20	20
N_G	38	38	38	38
d_P (mm)	40	40	50	60
d_G (mm)	76	76	95	114
V (m/s)	5.97	5.97	7.46	8.95
F (mm)	20	50	30	40
W_t (N)	838	838	670	558
H_{BP}	240	240	240	240
H_{BG}	200	200	200	200
S_{FP}	1.6	3.7	3.6	6.6
S_{FG}	1.7	3.9	3.8	7.0
S_{HP}	0.76	1.1	1.1	1.5
S_{HG}	0.96	1.4	1.4	1.9

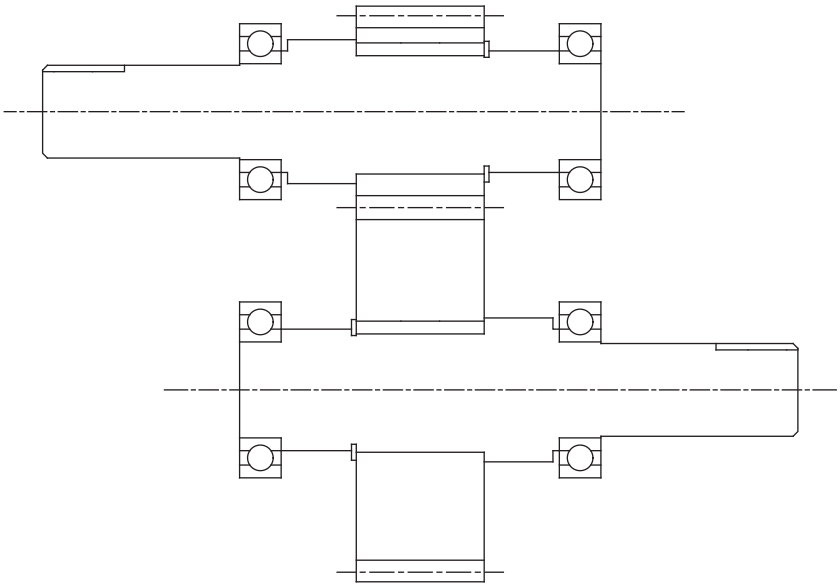


Figure 7.47
General arrangement proposal for the pinion and gear shafts.

The mass of the pinion is approximately 0.46 kg. The mass of the shaft is approximately 0.57 kg. Combining these values gives an approximate vertical load due to gravity of 10.1 N.

The mass of the gear is approximately 1.66 kg. The mass of the shaft is approximately 0.57 kg. Combining these values gives an approximate vertical load due to gravity of 21.9 N.

These various loads and the geometry proposed in Figure 7.47 are used to develop the shearing force and bending moment diagrams given in Figure 7.48. The maximum resultant bending moments for the pinion and gear shafts are 15.1 N m and 15.4 N m, respectively (see Figure 7.48).

The torque transmitted by the pinion shaft is given by

$$T = \frac{\text{power}}{\omega} = \frac{5000}{2850 \times \dfrac{2\pi}{60}} = 16.8 \text{ N m}$$

The torque transmitted by the gear shaft is given by

$$T = \frac{\text{power}}{\omega} = \frac{5000}{1500 \times \dfrac{2\pi}{60}} = 31.8 \text{ N m}$$

An estimate of the shaft diameter needs to be made in order to evaluate the endurance limit. Here, $d = 30$ mm will be assumed. 817M40 hot rolled alloy steel will be proposed for the

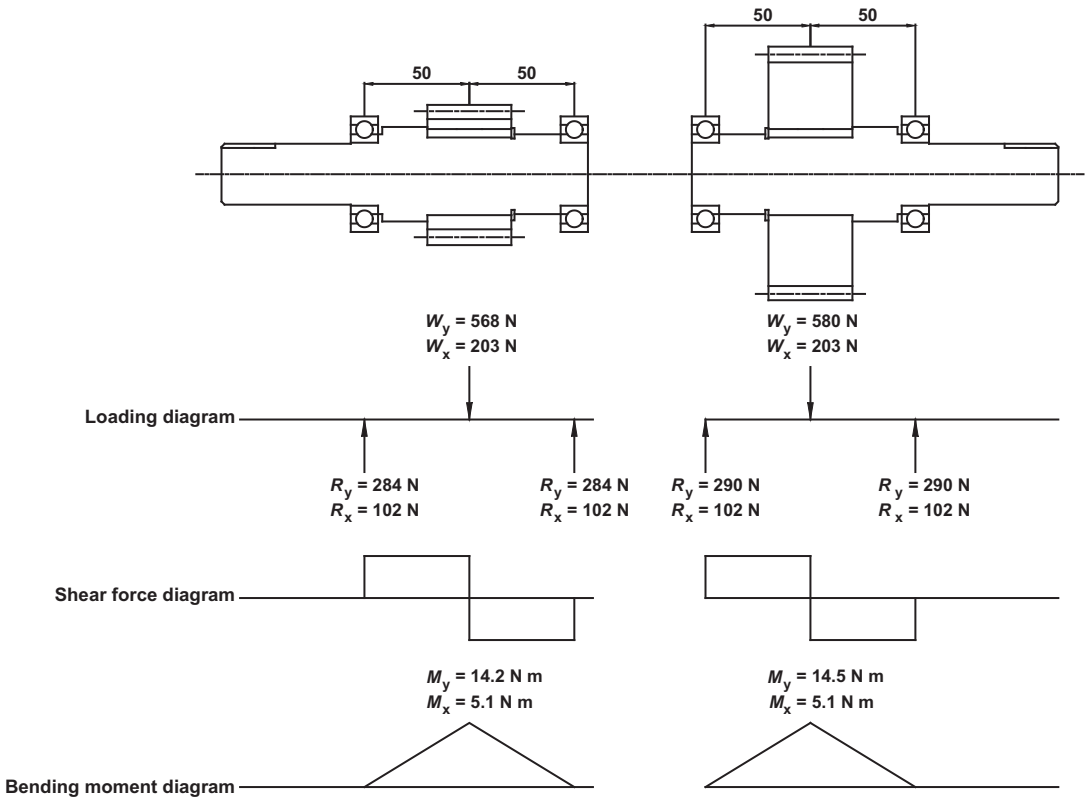


Figure 7.48
Shear and bending moment diagrams for the proposed shafts.

shaft. The ultimate tensile and yield strengths for this material are 1000 MPa and 770 MPa, respectively. Taking $k_a = 0.405$, $k_b = 0.856$, $k_c = 0.814$, $k_d = 1$, $k_e = 1$, $k_t = 2$, $q = 0.72$, $k_f = 0.538$, $k_g = 1$, and $\sigma'_e = 504$ MPa gives $\sigma_e = 66.1$ MPa.

The ASME equation for transmission shafting gives shaft diameters of 29.3 and 29.8 mm for the pinion and gear shafts, respectively. It should be noted that the diameter determined is particularly sensitive to the surface finish and the fatigue stress concentration factors.

It is necessary to check the critical frequency, deflection, and slope characteristics of the shaft. The deflection of the shaft can be estimated by considering the shaft to be a simply supported beam. The deflection at midspan of a simply supported beam with a midmounted load is given by

$$y = -\frac{WL^3}{48EI}$$

Taking the total mass of the gear and the shaft located at midspan the deflection is

$$y = -\frac{29.1 \times 0.1^3}{48 \times 200 \times 10^9 \pi 0.03^4/64} = 7.624 \times 10^{-8} \text{ m}$$

This is a very small deflection indicating that the slope of the shaft, at the bearings, and deflection, are unlikely to be a problem in this design.

Substitution in the Rayleigh–Ritz equation gives a first critical frequency of

$$\omega_c = \sqrt{g/y} = \sqrt{9.81/7.624 \times 10^{-8}} = 11340 \text{ rad/s}$$

This represents a rotating speed of 108,000 rpm, which is an order of magnitude higher than the operating speed. Even accounting for deflection in the bearings, the critical speed will still be significantly higher than the design speed of the gearbox. This is not surprising given the stocky nature of the shaft with its small span.

Deep groove ball bearings are proposed for the transmission, as these bearings provide support for the radial load and axial alignment. The design life for the bearings is 1.02×10^8 revolutions. The maximum load on the bearings is approximately 310 N. The basic dynamic load rating required is given by

$$C = 310\left(\frac{1.02 \times 10^8}{1 \times 10^6}\right)^{1/3} = 1448 \text{ N}$$

A 30 mm bore bearing with a basic dynamic load rating greater than this value and a speed capability with grease filled bearings of more than 2850 rpm would be suitable. The bearings from a number of specialist bearing manufacturers should be considered. ISO designation 6006 would be suitable with a considerable margin of safety. It is worth considering worse case usage. If the machine was operated continuously for the warranty period and other components did not fail, the design life would be

$$2850 \times 60 \times 24 \times 365 \times 1.5 = 2.25 \times 10^9 \text{ revolutions}$$

Using this figure the required basic dynamic load rating is 3798 N.

Selecting a low-carbon, cold drawn steel for the key with a yield strength of 350 MPa, assuming a square key of 4×4 mm and setting $N = 4$ for this application, the necessary key length is given by

$$L = \frac{4TN}{DW\sigma_y} = \frac{4 \times 31.8 \times 4}{0.03 \times 0.004 \times 350 \times 10^6} = 0.0121 \text{ m}$$

This length can readily be accommodated in the shaft and mating components. Because it is comparatively short, it would be possible to use the same type of key for the gears and couplings. This would have the advantage of a reduced parts inventory, simplifying tooling and economy of scale.

An almost finished engineering drawing for the pinion shaft is given in Figure 7.49. The drawing includes the results of a considerable number of decisions. The calculations outlined so far in this case study have resulted in the specification of some of the major dimensions. However, geometric tolerances need to be specified on certain features in order to control features such as location, form, profile, orientation, and runout, ensuring adequate assembly and operation of the machine. In addition, particular surface finishes must be specified on critical regions, such as mating surfaces and areas experiencing reversed bending. A typical surface finish for a ground surface is 0.4 μm and that for turning or milling is 1.6 μm. Figure 7.49 can be examined by the reader and each dimension and feature scrutinized.

The similarity in diameter required for the pinion shaft and gear shaft provides an opportunity to use a common design for both shafts. This has the advantage of reduced inventory, set-up costs, and economies of scale. This approach has been adopted for this design.

The options for the transmission support and enclosure realistically include fabricating, molding, or casting using metal or plastic. The temperature of operation indicates that a plastic material is a possibility provided rigidity, strength, and cost requirements are met. Metals are, however, an obvious choice. In order to provide rigid support use of a two-part casting assembly is explored in Figure 7.50. Turned inserts are proposed for the stationary bearing mountings. Although this solution appears adequate, fabrication and use of plastic molding should also be explored to identify whether they can meet the specification and provide a cost advantage (Figures 7.50 and 7.51).

Engineering drawings, similar to that given in Figure 7.49, should also be produced for all the components in the transmission. Here, however, just a general arrangement for the overall transmission is given in Figure 7.51.

Although the design developed is probably far from optimal, an estimate of the works' cost price should be developed in order to identify whether the proposal is likely to meet the specification in terms of price and which aspects are responsible for determining the majority of the costs. These can then be assessed and the use of expensive features justified or, if possible, removed from the design altogether. Estimating the cost of a product at the design stage is notoriously difficult. In the design proposal presented here, estimates for the costs of stock items such as the gears, bearings, keys, and fasteners can be obtained by examination of price lists if published or by contacting the original equipment manufacturer (OEM) or stock supplier concerned and obtaining a quotation. Ensure that the quotation provides the appropriate discount for the quantities concerned. For a production run of 1000 units per year,

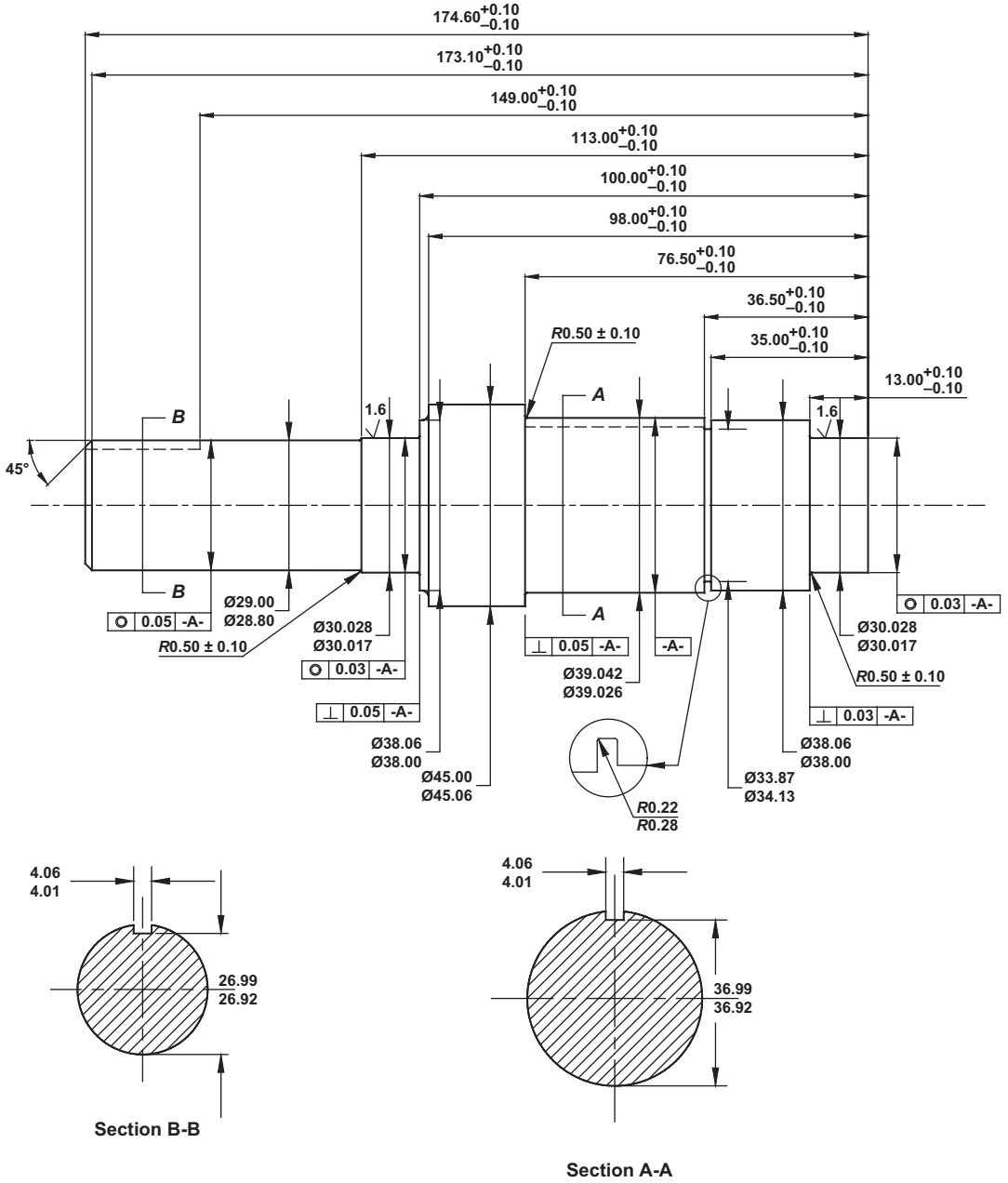


Figure 7.49
Pinion shaft.

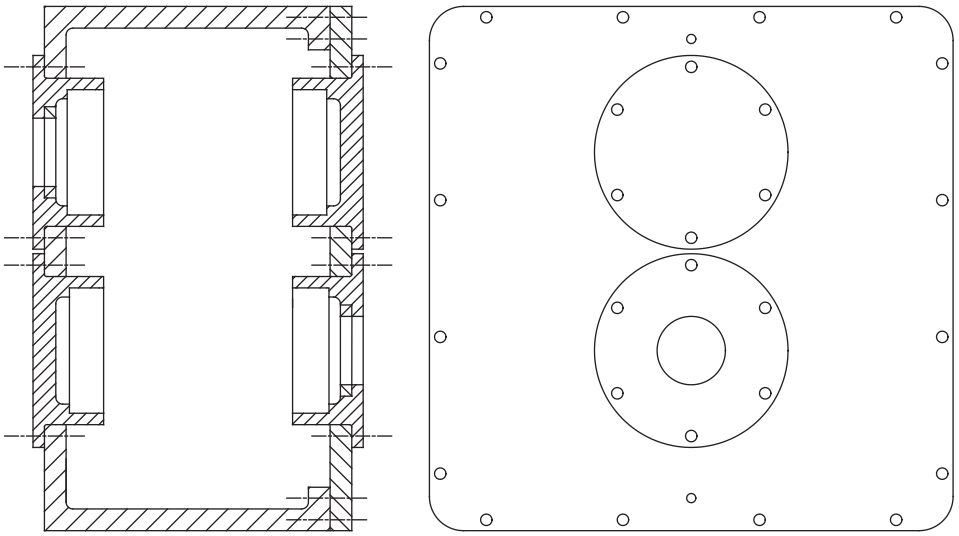


Figure 7.50
Casing.

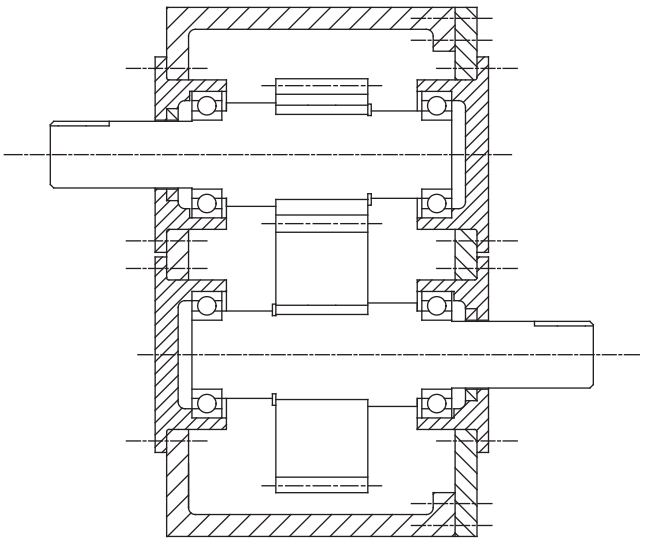


Figure 7.51
General arrangement for the compressor transmission.

it is likely that the transmissions will be manufactured in batches. In this case, a quotation for the necessary batch size would seem appropriate so that variations in demand will not tie up company capital unnecessarily. Estimates for the cost of the castings will need to be obtained by means of quotations from casting companies. In this design, machining of the shafts and finish machining of the castings need to be performed either in-house or bought in. Obvious

Table 7.9: Estimates for component costs based on a batch run of 100 gearboxes.

Item	Number	Time	Unit cost (Euro)	Total cost (Euro)	Source of estimate
Pinion	1 off		12	12	Stock gear manufacturer
Gear	1 off		18	18	Stock gear manufacturer
Keys	6 off		0.5	3	Stock supplier
Shaft material	2 off		10	20	Metal supplier
Bearings	4 off		10	40	Stock supplier
Split rings	2 off		2	4	Stock supplier
Main casting	1 off		60	60	Quotation from casting specialist
Casting cover	1 off		20	20	Quotation from casting specialist
Casting seal	1 off		2	2	Oring cording supplier
Bearing mountings material	4 off		5	20	Stock supplier
Shaft seals	2 off		8	16	Seal manufacturer
Couplings	2 off		20	40	Coupling manufacturer
Fasteners	20 off		0.25	5	Fastener stock supplier
Gear hardening	Batch of 200		100	1 Per transmission	Specialist service
Materials subtotal				261	
Keyway machining	6 off	2 min each	60/h	12	Machine shop
Turning	2 off	15 min/shaft	60/h	30	In-house estimate
Grinding	2 off	10 min/shaft	60/h	20	In-house estimate
Casting machining	1 off	30 min	60/h	30	In-house estimate
Bearing mountings machining	4 off	10 min/ mounting	60/h	40	In-house estimate

Continued

Table 7.9: Estimates for component costs based on a batch run of 100 gearboxes.—cont'd

Item	Number	Time	Unit cost (Euro)	Total cost (Euro)	Source of estimate
Assembly	1 off	30 min	40/h	20	In-house estimate
Packaging	1off	4 min	4	4	In-house estimate
Labor subtotal				156	

quotations from machine shops can be obtained to evaluate the possibility of buying the shafts directly. For in-house machining, reliance needs to be placed on experience or use of standard estimates for machining times. Standard estimates for a wide range of machining processes are outlined in Swift and Booker (2013).

From Table 7.9, taking the materials overhead as 130% and the labor overhead as 225%, the works cost price is given by

$$\text{works cost price} = (261 \times 1.3) + (156 \times 2.25) = 690.3 \text{ Euros}$$

The works cost price for the transmission system is close to the target price of 600 Euros. With careful control of procurement costs and in-house manufacturing costs, it should be possible to get this total cost well below the 600 Euro target. As such, the design is likely to meet the specification.

A proposal for the compressor transmission has been developed. The design appears adequate in terms of function and compliance with the specification, although the works cost price has exceeded the target. It should, however, be possible to refine the design of the shaft and casing and bearing mounts to reduce costs, and cheaper quotations for the stock items may be available. It is also possible that one of the alternative concepts, such as the belt drive, could be considerably cheaper to produce.

7.7 Conclusions

Shaft design involves consideration of the layout of features and components to be mounted on the shaft, specific dimensions and allowable tolerances, materials, deflection, frequency response, life, and manufacturing constraints. This chapter has introduced the use of steps and shoulders and miscellaneous devices to locate components. It has also presented methods to calculate the deflection of a shaft and its critical speeds and to determine the minimum safe diameter for shaft experiencing torque and bending for a given life.

References

Books and Papers

Beswarick, J., 1994a. Shaft for strength and rigidity. In: Hurst, K. (Ed.), Rotary Power Transmission Design. McGraw Hill, pp. 135–141.
Beswarick, J., 1994b. Shaft with fluctuating load. In: Hurst, K. (Ed.), Rotary Power Transmission Design. McGraw Hill, pp. 142–148.
Childs, P.R.N., 2004. Mechanical design, 2nd edition. Butterworth Heinemann.
Fuchs, H.O., Stephens, R.I., 1980. Metal Fatigue in Engineering. Wiley.
Hurst, K., 1994. Shaft/hub connections. In: Hurst, K. (Ed.), Rotary Power Transmission Design. McGraw Hill, pp. 55–65.
Juvinall, R.C., 1967. Engineering Considerations of Stress, Strain and Strength. McGraw Hill.
Kuguel, R., 1961. A relation between theoretical stress concentration factor and fatigue notch factor deduced from the concept of highly stressed volume. Proc. ASTM 61, 732–748.
Kuhn, P., Hardrath, H.F., 1952. An Engineering Method for Estimating Notch Size Effect on Fatigue Tests of Steel. National Advisory Committee for Aeronautics, NACA. TN2805.
Marin, J., 1962. Mechanical Behaviour of Engineering Materials. Prentice Hall.
Mischke, C.R., 1996. Shafts. In: Shigley, J.E., Mischke, C.R. (Eds.), Standard Handbook of Machine Design. McGraw Hill.
Neale, M.J., Needham, P., Horrell, R., 1998. Couplings and Shaft Alignment. Professional Engineering Publishing.
Noll, C.G., Lipson, C., 1946. Allowable working stresses. In: Society for Experimental Stress Analysis, vol. 3.
Peterson, R.E., 1974. Stress Concentration Factors. Wiley.
Piotrowski, J., 2006. Shaft Alignment Handbook, third ed. CRC Press.
Pyrhonen, J., Jokinen, T., Hrabovcova, V., 2008. Design of Rotating Electrical Machines. Wiley-Blackwell.
Reshetov, D.N., 1978. Machine Design. Mir Publishers.
Sclater, N., 2011. Mechanisms and Mechanical Devices Sourcebook, fifth ed. McGraw-Hill.
Swift, K.G., Booker, J.D., 2013. Manufacturing process selection handbook. Butterworth Heinemann.
Vidosek, J.P., 1957. Machine Design Projects. Ronald Press.

Further Reading

Juvinall, R.C., Marshek, K.M., 1991. Fundamentals of Machine Component Design. Wiley.
Juvinall, R.C., Marshek, K.M., 2012. Machine Component Design. Wiley.
Macreadys, 1995. Standard Stock Range of Quality Steels and Specifications. Glynwed Steels Ltd.
Mischke, C.R., 1987. Prediction of stochastic endurance strength. Trans. ASME, J. Vib. Acoust. Stress Reliab. Des., 113–122.
Mott, R.L., 1992. Machine Elements in Mechanical Design. Merrill.
Schubert, P.B. (Ed.), 1982. Machinery's Handbook. Industrial Press Inc.
Sines, G., Waisman, J.L., 1959. Metal Fatigue. McGraw Hill.

Standards

ANSI/ASME B106.1M-1985. Design of Transmission Shafting.
BSI, 1984. Manual of British Standards in Engineering Drawing and Design. BSI, Hutchinson.
British Standards Institution. BS 4235: Part 2: 1977. Specification for Metric Keys and Keyways: Woodruff Keys and Keyways.
British Standards Institution. BS 4235: Part 1: 1992. Specification for Metric Keys and Keyways. Parallel and Taper Keys.
British Standards Institution. BS 3550: 1963. Specification for Involute Splines.

British Standards Institute. BS 970: Part 1:1996. Specification for Wrought Steels for Mechanical and Allied Engineering Purposes. General Inspection and Testing Procedures and Specific Requirements for Carbon, Carbon Manganese, Alloy and Stainless Steels.

Web Sites

At the time of press, the world-wide-web contained useful information relating to this chapter at the following sites:

www.allpowertrans.com
www.ameridrives.com
www.couplingcorp.com
www.heli-cal.com
www.magnaloy.com
www.mayrcorp.com
www.peterstubs.com
www.rw-america.com
www.servometer.com
www.zero-max.com

Nomenclature

The following symbols were used in this chapter. Generally, preferred SI units have been stated:

a	influence coefficient (m)
d	diameter (m)
E	Young's modulus (N/m^2)
F	force (N)
g	acceleration due to gravity (m^2/s)
I	second moment of area (m^4)
k_a	surface factor
k_b	size factor
k_c	reliability factor
k_d	temperature factor
k_e	duty cycle factor
k_f	fatigue stress concentration factor
K_f	component fatigue stress concentration factor
k_g	miscellaneous effects factor
k_t	geometric stress concentration factor
L	length (m)
m	mass (kg)
M	moment (N m)
n_s	factor of safety
q	notch sensitivity

R	reaction (N)
T	torque (N m), tension (N)
U	strain energy (J)
W_i	mass (kg) or weight of node i (N)
x	coordinate (m)
y	static deflection (m)
y_i	static deflection of W_i (m)
σ_e	endurance limit of the item (N/m^2)
σ'_e	endurance limit of test specimen (N/m^2)
σ_{uts}	ultimate tensile strength (N/m^2)
σ_y	yield strength (N/m^2)
ω	frequency
ω_c	critical speed (rad/s)

CHAPTER 8

Gears

Chapter Outline

8.1 Introduction 317
8.2 Construction of Gear Tooth Profiles 329
8.3 Gear Trains 331
8.3.1 Manually Shifted Automotive Transmissions 337
8.3.2 Epicyclic Gear Trains 341
8.4 Tooth Systems 344
8.5 Force Analysis 345
8.5.1 Introduction to Gear Stresses 346
8.5.2 Bending Stresses 346
8.6 Simple Gear Selection Procedure 348
8.7 Conclusions 364
References 364
Further Reading 365
Nomenclature 365

Abstract

Gears are toothed cylindrical wheels used for transmitting mechanical power from one rotating shaft to another. Several types of gears are commonly used and are available as stock items from original equipment suppliers worldwide. This chapter introduces various types of gears and gear transmission and details the design, specification, and selection of spur gears, in particular, based on the consideration of failure due to bending using the Lewis equation.

8.1 Introduction

Many power producing machines, or prime movers, such as IC engines, industrial gas turbine engines (GTEs), and electric motors produce power in the form of rotary motion. The operating characteristics of prime movers vary according to their type and size. Typical examples are illustrated in Figure 8.1 for an IC engine, an alternating current electric motor and a series wound direct current electric motor. These characteristics may not match those of a given application. For example, the IC engine gives little torque at the low speeds associated with starting. It is for this reason that a car engine requires a clutch to decouple the engine from the load during starting and a gearbox to transform the characteristic of the prime mover to match that of the load.

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00008-3
Copyright © 2014 Elsevier Ltd. All rights reserved.

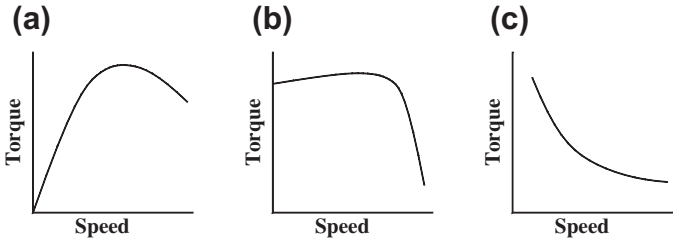


Figure 8.1
Typical torque speed characteristics. (a) IC engine. (b) Three-phase induction alternating current motor. (c) Series wound direct current motor.

A change of speed can be used to transform the torque speed characteristic of a prime mover to a useful output characteristic. An example of this is the use of a reduction gearbox to reduce the speed and increase the torque seen by a load, as illustrated in Figure 8.2. Recalling the relationship between torque, angular velocity, and power,

$$T_q\Omega = \text{Power} \tag{4.1}$$

or torque = power/angular velocity.

Examination of Eqn (8.1) indicates that as the speed reduces the torque rises for a given power. So if increased torque is required then a reduction gearbox, which decreases the speed of the output shaft relative to the input shaft, can be used to increase the torque delivered by the output shaft of the gearbox.

When transmitting power from a source to the required point of application, a series of devices are available including belts, pulleys, chains, hydraulic and electrical systems, and gears. Generally, if the distances of power transmission are large, gears are not suitable, and chains and belts can be considered. However, when a compact, efficient, or high-speed drive is required, gear trains offer a competitive and suitable solution.

Gears, belts, and chains are all available from specialist suppliers. The manufacture of gears requires specialist machinery, and the stresses in the region of contact between gears in mesh

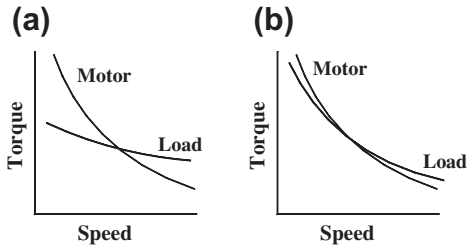


Figure 8.2
Transformation of the torque speed characteristic using a reduction gear ratio. (a) Mismatched system. (b) Transformed system.

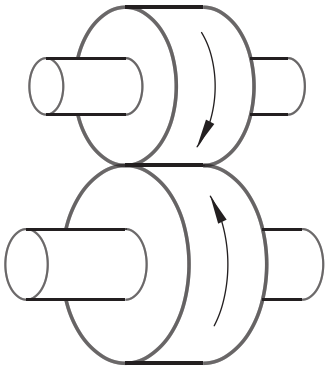


Figure 8.3
Disk or roller drives.

can be exceptionally high requiring the use of specialist materials. Similarly, the design of a roller chain and associated sprockets involves high contact stresses and, in addition, the design of an articulated joint and associated bearings. For these reasons, combined with financial expediency, leaving the cost of specialist manufacture infrastructure to other companies, gear, chain, and belt manufacturing is usually best left to the specialist, and the task of the machine designer becomes that of selecting and specifying appropriate components from the specialist stock suppliers and the original equipment manufacturers. A general introduction to gears is considered in this chapter. Chapters 9–11 provide further detailed consideration of spur and helical gears, bevel gears, and worm and wheels, respectively. The design and selection of belts and chains is considered in Chapter 12.

Simplistically, a speed change can be achieved by either running disks of different diameters together, illustrated in Figure 8.3, or alternatively cones for turning corners as well, shown in Figure 8.4. However, the torque capacity of disk or cone drives is limited by the frictional

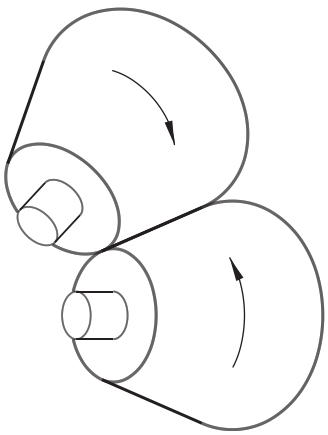


Figure 8.4
Cone drives.

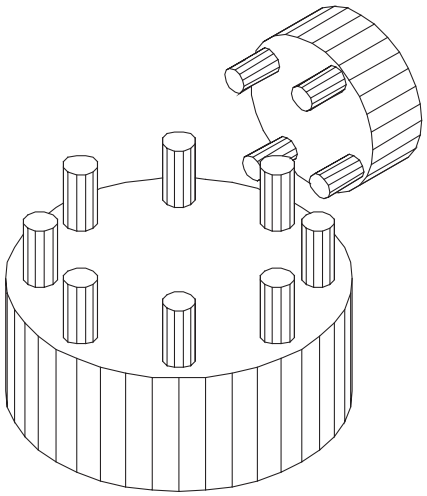


Figure 8.5
Primitive teeth gears.

properties of the surfaces. The addition of teeth to the surfaces of the disks or cones makes the drive positive, ensuring synchronization and increases the torque capacity substantially.

Primitive gear trains consisted of teeth or pegs located on disks, illustrated in Figures 8.5 and 8.6. The disadvantage of these simple teeth is that the velocity ratio is not constant and changes as the teeth go through the meshing cycle causing noise and vibration problems at elevated speeds. The solution to this problem can be achieved by using a profile on the gear teeth, which gives a constant velocity ratio throughout the meshing cycle. Several different geometrical forms can be used, but the full depth involute form is primarily used in current professional engineering practice.

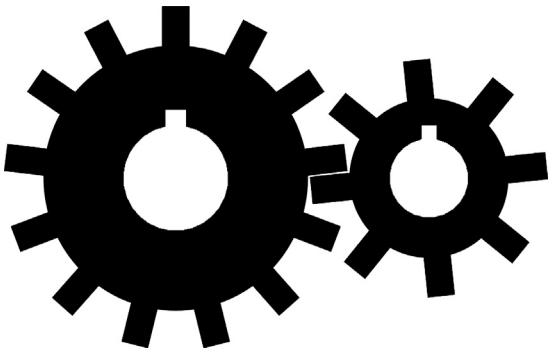


Figure 8.6
Primitive gears.



Figure 8.7
Spur gears. *Photograph courtesy of Hinchliffe Precision Components Ltd.*

Gears can be divided into several broad classifications.

1. Parallel axis gears:
   a. Spur gears (see Figure 8.7),
   b. Helical gears (see Figures 8.8 and 8.9), and
   c. Internal gears.
2. Nonparallel co-planar gears (intersecting axes):
   a. Bevel gears (see Figure 8.10),
   b. Face gears, and
   c. Conical involute gearing.

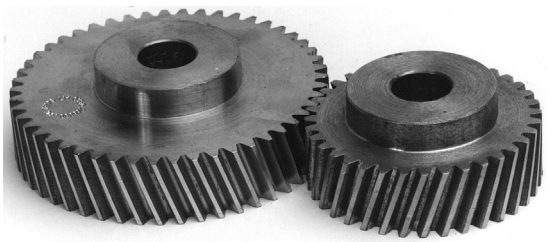


Figure 8.8
Helical gears. *Photograph courtesy of Hinchliffe Precision Components Ltd.*



Figure 8.9
Double helical gears.



Figure 8.10
Bevel gears.

3. Nonparallel noncoplanar gears (nonintersecting axes):
   a. Crossed axis helicals (see Figure 8.11),
   b. Cylindrical worm gearing (see Figure 8.12),
   c. Single enveloping worm gearing,



Figure 8.11
Crossed axis helical gears. *Photograph courtesy of Hinchliffe Precision Components Ltd.*

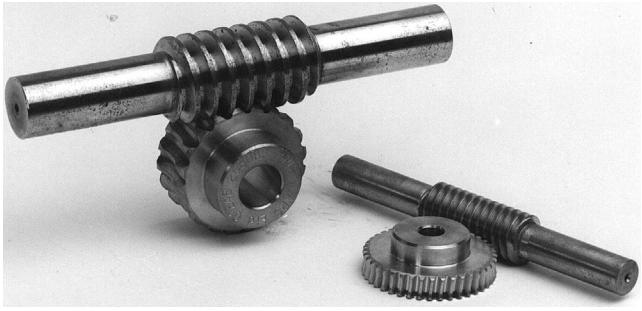


Figure 8.12
Worm gears. *Photograph courtesy of Hinchliffe Precision Components Ltd.*

   d. Double enveloping worm gearing,
   e. Hypoid gears,
   f. Spiroid and helicon gearing, and
   g. Face gears (off center).
4. Special gear types:
   a. Square and rectangular gears,
   b. Elliptical gears,
   c. Scroll gears, and
   d. Multiple sector gears.

Spur gears, illustrated in Figure 8.7, are the least expensive of all types for parallel shaft applications. Their straight teeth allow running engagement or disengagement using sliding shaft and clutch mechanisms. Typical applications of spur gears include manual and automatic motor vehicle gearboxes, machine tool drives, conveyor systems, electric motor gearboxes, timing mechanisms, and power tool drives. The majority of power gears are manufactured from hardened and case-hardened steel. Other materials used include iron, brass, bronze, and polymers such as polyamide (e.g. nylon), polyacetal (e.g. Delrin), and synthetic resin bonded fabric (SRBF) (e.g. Tufnol). Typical material matches are listed in Table 8.1. A harder material may sometimes be used for a pinion, rather than that for the mating gear wheel, as the pinion teeth will experience more use than those on the gear wheel. Steel gears are often supplied in unhardened form to allow for grub screws, keyways, or splines to be added before hardening and finishing.

A helical gear is a cylindrical gear whose tooth traces are helixes as shown in Figure 8.8. Common helix angles are $15°-30°$. Helical gears are typically used for heavy-duty high-speed (>3500 rpm) power transmissions, turbine drives, locomotive gearboxes, and machine tool drives. Helical gears are generally more expensive than spur gears. Noise levels are lower than those of spur gears because teeth in mesh make point contact rather than line contact. The forces arising from meshing of helical gears can be resolved into three component loads; radial, tangential, and axial (axial loads are often called thrust loads). Bearings used to support the gear on a shaft must be able to withstand the axial or thrust force component. A solution to this problem is the use of double helical gears, as illustrated in Figure 8.9.

Table 8.1: Typical material matches for gears and pinions.

Gear material	Pinion material
Cast iron	Cast iron
Cast iron	Carbon steel
Carbon steel	Alloy steel
Alloy steel	Alloy steel
Alloy steel	Case hardened steel

These gears eliminate the need for thrust bearings because the axial force components cancel each other. While spur gears are generally less expensive, a comparable helical gear will be smaller.

Bevel gears have teeth cut on conical blanks (see Figure 8.10), and a gear pair can connect nonparallel intersecting shafts. Bevel gears are used for motor transmission differential drives, valve control, and mechanical instruments.

A worm gear is a cylindrical helical gear with one or more threads (see Figure 8.12). A worm wheel is a cylindrical gear with flanks cut in such a way as to ensure contact with the flanks of the worm gear. Worm gears are used for steering gear, winch blocks, low-speed gearboxes, rotary tables, and remote valve control. Worm gear sets are capable of high-speed reduction and high load applications where nonparallel, noninteracting shafts are used. The 90° configuration is most common. Frictional heat generation is high in worm gears, so continuous lubrication and provision for heat dissipation are required.

Some gear axes can be allowed to rotate about others. In such cases, the gear trains are called planetary or epicyclic (see Figure 8.13). Planetary trains always include a sun gear, a planet carrier or arm, and one or more planet gears.

All gear mechanisms and gear trains demand continuous lubrication, which must be pressure fed for high-speed gears in order to counteract centrifugal effects on the oil. Plastic gears, made from nylon, for example, can be used in certain applications and have the advantage of

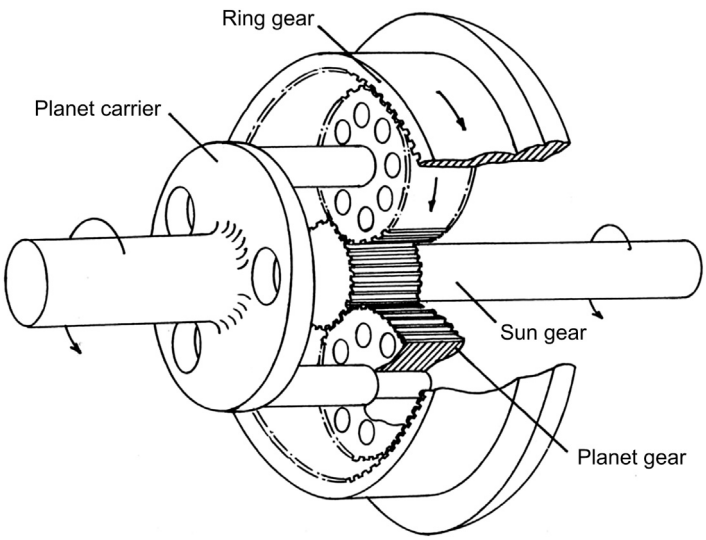


Figure 8.13
Epicyclic gears. *Reproduced from Townsend (1992).*

Table 8.2: Useful range of gear ratios.

Gear	Ratio range	Pitch line velocity (m/s)	Efficiency
Spur	1:1 to 6:1	25	98–99%
Helical	1:1 to 10:1	50	98–99%
Double helical	1:1 to 15:1	150	98–99%
Bevel	1:1 to 4:1	20	
Worm	5:1 to 75:1	30	20–98%
Crossed helical	1:1 to 6:1	30	

not needing lubrication; but they are only suitable for low-speed applications. Plastic gears can reduce noise levels significantly.

Generally, the pinion of a pair of gears should have the largest number of teeth consistent with adequate resistance to failure by bending stress in the teeth. For a given diameter, the larger the number of teeth, the finer the pitch, and consequently the weaker they are and the greater the liability to fracture. Table 8.2 lists the range of gear ratios and performance characteristics typically achievable.

When the number of teeth selected for a gear wheel is equal to the product of an integer and the number of teeth in the mating pinion, then the same tooth on the gear wheel will touch the same tooth on the pinion in each revolution of the gear wheel. If there is no common factor between the numbers of teeth on the gear wheel and pinion, then each tooth on the gear wheel will touch each tooth on the pinion in regular succession. The frequency of contact between a particular pair of teeth is the speed of the pinion divided by the number of teeth on the gear wheel. The avoidance of an integer ratio between the number of teeth on the gear wheel and the pinion can be achieved by the addition of an extra tooth to the gear wheel, provided that there is no operational need for an exact velocity ratio. This extra tooth is called a hunting tooth and can have advantages for wear equalization (see Tuplin (1962) for a fuller discussion).

Various definitions used for describing gear geometry are illustrated in Figure 8.14 and listed next. For a pair of meshing gears, the smaller gear is called the "pinion," the larger is called the "gear wheel" or simply the "gear."

Pitch circle: This is a theoretical circle on which calculations are based. Its diameter is called the pitch diameter.

$$d = mN \tag{8.1}$$

where: d is the pitch diameter (mm), m is the module (mm), and N is the number of teeth. Care must be taken to distinguish the module from the unit symbol for a meter;

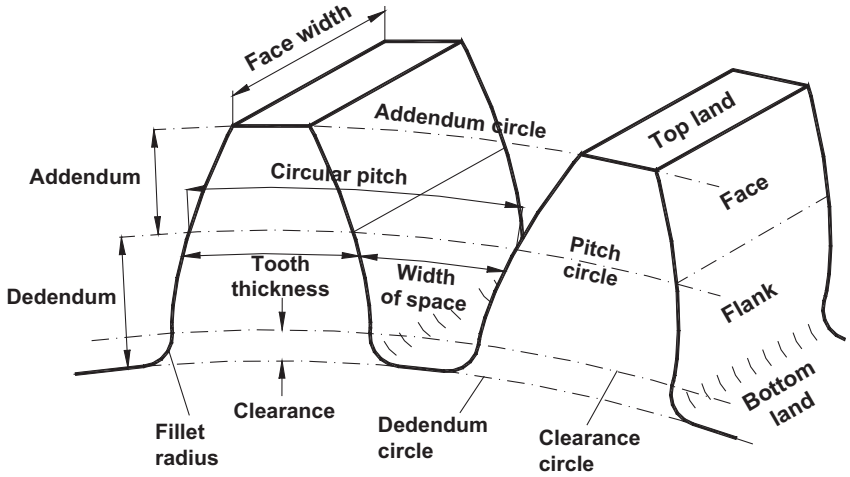


Figure 8.14
Spur gear schematic showing principal terminology.

Circular pitch: This is the distance from a point on one tooth to the corresponding point on the adjacent tooth measured along the pitch circle.

$$p = \pi m = \frac{\pi d}{N} \tag{8.2}$$

where p is the circular pitch (mm), m is the module, d is the pitch diameter (mm), and N is the number of teeth;
Module: This is the ratio of the pitch diameter to the number of teeth. The unit of the module should be in millimeters (mm). The module is defined by the ratio of pitch diameter and number of teeth. Typically the height of a tooth is about 2.25 times greater than the module. Various modules are illustrated in Figure 8.15.

$$m = \frac{d}{N} \tag{8.3}$$

Addendum, a: This is the radial distance from the pitch circle to the outside of the tooth;
Dedendum, b: This is the radial distance from the pitch circle to the bottom land; and



Figure 8.15
Gear tooth size as a function of the module.

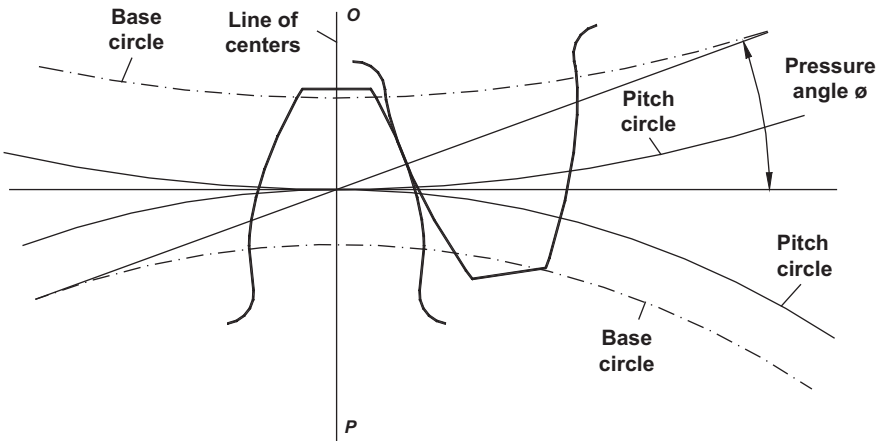


Figure 8.16
Schematic showing the pressure line and pressure angle.

Backlash: The amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circle.

Prior to use of the metric module, the term diametral pitch was commonly used. The diametral pitch is the ratio of the number of teeth in the gear to the pitch diameter, $P_d = N/d$ [usually in U.S./English units only, i.e. teeth per inch (tpi)]. To convert from diametral pitch P_d (tpi) to the module m (in mm) use $m = 25.4/P_d$.

Figure 8.16 shows the line of centers OP connecting the rotation axes of a pair of meshing gears. The angle ϕ is called the pressure angle. The pressure line (also called the generating line or line of action) is defined by the pressure angle. The resultant force vector between a pair of operating meshing gears acts along this line. The actual shape or form of the gear teeth depends on the pressure angle chosen. Figure 8.17 shows two gear teeth of differing pressure angle for a gear with identical number of teeth, pitch, and pitch diameter. Current standard pressure angles are 20° and 25°, with the 20° form most widely available.

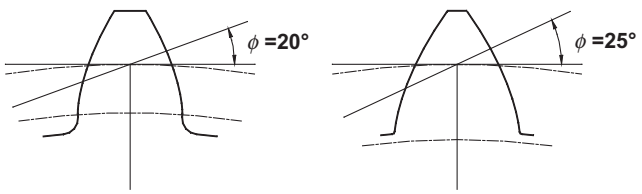


Figure 8.17
Full depth involute form for varying pressure angles.

8.2 Construction of Gear Tooth Profiles

The most widely used tooth form for spur gears is the full depth involute form, as illustrated in Figure 8.18. An involute is one of a class of curves called conjugate curves. When two involute form gear teeth are in mesh, there is a constant velocity ratio between them. From the moment of initial contact to the moment of disengagement, the speed of the gear is in constant proportion to the speed of the pinion. The resulting action of the gears is very smooth. If the velocity ratio were not constant, there would be accelerations and decelerations during the engagement and disengagement, causing vibration noise and potentially damaging torsional oscillations.

The layout and geometry for a pair of meshing spur gears can be determined by the procedure set out next. This procedure assumes access to a computer-aided design (CAD) drawing package. It should be noted that gears are commonly available as standard items

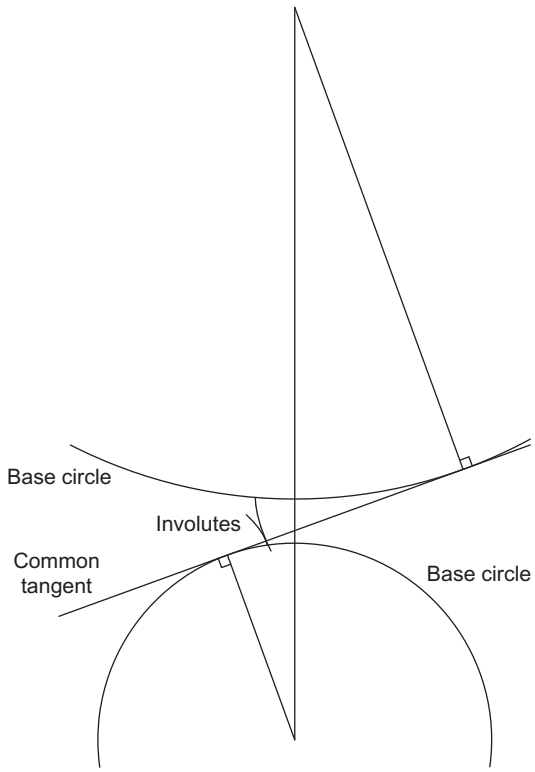


Figure 8.18
Schematic of the involute form.

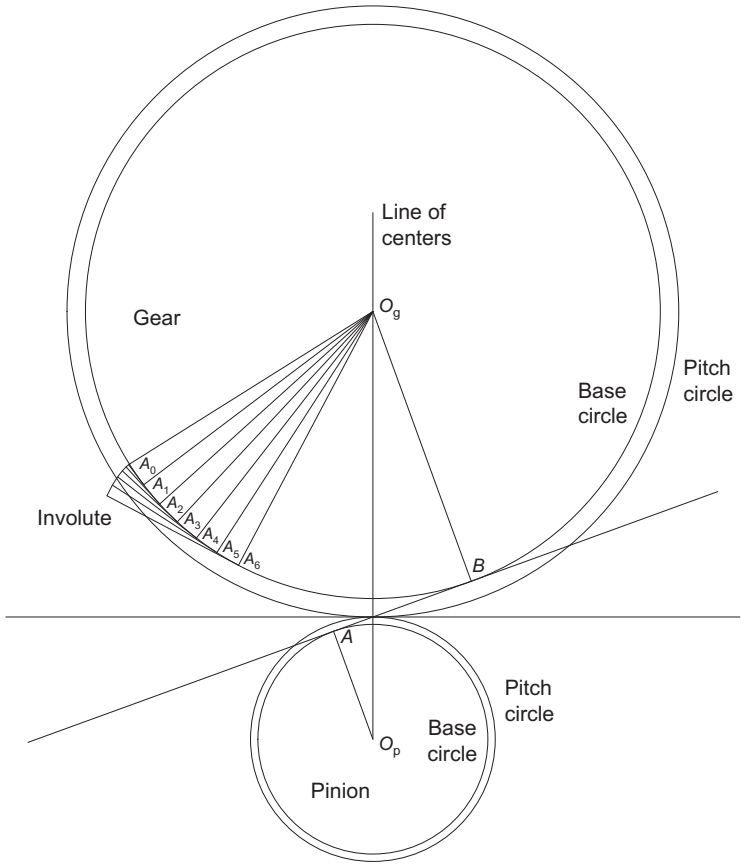


Figure 8.19
Construction of gear geometry.

from specialist manufacturers and suppliers and need not necessarily be designed from scratch.

1. Calculate the pitch diameter, and draw pitch circles tangential to each other (see Figure 8.19).

$$d = mN$$

$$d_p = mN_p$$

$$d_g = mN_g$$

   In the example shown, the module has been selected as $m = 2.5$, the number of teeth in the pinion 20, and in the gear 50. So $d_p = 2.5 \times 20 = 50$ mm and $d_g = 2.5 \times 50 = 125$ mm.
2. Draw a line perpendicular to the line of centers through the pitch point (this is the point of tangency of the pitch circles). Draw the pressure line at an angle equal to the pressure angle

from the perpendicular. It is called the pressure line because the resultant tooth force is along this line during meshing. Here, the pressure angle is 20°.

3. Construct perpendiculars O_pA and O_gB to the pressure line through the centers of each gear. The radial distance of each of these lines is the radii of the base circles of the pinion and gear, respectively. Draw the base circles.
4. Draw an involute curve on each base circle. This is illustrated on the gear. First divide the base circle into equal parts $A_0, A_1, A_2, A_3, A_4, A_5, \ldots, A_n$. Construct radial lines O_gA_0, O_gA_1, O_gA_2, O_gA_3, …, O_gA_n. Construct perpendiculars to these radial lines. The involute begins at A_0. The second point is obtained by measuring off the distance A_0A_1 on the perpendicular through A_1. The next point is found by measuring off twice the distance A_0A_1 on the perpendicular through A_2 and so on. The curve constructed through these points is the involute for the gear. The involute for the pinion is constructed in the same way on the base circle of the pinion.
5. Calculate the circular pitch $p = \pi m$. The width of the teeth and the width of the spaces are equal to half the circular pitch. Mark these distances off on the pitch circles. (Here $p = \pi \times 2.5$).
6. Draw the addendum and dedendum circles for the pinion and gear (see Figure 8.20). Here, a tooth system (see Table 8.3) has been selected with

$$a = m,$$

$$b = 1.25 \text{ m}.$$

7. Mirror the involute profile about a line constructed using a distance that is one-half of the tooth width along the pitch circle and the gear center. Using a polar array, generate all of the teeth for the gear. Construct the root fillets, as appropriate. Construct the tooth top and bottom lands.

Gears can be purchased as standard items from specialist manufacturers. The exact geometry for the blade teeth is not necessary for engineering drawings and general design purposes (unless you are designing gears, cutters/forming tools). However, the tooth geometry is necessary for detailed stress/reliability and dynamic analysis.

8.3 Gear Trains

A gear train is one or more pairs of gears operating together to transmit power. Figure 8.21 shows examples of a simple gear train, a reverted gear train, and a nonreverted gear train.

When two gears are in mesh, their pitch circles roll on each other without slippage. If r_1 = pitch radius of gear 1, r_2 = pitch radius of gear 2, ω_1 = angular velocity of gear 1, ω_2 = angular velocity of gear 2, then the pitch line velocity is given by

$$V = |r_1\omega_1| = |r_2\omega_2| \tag{8.4}$$

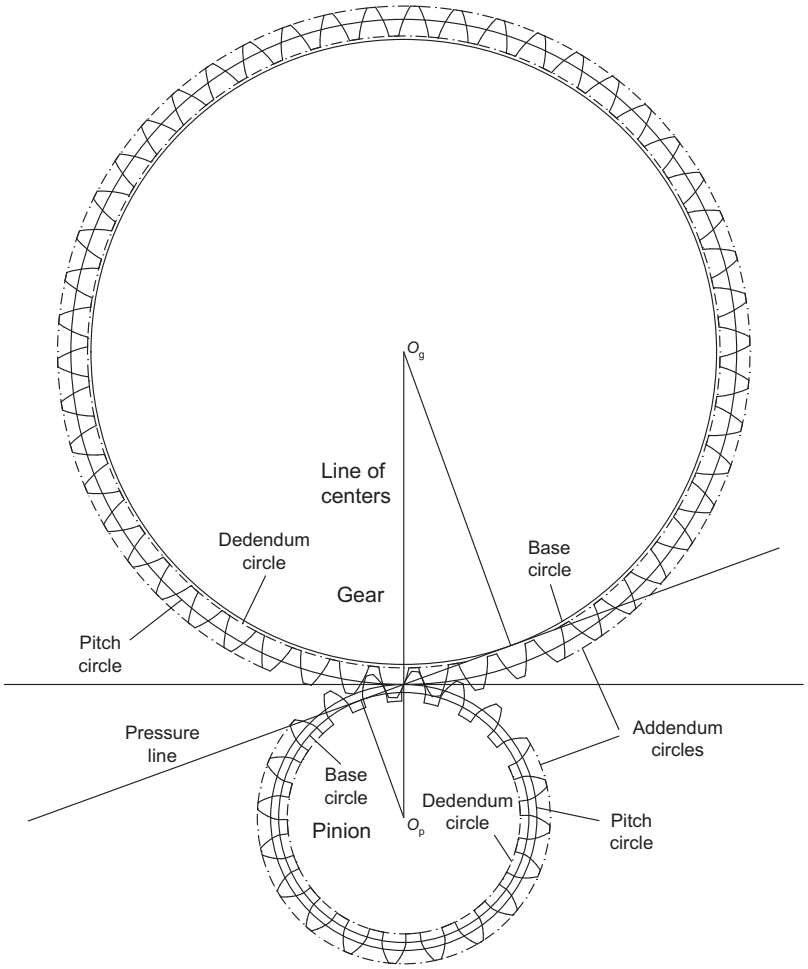


Figure 8.20
Layout of a pair of meshing spur gears.

The velocity ratio is

$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1} \tag{8.5}$$

It can be defined in any of the following ways:

$$\frac{\omega_P}{\omega_G} = \frac{n_P}{n_G} = -\frac{N_G}{N_P} = -\frac{d_G}{d_P} \tag{8.6}$$

where ω_P and ω_G are the angular velocities of the pinion and gear respectively (rad/s), n_P and n_G are the rotational speeds of the pinion and gear, respectively (rpm), N_P and N_G are the

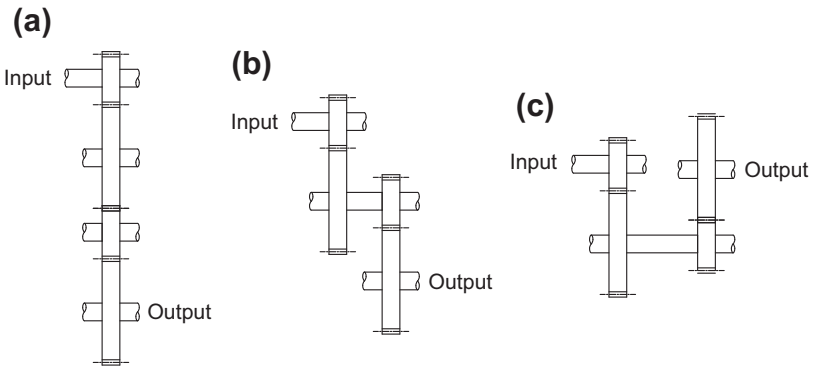


Figure 8.21
Gear trains. (a) Simple gear train. (b) Reverted compound gear train. (c) Nonreverted compound gear train.

number of teeth in the pinion and gear, respectively, and d_P and d_G are the pitch diameter of the pinion and gear, respectively (mm).

Consider a pinion 1, driving a gear 2. The speed of the driven gear is

$$n_2 = \left|\frac{N_1}{N_2} n_1\right| = \left|\frac{d_1}{d_2} n_1\right| \tag{8.7}$$

where $n =$ revolutions or rpm, $N =$ number of teeth, and $d =$ pitch diameter.

Equation (8.7) applies to any gear set (spur, helical, bevel, or worm). For spur and parallel helical gears, the convention for direction is positive for counterclockwise rotation, i.e.

$$n_2 = -\frac{N_1}{N_2} n_1 \tag{8.8}$$

Example 8.1

Consider the gear train shown in Figure 8.22. Calculate the speed of gear 5.

Solution

$$n_2 = -\frac{N_1}{N_2} n_1$$

$$n_3 = -\frac{N_2}{N_3} n_2$$

$$n_4 = n_3 \quad \text{(on the same shaft)}$$

$$n_5 = -\frac{N_4}{N_5} n_4$$

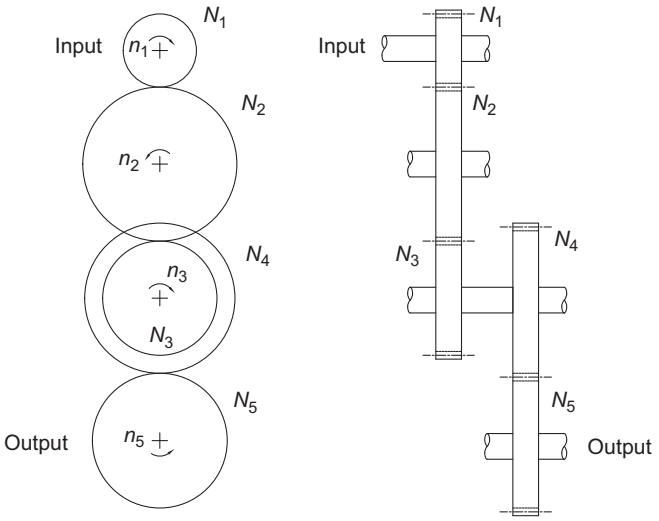


Figure 8.22
Example gear train.

$$n_5 = -\frac{N_4 N_2 N_1}{N_5 N_3 N_2} n_1$$

Example 8.2

For the double reduction gear train shown in Figure 8.23, if the input speed is 1750 rpm in a clockwise direction what is the output speed?

Solution

$$n_2 = -\frac{N_1}{N_2} n_1$$

$$n_3 = n_2 \quad \text{(on the same shaft)}$$

$$n_4 = -\frac{N_3}{N_4} n_3$$

$$n_4 = \frac{N_3}{N_4}\frac{N_1}{N_2} n_1 = \left(\frac{18}{54}\right)\left(\frac{20}{70}\right)(-1750) = -166.7 \text{ rpm}$$

Example 8.3

For the double reduction gear train with an idler shown in Figure 8.24, if the input speed is 1750 rpm in a clockwise direction, what is the output speed?

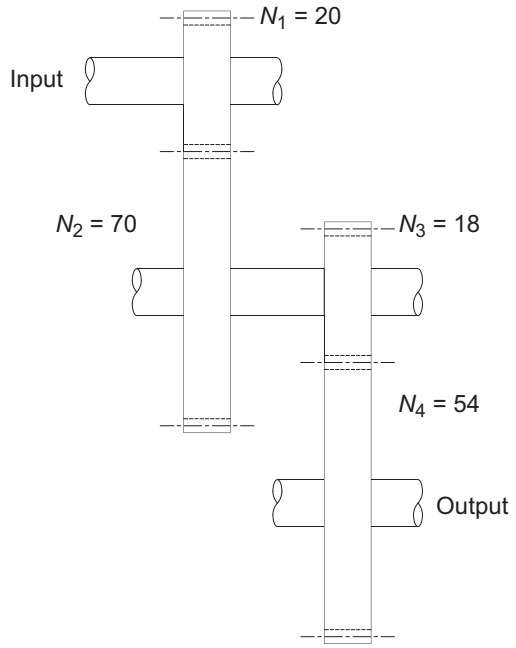


Figure 8.23
Double reduction gear train.

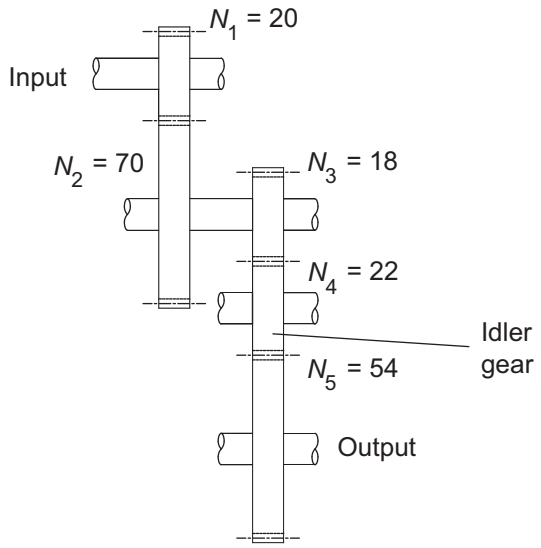


Figure 8.24
Double reduction gear with idler.

Solution

$$n_5 = -\frac{N_4}{N_5} n_4$$

$$n_4 = -\frac{N_3}{N_4} n_3$$

$$n_3 = n_2$$

$$n_2 = -\frac{N_1}{N_2} n_1$$

$$n_5 = -\frac{N_4}{N_5}\frac{N_3}{N_4}\frac{N_1}{N_2} n_1 = -\left(\frac{22}{54}\right)\left(\frac{18}{22}\right)\left(\frac{20}{70}\right)(-1750) = 166.7 \text{ rpm}$$

Notice that the presence of the idler gear has caused the gear train output to reverse direction but has not altered the gear ratio in comparison to example 8.2.

Example 8.4

For the gear train illustrated in Figure 8.25, determine the output speed and direction of rotation if the input shaft rotates at 1490 rpm in a clockwise direction. In Figure 8.25, gears *A* to *D* have a module of 1.5 and gears *E* to *H* a module of 2.

Solution

$d = mN$; $N = d/m$; $N_B = 67.5/1.5 = 45$; $N_C = 27/1.5 = 18$; and $N_F = 56/2 = 28$.

$$n_H = \frac{N_G}{N_H} n_G.$$

$$n_G = n_F.$$

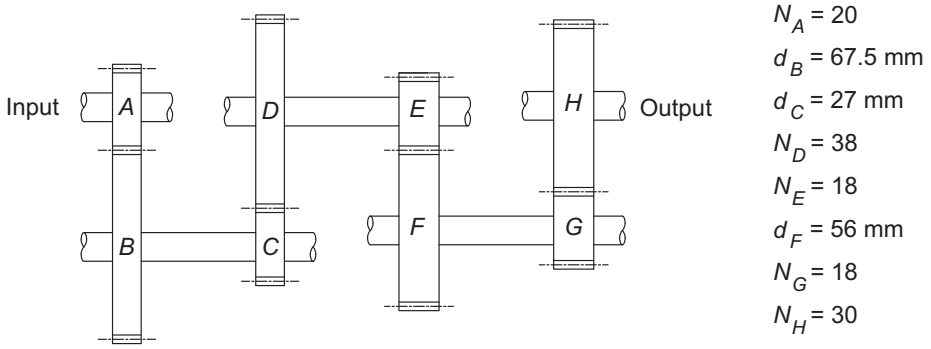


Figure 8.25
Gear train.

$$n_F = \frac{N_E}{N_F} n_E.$$

$$n_E = n_D.$$

$$n_D = \frac{N_C}{N_D} n_C.$$

$$n_C = n_B.$$

$$n_B = \frac{N_A}{N_B} n_A.$$

$$n_H = \frac{N_G}{N_H} \times \frac{N_E}{N_F} \times \frac{N_C}{N_D} \times \frac{N_A}{N_B} n_A = \frac{18}{30} \times \frac{18}{28} \times \frac{18}{38} \times \frac{20}{45} \times -1490 = -121 \text{ rpm}.$$

Gear	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Direction	CW	CCW	CCW	CW	CW	CCW	CCW	CW

Note: CW = clockwise. CCW = counterclockwise.

8.3.1 Manually Shifted Automotive Transmissions

The torque speed characteristic of a petrol IC engine delivers low torque at starting and at low speeds. This does not match the load, which—to enable a car to pull away from a stationary position—demands a high torque. The torque from an IC engine can be increased by means of gearing. Similarly, at other load conditions such as high acceleration, hills, and cruising, the torque speed characteristics of the engine may not be compatible with the load. A multiratio gearbox is the traditional solution to this problem.

A typical manually shifted gearbox for a passenger car is illustrated in Figure 8.26. This has five forward speeds and reverse. The basic elements of a manually shifted transmission are a single or multiplate clutch for engaging and disengaging the power from the load, a variable ratio transmission unit with permanent mesh gears, and a gear shift mechanism and lever. Clutches are presented more fully in Chapter 13, and their function here is to interrupt the power flow. The demands of the automobile product mean that harshness in gear changes is unacceptable to the majority of customers. A method of overcoming shocks from gears crashing into each other as they are engaged is to hold the gears in permanent mesh but not necessarily transmitting any power. This is illustrated for gears 2–4 which, although in mesh in Figure 8.26a, do not transmit torque because one of each of the pairs of gears concerned is free to rotate relative to its shaft. In order to transmit torque, a spline is moved on the shaft causing the gear to rotate at the same speed as the shaft; therefore, it is able to transmit power. This is the principle of the synchromesh illustrated in Figure 8.27. Depending on the gear train desired, the gear lever shift mechanism moves the synchromesh splines to engage the appropriate gears.

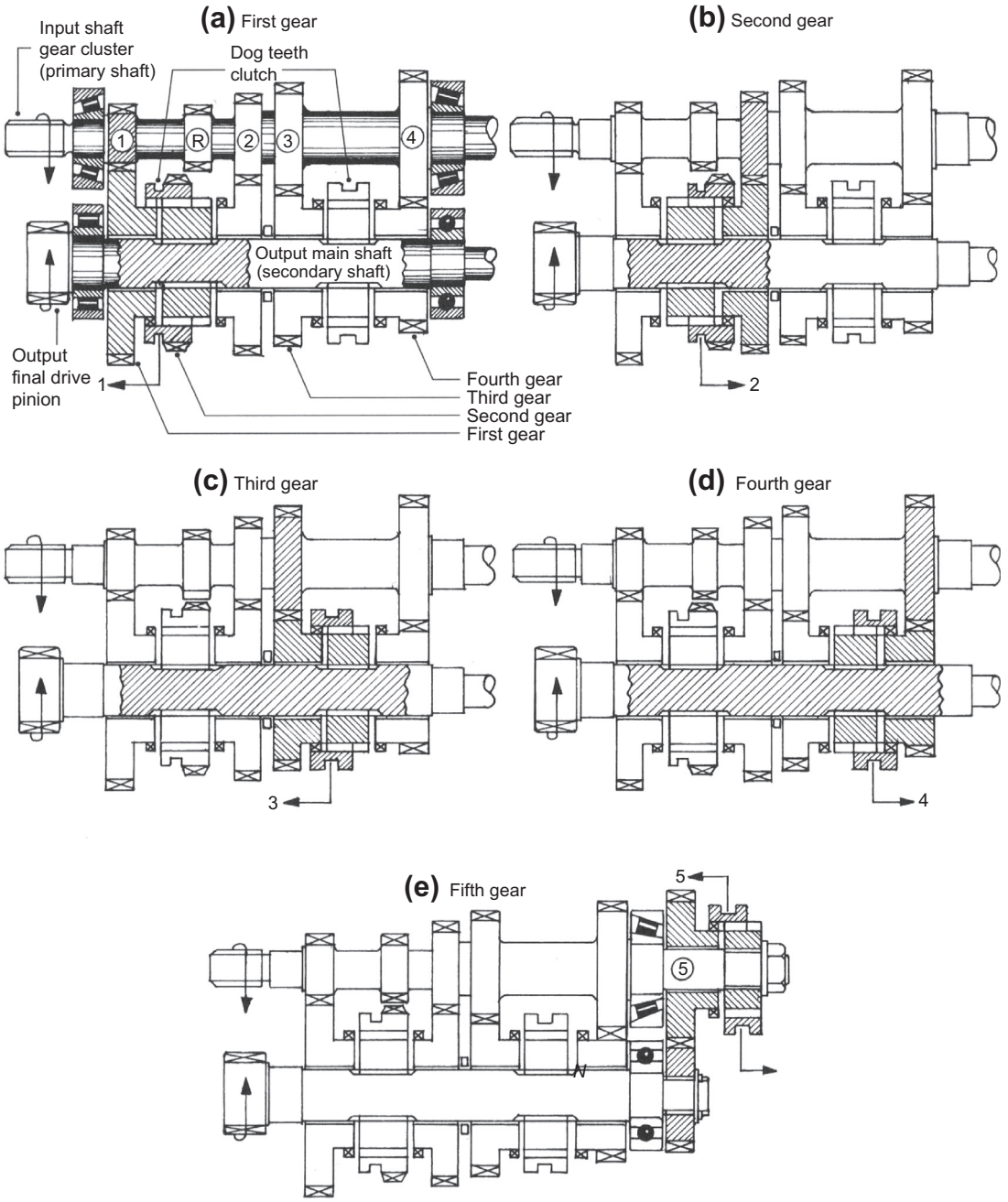


Figure 8.26

A five-speed and reverse single-stage constant mesh transmission ((a) to (e) correspond to 1st to 5th gear). *Figure reproduced from Heisler (1999).*

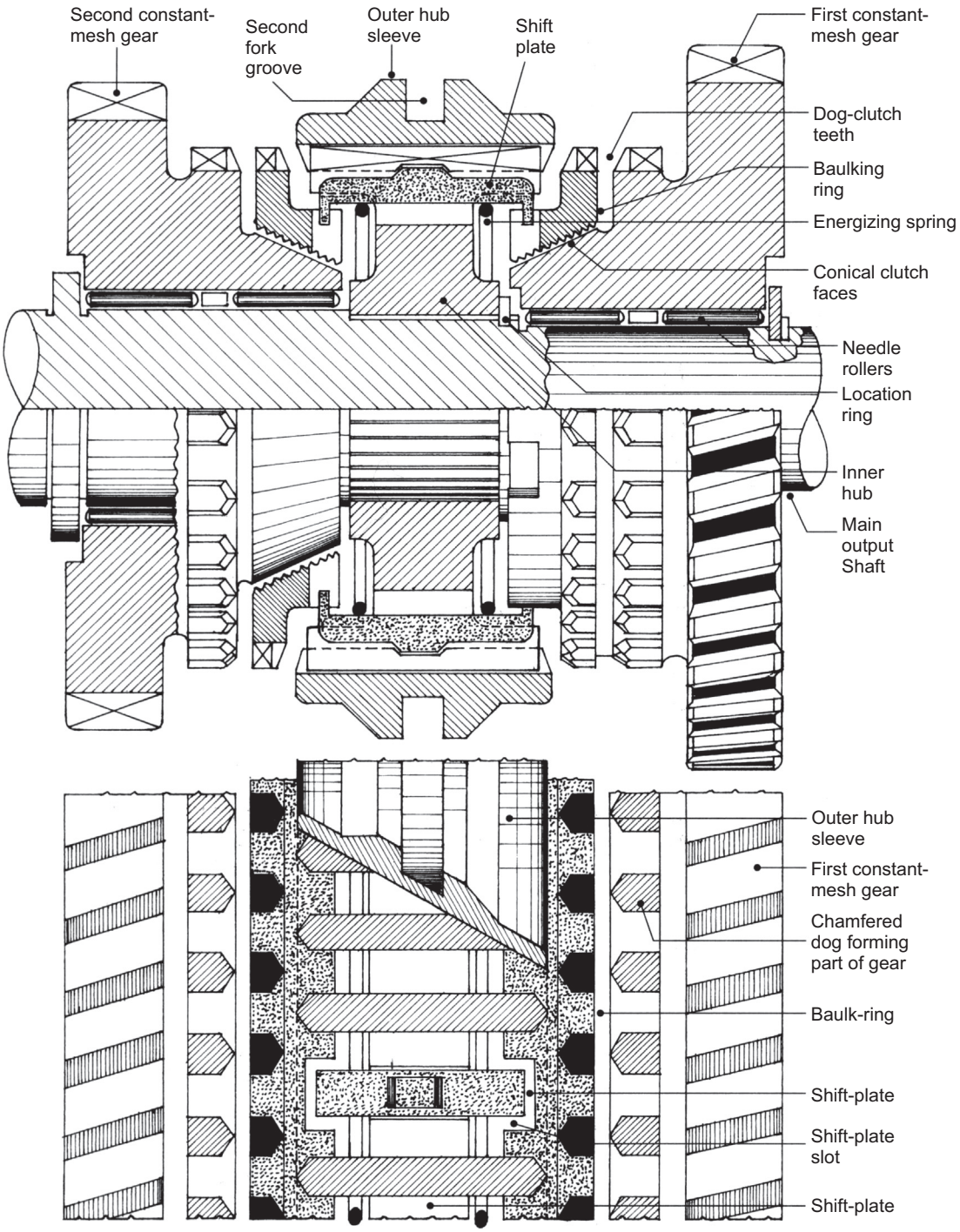
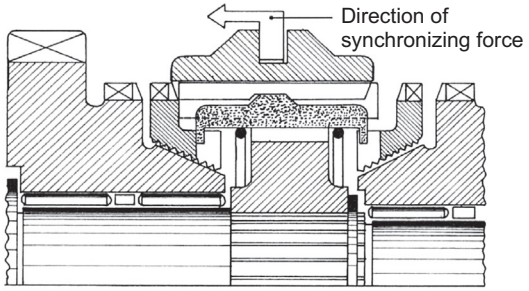
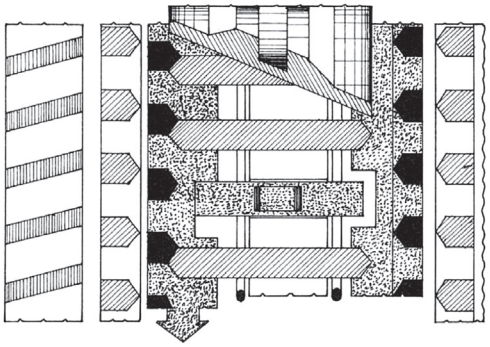
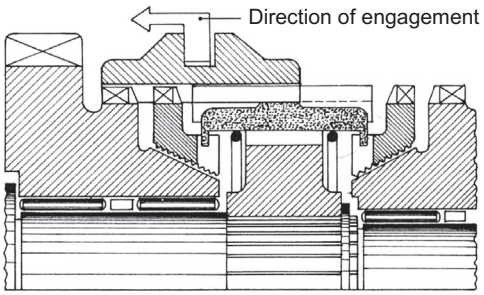


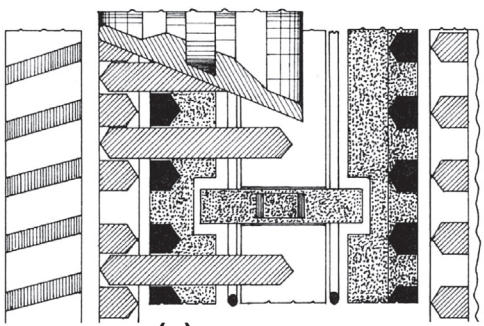
Figure 8.27
Positive baulk-ring synchromesh. (a) Disengaged position; (b) Synchronization position; (c) Engaged position. *Figure reproduced from Heisler (1999).*





(b) Synchronization position





(c) Engaged position

Figure 8.27
(Continued)

8.3.2 Epicyclic Gear Trains

Figure 8.13 illustrated an example of an epicyclic gear train. Epicyclic gear trains include a sun gear, a planet carrier, and one or more planetary gears. The planetary gears may also mesh with an internal ring gear. Epicyclic gear trains have two degrees of freedom and, in order to determine their motion, two inputs must be defined such as the speed of both the sun and the ring gear. For the example shown in Figure 8.13, the rotational speed of the sun gear is given by

$$n_{\text{sun}} = n_{\text{arm}} + n_{\text{sun/arm}} \tag{8.9}$$

If n_F represents the speed of the first gear in the train and n_L the speed of the last gear in the train,

$$n_{F/\text{arm}} = n_F - n_{\text{arm}} \tag{8.10}$$

$$n_{L/\text{arm}} = n_L - n_{\text{arm}} \tag{8.11}$$

Dividing Eqn (8.11) by (8.10) gives

$$\frac{n_{L/\text{arm}}}{n_{F/\text{arm}}} = \frac{n_L - n_{\text{arm}}}{n_F - n_{\text{arm}}} = e = \pm\frac{\text{Product of driving teeth numbers}}{\text{Product of driven teeth numbers}} = \frac{N_S}{N_P}\frac{N_P}{N_R} \tag{8.12}$$

where e is the train value. For spur and helical gears, the train value is positive if the last gear rotates in the same direction as the first gear.

Equation (8.12) can be rearranged to give the reduction ratio defined as $n_{\text{sun}}/n_{\text{arm}}$ which for an epicyclic gearbox, with the ring fixed stationary, $n_L = 0$, $n_F = n_{\text{sun}}$, is given by

$$\frac{n_{\text{sun}}}{n_{\text{arm}}} = \frac{N_R}{N_S} + 1 \tag{8.13}$$

For calculations of stress, it may also be necessary to determine the speed of the planetary gears. Velocity difference equations can be used for this purpose. For the gear train illustrated,

$$n_{\text{planet/arm}} = n_{\text{planet}} - n_{\text{arm}} \tag{8.14}$$

$$n_{\text{sun/arm}} = n_{\text{sun}} - n_{\text{arm}} \tag{8.15}$$

From knowledge of n_{arm}, N_P and N_S these equations can be solved to determine $n_{\text{P.}}$

Note that if equi-spacing of the planetary gears is desired, the ratio given in Eqn (8.16) must be an integer. Equi-spacing of planets is desirable to produce balanced forces on the pinion and ring gear.

$$\frac{N_S + N_R}{\text{Number of planets}} = \text{integer} \tag{8.16}$$

Example 8.5

For the epicyclic gearbox illustrated in Figure 8.28, determine the speed and direction of the final drive and also the speed and direction of the planetary gears. The teeth numbers of the sun, planets, and ring gear are 20, 30, and 80, respectively. The speed and direction of the sun gear is 1000 rpm clockwise, and the ring gear is held stationary.

Solution

From Eqn (8.13),

$$n_{\text{arm}} = \frac{n_{\text{sun}}}{(80/20) + 1} = \frac{-1000}{5} = -200 \text{ rpm}$$

The speed of the final drive is 200 rpm clockwise.

The reduction ratio for the gearbox is given by $n_{\text{sun}}/n_{\text{arm}} = 1000/200 = 5$.

To determine the speed of the planets use Eqns (8.14) and (8.15).

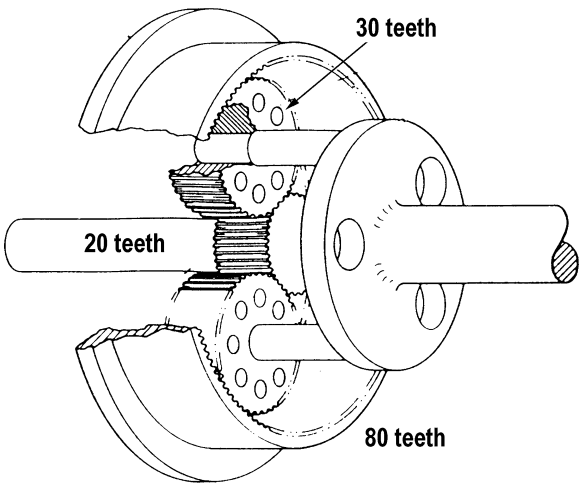


Figure 8.28
Epicyclic gear train.

The planets and sun are in mesh, so

$$\frac{n_{\text{planet}}/n_{\text{arm}}}{n_{\text{sun}}/n_{\text{arm}}} = -\frac{N_S}{N_P}$$

$$\frac{n_{\text{planet}} - n_{\text{arm}}}{n_{\text{sun}} - n_{\text{arm}}} = -\frac{N_S}{N_P}$$

$$\frac{n_{\text{planet}} - (-200)}{-1000 - (-200)} = -\frac{20}{30}$$

$$n_{\text{planet}} = -\frac{20}{30} \times (-800) - 200 = 333 \text{ rpm}$$

The speed of rotation of the planetary gears is 333 rpm counterclockwise.

Example 8.6

An epicyclic gearbox is required to transmit 12 kW from a shaft rotating at 3000 rpm. The desired output speed is approximately 500 rpm. Using a module 3 hardened steel gear, determine the number of teeth in each gear for a single-stage reduction gearbox.

Solution

Ratio sun/arm = 3000/500 = 6

$$\frac{n_{\text{sun}}}{n_{\text{arm}}} = \frac{3000}{500} = 6$$

$$\frac{n_L - n_{\text{arm}}}{n_F - n_{\text{arm}}} = -\frac{N_S}{N_P}\frac{N_P}{N_R}$$

Set $N_s = 18$ (minimum normally recommended)

$$\frac{0 - 500}{3000 - 500} = -\frac{18}{N_R}$$

$$N_R = 90$$

With $m = 3$, does this fit in?

$$d_R = mN = 3 \times 90 = 270 \text{ mm}$$

$$d_S = mN = 3 \times 18 = 54 \text{ mm}$$

This leaves 270/2 − 54/2 = 108 mm as the diameter of the planetary gears.

With $m = 3$, $N_P = 36$, i.e. $d = mN = 3 \times 36 = 108$ mm. This fits. Design OK, so far.

Does this combination give even spacing using three planets?

$$\frac{N_S + N_R}{\text{number of planets}} = \frac{18 + 90}{3} = \text{integer} = 3$$

Integer result: even spacing. Design OK.

A check on stresses, although not asked for here, would confirm the validity of the design using a module of 3.

8.4 Tooth Systems

Tooth systems are standards that define the geometric proportions of gear teeth. Table 8.3 lists the basic tooth dimensions for full depth teeth with pressure angles of 20° and 25°.

Table 8.4 lists preferred values for the module, m, which are used to minimize gear cutting tool requirements, and Table 8.5 lists the preferred standard gear teeth numbers.

The failure of gears can principally be attributed to tooth breakage, surface failure and scuffing, and scoring of the teeth surfaces. Section 8.5 outlines the forces involved on spur

Table 8.3: Tooth dimension formulas for $\phi = 20^\circ$ and 25°.

Addendum	$a = m$
Dedendum	$b = 1.25$ m
Working depth	$h_k = 2$ m
Whole depth (min)	$h_t = 2.25$ m
Tooth thickness	$T = \pi m/2$
Fillet radius of basic rack	$r_f = 0.3$ m
Clearance (min)	$c = 0.25$ m
Clearance for shaved or ground teeth	$c = 0.35$ m
Minimum number of pinion teeth $\phi = 20^\circ$	$N_P = 18$
Minimum number of pinion teeth $\phi = 25^\circ$	$N_P = 12$
Minimum number of teeth per pair $\phi = 20^\circ$	$N_P + N_G = 36$
Minimum number of teeth per pair $\phi = 25^\circ$	$N_P + N_G = 24$
Width of top land (min)	$t_o = 0.25$ m

Note: "m" stands for module here.

Table 8.4: Preferred values for the module m.

m	0.5	0.8	1	1.25	1.5	2	2.5	3	4	5	6	8	10	12	16	20	25	32	40	50

Table 8.5: Preferred standard gear teeth numbers.

12	13	14	15	16	18	20	22	24	25	28	30
32	34	38	40	45	50	54	60	64	70	72	75
80	84	90	96	100	120	140	150	180	200	220	250

gear teeth, and Section 8.6 defines methods for estimating gear stresses and the preliminary selection of standard stock gears.

8.5 Force Analysis

Figure 8.29 shows the forces involved for two spur gears in mesh. The force acting at the pressure angle ϕ can be subdivided into two components: The tangential component F_t and the radial component F_r. The radial component serves no useful purpose. The tangential component F_t transmits the load from one gear to the other gear. If W_t is defined as the transmitted load, $W_t = F_t$. The transmitted load is related to the power transmitted through the gears by the following equation:

$$W_t = \frac{P}{V} \tag{8.17}$$

where W_t = transmitted load (N), P = power (W), and V = pitch line velocity (m/s).

Alternatively, the pitch line velocity can be defined by

$$W_t = \frac{60 \times 10^3 H}{\pi d n} \tag{8.18}$$

where W_t = transmitted load (kN), H = power (kW), d = pitch diameter (mm), and n = speed (rpm).

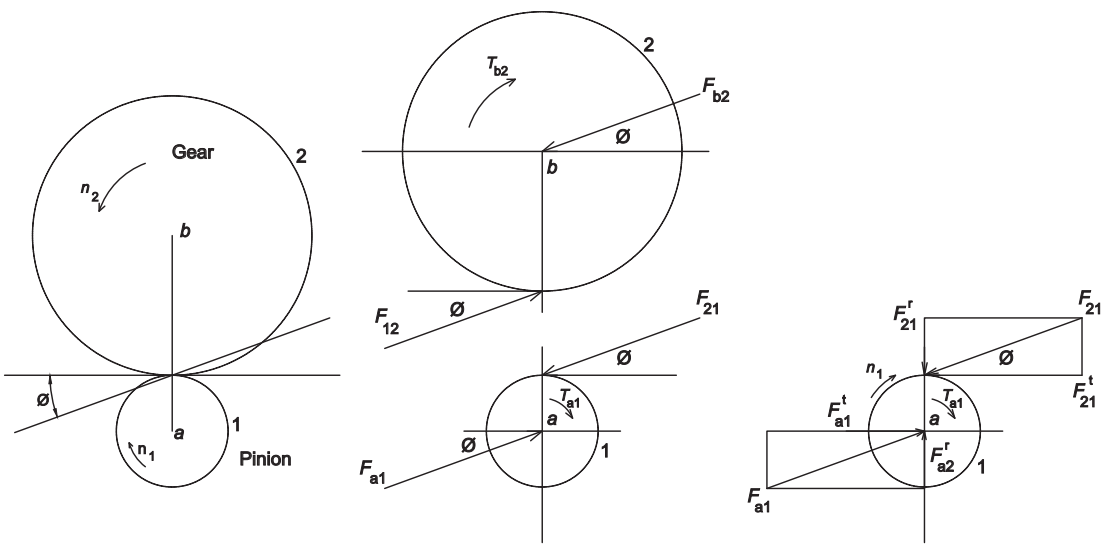


Figure 8.29
Forces for two spur gears in mesh.

8.5.1 Introduction to Gear Stresses

Gears experience two principal types of stresses; bending stress at the root of the teeth due to the transmitted load and contact stresses on the flank of the teeth due to repeated impact, or sustained contact, of one tooth surface against another. A simple method of calculating bending stresses is presented in Section 8.5.2, and this is utilized within a gear selection procedure given in Section 8.6. Although this methodology is suitable for determining the geometry for a gear train at the conceptual phase, it is inadequate for designing a gear train for highly stressed applications or where life of the components is important. For the determination of life critical parts, national and international standards for gears are available such as the American Gear Manufacturers Association (AGMA), British Standards (BS), and the International Standards Organization (ISO) standards for gears. These are considered in Chapter 9, and the AGMA equations recommended for bending and contact stresses for spur gears and helical gears are introduced.

8.5.2 Bending Stresses

The calculation of bending stress in gear teeth can be based on the Lewis formula

$$\sigma = \frac{W_t}{FmY} \tag{8.19}$$

where W_t = transmitted load (N), F = face width (m or mm), m = module (m or mm), and Y = the Lewis form factor and can be found from Table 8.6.

When teeth mesh, the load is delivered to the teeth with some degree of impact. The velocity factor is used to account for this and is given, in the case of cut or milled profile gears, by the Barth equation, (8.20).

$$K_v = \frac{6.1}{6.1 + V} \tag{8.20}$$

where V is the pitch line velocity which is given by

$$V = \frac{d}{2} \times 10^{-3} n \frac{2\pi}{60} \tag{8.21}$$

where d is in mm and n is in rpm.

Introducing the velocity factor into the Lewis equation gives

$$\sigma = \frac{W_t}{K_v FmY} \tag{8.22}$$

Equation (8.22) forms the basis of a simple approach to the calculation of bending stresses in gears.

Table 8.6: Values for the Lewis form factor *Y* defined for two different tooth standards (Mitchener, 1982).

N, Number of teeth	*Y* $\phi = 20^\circ$ $a = 0.8$ m $b = m$	*Y* $\phi = 20^\circ$ $a = m$ $b = 1.25$ m
12	0.33512	0.22960
13	0.34827	0.24317
14	0.35985	0.25530
15	0.37013	0.26622
16	0.37931	0.27610
17	0.38757	0.28508
18	0.39502	0.29327
19	0.40179	0.30078
20	0.40797	0.30769
21	0.41363	0.31406
22	0.41883	0.31997
24	0.42806	0.33056
26	0.43601	0.33979
28	0.44294	0.34790
30	0.44902	0.35510
34	0.45920	0.36731
38	0.46740	0.37727
45	0.47846	0.39093
50	0.48458	0.39860
60	0.49391	0.41047
75	0.50345	0.42283
100	0.51321	0.43574
150	0.52321	0.44930
300	0.53348	0.46364
Rack	0.54406	0.47897

Note: a = addendum; b = dedendum; ϕ = pressure angle, and m = module.

Example 8.7

A 20° full depth spur pinion is to transmit 1.25 kW at 850 rpm. The pinion has 18 teeth. Determine the Lewis bending stress if the module is 2 and the face width is 25 mm.

Solution

Calculating the pinion pitch diameter: $d_P = mN_P = 2 \times 18 = 36$ mm.

Calculating the pitch line velocity,

$$V = \frac{d_P}{2} \times 10^{-3} \times n\frac{2\pi}{60} = \frac{0.036}{2} \times 850 \times 0.1047 = 1.602 \text{ m/s}.$$

Calculating the velocity factor,

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 1.602} = 0.7920.$$

Calculating the transmitted load:

$$W_t = \frac{\text{Power}}{V} = \frac{1250}{1.602} = 780.2 \text{ N}.$$

From Table 8.6 for $N_P = 18$, the Lewis form factor $Y = 0.29327$.

The Lewis equation for bending stress gives

$$\sigma = \frac{W_t}{K_v F m Y} = \frac{780.2}{0.792 \times 0.025 \times 0.002 \times 0.29327} = 67.18 \times 10^6 \text{ N/m}^2 = 67.18 \text{ MPa}.$$

8.6 Simple Gear Selection Procedure

The Lewis formula, in the form given by Eqn (8.22) ($\sigma = W_t/(K_v F m Y)$), can be used in a provisional spur gear selection procedure for a given transmission power, input, and output speeds. The procedure is outlined as follows:

1. Select the number of teeth for the pinion and the gear to give the required gear ratio (observe the guidelines presented in Table 8.2 for maximum gear ratios). Note that the minimum number of teeth permissible when using a pressure angle of 20° is 18 (Table 8.3). Use either the standard teeth numbers as listed in Table 8.5, or as listed in a stock gear catalog.
2. Select a material. This will be limited to those listed in the stock gear catalogs.
3. Select a module, m from Table 8.4 or as listed in a stock gear catalog (see Tables 8.7–8.10, which give examples of a selection of stock gears available).
4. Calculate the pitch diameter, $d = mN$.
5. Calculate the pitch line velocity, $V = (d/2) \times n \times (2\pi/60)$. Ensure this does not exceed the guidelines given in Table 8.2.
6. Calculate the dynamic factor, $K_v = 6.1/(6.1 + V)$.
7. Calculate the transmitted load, $W_t = \text{Power}/V$.
8. Calculate an acceptable face width using the Lewis formula in the form

$$F = \frac{W_t}{K_v m Y \sigma_p} \tag{8.23}$$

The Lewis form factor, Y, can be obtained from Table 8.6.

Table 8.7: Example spur gears. 1.0 module. Heavy duty steel 817M40, 655M13. Face width 15 mm.

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG1-9	9	10.00	12.00	12	6
SG1-10	10	11.00	13.00	13	6
SG1-11	11	12.00	14.00	14	6
SG1-12	12	12.00	14.00	14	6
SG1-13	13	13.00	15.00	15	6
SG1-14	14	14.00	16.00	16	6
SG1-15	15	15.00	17.00	17	6
SG1-16	16	16.00	18.00	18	8
SG1-17	17	17.00	19.00	18	8
SG1-18	18	18.00	20.00	18	8
SG1-19	19	19.00	21.00	18	8
SG1-20	20	20.00	22.00	20	8
SG1-21	21	21.00	23.00	20	8
SG1-22	22	22.00	24.00	20	8
SG1-23	23	23.00	25.00	20	8
SG1-24	24	24.00	26.00	20	8
SG1-25	25	25.00	27.00	20	8
SG1-26	26	26.00	28.00	25	8
SG1-27	27	27.00	29.00	25	8
SG1-28	28	28.00	30.00	25	8
SG1-29	29	29.00	31.00	25	8
SG1-30	30	30.00	32.00	25	8
SG1-31	31	31.00	33.00	25	8
SG1-32	32	32.00	34.00	25	8
SG1-33	33	33.00	35.00	25	8
SG1-34	34	34.00	36.00	25	8
SG1-35	35	35.00	37.00	25	8
SG1-36	36	36.00	38.00	30	8
SG1-37	37	37.00	39.00	30	8
SG1-38	38	38.00	40.00	30	8
SG1-39	39	39.00	41.00	30	8
SG1-40	40	40.00	42.00	30	8
SG1-41	41	41.00	43.00	30	8
SG1-42	42	42.00	44.00	30	8
SG1-43	43	43.00	45.00	30	8
SG1-44	44	44.00	46.00	30	8
SG1-45	45	45.00	47.00	30	8
SG1-46	46	46.00	48.00	30	8
SG1-47	47	47.00	49.00	30	8
SG1-48	48	48.00	50.00	30	8
SG1-49	49	49.00	51.00	30	8
SG1-50	50	50.00	52.00	35	10
SG1-51	51	51.00	53.00	35	10
SG1-52	52	52.00	54.00	35	10
SG1-53	53	53.00	55.00	35	10
SG1-54	54	54.00	56.00	35	10

Continued

Table 8.7: Example spur gears. 1.0 module. Heavy duty steel 817M40, 655M13. Face width 15 mm.—cont'd

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG1-55	55	55.00	57.00	35	10
SG1-56	56	56.00	58.00	35	10
SG1-57	57	57.00	59.00	35	10
SG1-58	58	58.00	60.00	35	10
SG1-59	59	59.00	61.00	35	10
SG1-60	60	60.00	62.00	35	10
SG1-61	61	61.00	63.00	35	10
SG1-62	62	62.00	64.00	35	10
SG1-63	63	63.00	65.00	35	10
SG1-64	64	64.00	66.00	35	10
SG1-65	65	65.00	67.00	35	10
SG1-66	66	66.00	68.00	35	10
SG1-68	68	68.00	70.00	35	10
SG1-70	70	70.00	72.00	35	10
SG1-72	72	72.00	74.00	35	10
SG1-74	74	74.00	76.00	45	10
SG1-76	76	76.00	78.00	45	10
SG1-78	78	78.00	80.00	45	10
SG1-80	80	80.00	82.00	45	10
SG1-84	84	84.00	86.00	45	10
SG1-88	88	88.00	90.00	45	10
SG1-90	90	90.00	92.00	45	10
SG1-96	96	96.00	98.00	45	10
SG1-100	100	100.00	102.00	45	10
SG1-112	112	112.00	114.00	45	10
SG1-120	120	120.00	122.00	45	10
SG1-130	130	130.00	132.00	45	10
SG1-150	150	150.00	152.00	45	10

The permissible bending stress, σ_p, can be taken as σ_{uts}/factor of safety, where the factor of safety is set by experience but may range from 2 to 5. Alternatively, one can use values of σ_p as listed for the appropriate material in a stock gear catalog. Certain plastics are suitable for use as gear materials in application where low weight, low friction, high corrosion resistance, low wear, and quiet operation are beneficial. The strength of plastic is usually significantly lower than that of metals. Plastics are often formed using a filler to improve strength, wear, impact resistance, temperature performance, as well as other properties. It is, therefore, difficult to regulate or standardize properties for plastics and these need to be obtained from either the manufacturer or a traceable testing laboratory. Plastics used for gears include acrylonitrile-butadiene-styrene (ABS), acetal, nylon, polycarbonate, polyester, polyurethane, and styrene-acrylonitrile (SAN). Values of permissible bending stress for a few gear materials are listed in Table 8.11.

Table 8.8: Example spur gears. 1.5 module. Heavy duty steel 817M40, 655M13. Face width 20 mm.

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG1.5-9	9	15.00	18.00	18	8
SG1.5-10	10	16.50	19.50	19.5	8
SG1.5-11	11	18.00	21.00	21	8
SG1.5-12	12	18.00	21.00	21	8
SG1.5-13	13	19.50	22.50	20	8
SG1.5-14	14	21.00	24.00	20	8
SG1.5-15	15	22.50	25.50	20	8
SG1.5-16	16	24.00	27.00	25	10
SG1.5-17	17	25.50	28.50	25	10
SG1.5-18	18	27.00	30.00	25	10
SG1.5-19	19	28.50	31.50	25	10
SG1.5-20	20	30.00	33.00	25	10
SG1.5-21	21	31.50	34.50	25	10
SG1.5-22	22	33.00	36.00	30	10
SG1.5-23	23	34.50	37.50	30	10
SG1.5-24	24	36.00	39.00	30	10
SG1.5-25	25	37.50	40.50	30	10
SG1.5-26	26	39.00	42.00	30	10
SG1.5-27	27	40.50	43.50	30	10
SG1.5-28	28	42.00	45.00	30	10
SG1.5-29	29	43.50	46.50	30	10
SG1.5-30	30	45.00	48.00	30	10
SG1.5-31	31	46.50	49.50	30	10
SG1.5-32	32	48.00	51.00	30	10
SG1.5-33	33	49.50	52.50	30	10
SG1.5-34	34	51.00	54.00	30	10
SG1.5-35	35	52.50	55.50	50	15
SG1.5-36	36	54.00	57.00	50	15
SG1.5-37	37	55.50	58.50	50	15
SG1.5-38	38	57.00	60.00	50	15
SG1.5-39	39	58.50	61.50	50	15
SG1.5-40	40	60.00	63.00	50	15
SG1.5-41	41	61.50	64.50	50	15
SG1.5-42	42	63.00	66.00	50	15
SG1.5-43	43	64.50	67.50	50	15
SG1.5-44	44	66.00	69.00	50	15
SG1.5-45	45	67.50	70.50	50	15
SG1.5-46	46	69.00	72.00	50	15
SG1.5-47	47	70.50	73.50	50	15
SG1.5-48	48	72.00	75.00	50	15
SG1.5-49	49	73.50	76.50	50	15
SG1.5-50	50	75.00	78.00	50	15
SG1.5-51	51	76.50	79.50	50	15
SG1.5-52	52	78.00	81.00	50	15
SG1.5-53	53	79.50	82.50	60	15
SG1.5-54	54	81.00	84.00	60	15

Continued

Table 8.8: Example spur gears. 1.5 module. Heavy duty steel 817M40, 655M13. Face width 20 mm.—cont'd

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG1.5-55	55	82.50	85.50	60	15
SG1.5-56	56	84.00	87.00	60	15
SG1.5-57	57	85.50	88.50	60	15
SG1.5-58	58	87.00	90.00	60	15
SG1.5-59	59	88.50	91.50	60	15
SG1.5-60	60	90.00	93.00	60	15
SG1.5-62	62	93.00	96.00	60	15
SG1.5-64	64	96.00	99.00	60	15
SG1.5-65	65	97.50	100.50	60	15
SG1.5-66	66	99.00	102.00	60	15
SG1.5-68	68	102.00	105.00	60	15
SG1.5-70	70	105.00	108.00	60	15
SG1.5-71	71	106.50	109.50	60	15
SG1.5-72	72	108.00	111.00	60	15
SG1.5-73	73	109.50	112.50	60	15
SG1.5-74	74	111.00	114.00	60	15
SG1.5-75	75	112.50	115.50	60	15
SG1.5-76	76	114.00	117.00	60	15
SG1.5-78	78	117.00	120.00	75	15
SG1.5-80	80	120.00	123.00	75	15
SG1.5-86	86	129.00	132.00	75	15
SG1.5-90	90	135.00	138.00	75	15
SG1.5-96	96	144.00	147.00	75	15
SG1.5-98	98	147.00	150.00	75	15
SG1.5-100	100	150.00	153.00	75	15
SG1.5-105	105	157.50	160.50	75	15
SG1.5-110	110	165.00	168.00	75	15
SG1.5-115	115	175.50	178.50	75	15
SG1.5-120	120	180.00	183.00	75	15

The design procedure consists of proposing teeth numbers for the gear and pinion, selecting a suitable material, selecting a module, calculating the various parameters as listed, resulting in a value for the face width. If the face width is greater than that available in the stock gear catalog, or if the pitch line velocity is too high, repeat the process for a different module. If this does not provide a sensible solution, try a different material, etc. The process can be optimized taking cost and other performance criteria into account, if necessary, and can be programmed.

Example 8.8

A 20° full depth spur pinion is required to transmit 1.8 kW at a speed of 1100 rpm. If the pinion has 18 teeth and is manufactured from heavy-duty 817M40 steel select a suitable gear

Table 8.9: Example spur gears. 2.0 module. Heavy duty steel 817M40, 655M13. Face width 25 mm.

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG2-9	9	20.00	24.00	24	12
SG2-10	10	22.00	26.00	26	12
SG2-11	11	24.00	28.00	28	12
SG2-12	12	24.00	28.00	28	12
SG2-13	13	26.00	30.00	30	12
SG2-14	14	28.00	32.00	30	12
SG2-15	15	30.00	34.00	30	12
SG2-16	16	32.00	36.00	30	12
SG2-17	17	34.00	38.00	35	15
SG2-18	18	36.00	40.00	35	15
SG2-19	19	38.00	42.00	35	15
SG2-20	20	40.00	44.00	35	15
SG2-21	21	42.00	46.00	35	15
SG2-22	22	44.00	48.00	35	15
SG2-23	23	46.00	50.00	35	15
SG2-24	24	48.00	52.00	35	15
SG2-25	25	50.00	54.00	35	15
SG2-26	26	52.00	56.00	35	15
SG2-27	27	54.00	58.00	50	20
SG2-28	28	56.00	60.00	50	20
SG2-29	29	58.00	62.00	50	20
SG2-30	30	60.00	64.00	50	20
SG2-31	31	62.00	66.00	50	20
SG2-32	32	64.00	68.00	50	20
SG2-33	33	66.00	70.00	50	20
SG2-34	34	68.00	72.00	50	20
SG2-35	35	70.00	74.00	50	20
SG2-36	36	72.00	76.00	50	20
SG2-37	37	74.00	78.00	50	20
SG2-38	38	76.00	80.00	50	20
SG2-39	39	78.00	82.00	50	20
SG2-40	40	80.00	84.00	50	20
SG2-41	41	82.00	86.00	50	20
SG2-42	42	84.00	88.00	50	20
SG2-43	43	86.00	90.00	60	20
SG2-44	44	88.00	92.00	60	20
SG2-45	45	90.00	94.00	60	20
SG2-46	46	92.00	96.00	60	20
SG2-47	47	94.00	98.00	60	20
SG2-48	48	96.00	100.00	60	20
SG2-49	49	98.00	102.00	60	20
SG2-50	50	100.00	104.00	60	20
SG2-51	51	102.00	106.00	60	20
SG2-52	52	104.00	108.00	60	20
SG2-53	53	106.00	110.00	60	20
SG2-54	54	108.00	112.00	60	20

Continued

Table 8.9: Example spur gears. 2.0 module. Heavy duty steel 817M40, 655M13. Face width 25 mm.—cont'd

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG2-55	55	110.00	114.00	60	20
SG2-56	56	112.00	116.00	60	20
SG2-57	57	114.00	118.00	60	20
SG2-58	58	116.00	120.00	60	20
SG2-59	59	118.00	122.00	60	20
SG2-60	60	120.00	124.00	60	20
SG2-62	62	124.00	128.00	75	20
SG2-64	64	128.00	132.00	75	20
SG2-65	65	130.00	134.00	75	20
SG2-66	66	132.00	136.00	75	20
SG2-68	68	136.00	140.00	75	20
SG2-70	70	140.00	144.00	75	20
SG2-71	71	142.00	146.00	75	20
SG2-72	72	144.00	148.00	75	20
SG2-73	73	146.00	150.00	75	20
SG2-74	74	148.00	152.00	75	20
SG2-75	75	150.00	154.00	75	20
SG2-76	76	152.00	156.00	75	20
SG2-78	78	156.00	160.00	100	20
SG2-80	80	160.00	164.00	100	20
SG2-86	86	172.00	176.00	100	20
SG2-90	90	180.00	184.00	100	20
SG2-96	96	192.00	196.00	100	20
SG2-98	98	196.00	200.00	100	20
SG2-100	100	200.00	204.00	100	20
SG2-105	105	210.00	214.00	100	20
SG2-110	110	220.00	224.00	100	20
SG2-115	115	230.00	234.00	100	20
SG2-120	120	240.00	244.00	100	20

from the limited choice illustrated in Tables 8.7–8.10, specifying the module and face width based on the Lewis formula.

Solution

Power = 1.8 kW

$N_P = 18$

Try $m = 1$

Assuming a hardened material will be used to improve wear resistance, $\sigma_p = 183$ MPa.

$Y_{18} = 0.29327$

$n_P = 1100$ rpm

$d_P = mN = 0.001 \times 18 = 0.018$ m

Table 8.10: Example spur gears. 3.0 module. Heavy duty steel 817M40, 655M13. Face width 35 mm.

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG3-9	9	30.00	36.00	36	15
SG3-10	10	33.00	39.00	39	15
SG3-11	11	36.00	42.00	42	15
SG3-12	12	36.00	42.00	42	20
SG3-13	13	39.00	45.00	45	20
SG3-14	14	42.00	48.00	45	20
SG3-15	15	45.00	51.00	45	20
SG3-16	16	48.00	54.00	45	20
SG3-17	17	51.00	57.00	45	20
SG3-18	18	54.00	60.00	45	20
SG3-19	19	57.00	63.00	45	20
SG3-20	20	60.00	66.00	60	25
SG3-21	21	63.00	69.00	60	25
SG3-22	22	66.00	72.00	60	25
SG3-23	23	69.00	75.00	60	25
SG3-24	24	72.00	78.00	60	25
SG3-25	25	75.00	81.00	60	25
SG3-26	26	78.00	84.00	60	25
SG3-27	27	81.00	87.00	60	25
SG3-28	28	84.00	90.00	60	25
SG3-29	29	87.00	93.00	60	25
SG3-30	30	90.00	96.00	60	25
SG3-31	31	93.00	99.00	60	25
SG3-32	32	96.00	102.00	60	25
SG3-33	33	99.00	105.00	60	25
SG3-34	34	102.00	108.00	75	25
SG3-35	35	105.00	111.00	75	25
SG3-36	36	108.00	114.00	75	25
SG3-37	37	111.00	117.00	75	25
SG3-38	38	114.00	120.00	75	25
SG3-39	39	117.00	123.00	75	25
SG3-40	40	120.00	126.00	75	25
SG3-41	41	123.00	129.00	75	25
SG3-42	42	126.00	132.00	75	25
SG3-43	43	129.00	135.00	75	25
SG3-44	44	132.00	138.00	100	25
SG3-45	45	135.00	141.00	100	25
SG3-46	46	138.00	144.00	100	25
SG3-47	47	141.00	147.00	100	25
SG3-48	48	144.00	150.00	100	25
SG3-49	49	147.00	153.00	100	25
SG3-50	50	150.00	156.00	100	25
SG3-51	51	153.00	159.00	100	25
SG3-52	52	156.00	162.00	100	25
SG3-53	53	159.00	165.00	100	25
SG3-54	54	162.00	168.00	100	25

Continued

Table 8.10: Example spur gears. 3.0 module. Heavy duty steel 817M40, 655M13. Face width 35 mm.—cont'd

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG3-55	55	165.00	171.00	100	25
SG3-56	56	168.00	174.00	100	25
SG3-57	57	171.00	177.00	127	25
SG3-58	58	174.00	180.00	127	25
SG3-59	59	177.00	183.00	127	25
SG3-60	60	180.00	186.00	127	25
SG3-62	62	186.00	192.00	127	25
SG3-63	63	189.00	195.00	127	25
SG3-64	64	192.00	198.00	127	25
SG3-65	65	195.00	201.00	127	25
SG3-68	68	204.00	210.00	127	25
SG3-70	70	210.00	216.00	150	30
SG3-72	72	216.00	222.00	150	30
SG3-75	75	225.00	231.00	150	30
SG3-76	76	228.00	234.00	150	30
SG3-78	78	234.00	240.00	150	30
SG3-80	80	240.00	246.00	150	30
SG3-82	82	246.00	252.00	150	30
SG3-84	84	252.00	258.00	150	30
SG3-86	86	258.00	264.00	150	30
SG3-90	90	270.00	276.00	150	30
SG3-92	92	276.00	282.00	150	30
SG3-94	94	282.00	288.00	150	30
SG3-95	95	285.00	291.00	150	30
SG3-96	96	288.00	294.00	150	30

Source: Data adapted from HPC Gears Ltd.
PCD = pitch circle diameter, OD = outer diameter.

Table 8.11: Permissible bending stresses for various commonly used gear materials.

Material	Treatment	σ_{uts} (MPa)	Permissible bending stress σ_p (MPa)
Nylon		65 (20 °C)	27
Tufnol		110	31
080M40		540	131
080M40	Induction hardened	540	117
817M40		772	221
817M40	Induction hardened	772	183
045M10		494	117
045M10	Case hardened	494	276
655M13	Case hardened	849	345

$$V = \frac{d_P}{2} \times 10^{-3} \times n\frac{2\pi}{60} = \frac{0.018}{2} \times 1100 \times 0.1047 = 1.037 \text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 1.037} = 0.8548$$

$$W_t = \frac{\text{Power}}{V} = \frac{1800}{1.037} = 1737 \text{ N}$$

$$F = \frac{W_t}{K_v m Y \sigma} = \frac{1737}{0.8548 \times 0.001 \times 0.29327 \times 183 \times 10^6} = 0.0379 \text{ m}$$

This value of face width is greater than the stock value available as listed in Table 8.7 for a module 1 gear. Therefore, an alternative design needs to be considered.

Trying $m = 1.5$ mm
$d_P = mN = 0.0015 \times 18 = 0.027$ m
$V = 1.555$ m/s
$K_v = 0.7969$
$W_t = 1158$ N

$$F = \frac{W_t}{K_v m Y \sigma} = \frac{1158}{0.7969 \times 0.0015 \times 0.29327 \times 183 \times 10^6} = 0.01805 \text{ m}$$

This is less than the $F = 20$ mm available for the 1.5 module stock gears listed in Table 8.8. This gear is, therefore, likely to be acceptable in terms of bending stress capability.

The gear specification is therefore: $N_P = 18$, $m = 1.5$ mm, $F = 20$ mm, 817M40 induction hardened.

Example 8.9

A gearbox is required to transmit 18 kW from a shaft rotating at 2650 rpm. The desired output speed is approximately 12,000 rpm. For space limitation and standardization reasons, a double step-up gearbox is requested with equal ratios. Using the limited selection of gears presented in Tables 8.8–8.11, select suitable gears for the gear wheels and pinions.

Solution

The overall ratio = 12,000/2650 = 4.528.

First stage ratio = $\sqrt{4.528} = 2.128$.

This could be achieved using a gear with 38 teeth and pinion with 18 teeth (ratio = 38/18 = 2.11).

The gear materials listed in Tables 8.7–8.10 are 817M40 and 655M13 steels.

From Table 8.11, the 655M13 is the stronger steel, and this is selected for this example prior to a more detailed consideration.

For 655M13 case hardened steel gears, the permissible stress $\sigma_p = 345$ MPa.

Calculations for gear 1: $Y_{38} = 0.37727$, $n = 2650$ rpm.

m	1.5	2.0
$D = mN$ (mm)	57	76
$V = \frac{d}{2} \times 10^{-3} \times n\frac{2\pi}{60}$ (m/s)	7.9	10.5
$W_t = \frac{\text{Power}}{V}$ (N)	2276	1707
$K_v = \frac{6.1}{6.1 + V}$	0.4357	0.3675
$F = \frac{W_t}{K_v m Y \sigma_p}$ (m)	0.027	0.018

Note: $m = 1.5$ gives a face width greater than the catalog value of 20 mm, so try $m = 2$.

$m = 2$ gives a face width less than the catalog value of 25 mm, so OK.

Calculations for pinion 1: $Y_{18} = 0.29327$, $n = 5594$ rpm.

(No need to do calculations for $m = 1.5$ because it has now been rejected.)

m	2.0
d (mm)	36
V (m/s)	10.5
W_t (N)	1707
K_v	0.3676
F (m)	0.023

Note: $m = 1.5$ gives a face width greater than the catalog value of 20 mm, so try $m = 2$.

$m = 2$ gives a face width less than the catalog value of 25 mm, so OK.

Calculations for gear 2: $Y_{38} = 0.37727$, $n = 5594$ rpm.

m	2.0
d (mm)	76
V (m/s)	22.26
W_t (N)	808.6
K_v	0.215
F (m)	0.0144

Note: $m = 2$ gives value for face width lower than catalog specification, so the design is OK.

Calculations for pinion 2: $Y_{18} = 0.29327$, $n = 11{,}810$ rpm.

m	2.0
d (mm)	36
V (m/s)	22.26
W_t (N)	808.6
K_v	0.215
F (m)	0.0186

Note: $m = 2$ gives value for face width lower than catalog specification, so the design is OK.

The overall design for Example 8.9 is illustrated in Figure 8.30. Note in this example common shafts, bearings and machine features have been implemented throughout in order to minimise the parts inventory.

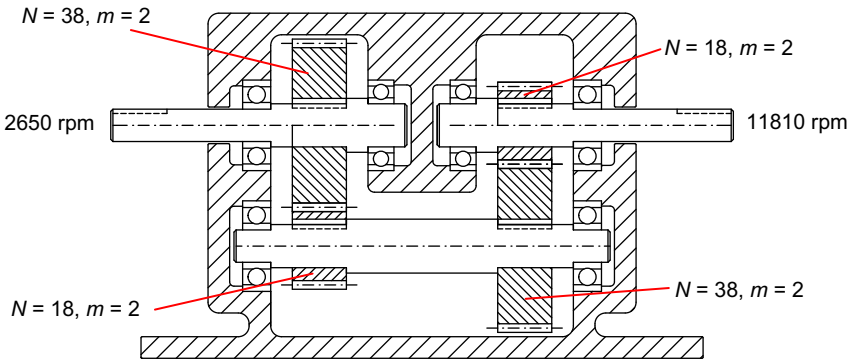


Figure 8.30
Step up gearbox solution schematic.

Example 8.10

A gearbox for a cordless hand tool, Figure 8.31, is required to transmit 5 W from an electric motor running at 3000 rpm to an output drive running at approximately 75 rpm. The specification for the gearbox is that it should be reliable, efficient, maintenance free for the life of the device, light, and fit within an enclosure of 50 mm diameter. The length is not critical. The tolerance on the output speed acceptable is $\pm 10\%$. Specify an appropriate gear train including values for the module, teeth numbers, and gear materials.

Solution

The reduction ratio 3000/75 is quite large and could be achieved by a compound gear train, a worm and wheel, or a two-stage reduction epicyclic. The restriction on the maximum diameter precludes the use of compound gear trains, especially if the drive input is axial, as would be expected from an electric motor. The high efficiency demand rules out the use of a worm and wheel.

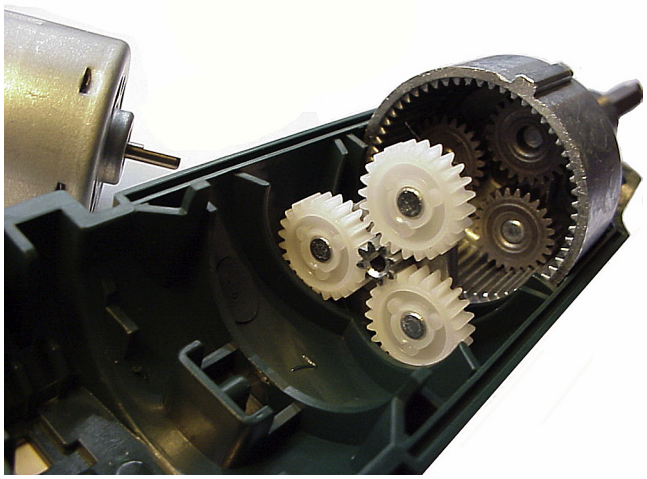


Figure 8.31
Cordless hand tool molding, electric motor, and gearbox.

We want an overall ratio of 40 (=3000/75). This could be achieved in a two-stage reduction epicyclic gearbox, as illustrated in Figure 8.32. For identical stages, the gear ratio of each stage would be $\sqrt{40} = 6.3$ ($6.3 \times 6.3 \approx 39.7$).

Try $N_s = 18$, $N_R = 72$. This gives a reduction ratio per stage of $N_R/N_s + 1 = 72/18 + 1 = 5$. Overall reduction ratio of the two stage gear train $= 5 \times 5 = 25$; no good.

Try $N_s = 12$, $N_R = 72$. This gives a reduction ratio of 7. Overall reduction ratio $7 \times 7 = 49$; no good.

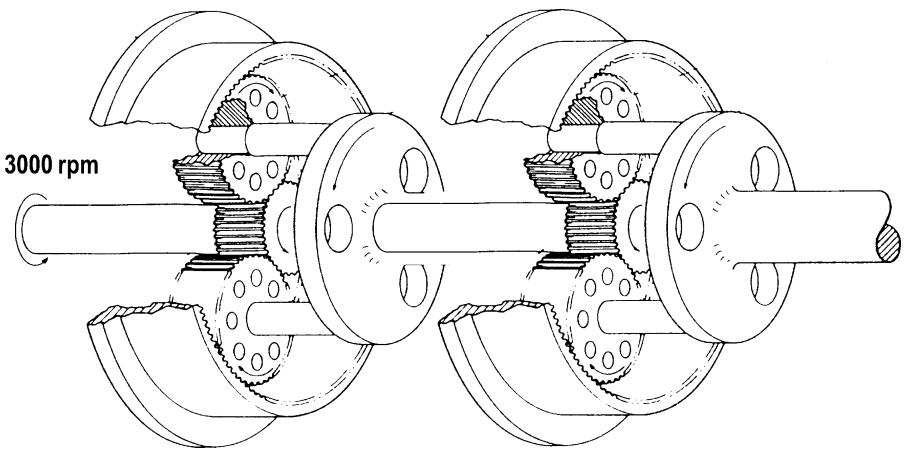


Figure 8.32
Two-stage reduction epicyclic gear train. *After Townsend (1992).*

Try $N_s = 12$, $N_R = 63$. This gives a reduction ratio of 6.25. Overall reduction ratio $6.25 \times 6.25 = 39.1$; OK.

Can this fit in the space?

Propose a module. Say $m = 0.5$.

With $N_S = 12$, $d_{\text{sun}} = \text{mN} = 0.5 \times 12 = 6$ mm.

With $N_R = 63$, $d_{\text{ring}} = \text{mN} = 0.5 \times 63 = 31.5$ mm.

Room available for planet gear is $31.5/2 - 6/2 = 12.75$ mm. With a module of 0.5 there are no gears with this diameter. Impossible solution.

Try $N_s = 12$, $N_R = 66$. This gives a reduction ratio of 6.5. Reduction ratio $6.5 \times 6.5 = 42.3$.

This would give an output speed of 71 rpm.

This is within the tolerance allowable on the output speed. OK.

Can this fit in the space?

Propose a module. Say $m = 0.5$.

With $N_S = 12$, $d_{\text{sun}} = mN = 0.5 \times 12 = 6$ mm.

With $N_R = 66$, $d_{\text{ring}} = mN = 0.5 \times 66 = 33$ mm.

Room available for planet gear is $33/2 - 6/2 = 13.5$ mm.

With a module of 0.5, the planetary gears would have $13.5/0.5 = 27$ teeth. $N_p = 27$.

Proposed design: $N_s = 12$, $N_p = 27$, $N_R = 66$, module 0.5.

What face width should the gears have? For a module of 0.5 HPC Gears Ltd. offers standard 0.5 module gears with a face width of 5 mm available in Tufnol, Delrin, brass, and 045M10 steel.

The Lewis formula can be used as an approximate check to determine whether the gears with a face width of 5 mm manufactured from a particular material are strong enough.

Analyzing the input sun gear

$$V_s = \frac{6 \times 10^{-3}}{2} \times 3000 \times \frac{2\pi}{60} = 0.942 \text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.942} = 0.866$$

$$W_t = \frac{\text{power}}{V} = \frac{5}{0.942} = 5.3 \text{ N}$$

From Table 8.6, $Y_{12} = 0.2296$.

Try Tufnol, $\sigma_p = 31$ MPa

$$F = \frac{W_t}{K_v m Y \sigma_p} = \frac{5.3}{0.866 \times 0.0005 \times 0.2296 \times 31 \times 10^6} = 1.72 \times 10^{-3}\text{m}$$

This face width of 1.7 mm is less than the 5 mm available, indicating that the gear will be more than strong enough. If this figure had been greater than 5 mm, then another material such as steel or brass could be considered.

Analyzing the planets:

$$n_{\text{arm}} = 3000/6.5 = 461.5 \text{ rpm}$$

To determine the speed of the planets use Eqns (8.12) and (8.13)

$$\frac{n_P - n_{\text{arm}}}{n_S - n_{\text{arm}}} = -\frac{N_S}{N_P}$$

$$\frac{n_P - (-461.5)}{-3000 - (-461.5)} = -\frac{12}{27}$$

$$n_P = -\frac{12}{27} \times (-2538) - 461.5 = 666.7 \text{ rpm}$$

$$V_p = \frac{13.5 \times 10^{-3}}{2} \times 666.7 \times \frac{2\pi}{60} = 0.47 \text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.47} = 0.928$$

The power is divided between three planets, so:

$$W_t = \frac{\text{power}}{V} = \frac{5/3}{0.47} = 3.55 \text{ N}$$

From Table 8.6, $Y_{27} = 0.34385$.

Try Tufnol, $\sigma_p = 31$ MPa

$$F = \frac{W_t}{K_v m Y \sigma_p} = \frac{3.55}{0.928 \times 0.0005 \times 0.34385 \times 31 \times 10^6} = 0.72 \times 10^{-3} \text{ m}$$

This face width of 0.72 mm is less than the 5 mm available, indicating that the gear will be more than strong enough.

Analyzing the ring gear: The stress on the ring gear will be less than on the planets so it does not necessarily need to be analyzed at this stage.

Analyzing the stage 2 sun gear ($n_{\text{sun}} = n_{\text{planet carrier}}$)

$$V_s = \frac{6 \times 10^{-3}}{2} \times 461.5 \times \frac{2\pi}{60} = 0.145 \text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.145} = 0.98$$

$$W_t = \frac{\text{power}}{V} = \frac{5}{0.145} = 34.5 \text{ N}$$

From Table 8.6, $Y_{12} = 0.2296$.

Try Tufnol, $\sigma_p = 31$ MPa

$$F = \frac{W_t}{K_v m Y \sigma_p} = \frac{34.5}{0.98 \times 0.0005 \times 0.2296 \times 31 \times 10^6} = 0.0099 \text{ m}$$

This is larger than the available standard size (5 mm). So try a stronger material.

Using 045M10 steel, $\sigma_p = 117$ MPa

$$F = \frac{W_t}{K_v m Y \sigma_p} = \frac{34.5}{0.98 \times 0.0005 \times 0.2296 \times 117 \times 10^6} = 2.62 \times 10^{-3} \text{m}$$

This face width of 2.62 mm is less than the 5 mm available, indicating that the gear will be more than strong enough. So the sun gear must be manufactured from 045M10 steel. Manufacturing expediency might dictate that the planet carrier should also be made from the same steel.

Analyzing the planets:

$$\frac{n_P - (-71)}{-461.5 - (-71)} = -\frac{12}{27}$$

$$n_P = -\frac{12}{27} \times (-390.5) - 71 = 102.6 \text{ rpm}$$

$$V_p = \frac{13.5 \times 10^{-3}}{2} \times 102.6 \times \frac{2\pi}{60} = 0.0725 \text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.0725} = 0.99$$

The power is divided between three planets, so

$$W_t = \frac{\text{power}}{V} = \frac{5/3}{0.0725} = 23.0 \text{ N}$$

$Y_{27} = 0.34385$.

Try Tufnol, $\sigma_p = 31$ MPa

$$F = \frac{W_t}{K_v m Y \sigma_p} = \frac{23.0}{0.99 \times 0.0005 \times 0.34285 \times 31 \times 10^6} = 4.4 \times 10^{-3} \text{ m}$$

This face width of 4.4 mm is less than the 5 mm available, indicating that the gear will be more than strong enough.

Design summary:

Stage 1: module 0.5, $F = 5$ mm, $N_s = 12$ (Tufnol), $N_p = 27$ (three planets, Tufnol), $N_R = 66$ (Tufnol).

Stage 2: module 0.5, $F = 5$ mm, $N_s = 12$ (steel), $N_p = 27$ (three planets, Tufnol), $N_R = 66$ (Tufnol).

8.7 Conclusions

For transmissions where compact size, high efficiency, or high speed are required, gears offer a competitive solution in comparison to other types of drives such as belts and chains. This chapter has introduced a wide range of types of gears and calculations enabling simple gear trains to be analyzed. A preliminary selection procedure for spur gears, based on the Lewis formula for bending stresses, has been presented. This serves as the starting point in proposing the details for a gear train, but more sophisticated analysis is necessary if a robust gearbox is required. Analysis for determining bending and contact stresses and associated safety factors based on national standards for spur and helical gears is outlined in Chapter 9.

References

Books and Papers

HPC Gears Ltd. Gears catalogue.
Heisler, H., 1999. Vehicle and Engine Technology, second ed. Butterworth Heinemann.
Mitchener, R.G., Mabie, H.H., 1982. The determination of the Lewis form factor and the AGMA geometry factor J for external spur gear teeth. ASME J. Mech. Des. 104, 148–158.
Townsend, D.P., 1992. Dudley's Gear Handbook. McGraw Hill.
Tuplin, W.A., 1962. Gear Design. The Machinery Publishing Co.

Further Reading

David Brown Special Products Ltd, 1995. David Brown Basic Gear Book.
Hofmann, D.A., Kohler, H.K., Munro, R.G., 1991. Gear Technology Teaching Pack. British Gear Association.

Standards

British Standards Institution. BS436: Part 1:1967 Specification for spur and helical gears. Part 1 Basic rack form, pitches and accuracy (diametral pitch series).
British Standards Institution. BS436: Part 2:1970 Specification for spur and helical gears. Part 2 Basic rack form, modules and accuracy (1 to 50 metric module).
British Standards Institution. BS2519: Part 1, 1976. Glossary for gears. Geometrical definitions.
British Standards Institution. BS2519: Part 2, 1976. Glossary for gears. Notation.
British Standards Institution. BS 6168: 1987. Specification for non-metallic spur gears.
British Standards Institution. BS ISO 9083:2001 Calculation of load capacity of spur and helical gears. Application to marine gears.

Web Sites

At the time of press, the world-wide-web contained useful information relating to this chapter at the following sites:

moorecustomgear.com
www.bga.org.uk
www.cross-morse.co.uk
www.davall.co.uk
www.drgears.com
www.gearing-hk.com
www.geartechnology.com
www.hpcgears.com
www.omnigearandmachine.com
www.penfold.co.nz
www.powertransmission.com
www.precipart.com
www.qtcgears.com
www.rapidgear.com
www.reliance.co.uk
www.shanthigears.com
www.textronpt.com
www.ugaco.com
www.wgear.com

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

a	addendum (mm)
b	dedendum (mm)
c	clearance (mm)

d pitch diameter (mm)
d_G pitch diameter of the gear (mm)
d_P pitch diameter of the pinion (mm)
d_{ring} pitch diameter of the ring gear (mm)
d_{sun} pitch diameter of the sun gear (mm)
e train value
F face width (mm)
F_r radial force component (N)
F_t tangential force component (N)
h_k working depth (mm)
h_t whole depth (mm)
H power (kW)
K_v velocity factor
M module (mm)
N rotational speed (rpm)
n_P rotational speeds of the pinion (rpm)
n_G rotational speeds of the gear (rpm)
N number of teeth
N_G number of teeth in the gear
N_P number of teeth in the pinion or planet gear
N_R number of teeth in annulus gear
N_s number of teeth in sun gear
OD outside diameter (mm)
p circular pitch (mm)
PCD pitch circle diameter (mm)
P_d diametral pitch (teeth per inch)
R pitch radius (mm)
r_f fillet radius of basic rack (mm)
t tooth thickness (mm)
t_o width of top land (mm)
V velocity (m/s)
W_t transmitted load (N)
Y Lewis form factor
ϕ pressure angle (°)
σ bending stress (N/m^2)
σ_p permissible stress (MPa)
σ_{uts} ultimate tensile strength (MPa)
ω angular velocity (rad/s)
ω_G angular velocities of the gear (rad/s)
ω_P angular velocities of the pinion (rad/s)

CHAPTER 9

Spur and Helical Gear Stressing

Chapter Outline

9.1 Introduction 367
9.2 Wear Failure 368
9.3 AGMA Equations for Bending and Contact Stress 370
9.4 Gear Selection Procedure 397
9.5 Conclusions 399
References 399
Further Reading 399
Nomenclature 400

Abstract

Although suitable for initial selection of a gear, the Lewis equation is inadequate for detailed design calculations when the life of the machine is critical or when the gears are highly loaded. National and international standards are available detailing the calculation of bending and contact stresses for gears. This chapter outlines the American Gear Manufacturers Association standards for the calculation of bending and contact stresses for spur and helical gears.

9.1 Introduction

The Lewis equation introduced in Chapter 8 provides a useful starting point allowing the configuration of a gear set that is to be developed. The Lewis equation takes into account the bending stress on a gear. Gears, however, can fail due to excessive bending stress or wear. Wear occurs because as the teeth move in and out of contact with each other, there is accompanying local deformation of the gear teeth surfaces in the region of contact. The stresses resulting from the surface deformation are known as contact stresses. If the stresses are too high, then material failure can take the form of a loss of material from the surfaces, which is also known as pitting. Pitting is surface fatigue failure due to too many repetitions of high contact stresses. Wear can also occur due to scoring, failure of lubrication, or abrasion due to the presence of a foreign material such as grit or powder.

The modeling of gear stresses has been the subject of extensive research. The analysis is complex because of the dynamic nature of the loads and the very high stresses involved for some gear designs. While techniques such as finite element analysis enable detailed modeling

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00009-5
Copyright © 2014 Elsevier Ltd. All rights reserved.

of specific gear geometry, the results are labor-intensive in terms of setting up the model, and uncertainty often remains concerning the validity of boundary conditions and material properties. An alternative to finite element analysis is the use of procedural approaches such as those outlined in standards developed by the American Gear Manufacturers Association (AGMA), the British Standards Institution (BSI), and the International Organization for Standardization (ISO).

This chapter concentrates on analyzing whether failure due to bending and contact stresses in a gear set is likely. In Section 9.2 a relatively simple relationship for contact stresses is outlined prior to the introduction of the current AGMA equations for bending and contact stresses in Section 9.3. The AGMA equations rely on the evaluation of a significant number of factors and geometrical parameters, which depend on the specific geometry and materials concerned. In this chapter, the basic approach is outlined for 20° pressure angle, full depth teeth spur and helical gears only. Many of the detailed charts and equations given in the source standards have been omitted and the reader is encouraged to view the most recent standards as necessary. A general strategy for gear design is outlined in Section 9.4.

9.2 Wear Failure

In addition to failure due to bending stresses in gears, failure due to wear on the surface of gear teeth should also be considered. Possible surface failures include pitting, which is a surface fatigue failure due to many repetitions of high contact stresses, scoring due to failure of lubrication, and abrasion due to the presence of foreign particles.

The surface compressive, Hertzian, or contact stress for a gear can be modeled by

$$\sigma_c = -C_p \left[\frac{K_v' W_t}{F \cos \phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{0.5} \tag{9.1}$$

where

C_p is an elastic coefficient,
r_1 and r_2 are radii of curvature,
K_v' is the velocity factor,
W_t is the transmitted load (N),
F is the face width (m)
ϕ is the pressure angle (°).

If the units of C_p are in $\sqrt{\text{MPa}}$, W_t is in N, and r_1,r_2 are in meters, then Eqn (9.1) gives the contact stress σ_c in kPa.

The radii of curvature are given by

$$r_1 = \frac{d_P \sin \phi}{2} \tag{9.2}$$

$$r_2 = \frac{d_G \sin \phi}{2} \tag{9.3}$$

where d_P and d_G are the pitch diameters of the pinion and gear, respectively.

The velocity factor K'_v for cut or milled profile gears is given by

$$K'_v = \frac{6.1 + V}{6.1} \tag{9.4}$$

where V is the pitch line velocity (m/s).

Note that this definition for the velocity factor, now used in the latest AGMA standards, is the reciprocal of that previously defined. For this reason, following the notation suggested by Mischke in Shigley and Mischke (2001), a superscript prime has been added to distinguish it from previous definitions.

The elastic coefficient C_p can be calculated from Eqn (9.5) or obtained from Table 9.1.

$$C_p = \left[\frac{1}{\pi \left(\frac{1 - \nu_P^2}{E_P} + \frac{1 - \nu_G^2}{E_G} \right)} \right]^{0.5} \tag{9.5}$$

where

ν_P is Poisson's ratio for the pinion,
ν_G is Poisson's ratio for the gear,
E_P is Young's modulus for the pinion,
E_G is Young's modulus for the gear.

Table 9.1: Values of the elastic coefficient (C_p, $Ze\sqrt{\text{MPa}}$).

		Gear material					
Pinion material	**E_{pinion} (GPa)**	**Steel**	**Malleable iron**	**Nodular iron**	**Cast iron**	**Aluminum bronze**	**Tin bronze**
Steel	200	191	181	179	174	162	158
Malleable iron	170	181	174	172	168	158	154
Nodular iron	170	179	172	170	166	156	152
Cast iron	150	174	168	166	163	154	149
Aluminum bronze	120	162	158	156	154	145	141
Tin bronze	110	158	154	152	149	141	137

Source: AGMA 218.01

Example 9.1

A speed reducer has a 22-tooth spur pinion made of steel, driving a 60-tooth gear made of cast iron. The transmitted power is 10 kW. The pinion speed is 1200 rpm, module 4, and face width 50 mm. Determine the contact stress.

Solution

$N_P = 22$; steel pinion, $N_G = 60$; cast iron gear. $n = 1200$ rpm, $m = 4$, $F = 50$ mm, $H = 10$ kW.

$$d_P = mN_P = 4 \times 22 = 88 \text{ mm}, \quad d_G = 4 \times 60 = 240 \text{ mm}.$$

$$V = \frac{0.088}{2} \times 1200 \times \frac{2\pi}{60} = 5.529 \text{ m/s}.$$

$$K_v' = \frac{6.1 + 5.529}{6.1} = 1.906.$$

$$W_t = \frac{10{,}000}{5.529} = 1809 \text{ N}.$$

From Table 9.1 for a steel pinion and cast iron gear, $C_p = 174 \sqrt{\text{MPa}}$.

From Eqns 9.2 and 9.3,

$$r_1 = \frac{88 \times 10^{-3} \sin 20}{2} = 0.01505 \text{ m}, \quad r_2 = \frac{240 \times 10^{-3} \sin 20}{2} = 0.04104 \text{ m}.$$

From Eqn (9.1):

$$\sigma_c = -174 \left[\frac{1.906 \times 1809}{0.05 \cos 20} (90.81) \right]^{0.5} = -174 \times 2581 \text{ kPa} = -449{,}200 \text{ kPa} \cong -449 \text{ MPa}.$$

9.3 AGMA Equations for Bending and Contact Stress

The calculation of bending and contact stresses in spur and helical gears can be determined using standardized methods presented by the BSI, the ISO, the Deutsches Institut für Normung, and the AGMA. The AGMA standards have recently been reaffirmed (ANSI/AGMA 2101-D04) and are widely used and have therefore been selected for presentation here. The procedures make extensive use of a series of geometry and design factors, which can be determined from design charts and tables.

The AGMA formula for bending stress for spur gears is given in SI units by

$$\sigma = W_t K_O K_V' K_s \frac{1}{Fm} \frac{K_H K_B}{Y_J} \tag{9.6}$$

or U.S. customary units by:

$$\sigma = W_t K_O K'_V K_s \frac{P_d}{F} \frac{K_m K_B}{J} \tag{9.7}$$

where

σ is the bending stress (N/m^2),
W_t is the transmitted tangential load (N),
K_O is the overload factor,
K'_V is the dynamic factor,
K_s is the size factor,
P_d is the transverse diametral pitch (in.),
F is the face width (m or in.),
m is the normal metric module for spur gears and the transverse metric module, m_t, for helical gears (in units of meter),
m_t is the transverse metric module (in units of meter),
K_H or K_m is the load distribution factor,
K_B is the rim thickness factor,
Y_J or J is the geometry factor for bending strength.

The transverse metric module for a helical gear is given by

$$m_t = \frac{m_n}{\cos \psi} \tag{9.8}$$

where

ψ is the helix angle at the standard pitch diameter (°) and m_n is the normal module (m or mm).

The circular pitch for helical gears is measured in two ways. The transverse circular pitch, p_t, is measured along the pitch circle, and the normal circular pitch, p_n, is measured normal to the helix of the gear. For a helical gear, the normal and transverse circular pitch are given by Eqns (9.9) and (9.10), respectively,

$$p_n = \pi m_n \tag{9.9}$$

$$p_t = \pi m_t \tag{9.10}$$

The pitch circle diameter for a helical gear is given by

$$d = m_t N = \frac{m_n}{\cos \psi} N \tag{9.11}$$

The transverse pressure angle, ϕ_t, is given by

$$\phi_t = \tan^{-1}\left(\frac{\tan \phi_n}{\cos \psi}\right) \tag{9.12}$$

The radial, axial, and resultant loads, W_r, W_a, and W, for a helical gear are given, respectively, by

$$W_r = W_t \tan \phi_t \tag{9.13}$$

$$W_a = W_t \tan \psi \tag{9.14}$$

$$W = \frac{W_t}{\cos \phi_n \cos \psi} \tag{9.15}$$

The AGMA equation for pitting resistance is given in SI units by

$$\sigma_c = Z_e \left(W_t K_O K_V' K_s \frac{K_H}{Fd} \frac{Z_R}{Z_I} \right)^{0.5} \tag{9.16}$$

or in U.S. customary units by

$$\sigma_c = C_p \left(W_t K_O K_V' K_s \frac{K_m}{Fd} \frac{C_f}{I} \right)^{0.5} \tag{9.17}$$

where

σ_c is the absolute value of contact stress (kPa),
Z_e is an elastic coefficient $(\text{MPa})^{0.5}$,
W_t is the transmitted tangential load (N),
K_O is the overload factor,
K_V' is the velocity factor,
K_s is the size factor,
K_H or K_m is the load distribution factor,
F is the face width (m),
d is the operating pitch diameter of the pinion (m),
Z_R or C_f is the surface condition factor for pitting resistance,
Z_I or I is the geometry factor for pitting resistance.

For Eqn (9.16), if W_t is in N, F in m, d in m, and C_p in $\sqrt{\text{MPa}}$, then the units of the contact stress σ_c are in kPa.

Based on safe working practices, the AGMA has defined allowable stress equations for gears for the allowable bending stress and for the allowable contact stress. The AGMA equation for determining a safe value for the allowable bending stress in SI units is given by

$$\sigma_{\text{all}} = \frac{\sigma_{FP} Y_N}{S_F Y_\theta Y_Z} \tag{9.18}$$

or in U.S. customary units by

$$\sigma_{\text{all}} = \frac{S_t Y_N}{S_F K_T K_R} \tag{9.19}$$

where

S_t or σ_{FP} is the allowable bending stress (lb/in.^{2} or MPa),
Y_N is the stress cycle factor for bending stress,
S_F is the AGMA factor of safety,
Y_θ or K_T is the temperature factor,
Y_Z or K_R is the reliability factor.

The AGMA equation for determining a safe value for the allowable contact stress in SI units is given by

$$\sigma_{c,\text{all}} = \frac{\sigma_{HP}}{S_H} \frac{Z_N Z_W}{Y_\theta Y_Z} \tag{9.20}$$

or in U.S. customary units by

$$\sigma_{c,\text{all}} = \frac{S_c}{S_H} \frac{Z_N C_H}{K_T K_R} \tag{9.21}$$

where

S_c or σ_{HP} is the allowable contact stress (lb/in.^{2} or MPa),
Z_N is the stress cycle life factor,
Z_W or C_H is a hardness ratio factor,
Y_θ or K_T is the temperature factor,
Y_Z or K_R is the reliability factor,
S_H is the AGMA factor of safety.

The purpose of the overload factor, K_O, is to account for externally applied loads in excess of the nominal tangential load W_t. A typical example is the variation in torque from an internal combustion engine. Values for K_O tend to be derived from extensive experience of a particular application. Values for the overload factor can be estimated from Table 9.2. Within this text, the value for the overload factor will generally be assumed to be unity.

The dynamic factor, K'_V, is used to account for the effect of tooth spacing and profile errors, the magnitude of the pitch line velocity, the inertia and stiffness of the rotating components, and the transmitted load per unit face width. The AGMA has defined a set of quality control numbers that can be taken as equal to the transmission accuracy level number, Q_v, to quantify these parameters (AGMA 390.01). Classes $3 \leq Q_v \leq 7$ include most

Table 9.2: Estimate for overload factors.

	Driven machine			
Driving machine	Uniform. For example, continuous duty generator sets.	Light shock. For example, fans, low-speed centrifugal pumps, variable duty generators, uniformly loaded conveyors, positive displacement pumps.	Moderate shock. For example, high-speed centrifugal pumps, reciprocating pumps and compressors, heavy duty conveyors, machine tools drives, and saws.	Heavy shock. For example, punch presses and crushers.
Uniform. For example, electric motors or constant speed gas turbines.	1	1.25	1.5	1.75
Light shock. For example, variable speed drives.	1.2	1.4	1.75	2.25
Moderate shock. For example, multicylinder engines.	1.3	1.7	2	2.75

Source: After Mott (1998).

commercial quality gears and are generally suitable for applications such as agricultural and plant machinery, presses, and conveyors. Classes of $8 \leq Q_v \leq 12$ are of precision quality and are generally suitable for power tools, washing machines, and automotive transmissions. Generally the higher the peripheral speed and the smaller the specific tooth load, the more accurate the gear manufacture must be, and a gear with a higher quality control number should be used. The dynamic factor K'_V can be determined from

$$K'_V = \left(\frac{A + \sqrt{200V}}{A}\right)^B \tag{9.22}$$

where:

$$B = \frac{(12 - Q_v)^{2/3}}{4} \tag{9.23}$$

and

$$A = 50 + 56(1 - B) \tag{9.24}$$

The size factor K_s is usually taken as unity. Higher values can be used if there is any nonuniformity in the material properties due to size.

The load distribution factor K_H is used to account for nonuniform distribution of load across the line of contact. Ideally gears should be sited at a location where there is zero slope on the shaft under the load. This is normally at midspan.

For $F/d \leq 2$, gears mounted between bearings, $F \leq 1.016$ m, come into contact across the full width of the narrowest member,

$$K_H = K_m = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) \tag{9.25}$$

For uncrowned teeth $C_{mc} = 1$ and for crowned teeth $C_{mc} = 0.8$.

If $F \leq 0.0254$ m, then

$$C_{pf} = \frac{F}{10d} - 0.025 \tag{9.26}$$

If $0.0254 \text{ m} < F \leq 0.4318$ m, then

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.4921F \tag{9.27}$$

If $0.4318 \text{ m} < F \leq 1.016$ m, then

$$C_{pf} = \frac{F}{10d} + 0.815F - 0.3534F^2 \tag{9.28}$$

For values of $F/10d < 0.05$, set $F/10d = 0.05$ in Eqns (9.26)–(9.28).

For a straddle mounted pinion with $S_1/S < 0.175$, use $C_{pm} = 1$.

For a straddle mounted pinion with $S_1/S \geq 0.175$, use $C_{pm} = 1.1$.

The definition for S_1 and S is given in Figure 9.1.

$$C_{ma} = A + BF + CF^2 \tag{9.29}$$

where the coefficients A, B, and C can be found from Table 9.3.

For gearing adjusted at assembly or if compatibility is improved by lapping or both, $C_e = 0.8$. For all other conditions, take $C_e = 1$.

If the rim thickness is insufficient to provide full support for the bearing, then this can be accounted for by the rim thickness factor K_B. This is necessary when the ratio $t_R/h_t < 1.2$. In the examples given here, it will be assumed that $t_R/h_t \geq 1.2$, in which case $K_B = 1$.

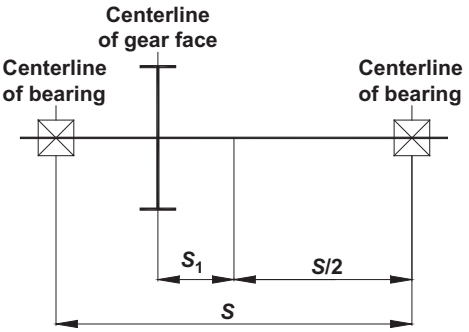


Figure 9.1
Definition of distances S_1 and S used for evaluating C_{pm}. *ANSI/AGMA 2001/C95.*

The bending strength geometry factor, Y_J, for 20° pressure angle full depth teeth spur gears can be obtained from Figure 9.2. The bending strength geometry factor, Y_J, for helical gears is given in AGMA Information Sheet 908-B89. A selection of these is given in Tables 9.4–9.8.

The surface condition factor Z_R is used to account for surface finish, residual stress, and work hardening. A value for Z_R greater than unity is suggested when a detrimental surface finish is known to exist.

For spur and helical gears, the surface strength geometry factor, Z_I, I, can be determined from Eqn (9.22) or Eqn (9.23).

For external gears:

$$Z_I = I = \frac{\cos\phi_t \sin\phi_t}{2m_N} \frac{m_G}{m_G + 1} \tag{9.30}$$

For internal gears:

$$Z_I = I = \frac{\cos\phi_t \sin\phi_t}{2m_N} \frac{m_G}{m_G - 1} \tag{9.31}$$

where $m_N = 1$ for spur gears. (ϕ_t is the transverse pressure angle used for helical gears. Replace ϕ_t by ϕ for spur gears.) For helical gears, the load sharing ratio can be taken as

Table 9.3: Coefficients used in determining C_{ma}, Eqn (9.29).

Condition	A	B	C
Open gearing	0.247	0.657	−0.119
Commercial enclosed gear units	0.127	0.622	−0.0144
Precision enclosed gear units	0.0675	0.504	−0.144
Extra-precision enclosed gear units	0.00360	0.402	−0.127

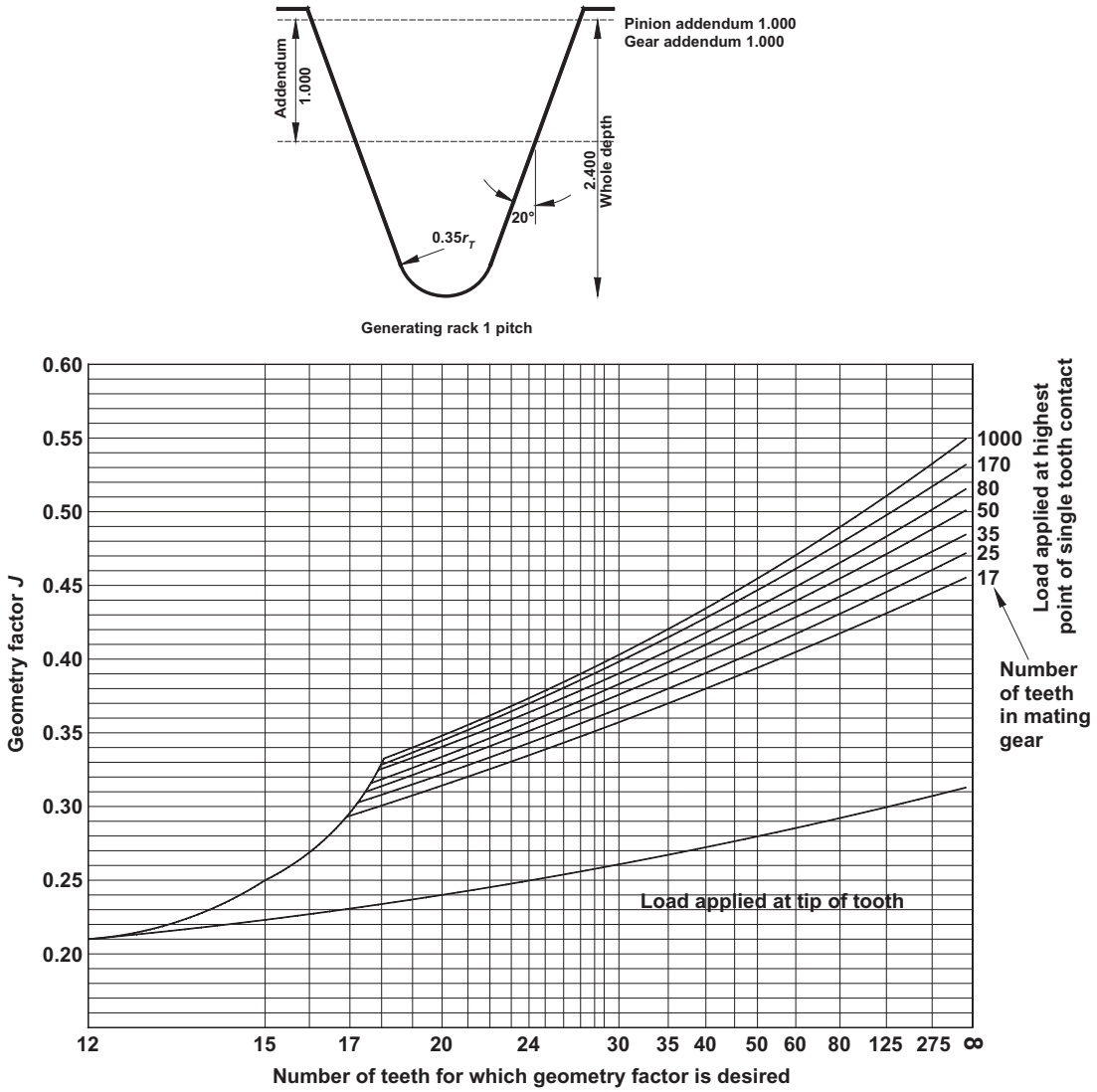


Figure 9.2

Bending strength geometry factor Y_J or J for 20° pressure angle full depth teeth spur gears.

$$m_N = p_N / 0.95Z \tag{9.32}$$

where p_N is the base pitch and Z is the length of the line of action in the transverse plane. The line of action in the transverse plane can be obtained from Eqn (9.33) (see Shigley and Uicker, 1980).

$$Z = \left[(r_P + a)^2 - r_{bP}^2\right]^{0.5} + \left[(r_G + a)^2 - r_{bG}^2\right]^{0.5} - (r_P + r_G)\sin\phi_t \tag{9.33}$$

Table 9.4: AGMA bending geometry factor *J* and pitting resistance geometry factor *I*. Pressure angle = 20°, helix angle = 0°, full depth teeth with tip loading.

		Pinion teeth															
		12		14		17		21		26		35		55		135	
Gear teeth		*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*
12	*I*																
	J	*U*	*U*														
14	*I*																
	J	*U*	*U*	*U*	*U*												
17	*I*																
	J	*U*	*U*	*U*	*U*	*U*	*U*										
21	*I*							0.078									
	J	*U*	*U*	*U*	*U*	*U*	*U*	0.24	0.24								
26	*I*							0.084		0.079							
	J	*U*	*U*	*U*	*U*	*U*	*U*	0.24	0.25	0.25	0.25						
35	*I*							0.091		0.088		0.080					
	J	*U*	*U*	*U*	*U*	*U*	*U*	0.24	0.26	0.25	0.26	0.26	0.26				
55	*I*							0.102		0.101		0.095		0.080			
	J	*U*	*U*	*U*	*U*	*U*	*U*	0.24	0.28	0.25	0.28	0.26	0.28	0.28	0.28		
135	*I*							0.118		0.121		0.120		0.112		0.080	
	J	*U*	*U*	*U*	*U*	*U*	*U*	0.24	0.29	0.25	0.29	0.26	0.29	0.28	0.29	0.29	0.29

Source: AGMA 908-B89.

Table 9.5: AGMA bending geometry factor *J* and pitting resistance geometry factor *I*. Pressure angle = 20°, helix angle = 15°, full depth teeth with tip loading.

		Pinion teeth															
		12		14		17		21		26		35		55		135	
Gear teeth		*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*
12	*I*																
	J	*U*	*U*														
14	*I*																
	J	*U*	*U*	*U*	*U*												
17	*I*					0.0124											
	J	*U*	*U*	*U*	*U*	0.43	0.43										
21	*I*					0.139											
	J	*U*	*U*	*U*	*U*	0.44	0.46	0.47	0.47								
26	*I*					0.154		0.143		0.132							
	J	*U*	*U*	*U*	*U*	0.45	0.49	0.48	0.50	0.50	0.50						
35	*I*					0.175											
	J	*U*	*U*	*U*	*U*	0.46	0.52	0.49	0.53	0.51	0.53	0.54	0.54				
55	*I*					0.204											
	J	*U*	*U*	*U*	*U*	0.47	0.55	0.50	0.56	0.53	0.57	0.56	0.58	0.59	0.59		
135	*I*					0.244		0.241		0.237		0.229		0.209		0.151	
	J	*U*	*U*	*U*	*U*	0.48	0.59	0.51	0.60	0.54	0.61	0.57	0.62	0.61	0.64	0.65	0.65

Source: AGMA 908-B89.

Table 9.6: AGMA bending geometry factor *J* and pitting resistance geometry factor *I*. Pressure angle = 20°, helix angle = 20°, full depth teeth with tip loading.

		Pinion teeth															
		12		14		17		21		26		35		55		135	
Gear teeth		*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*
12	*I*																
	J	*U*	*U*														
14	*I*																
	J	*U*	*U*	*U*	*U*												
17	*I*					0.125											
	J	*U*	*U*	*U*	*U*	0.44	0.44										
21	*I*					0.140		0.129									
	J	*U*	*U*	*U*	*U*	0.45	0.46	0.47	0.47								
26	*I*					0.156		0.145		0.133							
	J	*U*	*U*	*U*	*U*	0.45	0.49	0.48	0.49	0.50	0.50						
35	*I*					0.177		0.167		0.155		0.138					
	J	*U*	*U*	*U*	*U*	0.46	0.51	0.49	0.52	0.51	0.53	0.54	0.54				
55	*I*					0.205		0.197		0.188		0.172		0.144			
	J	*U*	*U*	*U*	*U*	0.47	0.54	0.50	0.55	0.52	0.56	0.55	0.57	0.58	0.58		
135	*I*					0.245		0.242		0.238		0.229		0.209		0.151	
	J	*U*	*U*	*U*	*U*	0.48	0.58	0.51	0.59	0.54	0.60	0.57	0.61	0.60	0.62	0.64	0.64

Source: AGMA 908-B89.

Table 9.7: AGMA bending geometry factor *J* and pitting resistance geometry factor *I*. Pressure angle = 20°, helix angle = 25°, full depth teeth with tip loading.

		Pinion teeth															
		12		14		17		21		26		35		55		135	
Gear teeth		*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*
12	*I*																
	J	*U*	*U*														
14	*I*			0.123													
	J	*U*	*U*	0.40	0.40												
17	*I*			0.137		0.126											
	J	*U*	*U*	0.41	0.43	0.43	0.43										
21	*I*			0.152		0.142		0.130									
	J	*U*	*U*	0.41	0.45	0.44	0.45	0.46	0.46								
26	*I*			0.167		0.157		0.146		0.134							
	J	*U*	*U*	0.42	0.47	0.44	0.47	0.47	0.48	0.49	0.49						
35	*I*			0.187		0.178		0.168		0.156		0.138					
	J	*U*	*U*	0.43	0.49	0.45	0.50	0.48	0.50	0.50	0.51	0.52	0.52				
55	*I*			0.213		0.207		0.199		0.189		0.173		0.144			
	J	*U*	*U*	0.44	0.52	0.46	0.52	0.49	0.53	0.51	0.54	0.53	0.55	0.56	0.56		
135	*I*			0.248		0.247		0.244		0.239		0.230		0.210		0.151	
	J	*U*	*U*	0.45	0.55	0.47	0.56	0.50	0.56	0.52	0.57	0.54	0.58	0.57	0.59	0.61	0.61

Source: AGMA 908-B89.

Table 9.8: AGMA bending geometry factor *J* and pitting resistance geometry factor *I*. Pressure angle = 20°, helix angle = 30°, full depth teeth with tip loading.

		Pinion teeth															
		12		14		17		21		26		35		55		135	
Gear teeth		*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*	*P*	*G*
12	*I*																
	J	*U*	*U*														
14	*I*			0.125													
	J	*U*	*U*	0.39	0.39												
17	*I*			0.139		0.128											
	J	*U*	*U*	0.39	0.41	0.41	0.41										
21	*I*			0.154		0.144		0.132									
	J	*U*	*U*	0.40	0.43	0.42	0.43	0.44	0.44								
26	*I*			0.169		0.159		0.148		0.135							
	J	*U*	*U*	0.41	0.44	0.43	0.45	0.45	0.46	0.46	0.46						
35	*I*			0.189		0.180		0.170		0.158		0.139					
	J	*U*	*U*	0.41	0.46	0.43	0.47	0.45	0.48	0.47	0.48	0.49	0.49				
55	*I*			0.215		0.208		0.200		0.190		0.174		0.145			
	J	*U*	*U*	0.42	0.49	0.44	0.49	0.46	0.50	0.48	0.50	0.50	0.51	0.52	0.52		
135	*I*			0.250		0.248		0.245		0.240		0.231		0.210		0.151	
	J	*U*	*U*	0.43	0.51	0.45	0.52	0.47	0.53	0.49	0.53	0.51	0.54	0.53	0.55	0.56	0.56

Source: AGMA 908-B89.

where depending on the application, $a = m_n$, $r_{bP} = r_p \cos \phi_t$, and $r_{bG} = r_G \cos \phi_t$. The speed ratio m_G is

$$m_G = \frac{N_G}{N_P} \tag{9.34}$$

The elastic coefficient Z_e or C_p can be found from Table 9.1 or Eqn (9.5).

The life factors Y_N and Z_N are used to modify the AGMA strengths for lives other than 10^7 cycles. The life required for a design is a critical decision. Consideration should be given to the maximum number of cycles that a machine in service will see. Obviously different customers may use machines in different ways. A common approach is to consider 95% of the machines in use. This allows for exclusion of extreme cases. Take, for instance, a washing machine. Market analysis would be required to identify what the maximum use of the machine would be. The 95th percentile excludes the tail of extreme users who might only rarely use their machines or use them 24 h a day. Such high use may well be excluded from warrantee claims in any case. An estimate of use can also be made. It is conceivable that a washing machine in a family household might be used twice a day, every day of the year. Assuming that the washing process involves operation at 30 rpm for 1 h and 800 rpm for 15 min, then the total number of cycles in a year is given by $365 \times 2 \times ((30 \times 60) + (800 \times 15)) = 10.074 \times 10^6$ or about 10^7 cycles. If a one-year warranty is to be considered, then the number of cycles to be used in calculations should be 10^7. The life factor Y_N can be determined from Figure 9.3 and Z_N from Figure 9.4. Note that if the pinion and gear have different teeth numbers, then they will experience different numbers of cycles.

For oil or gear temperatures up to 120 °C, the temperature factor Y_θ can be taken as unity. Operation at temperatures above this value is not recommended due to degradation of the lubricant, and use of a heat exchanger to reduce the lubricant temperature is advised in such circumstances. If the temperature exceeds 120 °C, the temperature factor Y_θ needs to be increased above unity.

The reliability factor, K_R or Y_Z, is used to account for the statistical distributions of failure of material due to fatigue. Depending on the level of reliability desired, the factor Y_Z should be adjusted accordingly as identified in Table 9.4. For values other than those listed in Table 9.9, use logarithmic interpolation.

The allowable bending stress for through hardened steels for grade 1 components is given by

$$\sigma_{FP} = 0.533 H_B + 88.3 \text{ MPa} \tag{9.35}$$

and for grade 2,

$$\sigma_{FP} = 0.703 H_B + 113 \text{ MPa} \tag{9.36}$$

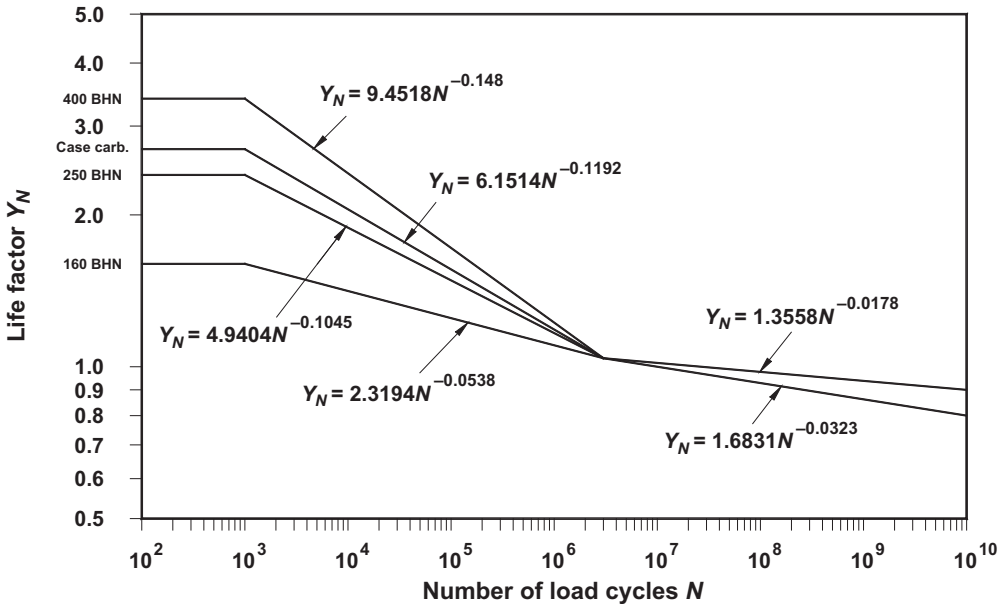


Figure 9.3
Bending strength life factor Y_N. *Reproduced from ANSI/AGMA 2001-C95.*

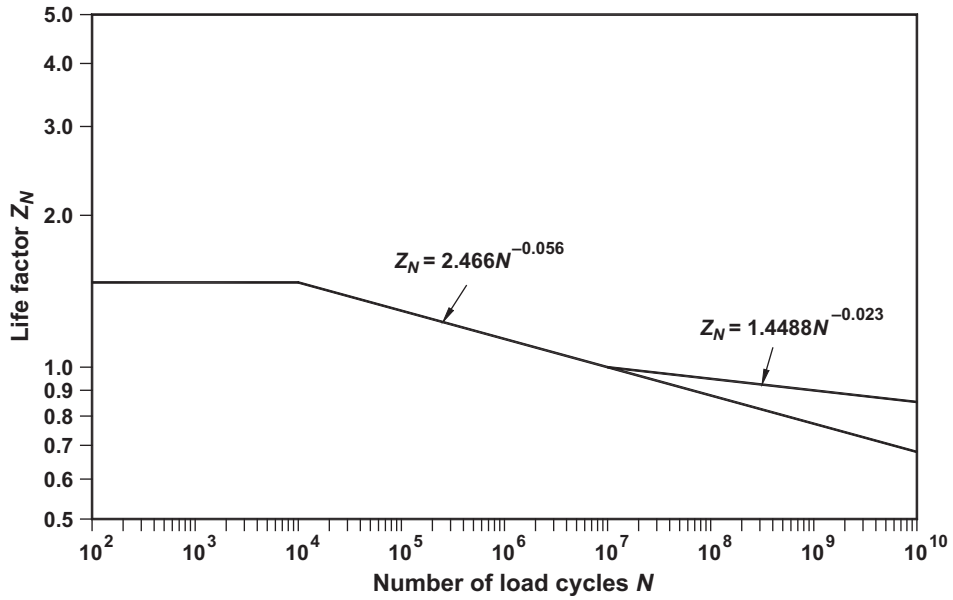


Figure 9.4
Pitting resistance life factor Z_N. *Reproduced from ANSI/AGMA 2001-C95.*

Table 9.9: AGMA reliability factor Y_Z or K_R.

Reliability	Y_Z, K_R
0.5	0.70
0.90	0.85
0.99	1.00
0.999	1.25
0.9999	1.50

Source: After ANSI/AGMA, 2001-C95.

The allowable bending stress for nitrided through hardened steels for grade 1 components is given by

$$\sigma_{FP} = 0.568H_B + 83.8 \text{ MPa} \tag{9.37}$$

and for grade 2,

$$\sigma_{FP} = 0.749H_B + 110 \text{ MPa} \tag{9.38}$$

Values for the allowable bending stress can be found from Eqns (9.35)–(9.38) for through hardened and nitrided through hardened steels and from Table 9.10 for iron and bronze gears. Alternatively, an estimate for σ_{FP} can be obtained from Eqn (9.39).

$$\sigma_{FP} \approx 20.55 + 1.176H_B - 9.584 \times 10^{-4}H_B^2 \tag{9.39}$$

where σ_{FP} is the allowable bending stress in MPa.

Table 9.10: Repeatedly applied allowable bending stress and allowable contact stress for a selection of iron and bronze gear materials at 10^7 cycles and 99% reliability.

Material	Designation	Heat treatment	Typical minimum surface hardness	Allowable bending stress (MPa)	Allowable contact stress (MPa)
ASTM A48 gray cast iron	Class 20	As cast	–	34	345–414
	Class 30	As cast	$174H_B$	59	448–517
	Class 40	As cast	$201H_B$	90	517–586
ASTM A536 ductile (nodular) iron	Grade 60-40-18	Annealed	$140H_B$	152–228	530–634
	Grade 80-55-06	Quenched and tempered	$179H_B$	152–228	530–634
	Grade 100-70-03	Quenched and tempered	$229H_B$	186–276	634–772
	Grade 120-90-02	Quenched and tempered	$269H_B$	214–303	710–869
Bronze		Sand cast	$\sigma_{\text{uts min}} = 275$ MPa	39	207
	ASTM B-148 alloy 954	Heat treated	$\sigma_{\text{uts min}} = 620$ MPa	163	448

Source: ANSI/AGMA 2001/C95.

The allowable contact stress, σ_{HP}, assuming unidirectional loading, 10^7 cycles, and 99% reliability for grade 1 through hardened steel gears is given by

$$\sigma_{HP} = 2.22H_B + 200 \tag{9.40}$$

and for grade 2 through hardened steel gears by

$$\sigma_{HP} = 2.41H_B + 237 \tag{9.41}$$

Values for the allowable contact stress can be found from Eqns (9.40) and (9.41) for grades 1 and 2 through hardened steels and from Table 9.10 for iron and bronze gears. Alternatively, an estimate for σ_{HP} can be obtained from Eqn (9.42).

$$\sigma_{HP} \approx 182.7 + 2.382H_B \tag{9.42}$$

where σ_{HP} is the allowable contact stress in MPa.

The hardness ratio factor is used only for the gear. Its purpose is to adjust the surface strengths to account for different hardness of the gear and pinion. The hardness ratio factor for a gear is given by

$$Z_W = C_H = 1 + A'(m_G - 1) \tag{9.43}$$

where:

$$A' = 8.98 \times 10^{-3}\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29 \times 10^{-3} \tag{9.44}$$

for

$$1.2 \leq \frac{H_{BP}}{H_{BG}} \leq 1.7 \tag{9.45}$$

where H_{BP} and H_{BG} are the Brinell hardness of the pinion and gear, respectively. $m_G = N_G/N_P$.

Example 9.2

A gear drive proposal consists of a 20° pressure angle spur pinion with 18 teeth driving a 50-tooth gear. The module of the gears is 2.5 and the face width is 30 mm. The pinion runs at 1425 rpm. The gear set is to transmit 3 kW and the load and drive can be considered smooth. The gear material proposed is grade 1 steel and the Brinell hardness of the pinion and gear is 240 and 200, respectively. The gears are to be manufactured to number 6 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 90%. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear.

Solution

The pinion diameter, $d_P = 18 \times 2.5 = 45$ mm.

The gear diameter, $d_G = 50 \times 2.5 = 125$ mm.

The speed ratio, Eqn (9.34), is given by

$$m_G = \frac{N_G}{N_P} = \frac{50}{18} = 2.778$$

Assume the overload factor, $K_O = 1$.

$$V = \omega r = 1425 \times \frac{2\pi}{60} \times \frac{0.045}{2} = 3.358 \text{ m/s}$$

$$W_t = \frac{\text{Power}}{V} = \frac{3000}{3.358} = 893.5 \text{ N}$$

From Eqns (9.15) and (9.16),

$$B = \frac{(12 - Q_v)^{2/3}}{4} = \frac{(12 - 6)^{2/3}}{4} = 0.8255$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$$

The dynamic factor, Eqn (9.22), is given by

$$K'_V = \left(\frac{A + \sqrt{200V}}{A}\right)^B = \left(\frac{59.77 + \sqrt{200 \times 3.358}}{59.77}\right)^{0.8255} = 1.346$$

Assume the size factor $K_s = 1$.

The teeth are uncrowned, so $C_{mc} = 1$.

$F = 30$ mm, so from Eqn (9.27),

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.4921F = \frac{0.03}{10 \times 0.045} - 0.0375 + 0.4921 \times 0.03 = 0.04392$$

$$C_{pm} = 1$$

$$C_e = 1$$

For commercially enclosed units, from Eqn (9.29),

$$C_{ma} = 0.127 + 0.622F - 0.01442F^2 = 0.0127 + 0.622 \times 0.03 - 0.01442 \times 0.03^2 = 0.1456$$

The load distribution factor K_H, Eqn (9.25), is given by

$$K_H = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) = 1 + 1(0.04392 \times 1 + 0.1456 \times 1) = 1.190$$

The repeatedly applied bending strength stress cycle factor for the pinion, Y_{NP}, from Figure 9.3 taking the worst case line for 10^8 cycles is given by

$$Y_{NP} = 1.6831\left(1 \times 10^8\right)^{-0.0323} = 0.9284$$

The gear does comparatively fewer cycles, and the repeatedly applied bending strength stress cycle factor for the gear, Y_{NG}, is given by

$$Y_{NG} = 1.6831\left(\frac{1 \times 10^8}{m_G}\right)^{-0.0323} = 1.6831\left(\frac{1 \times 10^8}{2.778}\right)^{-0.0323} = 0.9284$$

The pitting resistance stress cycle factor for the pinion, Z_{NP}, from Figure 9.4 taking the worst case line for 10^8 cycles is given by

$$Z_{NP} = 2.466N^{-0.056} = 2.466\left(1 \times 10^8\right)^{-0.056} = 0.879$$

The gear does comparatively fewer cycles and the pitting resistance stress cycle factor for the gear, Z_{NG}, is given by

$$Z_{NG} = 2.466\left(\frac{1 \times 10^8}{m_G}\right)^{-0.056} = 2.466\left(\frac{1 \times 10^8}{2.778}\right)^{-0.056} = 0.9308$$

From Table 9.9, the reliability factor $Y_Z = 0.85$ for a reliability of 90%.

Assume the temperature factor $Y_\theta = 1$.

Assume the surface condition factor $Z_R = 1$.

For a steel pinion in mesh with a steel gear, the elastic coefficient $Z_e = 191\ (\text{MPa})^{0.5}$ from Table 9.1.

The allowable bending stress for through hardened grade 1 steel, in the case of the pinion from Eqn (9.35), is given by

$$\sigma_{FP} = 0.533H_B + 88.3 = 0.533 \times 240 + 88.3 = 216\ \text{MPa}$$

For the gear,

$$\sigma_{FG} = 0.533 \times 200 + 88.3 = 195\ \text{MPa}$$

The allowable contact stress for through hardened steel gears, in the case of the pinion, Eqn (9.40), is given by

$$\sigma_{HP} = 2.22H_B + 200 = 2.22 \times 240 + 200 = 733 \text{ MPa}$$

For the gear,

$$\sigma_{HG} = 2.22 \times 200 + 200 = 644 \text{ MPa}$$

From Eqn (9.44),

$$A' = 8.98 \times 10^{-3}\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29 \times 10^{-3} = 8.98 \times 10^{-3}\left(\frac{240}{200}\right) - 8.29 \times 10^{-3}$$
$$= 2.486 \times 10^{-3}$$

The hardness ratio factor, Z_W, Eqn (9.43), is given by

$$Z_W = 1 + A\prime(m_G - 1) = 1 + 2.486 \times 10^{-3}(2.778 - 1) = 1.004$$

From Figure 9.2, the bending strength geometry factors $Y_{JP} = 0.316$ and $Y_{JG} = 0.396$.

The geometry factor Z_I, Eqn (9.30), is given by

$$Z_I = \frac{\cos\phi_t \sin\phi_t}{2m_N}\ \frac{m_G}{m_G + 1} = \frac{\cos 20 \sin 20}{2}\ \frac{2.778}{2.778 + 1} = 0.1182$$

The AGMA bending stress, Eqn (9.6), is given by

$$\sigma = W_t K_O K_V' K_s \frac{1}{Fm}\ \frac{K_H K_B}{Y_J}$$

Substitution for the factors and geometrical variables gives, for the pinion,

$$\sigma_P = 893.5 \times 1 \times 1.346 \times 1 \times \frac{1}{0.03 \times 0.0025} \times \frac{1.190 \times 1}{0.316} = 60.39 \times 10^6 \text{ Pa}$$

For the gear,

$$\sigma_G = 893.5 \times 1 \times 1.346 \times 1 \times \frac{1}{0.03 \times 0.0025} \times \frac{1.190 \times 1}{0.396} = 48.18 \times 10^6 \text{ Pa}$$

The allowable bending stress, Eqn (9.18), is given by

$$\sigma_{\text{all}} = \frac{\sigma_{FP} Y_N}{S_F Y_\theta Y_Z}$$

Setting $S_F = 1$ and determining the allowable bending stress for the pinion gives

$$\sigma_{\text{all}} = \frac{216 \times 0.9284}{1 \times 1 \times 0.85} = 236 \text{ MPa}$$

The factor of safety for the pinion, S_{FP}, is given by

$$S_{FP} = \frac{236}{60.4} = 3.91$$

Setting $S_F = 1$ and determining the allowable bending stress for the gear gives

$$\sigma_{\text{all}} = \frac{195 \times 0.9583}{1 \times 1 \times 0.85} = 220 \text{ MPa}$$

The factor of safety for the gear, S_{FG}, is given by

$$S_{FG} = \frac{220}{48.2} = 4.56$$

The values for S_{FP} and S_{FG} are greater than 1, indicating that the design seems acceptable at this stage.

The contact stress, Eqn (9.16), is given by

$$\sigma_c = Z_e \left(W_t K_O K'_V K_s \frac{K_H}{Fd} \frac{Z_R}{Z_I} \right)^{0.5}$$

For the pinion,

$$\begin{aligned} \sigma_c &= 191 \sqrt{893.5 \times 1 \times 1.346 \times 1 \times \frac{1.190}{0.045 \times 0.03} \times \frac{1}{0.1182}} \text{ kPa} \\ &= 191 \times 2994 \text{ kPa} = 572 \text{ MPa} \end{aligned}$$

For the gear,

$$\begin{aligned} \sigma_c &= 191 \sqrt{893.5 \times 1 \times 1.346 \times 1 \times \frac{1.190}{0.125 \times 0.03} \times \frac{1}{0.1182}} \text{ kPa} \\ &= 191 \times 1796 \text{ kPa} = 342 \text{ MPa} \end{aligned}$$

The allowable contact stress, Eqn (9.20), is given by

$$\sigma_{c,\text{all}} = \frac{\sigma_{HP}}{S_H} \frac{Z_N Z_W}{Y_\theta Y_Z}$$

For the pinion, omitting Z_W and setting $S_H = 1$ gives

$$\sigma_{c,\text{all}} = \frac{\sigma_{HP}}{S_H}\,\frac{Z_N}{Y_\theta Y_Z} = \frac{732.8}{1} \times \frac{0.879}{1 \times 0.85} = 758 \text{ MPa}$$

$$S_{HP} = \frac{758}{572} = 1.33$$

For the gear,

$$\sigma_{c,\text{all}} = \frac{644}{1} \times \frac{0.9308 \times 1.004}{1 \times 0.85} = 708 \text{ MPa}$$

$$S_{HG} = \frac{708}{342} = 2.06$$

The values for S_{HP} and S_{HG} are greater than 1 indicating that the design seems acceptable at this stage.

Care needs to be taken when comparing values for the factors of safety for bending and contact stress. S_F should be compared with S_H^2 if the teeth are uncrowned or S_F with S_H^3 if the teeth are crowned. The exponent linearizes the safety factor for contact stress with respect to the transmitted load.

In Example 9.2, $S_{FP} = 3.91$ and $S_{HP} = 1.33$. Comparing $S_{FP} = 3.91$ with $S_{HP}^2 = 1.77$ shows that the risk of failure for the pinion is more likely due to wear. Similarly for the gear, comparing $S_{FP} = 4.56$ with $S_{HG}^2 = 4.24$ shows that the risk of failure is evenly balanced between bending and wear. In conclusion, failure due to wear is most likely to cause retirement of the gear set.

Example 9.3

A gear drive proposal consists of a 20° pressure angle helical pinion with 17 teeth and a right-hand helix angle of 30°, driving a 55-tooth helical gear. The normal module of the gears is 2.5 and the face width is 30 mm. The pinion runs at 1740 rpm. The gear set is to transmit 2.2 kW and the load and drive can be considered smooth. The gear material proposed is grade 1 steel and the Brinell hardness of the pinion and gear is 240 and 200, respectively. The gears are to be manufactured to number 6 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 90%. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear.

Solution

The speed ratio, Eqn (9.34), is given by

$$m_G = \frac{N_G}{N_P} = \frac{55}{17} = 3.235$$

The transverse module is given by

$$m_t = \frac{m_n}{\cos\psi} = \frac{2.5}{\cos 30} = 2.887$$

Assume the overload factor, $K_O = 1$.

Assume the size factor, $K_s = 1$.

The pitch diameter for the helical pinion is given by $d = m_t N$,

$$d_P = m_t N_P = 2.887 \times 17 = 49.07 \text{ mm}$$

and for the gear,

$$d_G = m_t N_G = 2.887 \times 55 = 158.8 \text{ mm}$$

$$V = \omega r = 1740 \times \frac{2\pi}{60} \times \frac{0.04907}{2} = 4.471 \text{ m/s}$$

$$W_t = \frac{\text{Power}}{V} = \frac{2200}{4.471} = 492.1 \text{ N}$$

The transverse pressure angle is given by

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20}{\cos 30}\right) = 22.80^\circ$$

The radial, axial, and resultant loads are given, respectively, by

$$W_r = W_t \tan\phi_t = 492.1 \tan 20 = 206.8 \text{ N}$$

$$W_a = W_t \tan\psi = 492.1 \tan 30 = 284.1 \text{ N}$$

$$W = \frac{W_t}{\cos\phi_n \cos\psi} = \frac{492.1}{\cos 20 \cos 30} = 604.6 \text{ N}$$

From Eqns (9.23) and (9.24),

$$B = \frac{(12 - Q_v)^{2/3}}{4} = \frac{(12 - 6)^{2/3}}{4} = 0.8255$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$$

The dynamic factor, Eqn (9.22), is given by

$$K'_V = \left(\frac{A + \sqrt{200V}}{A}\right)^B = \left(\frac{59.77 + \sqrt{200 \times 4.471}}{59.77}\right)^{0.8255} = 1.398$$

The teeth are uncrowned, so $C_{mc} = 1$.

$F = 30$ mm, so from Eqn (9.27),

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.4921F = \frac{0.03}{10 \times 0.04908} - 0.0375 + 0.4921 \times 0.03 = 0.03839$$

$$C_{pm} = 1$$

$$C_e = 1$$

For commercially enclosed units, from Eqn (9.29),

$$C_{ma} = 0.127 + 0.622F - 0.01442F^2 = 0.0127 + 0.622 \times 0.03 - 0.01442 \times 0.03^2 = 0.1456$$

The load distribution factor K_H, Eqn (9.25), is given by

$$K_H = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) = 1 + 1(0.03839 \times 1 + 0.1456 \times 1) = 1.184$$

The repeatedly applied bending strength stress cycle factor for the pinion, Y_{NP}, from Figure 9.3 taking the worst case line for 10^8 cycles is given by

$$Y_{NP} = 1.6831\left(1 \times 10^8\right)^{-0.0323} = 0.9283$$

The gear does comparatively fewer cycles and the repeatedly applied bending strength stress cycle factor for the gear, Y_{NG}, is given by

$$Y_{NG} = 1.6831\left(\frac{1 \times 10^8}{m_G}\right)^{-0.0323} = 1.6831\left(\frac{1 \times 10^8}{3.325}\right)^{-0.0323} = 0.9642$$

The pitting resistance stress cycle factor for the pinion, Z_{NP}, from Figure 9.4 taking the worst case line for 10^8 cycles is given by

$$Z_{NP} = 2.466N^{-0.056} = 2.466\left(1 \times 10^8\right)^{-0.056} = 0.879$$

The gear does comparatively fewer cycles and the pitting resistance stress cycle factor for the gear, Z_{NG}, is given by

$$Z_{NG} = 2.466\left(\frac{1 \times 10^8}{m_G}\right)^{-0.056} = 2.466\left(\frac{1 \times 10^8}{3.325}\right)^{-0.056} = 0.9642$$

From Table 9.9, the reliability factor $Y_Z = 0.85$ for a reliability of 90%.

Assume the temperature factor $Y_\theta = 1$.

Assume the surface condition factor $Z_R = 1$.

For a steel pinion in mesh with a steel gear, the elastic coefficient $Z_e = 191\ (\text{MPa})^{0.5}$ from Table 9.1.

The allowable bending stress for through hardened grade 1 steel, in the case of the pinion from Eqn (9.35), is given by

$$\sigma_{FP} = 0.533H_B + 88.3 = 0.533 \times 240 + 88.3 = 216\ \text{MPa}$$

For the gear,

$$\sigma_{FG} = 0.533 \times 200 + 88.3 = 195\ \text{MPa}$$

The allowable contact stress for through hardened steel gears, in the case of the pinion, Eqn (9.40), is given by

$$\sigma_{HP} = 2.22H_B + 200 = 2.22 \times 240 + 200 = 733\ \text{MPa}$$

For the gear,

$$\sigma_{HG} = 2.22 \times 200 + 200 = 644\ \text{MPa}$$

From Eqn (9.44),

$$A' = 8.98 \times 10^{-3}\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29 \times 10^{-3} = 8.98 \times 10^{-3}\left(\frac{240}{200}\right) - 8.29 \times 10^{-3}$$
$$= 2.486 \times 10^{-3}$$

The hardness ratio factor, Z_W, Eqn (9.43), is given by

$$Z_W = 1 + A'(m_G - 1) = 1 + 2.486 \times 10^{-3}(3.235 - 1) = 1.006$$

From Table 9.8, the bending strength geometry factors $Y_{JP} = 0.44$ and $Y_{JG} = 0.49$.

The load sharing ratio, m_N, Eqn (9.32), is given by

$$m_N = \frac{p_N}{0.95Z}$$

The length of the line of action in the transverse plane, Eqn (9.33), is given by

$$Z = \left[(r_P + a)^2 - r_{bP}^2\right]^{0.5} + \left[(r_G + a)^2 - r_{bG}^2\right]^{0.5} - (r_P + r_G)\sin\phi_t$$

$$Z = 11.28 \text{ mm}$$

Hence $m_N = 0.6889$.

The geometry factor Z_I, Eqn (9.30), is given by

$$Z_I = \frac{\cos\phi_t \sin\phi_t}{2m_N} \frac{m_G}{m_G + 1} = \frac{\cos 22.80 \sin 22.80}{2 \times 0.6889} \frac{3.235}{3.235 + 1} = 0.1980$$

The AGMA bending stress, Eqn (9.6), is given by

$$\sigma = W_t K_O K_V' K_s \frac{1}{Fm} \frac{K_H K_B}{Y_J}$$

Substitution for the factors and geometrical variables gives, for the pinion,

$$\sigma_P = 492.1 \times 1 \times 1.398 \times 1 \times \frac{1}{0.03 \times 0.002887} \times \frac{1.184 \times 1}{0.44} = 21.05 \times 10^6 \text{ Pa}$$

For the gear,

$$\sigma_G = 492.1 \times 1 \times 1.398 \times 1 \times \frac{1}{0.03 \times 0.002887} \times \frac{1.184 \times 1}{0.49} = 19.05 \times 10^6 \text{ Pa}$$

The allowable bending stress, Eqn (9.18), is given by

$$\sigma_{\text{all}} = \frac{\sigma_{FP} Y_N}{S_F Y_\theta Y_Z}$$

Setting $S_F = 1$ and determining the allowable bending stress for the pinion gives

$$\sigma_{\text{all}} = \frac{216 \times 0.9283}{1 \times 1 \times 0.85} = 236 \text{ MPa}$$

The factor of safety for the pinion, S_{FP}, is given by

$$S_{FP} = \frac{236}{21.05} = 11.2$$

Setting $S_F = 1$ and determining the allowable bending stress for the gear gives

$$\sigma_{\text{all}} = \frac{195 \times 0.9642}{1 \times 1 \times 0.85} = 221 \text{ MPa}$$

The factor of safety for the gear, S_{FG}, is given by

$$S_{FG} = \frac{221}{19.06} = 11.6$$

The values for S_{FP} and S_{FG} are greater than 1, indicating that the design seems acceptable at this stage.

The contact stress, Eqn (9.16), is given by

$$\sigma_c = Z_e \left(W_t K_O K'_V K_s \frac{K_H}{Fd} \frac{Z_R}{Z_I} \right)^{0.5}$$

For the pinion,

$$\sigma_c = 191 \sqrt{492.1 \times 1 \times 1.398 \times 1 \times \frac{1.184}{0.04907 \times 0.03} \times \frac{1}{0.1980}} \text{ kPa} = 316.8 \text{ MPa}$$

For the gear,

$$\sigma_c = 191 \sqrt{492.1 \times 1 \times 1.398 \times 1 \times \frac{1.184}{0.1588 \times 0.03} \times \frac{1}{0.1980}} \text{ kPa} = 318.1 \text{ MPa}$$

The allowable contact stress, Eqn (9.20), is given by

$$\sigma_{c,\text{all}} = \frac{\sigma_{HP}}{S_H} \frac{Z_N Z_W}{Y_\theta Y_Z}$$

For the pinion, omitting Z_W and setting $S_H = 1$ gives

$$\sigma_{c,\text{all}} = \frac{\sigma_{HP}}{S_H} \frac{Z_N Z_W}{Y_\theta Y_Z} = \frac{732}{1} \times \frac{0.8790 \times 1.006}{1 \times 0.85} = 762 \text{ MPa}$$

$$S_{HP} = \frac{762}{318.1} = 2.39$$

For the gear,

$$\sigma_{c,\text{all}} = \frac{644}{1} \times \frac{0.9387 \times 1.006}{1 \times 0.85} = 715 \text{ MPa}$$

$$S_{HG} = \frac{715}{318.1} = 2.24$$

The values for S_{HP} and S_{HG} are greater than 1, indicating that the design seems acceptable at this stage.

9.4 Gear Selection Procedure

Figure 9.5 illustrates a general selection procedure for gears. Normally the required rotational speeds for the gear and pinion are known from the design requirements, as is the power the drive must transmit. A designer has to decide on the type of gears to be used, their arrangements on the shafts, materials, heat treatments, and gear geometry, including the number of teeth, module, tooth form, face width, and quality. There is no one solution to a gear design requirement. Several "good" or optimum designs are possible. The designer's judgment and creativity are important aspects. While Figure 9.5 shows the general selection procedure, the steps given below outline a general calculation procedure for a pinion and gear.

Select the number of teeth for the pinion and the gear to give the required gear ratio.

Propose a material for the pinion and gear.

Select a module, m. Figure 8.15 can be used as a visual guide to the size of gear teeth for a given module.

Calculate the pitch diameters,

$$d_P = mN_P \quad d_G = mN_G$$

Calculate the pitch line velocity, $V = \omega r$.

Calculate the transmitted load, $W_t = \frac{\text{Power}}{V}$.

Specify the face width or calculate using the procedure outlined in Section 8.6 from

$$F = \frac{W_t}{K_v m Y \sigma_p}$$

As a general rule, spur gears should be designed with a face width between three and five times the circular pitch.

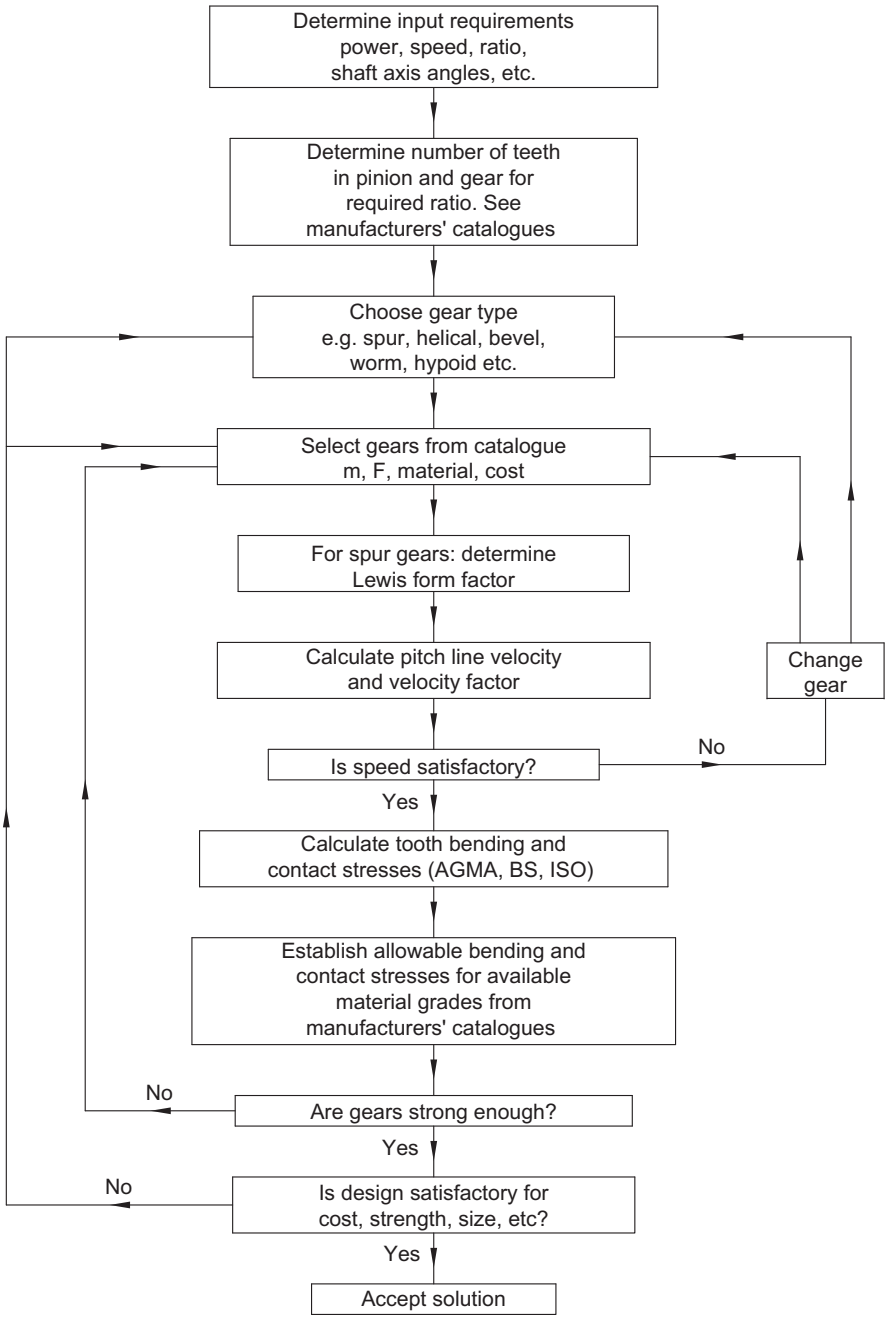


Figure 9.5
General selection procedure for gears.

Check the proposed gears for bending and contact stress. Determine the AGMA factors K_O, K'_v, K_H, Y_N, Z_N, Z_W, Z_I, and Y_J.

Calculate the AGMA bending stress (Eqn (9.6)) $\sigma = W_t K_O K'_V K_s \dfrac{1}{Fm} \dfrac{K_H K_B}{Y_J}$.

Calculate the AGMA contact stress (Eqn (9.16)) $\sigma_c = Z_e \left(W_t K_O K'_V K_s \dfrac{K_H}{Fd} \dfrac{Z_R}{Z_I} \right)^{0.5}$.

Calculate the allowable bending stress (Eqn (9.18)) $\sigma_{\text{all}} = \dfrac{\sigma_{FP} Y_N}{S_F Y_\theta Y_Z}$.

Calculate the allowable contact stress (Eqn (9.20)) $\sigma_{c,\text{all}} = \dfrac{\sigma_{HP}}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z}$.

If the values for the factors of safety S_F and S_H are unsatisfactory, change either the initial geometry or the materials.

This procedure can be reordered, say, if a particular gear material is to be used and stress is to be limited to a certain value, or if a particular factor of safety is required.

9.5 Conclusions

Gears can fail due to excessive bending stresses or from wear. Wear can take the form of pitting of the surface due to excessive contact stresses, scoring due to failure of lubrication, or abrasion due to the presence of foreign particulates. This chapter has introduced the AGMA equations for modeling bending and contact stresses in spur gears. The calculations involved in evaluating the AGMA bending and contact stresses are lengthy and rely on a large number of factors and geometrical parameters. These can be readily embedded in a program or spreadsheet calculation procedure if required.

References

Books and Papers

Mott, R.L., 1998. Machine Elements in Mechanical Design, third ed. Prentice Hall.
Shigley, J.E., Mischke, C.R., 2001. Mechanical Engineering Design, sixth ed. McGraw Hill.
Shigley, J.E., Uicker Jr, J.J., 1980. Theory of Machines and Mechanisms. McGraw Hill.

Further Reading

Standards

AGMA 908-B89. Geometry factors for determining the pitting resistance and bending strength of spur, helical and herringbone gear teeth. AGMA Information Sheet.
ANSI/AGMA 2001-D04. Fundamental rating factors and calculation methods for involute spur and helical gear teeth.

ANSI/AGMA 2101-D04. Fundamental rating factors and calculation methods for involute spur and helical gear teeth.
BS 2519-2:1976, ISO 701:1976. Glossary for gears. Notation.
BS 436-5:1997, ISO 1328-2:1997. Spur and helical gears. Definitions and allowable values of deviations relevant to radial composite deviations and runout information.
BS 6168:1987. Specification for non-metallic spur gears.
BS ISO 6336-1:2006. Calculation of load capacity of spur and helical gears. Basic principles, introduction and general influence factors.
BS ISO 6336-2:2006. Calculation of load capacity of spur and helical gears. Calculation of surface durability (pitting).
BS ISO 6336-3:2006. Calculation of load capacity of spur and helical gears. Calculation of tooth bending strength. Corrigendum 1.
BS ISO 6336-5:2003. Calculation of load capacity of spur and helical gears. Strength and quality of materials.
BS ISO 6336−6:2006. Calculation of load capacity of spur and helical gears. Calculation of service life under variable load.
BS ISO 9084:2000. Calculation of load capacity of spur and helical gears. Application to high speed gears and gears of similar requirements.
PD 6457:1970. Guide to the application of addendum modification to involute spur and helical gears.

Web Sites

At the time of going to press, the world-wide-web contained useful information relating to this chapter at the following sites.

www.agma.org
bsol.bsigroup.com
www.esdu.com
www.iso.org

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

A	used in evaluating the dynamic factor
A'	used in evaluating the hardness ratio factor
B	used in evaluating the dynamic factor
C_e	used in evaluating the load distribution factor K_H
C_f	surface condition factor for pitting resistance
C_H	hardness ratio factor
C_{ma}	used in evaluating the load distribution factor K_H
C_{mc}	used in evaluating the load distribution factor K_H,
C_p	elastic coefficient ((MPa)$^{0.5}$)
C_{pf}	used in evaluating the load distribution factor K_H
C_{pm}	used in evaluating the load distribution factor K_H
d	pitch diameter (m)
d_G	pitch diameter of the gear (m)

d_P pitch diameter of the pinion (m)
E Young's modulus (N/m^2)
E_G Young's modulus for the gear (N/m^2)
E_P Young's modulus for the pinion (N/m^2)
F face width (m)
h_t gear tooth whole depth (m)
H power
H_B Brinell hardness
H_{BG} Brinell hardness of the gear
H_{BP} Brinell hardness of the pinion
I geometry factor for pitting resistance
J geometry factor for bending strength
K_B rim thickness factor
K_H load distribution factor
K_m load distribution factor
K_O overload factor
K_R reliability factor
K_s size factor
K_T temperature factor
K_v' velocity factor
K_V' velocity factor
m module (mm or m)
m_n normal module (mm or m)
m_N load sharing ratio
m_G speed ratio (mm or m)
n speed (rpm)
N number of teeth
N_G number of teeth in the gear
N_P number of teeth in the pinion
P_d transverse diametral pitch (in)
p_n normal circular pitch (m)
p_N base pitch (m)
p_t transverse circular pitch (m)
Q_v quality control numbers, transmission accuracy level number
r_1 radius of curvature (m)
r_2 radius of curvature (m)
S distance (m)
S_c allowable contact stress (lb/in.^{2})
S_F AGMA factor of safety

S_H AGMA factor of safety
S_{HG} AGMA factor of safety for the gear
S_{HP} AGMA factor of safety for the pinion
S_t allowable bending stress (lb/in.^{2})
S_1 distance (m)
t_R rim thickness below the tooth (m)
V velocity (m/s)
W resultant load (N)
W_a axial load (N)
W_r radial load (N)
W_t transmitted load (N)
Y Lewis form factor
Y_J geometry factor for bending strength
Y_N stress cycle factor for bending stress
Y_Z reliability factor
Y_θ temperature factor
Z length of the line of action in the transverse plane (m)
Z_e elastic coefficient (MPa)$^{0.5}$
Z_I geometry factor for pitting resistance
Z_N stress cycle life factor
Z_R surface condition factor for pitting resistance
Z_W hardness ratio factor
ϕ pressure angle (°)
ϕ_t transverse pressure angle (°)
ν_G Poissons ratio for the gear
ν_P Poissons ratio for the pinion
σ bending stress (N/m^2)
σ_{all} allowable bending stress (N/m^2)
$\sigma_{c,\text{all}}$ allowable contact stress (N/m^2)
σ_G bending stress for the gear (N/m^2)
σ_P bending stress for the pinion (N/m^2)
σ_c contact stress (N/m^2)
σ_{FP} allowable bending stress (MPa)
σ_{HP} allowable contact stress (MPa)
σ_p permissible stress (N/m^2)
ψ helix angle at the standard pitch diameter (°).

CHAPTER 10

Bevel Gears

Chapter Outline

10.1 Introduction 403
10.2 Force Analysis 405
10.3 Stress Analysis 407
10.4 Calculation Procedure Summary 425
10.5 Conclusions 436
References 436
Further Reading 436
Nomenclature 436

Abstract

Bevel gears have a conical form and can be used to transmit rotational power through shafts that are typically at an angle of 90° to each other. This chapter introduces bevel gears and the ANSI/AGMA 2003-B97 standard that provides a conservative means for estimating the bending and contact stress in straight, zerol, and spiral bevel gears and comparing the merits of different design proposals.

10.1 Introduction

Bevel gears have teeth cut on conical blanks (see Figures 10.1 and 10.2), and a gear pair can connect nonparallel intersecting shafts. Bevel gears are used for motor transmission differential drives, valve control, and mechanical instruments. A variety of tooth forms are possible, including

- straight bevel gears,
- spiral bevel gears, and
- zerol bevel gears.

Straight bevel gears have, as implied in their name, a straight tooth form cut parallel to the cone axis, which if extended would pass through a point of intersection on the shaft axis. Bevel gears are normally mounted on shafts that are at 90° to each other, but bevel gears can be designed for other angles. Straight bevel gears can be noisy due to the abrupt line or flank contact between teeth as they mesh. This abrupt meshing means that there can be a high impact stress if the teeth are used under heavy loads or at high speeds, which can lead to the possibility of tooth breakages and gear failure. Straight bevel gears are usually only suitable for speeds up to 5 m/s.

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00010-1
Copyright © 2014 Elsevier Ltd. All rights reserved.



Figure 10.1
Straight cut bevel gears.



Figure 10.2
Spiral bevel gears.

Spiral bevel gears have curved teeth that are formed along a spiral angle to the cone axis. The most common spiral angle is 35° with a pressure angle of 20°. The advantage of spiral bevel gears over straight teeth is that the gears engage more gradually, with contact commencing at one end of the tooth which increases until there is contact across the whole length of

the tooth. This enables a smoother transmission of power and reduces the risk of tooth breakage. As a result of the smoother operation, spiral bevel gears tend to be quieter and can have a smaller diameter for the same load capacity than straight bevel gears. Spiral bevel gears are recommended for pitch line speeds in the range from 5 to 40 m/s. Higher speeds, in the region of 50 m/s, are possible with high precision finished gears.

Zerol bevel gears have a tooth form that is curved but with a zero spiral angle. They represent an intermediate category between straight and spiral bevel gears. Zerol gears are generally smoother and quieter than straight bevel gears, but they do produce side loads.

In general, a maximum reduction ratio of 10:1 is recommended for a bevel gear set. A maximum step ratio of 5:1 is recommended when using a bevel gear set for increasing the speed of a transmission. Bevel gears tend to be made as a paired set and are in general not interchangeable.

The mountings for bevel gears need to be rigid in order to maintain the displacement of the gears within specified limits. In order to align bevel gears, attention should be given to the accuracy of the mountings and careful design and fitting of keys and keyways.

The design procedure for a bevel gear set can be approached in a number of ways. If the power and speed are known, the procedure can be approached by proposing values for the tooth system, module, teeth numbers, face width, gear material and case hardness, quality number, mounting arrangement, and associated precision. These factors need to be considered in accordance with the required or desired reliability, design life, available space, and cost. The process involves determining the American Gear Manufacturers Association (AGMA) contact and bending stresses, comparing these to the respective permitted stresses for the proposed gears, and assessing the suitability of the safety factors. The process is typically iterative involving an initial proposal and then reconsideration of a series of factors.

Alternatively, the power rating, based on the AGMA equations for contact and bending stress, can be determined for a given proposal for the input speed, tooth system, module, teeth numbers, face width, gear material and case hardness, quality number, mounting arrangement, reliability, and design life.

10.2 Force Analysis

Usual practice for bevel gears is to approximate the tangential or transmitted load by the load that would occur if all the forces were concentrated at the midpoint of the tooth.

$$W_t = \frac{T_q}{r_{av}} \tag{10.1}$$

where W_t is the transmitted load (N), r_{av} is the pitch radius at the midpoint of the tooth for the gear or pinion under consideration (m), and T_q is the torque (N m).

The resultant force has three components: W_a; W_t; and W_r,

$$W_r = W_t \tan \phi \cos \alpha \tag{10.2}$$

$$W_a = W_t \tan \phi \sin \alpha \tag{10.3}$$

where W_a is the axial force (N), W_t is the tangential force or transmitted load (N), W_r is the radial force (N), ϕ is the pressure angle (°), and α is the pitch cone angle (°).

For the pitch cone angle, see Figure 10.3, the relevant angle for the pinion, α_P, or the gear, α_G, should be substituted, depending on which element is under analysis.

$$\alpha_G = \tan^{-1}\left(\frac{N_G}{N_P}\right) \tag{10.4}$$

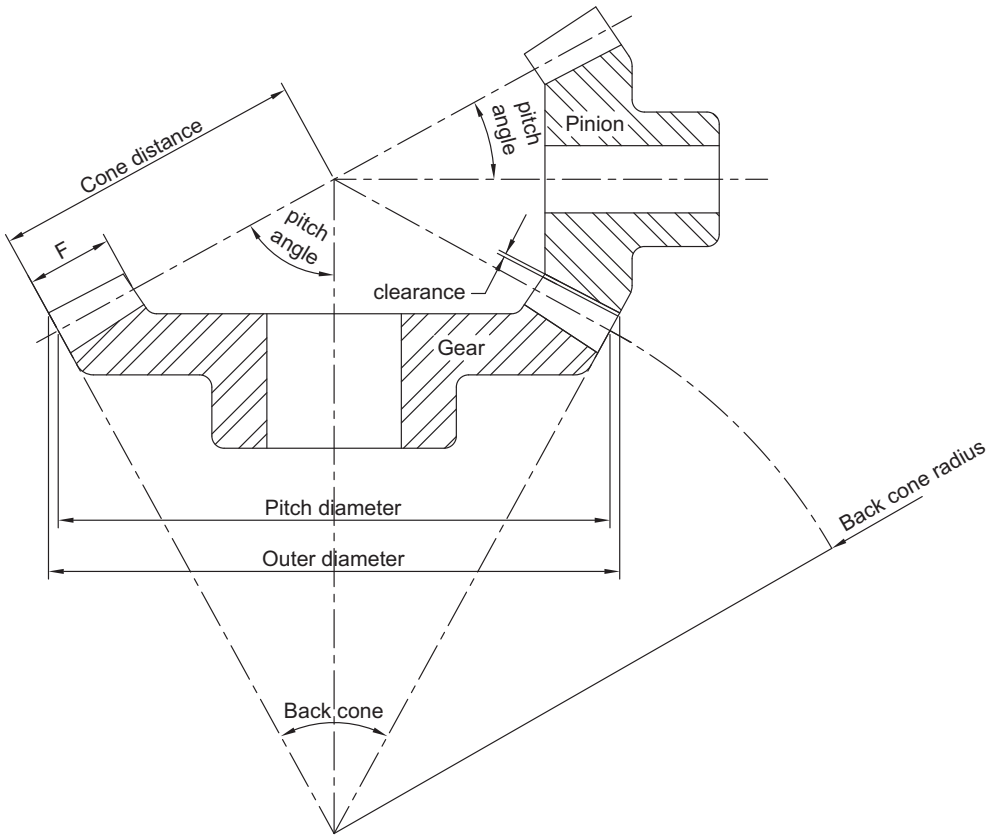


Figure 10.3
Bevel gear nomenclature.

$$\alpha_P = 90 - \alpha_G \tag{10.5}$$

where N_G is the number of teeth in the gear and N_P is the number of teeth in the pinion.

For a spiral bevel gear,

$$W_r = \frac{W_t}{\cos \psi}(\tan \phi_n \cos \alpha \pm \sin \psi \sin \alpha) \tag{10.6}$$

$$W_a = \frac{W_t}{\cos \psi}(\tan \phi_n \sin \alpha \mp \sin \psi \cos \alpha) \tag{10.7}$$

The hand of a spiral bevel gear is defined so that right hand teeth incline away from the axis in a clockwise direction looking at the small end. As with helical gears, the hand of a gear is opposite to that of its mate, with the hand of the gear pair defined as that of the pinion. The upper signs in Eqns (10.6) and (10.7) are used for a driving pinion with a right hand spiral rotating clockwise viewed from its large end, or for a driving pinion with a left hand spiral rotating counterclockwise viewed from its large end. The lower signs are used for the corresponding opposite directions.

10.3 Stress Analysis

Bevel gear sets are often mounted with one of the gears outboard of the bearings. As a result, shaft deflections can be more significant than for a midmounted gear. To compound the challenge in modeling gear stresses, the teeth are tapered. In order to achieve perfect line contact through the cone center, the teeth would need to bend more at the larger end of the cone than at the smaller end. Such conditions dictate proportionally greater loads at the large end. In general, as a result of the varying load, it is desirable to have a fairly short face width for bevel gears. In general the face width, F, is limited to a maximum of $L/3$ where

$$L = \frac{d_P}{2 \sin \alpha_P} = \frac{d_G}{2 \sin \alpha_G} \tag{10.8}$$

where L is the orthogonal length from the gear back cone to the axis (mm) and is also known as the back cone distance.

The ANSI/AGMA 2003-B97 standard provides a conservative means for estimating the bending and contact stress in straight, zerol, and spiral bevel gears and comparing the merits of different design proposals.

Use of the standard requires determination of a significant number of factors. The information provided in this section provides a summary of these.

Bevel gear teeth can be formed by cutting, hard-finish cutting, or grinding. There are several methods for cutting the teeth, dependent upon the volume of parts to be produced, the desired quality, and the availability of equipment. Hard-finish cutting and grinding of the bevel gear teeth after heat treatment can produce machine elements of superior quality with good profile and root fillet finishes, tooth position, and concentricity. Precision finish, as used in defining the AGMA standard, refers to a secondary machine finish operation that includes grinding, skiving, or hard cut finishing.

The teeth of most bevel gears are crowned in profile and lengthwise directions during the manufacturing in order to accommodate deflection and tooth variations. Crowning involves modifying the surface profile so that the teeth have convex surfaces along the face width. Under design load, the contact should spread over the tooth flank without any concentration of the pattern at the edges.

As most bevel gears are made from carburized case-hardened steel, the factors incorporated in the AGMA standard are principally based on tests performed on gears made using this heat treating process.

The dynamic response of the system results in additional gear tooth loads due to the characteristic relative motions of the driver and the driven equipment. The overload factor, K_A, is used to account for the operating characteristics of the driving and driven equipment.

Failure of gear teeth by breakage can be caused by severe instantaneous overloads, excessive pitting, case crushing, or bending fatigue. Assuming that the gears have been designed to avoid pitting and case crushing, the formulas for bending strength rating can be used. The strength ratings determined by the AGMA standard are based on cantilever beam theory modified to consider

- the compressive stress at the tooth roots caused by the radial component of the tooth load,
- nonuniform moment distribution of the load resulting from the inclined contact lines on the teeth of spiral bevel gears,
- stress concentration at the tooth root fillet,
- load sharing between adjacent contacting teeth, and
- the lack of smoothness due to low contact ratio.

The capacity of a gear tooth is determined by considering the following values:

1. The calculated stress number based on the geometry of the gear tooth; manufacturing tolerances; the rigidity of the gear blank, bearings, housing, and foundation; and the operating torque;
2. The permissible stress number based on the allowable stress and the effect of the working conditions under which the gears operate;

3. The maximum power that the gear is capable of transmitting at the operating speeds and loads and under the working conditions; the maximum power is based on the allowable stress.

The fundamental ANSI/AGMA 2003-B97 contact stress for pitting resistance of bevel gear teeth can be calculated by

$$\sigma_H = Z_E \sqrt{\frac{2000 T_{q1}}{F d_{e1}^2 Z_I} K_A K_V K_{H\beta} Z_x Z_{xc}} \tag{10.9}$$

where σ_H is the contact stress (N/mm^2, MPa), Z_E is the elastic coefficient ($\sqrt{\text{MPa}}$), Z_x is the size factor, T_{q1} is the operating pinion torque (N m), K_A is the overload factor, K_v is the dynamic factor, F is the net face width (mm), d_{e1} is the pinion outer pitch diameter (mm), $K_{H\beta}$ is the load distribution factor, Z_{xc} is the crowning factor, and Z_I is the pitting resistance geometry factor.

It should be noted that Eqn (10.9) has lengths expressed in terms of mm and produces a result in MPa. The various factors required for Eqn (10.9) are considered in this section, and their use illustrated in two examples.

The permissible ANSI/AGMA 2003-B97 contact stress is given by

$$\sigma_{HP} = \frac{\sigma_{H\,\text{lim}} Z_{NT} Z_W}{S_H K_\theta Z_Z} \tag{10.10}$$

where σ_{HP} is the permissible contact stress (N/mm^2, MPa), $\sigma_{H\,\text{lim}}$ is the allowable contact stress (N/mm^2, MPa), Z_{NT} is the stress cycle factor, S_H is the contact safety factor, Z_W is hardness ratio factor, K_θ is the temperature factor, and Z_Z is the reliability factor.

The various factors required for Eqn (10.10) are considered in this section, and their use illustrated in the two examples.

The calculated contact stress must be less than or equal to the permissible contact stress, i.e. $\sigma_H \leq \sigma_{HP}$.

The rated power at a specific speed to resist pitting of the gear teeth, according to ANSI/AGMA 2003-B97, is given by

$$P_{az} = \frac{n_1 F}{1.91 \times 10^7} \frac{Z_I}{K_v K_{H\beta} K_A Z_x Z_{xc}} \left(\frac{\sigma_{H\,\text{lim}} d_{e1} Z_{NI} Z_W}{S_H Z_E K_\theta Z_Z} \right)^2 \tag{10.11}$$

where P_{az} is the allowable transmitted power (kW) and n_1 is the pinion speed (rpm).

The values of $\sigma_{H\,\text{lim}}$ and Z_{NT} may be different for the pinion and the gear. The rating should be based on the machine element that has the lowest value of $\sigma_{H\,\text{lim}}$.

The bending stress of the gear teeth according to ANSI/AGMA 2003-B97 is given by

$$\sigma_F = \frac{2000T_{q1}}{Fd_{e1}} \frac{K_A K_v}{m_{et}} \frac{Y_x K_{H\beta}}{Y_\beta Y_J} \tag{10.12}$$

where σ_F is the bending stress at the root of the tooth (N/mm^2, MPa), m_{et} is the metric outer transverse module (mm), Y_x is the size factor, Y_β is the tooth lengthwise curvature factor, and Y_J is the bending strength geometry factor.

It should be noted that Eqn (10.12) has lengths expressed in terms of mm and produces a result in MPa.

The permissible bending stress according to ANSI/AGMA 2003-B97 is given by

$$\sigma_{FP} = \frac{\sigma_{F\,\lim} Y_{NT}}{S_F K_\theta Y_z} \tag{10.13}$$

where σ_{FP} is the bending stress (N/mm^2, MPa), $\sigma_{F\,\lim}$ is the allowable bending stress (N/mm^2, MPa), S_F is the bending safety factor, Y_{NT} is the stress cycle factor, and Y_Z is the reliability factor.

The tensile bending stress must be less than or equal to the permissible bending stress, i.e. $\sigma_F \leq \sigma_{FP}$.

The rated power to resist teeth breakage, according to ANSI/AGMA 2003-B97, is given by

$$P_{ay} = \frac{n_1 F}{1.91 \times 10^7} \frac{Y_J Y_\beta}{Y_x K_{H\beta} K_A K_v} \frac{\sigma_{F\,\lim} d_{e1} m_{et}}{1} \frac{Y_{NT}}{K_\theta Y_z S_F} \tag{10.14}$$

where P_{ay} is the allowable transmitted power (kW).

The values for the stress cycle factor, K_L, may be different for the pinion and the gear; the rating should therefore be based on the lower value of the product $\sigma_{F\,\lim} Y_{NT}$.

In most gear applications, the load is not constant. In order to obtain a nominal value for the operating torque, the following equation can be used based on the value of the power and speed of operation that will be in effect over significant periods of time and under the most severe operating conditions:

$$T_{q1} = \frac{9550P}{n_1} \tag{10.15}$$

where P is the power (kW) and n_1 is the pinion speed (rpm).

The overload factor, K_A, makes allowance for any externally applied loads in excess of the nominal tangential load. The overload factor is typically between 1 and 2.25 and depends on the characteristic motion of the driven and driving machinery. Factors can only be established

Table 10.1: Estimated overload factors, K_A, based on the driving and driven machine characteristics.

	Driven machine characteristics			
Prime mover characteristics	**Uniform**	**Light shocks**	**Medium shocks**	**Heavy shocks**
Uniform	1.00	1.25	1.50	1.75 or higher
Light shocks	1.10	1.35	1.60	1.85 or higher
Medium shocks	1.25	1.50	1.75	2.00 or higher
Heavy shocks	1.50	1.75	2.00	2.25 or higher

Source: Data sourced from ANSI/AGMA 2003-B97.

with confidence after considerable field experience is gained for a particular application. Table 10.1, however, can be used to provide an estimate for the overload factor, based on a reasonable assessment of the prime mover and driven machine characteristics.

A safety factor can be used to account for uncertainties in the design analysis, material properties, and manufacturing quality as well as the consequences of failure.

Care needs to be taken when comparing values for the factors of safety for bending and contact stress. S_F should be compared with S_H^2 if the teeth are uncrowned or S_F with S_H^3 if the teeth are crowned. The exponent linearizes the safety factor for contact stress with respect to the transmitted load.

The dynamic factor accounts for the effects of gear tooth quality and the associated variation of stress with speed and load. The AGMA standards use a quality number rating factor, Q_V, that varies according to the expected variation of component pitch and profile variations in a production run. High-accuracy gearing requires less derating than low-accuracy gearing. Heavily loaded gearing requires less derating than lightly loaded gearing. The dynamic factor, K_v, thus decreases with improving gear quality. In the absence of specific knowledge of the dynamic loads, estimates for the approximate dynamic factor, K_v, can be obtained using Figure 10.4, or by means of the curve fit given in Eqn (10.16).

$$K_v = \left(\frac{A}{A + \sqrt{200 v_{et}}}\right)^{-B} \tag{10.16}$$

The constants A and B and the pitch line velocity, v_{et}, can be determined by Eqns (10.17)–(10.19).

$$A = 50 + 56(1 - B) \tag{10.17}$$

$$B = 0.25(12 - Q_v)^{0.667} \tag{10.18}$$

$$v_{et} = 5.236 \times 10^{-5} d_1 n_1 \tag{10.19}$$

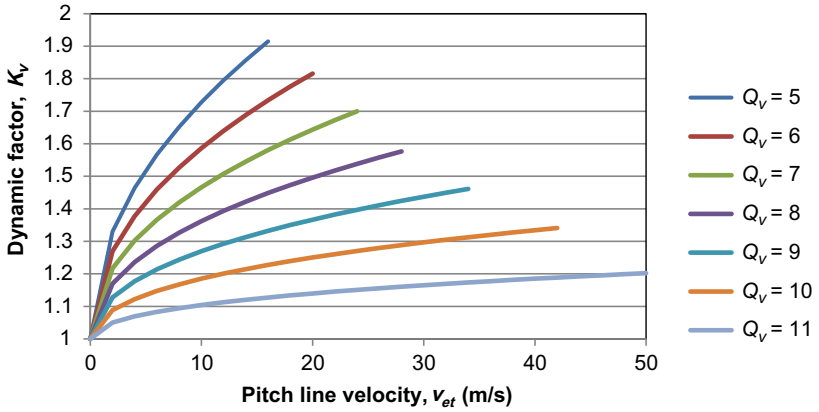


Figure 10.4
Dynamic factor, K_v, as a function of pitch line velocity and quality number.

where Q_v is the transmission accuracy number and v_{et} is the pitch line velocity at the outside pitch diameter (m/s).

The maximum recommended pitch line velocity for a given transmission accuracy number can be determined from

$$v_{et\ \max} = \frac{[A + (Q_v - 3)]^2}{200} \tag{10.20}$$

The size factor accounts for the nonuniformity of material properties and depends on tooth size, diameter, ratio of tooth size to diameter, face width, area of the stress pattern, and material characteristics.

The size factor for pitting resistance can be determined from Figure 10.5.

For $F < 12.7$ mm, $Z_x = 0.5$,

For 12.7 mm $\leq F \leq$ 114.3 mm,

$$Z_x = 0.00492F + 0.4375 \tag{10.21}$$

where Z_x is the size factor for pitting resistance.

For $F \leq 12.7$, $Z_x = 0.5$. For $F \geq 114.5$ mm, $Z_x = 1$.

The size factor for the bending strength of bevel gears can be determined using Figure 10.6.

For $m_{et} < 1.6$, $Y_x = 0.5$.

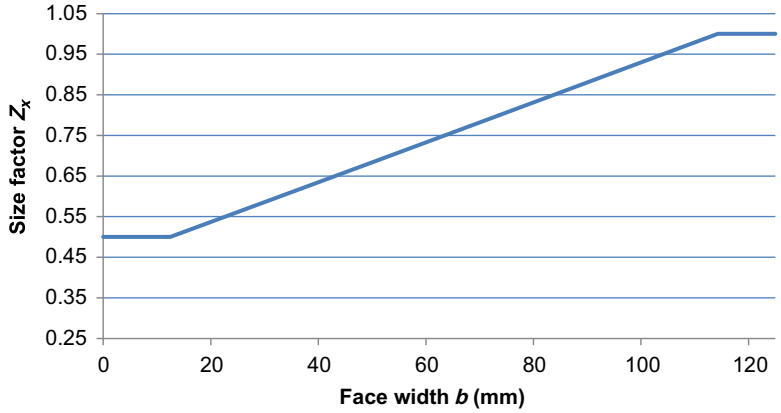


Figure 10.5
Size factor for pitting resistance, Z_x.

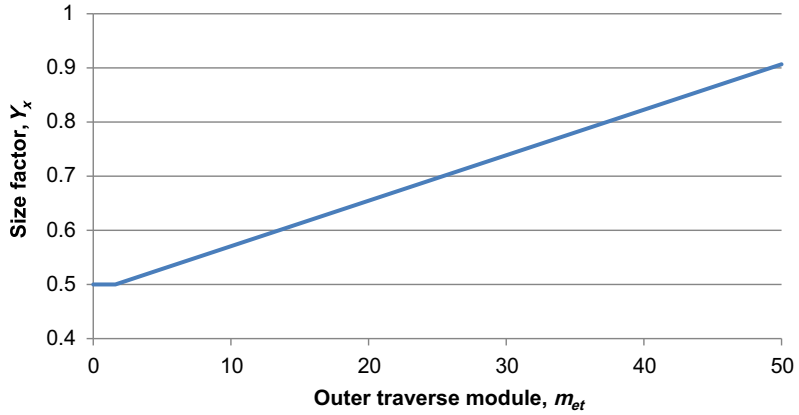


Figure 10.6
Size factor for bending strength, Y_x.

For $1.6 \leq m_{et} \leq 50$ the size factor for the bending strength, Y_x, is given by

$$Y_x = 0.4867 + 0.008399 m_{et} \tag{10.22}$$

The load distribution factor, $K_{H\beta}$, is used to modify the rating formulas to account for the nonuniform load along the tooth length. Figure 10.7 can be used to estimate the load distribution factor for gears with properly crowned teeth.

The load distribution factor, $K_{H\beta}$, for $F \leq 356$ mm, can be calculated from

$$K_{H\beta} = K_{mb} + 5.6 \times 10^{-6} F^2 \tag{10.23}$$

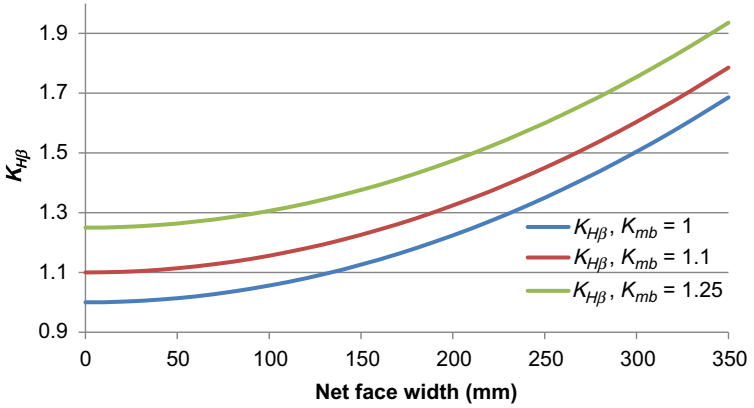


Figure 10.7
Load distribution factor, $K_{H\beta}$, for $F \leq 356$ mm.

where K_{mb} is the load distribution modifier. In general, straddle mounting with bearings on both sides of the gear is preferable for providing the best support for the gear. This arrangement is however often difficult to arrange in practice for intersecting shafts. Typically, in most applications, the gear is usually straddle mounted and the pinion is cantilevered.

$K_{mb} = 1$ for both gear and pinion straddle mounted.
$K_{mb} = 1.1$ if one member is straddle mounted.
$K_{mb} = 1.2$ if neither the gear nor the pinion is straddle mounted.

In order to accommodate deflection of the mountings, the bevel teeth are crowned in the lengthwise direction during the manufacture process. In order to account for the variation in contact pattern the crowning factor, Z_{xc}, can be used.

For properly crowned teeth, $Z_{xc} = 1$.

For large or noncrowned teeth, $Z_{xc} = 2$ or higher.

The lengthwise curvature factor for bending strength, Y_β, is given, for spiral bevel gears, by

$$Y_\beta = 0.211\left(\frac{r_{c0}}{R_m}\right)^q + 0.789 \tag{10.24}$$

where r_{c0} is the cutter radius (mm) and R_m is the mean cone distance (mm).

$$q = \frac{0.279}{\log_{10}(\sin \beta_m)} \tag{10.25}$$

β_m is the mean spiral angle (°).

For straight bevel, zerol bevel, and skew bevel gears, $Y_\beta = 1$.

The geometry factor, Z_I, is used to evaluate the effects of the gear tooth form on the gear tooth stresses. It accounts for the relative radius of curvature of the mating tooth surfaces, and the load sharing between adjacent pairs of teeth at the point on the tooth surfaces where the calculated contact pressure will reach its maximum value.

$$Z_I = \frac{g_c \rho_{yo} \cos \beta_m \cos \alpha_n}{b d_{e1} Z_i \varepsilon_{NI}} \frac{m_{mt}}{m_{et}} \tag{10.26}$$

where g_c is the length of line contact (mm), ρ_{yo} is the relative radius of profile curvature (mm), α_n is the normal pressure angle (°), Z_i is an inertia factor for gears with a low contact ratio, ε_{NI} is the load sharing ratio, m_{et} is the outer transverse module (mm), and m_{mt} is the mean transverse module (mm).

The geometry factor, Y_J, is used to evaluate the shape of the tooth, the position at which the most damaging load is applied, the stress concentration due to the geometric shape of the root fillet, the sharing of load between adjacent pairs of teeth, the tooth thickness balance between the gear and mating pinion, the effective face width due to lengthwise crowning of the teeth, and the buttressing effect of an extended face width on one member of the pair. Both tangential (bending) and radial (compressive) components of the tooth load are included.

As spiral bevel gear teeth are not symmetrical in the lengthwise direction, the stresses will be different on the concave and convex sides of the teeth. Normal practice, for machines driven in one direction, is to calculate the stresses on the concave side of the pinion and the convex side of the gear tooth, as these are the usual driving surfaces. For bidirectional operation it would be necessary to calculate the stresses for both sides for both the gear and the pinion.

The bending strength geometry factors, Y_{J1} and Y_{J2}, for the pinion and gear, respectively, can be calculated from

$$Y_{J1} = \frac{Y_{K1}}{\varepsilon_{NJ} Y_i} \frac{r_{myo1}}{r_{mpt1}} \frac{F'_1}{F} \frac{m_{mt}}{m_{et}} \tag{10.27}$$

$$Y_{J2} = \frac{Y_{K2}}{\varepsilon_{NJ} Y_i} \frac{r_{myo2}}{r_{mpt2}} \frac{F'_2}{F} \frac{m_{mt}}{m_{et}} \tag{10.28}$$

where Y_{K1} is a tooth form factor for the pinion, Y_{K2} is a tooth form factor for the gear, ε_{NJ} is the load sharing ratio, Y_i is the inertia ratio for gears with low contact ratio, r_{myo1} is the mean transverse radius to the point of load application for the pinion (mm), r_{myo2} is the mean transverse radius to the point of load application for the gear (mm), r_{mpt1} is the mean transverse pitch radius for the pinion (mm), r_{mpt2} is the mean transverse pitch radius for the gear (mm), F'_1 is the effective face width of the pinion (mm), F'_2 is the effective face width of the gear (mm), and F is the face width (mm).

Alternatively the geometry factors Z_I and Y_J can be determined from graphs presented in the ANSI/AGMA 2003-B97 standard. A selection of these are reproduced in Figures 10.8–10.13. It should be noted that the geometry factor varies as a function of the spiral angle and shaft angle. Figures 10.8 and 10.9 are for a straight bevel gear with 20° pressure angle and 90° shaft angle. Figures 10.10 and 10.11 are for a 20° pressure angle, 35° spiral angle, and 90° shaft angle. Figures 10.12 and 10.13 are for a 20° pressure angle, 25° spiral angle, and 90° shaft angle.

The stress cycle factors Z_{NT} and Y_{NT} account for the number of cycles of the gears in operation during the design life. A stress cycle factor is used in the equations for the permissible contact and bending stress. The stress cycle factors for pitting resistance and bending strength can be determined using Figures 10.14 and 10.15, respectively.

The stress cycle factor for pitting resistance:

$$Z_{NT} = 2 \text{ for } 10^3 \leq n_L \leq 10^4$$

$$Z_{NT} = 3.4822 n_L^{-0.0602} \text{ for } 10^4 \leq n_L \leq 10^{10}$$

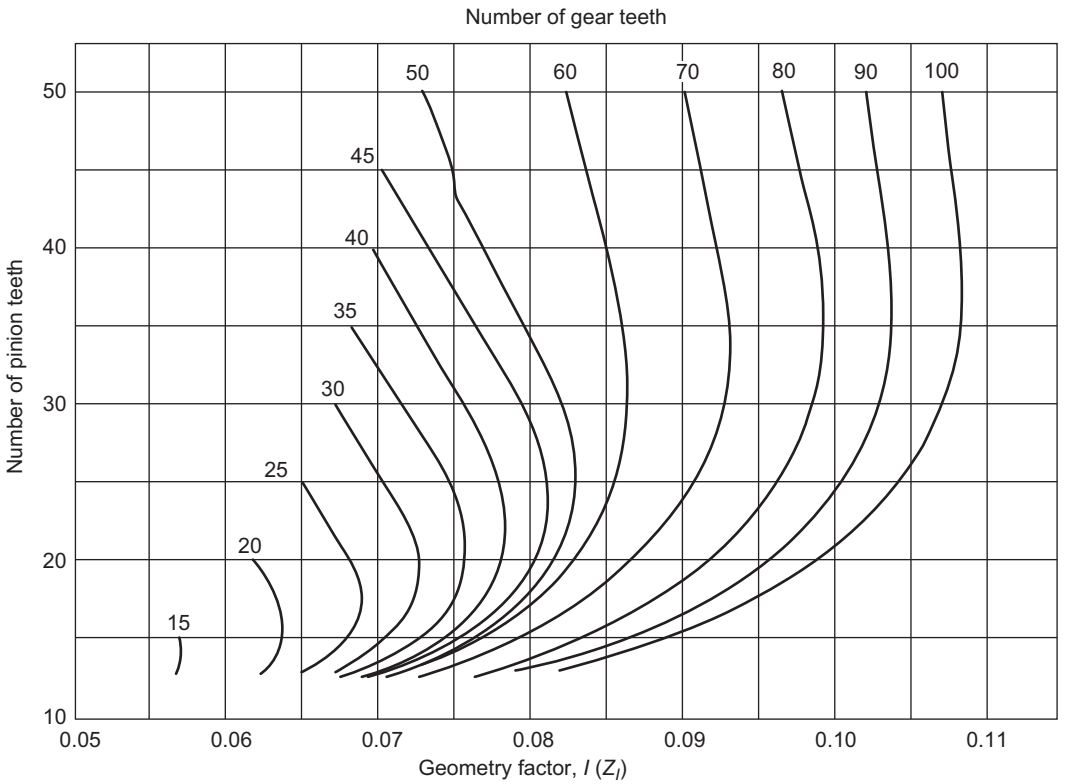


Figure 10.8
Geometry factor Z_I for coniflex straight bevel gears with 20° pressure angle and 90° shaft angle.
Image courtesy ANSI/AGMA 2003-B97.

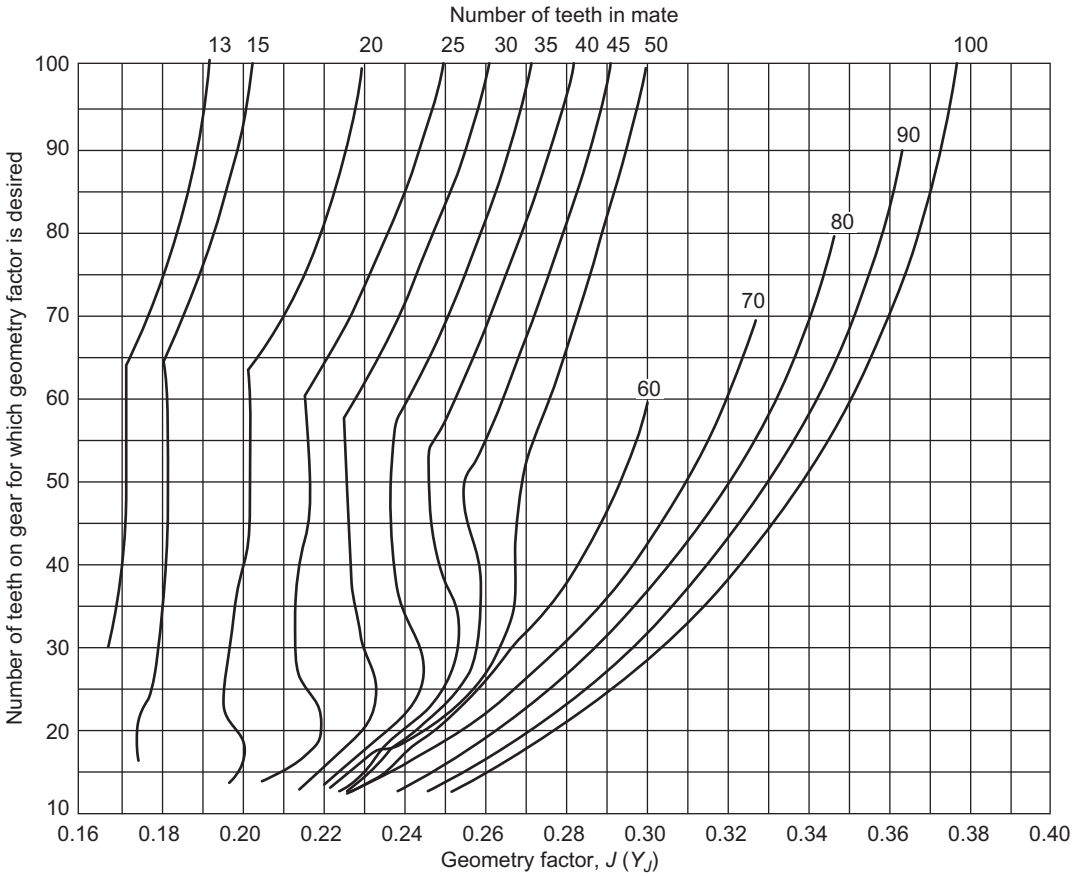


Figure 10.9
Geometry factor Y_J for coniflex straight bevel gears with 20° pressure angle and 90° shaft angle. *Image courtesy ANSI/AGMA 2003-B97.*

The stress cycle factor for bending strength:

$$Y_{NT} = 2.7 \text{ for } 10^2 \leq n_L \leq 10^3$$

$$Y_{NT} = 6.1514 n_L^{-0.1182} \text{ for } 10^3 \leq n_L \leq 3 \times 10^6$$

$$Y_{NT} = 1.6831 n_L^{-0.0323} \text{ for } 3 \times 10^6 \leq n_L \leq 10^{10} \text{ for general conditions}$$

$$Y_{NT} = 1.3558 n_L^{-0.0323} \text{ for } 3 \times 10^6 \leq n_L \leq 10^{10} \text{ for critical conditions}$$

The hardness ratio factor for pitting resistance is a function of the gear ratio, and the relative hardness of the materials used for the gear and the pinion. The hardness ratio factor for pitting resistance can be calculated using Eqn (10.29) and is plotted in Figure 10.16.

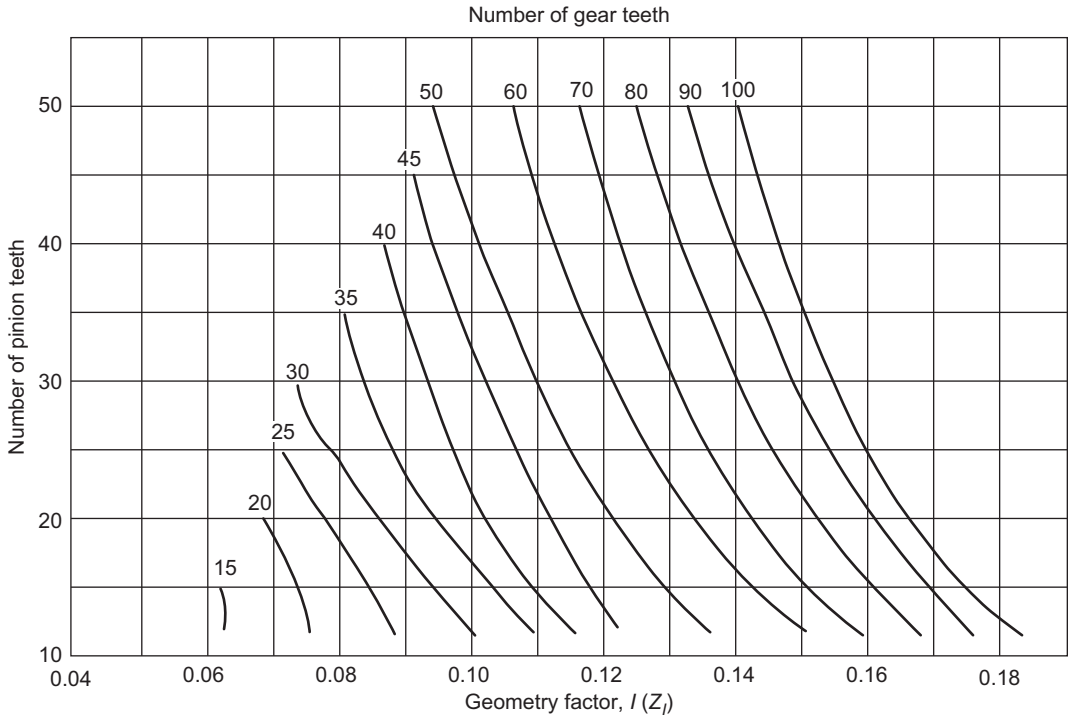


Figure 10.10
Geometry factor Z_I for spiral bevel gears with 20° pressure angle, 35° spiral angle, and 90° shaft angle. *Image courtesy ANSI/AGMA 2003-B97.*

$$Z_W = 1 + B_1\left(\frac{N_G}{N_P} - 1\right) \tag{10.29}$$

where

$$B_1 = 0.00898\left(\frac{H_{B1}}{H_{B2}}\right) - 0.00829 \tag{10.30}$$

Equations (10.29) and (10.30) are valid for the ratio of hardnesses for the pinion and gear in the range $1.2 \leq H_{B1}/H_{B2} \leq 1.7$, where H_{B1} is the minimum Brinell hardness for the pinion (HB, MPa) and H_{B2} is the minimum Brinell hardness for the gear (HB, MPa).

It should be noted that the units of hardness are the same as for stress but are seldom used.

For a surface hardened pinion (48 HRC or harder) run with a through-hardened gear (180−400 HB), a work hardening effect takes place and the hardness ratio factor for pitting resistance can be calculated using Eqn (10.31).

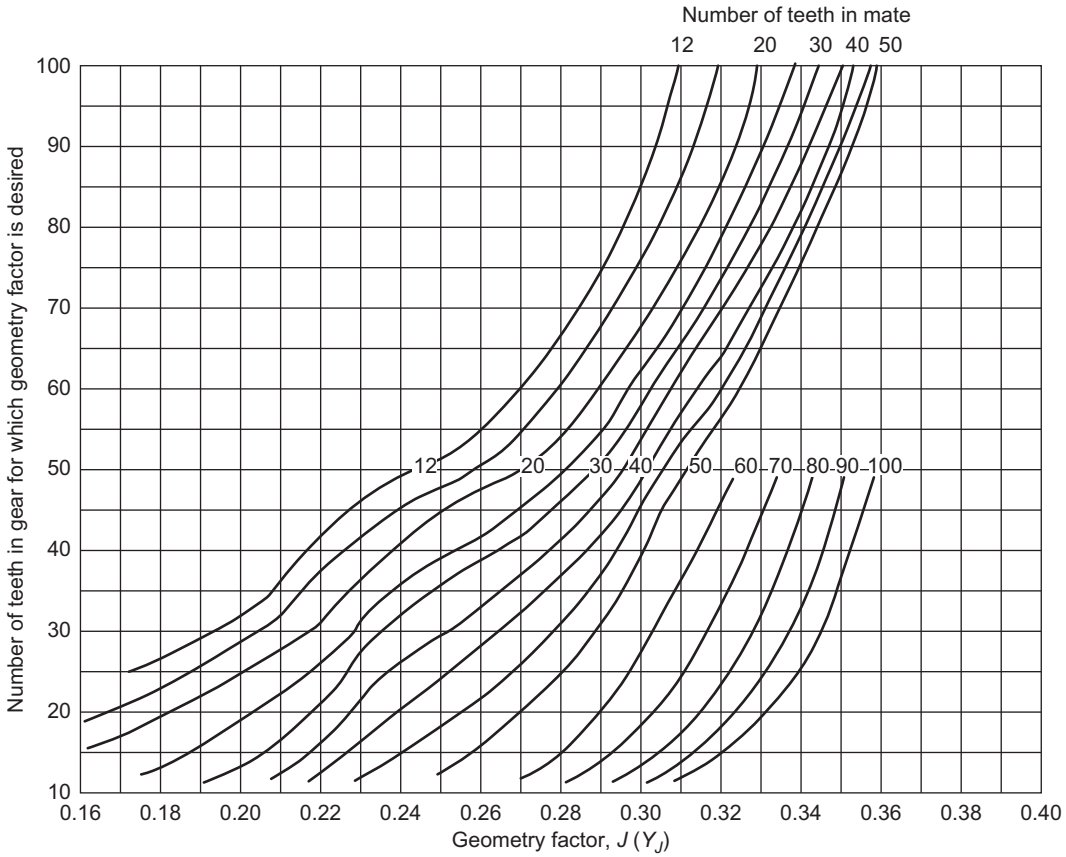


Figure 10.11
Geometry factor Y_J for spiral bevel gears with 20° pressure angle, 35° spiral angle, and 90° shaft angle. *Image courtesy ANSI/AGMA 2003-B97.*

$$Z_W = 1 + B_2(450 - H_{B2}) \tag{10.31}$$

where

$$B_2 = 0.00075e^{-0.52R_{a1}} \tag{10.32}$$

where R_{a1} is the pinion surface roughness (μm).

Under normal conditions the temperature factor, K_θ, can be taken as $K_\theta = 1$. For high temperature operation the temperature factor can be determined using Eqn (10.33).

$$K_\theta = \frac{T}{393} \tag{10.33}$$

where T is the peak operating gear temperature (K).

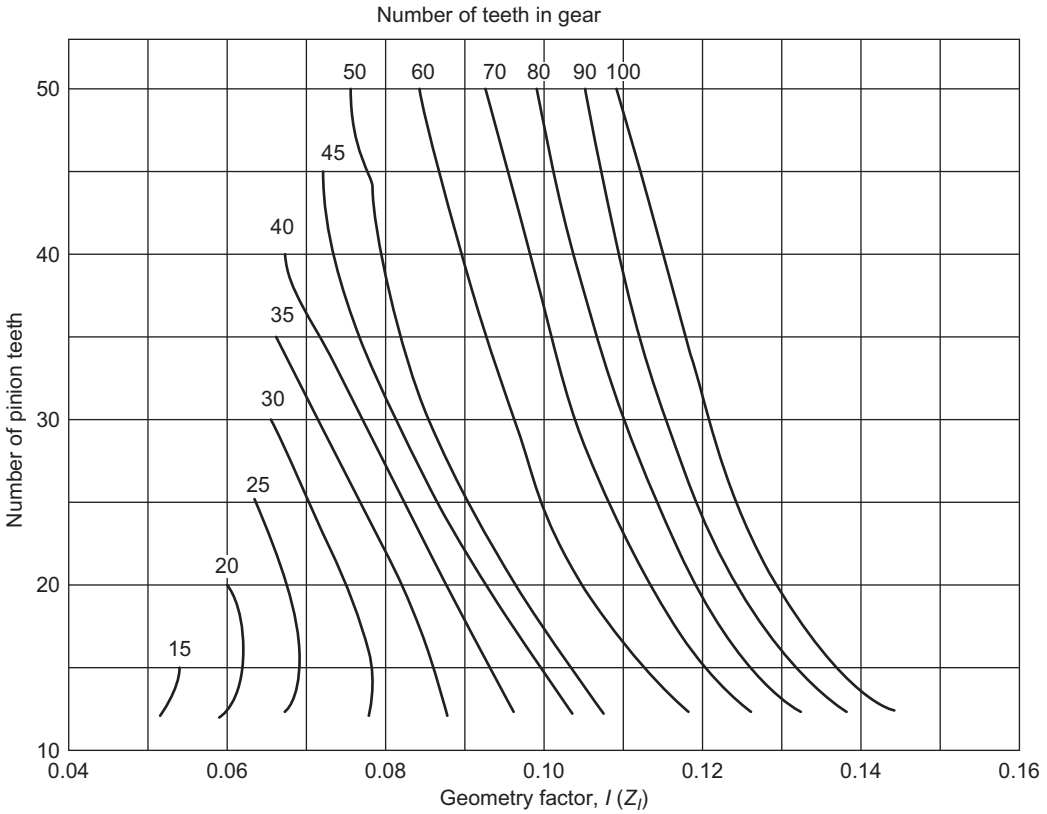


Figure 10.12
Geometry factor Z_I for spiral bevel gears with 20° pressure angle, 25° spiral angle, and 90° shaft angle. *Image courtesy ANSI/AGMA 2003-B97.*

The reliability factors Z_Z and Y_Z account for the effects of the statistical failure in material testing that is assumed to follow a normal distribution. If fewer than 1 failure in 100 is acceptable (99% reliability), then $Z_Z = Y_Z = 1$. For other requirements the values summarized in Table 10.2 can be used.

The elastic coefficient for pitting resistance, Z_E, is a function of Poisson's ratio and the Young's modulus and can be determined using Eqn (10.34) or from Table 9.1.

$$Z_E = \sqrt{\frac{1}{\pi\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)}} \tag{10.34}$$

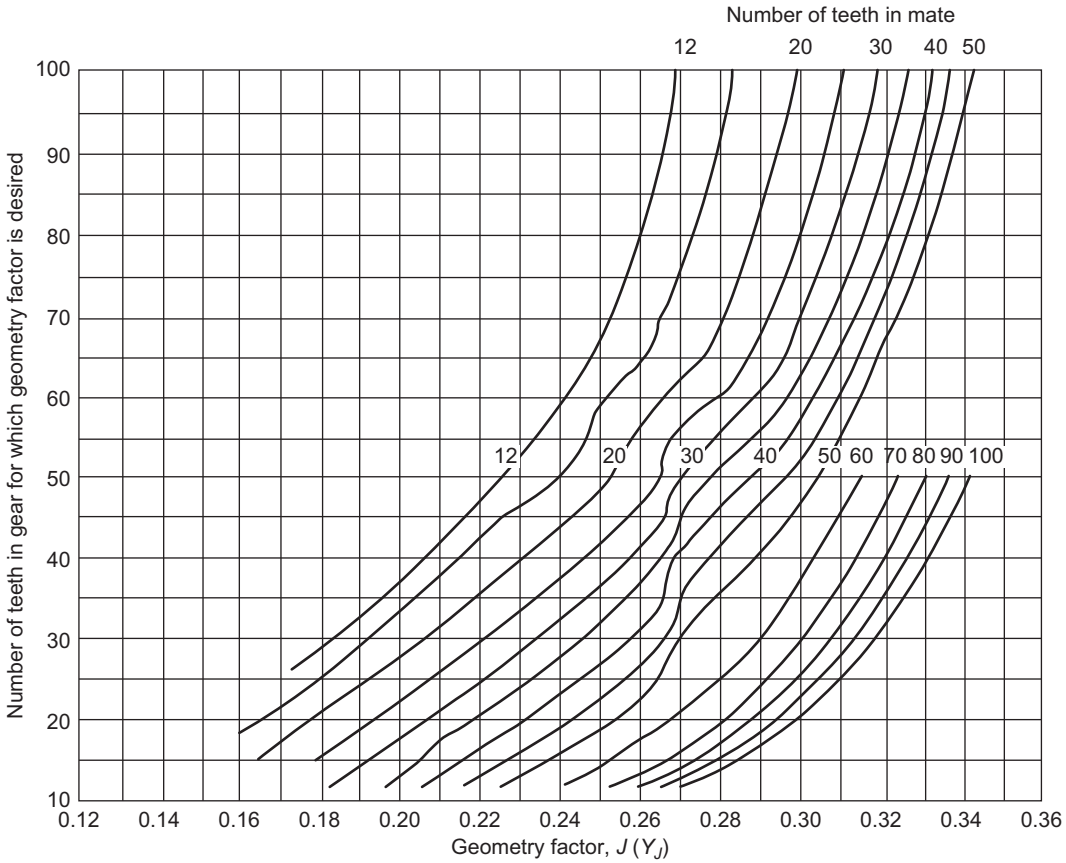


Figure 10.13
Geometry factor Y_J for spiral bevel gears with 20° pressure angle, 25° spiral angle, and 90° shaft angle. *Image courtesy ANSI/AGMA 2003-B97.*

where Z_E is the elastic coefficient for pitting resistance $\left(\sqrt{\text{MPa}}\right)$, ν_1 is the Poisson's ratio for the pinion, ν_2 is the Poisson's ratio for the gear, E_1 is the Young's modulus for the pinion (MPa), and E_2 is the Young's modulus for the gear (MPa).

The allowable stresses for pitting resistance and bending strength, $\sigma_{H\ \text{lim}}$ and $\sigma_{F\ \text{lim}}$, respectively, for unity values of the overload and life factors, need to be determined by field experience and laboratory trials. Testing of bevel gears has been principally on carburized case-hardened steels and the available data for the allowable contact stress for other materials are limited or based on estimates from data for other types of gear.

The AGMA standards have established allowable stress numbers for a range of material types and grades, defined by specific quality control requirements (see ANSI/AGMA

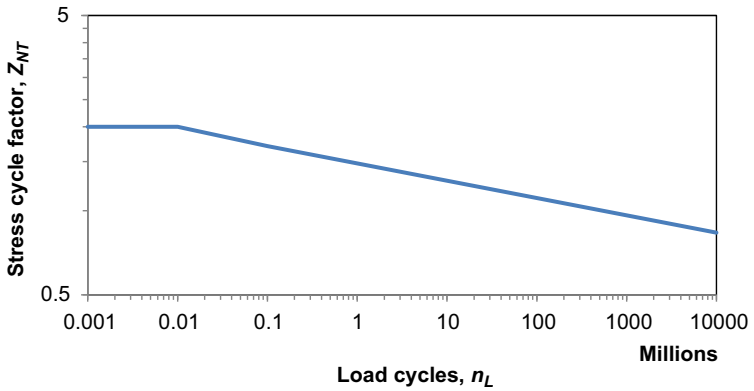


Figure 10.14
Stress cycle factor for pitting resistance, Z_{NT}, for carburized case-hardened steel bevel gears.

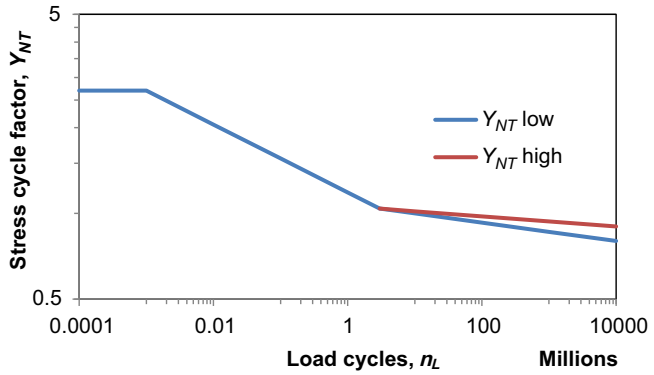


Figure 10.15
Stress cycle factor for bending strength, Y_{NT}, for carburized case-hardened steel bevel gears.

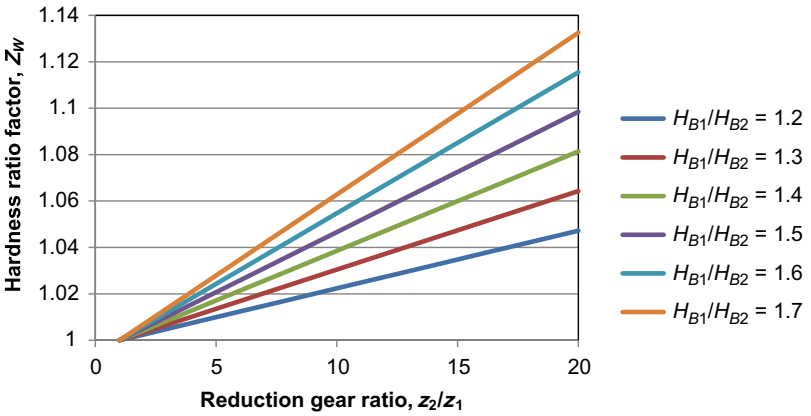


Figure 10.16
Hardness ratio factor, Z_W, for a through-hardened pinion and gear.

Table 10.2: Reliability factors.

Failure requirement	Z_Z	Y_Z
Fewer than one failure in 10,000	1.22	1.50
Fewer than one failure in 1000	1.12	1.25
Fewer than one failure in 100	1.00	1.00
Fewer than one failure in 10	0.92	0.85
Fewer than one failure in 2	0.84	0.70

Source: After ANSI/AGMA 2003-B97.

Table 10.3: Allowable bending stress number for steel gears.

Material	Heat treatment	Minimum surface hardness	$\sigma_{F\ \text{lim}}$ (MPa)		
			Grade 1	Grade 2	Grade 3
Steel	Through-hardened		See Eqn (10.35)	See Eqn (10.36)	
	Flame or induction hardened				
	Unhardened roots	50 HRC	85	95	
	Hardened roots		154		
	Carburized and case-hardened		205	240	275
AISI 4140	Nitrided	84.5 HR15N		150	
Nitralloy	Nitrided	90.0 HR15N		165	

Source: Data sourced from ANSI/AGMA 2003-B97.

2001-D04). The allowable bending stress for through-hardened steels for grade 1 quality components can be approximated by

$$\sigma_{F\ \text{lim}} = 0.533H_B + 88.3\ \text{MPa} \tag{10.35}$$

and for grade 2:

$$\sigma_{F\ \text{lim}} = 0.703H_B + 113\ \text{MPa} \tag{10.36}$$

The allowable bending stress for nitrided through-hardened steels for grade 1 quality components can be approximated by

$$\sigma_{F\ \text{lim}} = 0.568H_B + 83.8\ \text{MPa} \tag{10.37}$$

and for grade 2:

$$\sigma_{F\ \text{lim}} = 0.749H_B + 110\ \text{MPa} \tag{10.38}$$

Values for the allowable bending stress can be found from Eqns (10.35) to (10.38) for through-hardened and nitrided through-hardened steels or using the values given in Table 10.3. Alternatively, an estimate for $\sigma_{F\ \mathrm{lim}}$ can be obtained from Eqn (10.39).

$$\sigma_{F\ \mathrm{lim}} \approx 20.55 + 1.176H_B - 9.584 \times 10^{-4} H_B^2 \tag{10.39}$$

where $\sigma_{F\ \mathrm{lim}}$ is the allowable bending stress in MPa.

The allowable contact stress, $\sigma_{H\ \mathrm{lim}}$, assuming unidirectional loading, 10^7 cycles and 99% reliability for grade 1 through-hardened steel gears can be approximated by

$$\sigma_{H\ \mathrm{lim}} = 2.22H_B + 200 \tag{10.40}$$

and for grade 2 through-hardened steel gears by

$$\sigma_{H\ \mathrm{lim}} = 2.41H_B + 237 \tag{10.41}$$

Values for the allowable contact stress can be found from Eqns (10.40) to (10.41) for grade 1 and 2 through-hardened steels or using the values given in Table 10.4. Alternatively, an estimate for $\sigma_{H\ \mathrm{lim}}$ can be obtained from Eqn (10.42).

$$\sigma_{H\ \mathrm{lim}} \approx 182.7 + 2.382H_B \tag{10.42}$$

where $\sigma_{H\ \mathrm{lim}}$ is the allowable contact stress in MPa.

Table 10.4: Allowable contact stress number for steel gears.

Material	Heat treatment	Minimum surface hardness	$\sigma_{H\ \mathrm{lim}}$ (MPa)		
			Grade 1	Grade 2	Grade 3
Steel	Through-hardened		See Eqn (10.40)	See Eqn (10.41)	
	Flame or induction hardened	50 HRC	1210	1310	
	Carburized and case-hardened		1380	1550	1720
AISI 4140	Nitrided	84.5 HR15N		1000	
Nitralloy 135M	Nitrided	90.9 HR15N		1100	

Source: Data sourced from ANSI/AGMA 2003-B97.

10.4 Calculation Procedure Summary

Determine the power, speed, and ratio for the proposed transmission.

Select a tooth system.

Set values for the reliability and design life.

Propose a mounting arrangement for the gears.

Propose values for the module, teeth numbers, and gear materials as appropriate.

Determine the face width, $F = L/3$.

The gear ratio, m_G, can be determined from any of

$$m_G = \frac{\omega_P}{\omega_G} = \frac{n_P}{n_G} = -\frac{N_G}{N_P} = -\frac{d_G}{d_P} = \tan \alpha_G = \cot \alpha_P$$

(Note that a negative sign is included to account for a reversal of direction for some parameters while the direction is implied for rotational speed and angular velocity according to a sign convention.)

Calculate the pitch diameter for the pinion and gear, $d = mN$

Calculate the transmitted load, $W_t = T_q / r_{av}$

Determine the pitch angle, $\alpha_G = \tan^{-1}\left(\frac{N_G}{N_P}\right)$

Determine the pitch angle, $\alpha_P = 90 - \alpha_G$

Calculate the average pitch diameter, $d_{Pav} = d_P - F \cos \alpha_G$

Calculate the average pitch diameter, $d_{Gav} = d_G - F \cos \alpha_P$

Calculate the radial force component. For a straight bevel, $W_r = W_t \tan \phi \cos \gamma$

Calculate the axial force component. For a straight bevel, $W_a = W_t \tan \phi \sin \gamma$

Estimate the contact stress, $\sigma_H = Z_E \sqrt{\frac{2000 T_{q1}}{F d_{e1}^2 Z_I}} K_A K_V K_{H\beta} Z_x Z_{xc}$

Estimate the allowable contact stress, $\sigma_{HP} = \frac{\sigma_{H\,\lim} Z_{NT} Z_W}{S_H K_\theta Z_Z}$

Determine the contact safety number, $n_{sH} = \frac{\sigma_{HP}}{\sigma_H}$

Estimate the bending stress, $\sigma_F = \dfrac{2000T_1}{Fd_{e1}} \dfrac{K_A K_v}{m_{et}} \dfrac{Y_x K_{H\beta}}{Y_\beta Y_J}$

Estimate the allowable bending stress, $\sigma_{FP} = \dfrac{\sigma_{F\ \lim} Y_{NT}}{S_F K_\theta Y_z}$

Determine the bending safety number, $n_{sF} = \dfrac{\sigma_{FP}}{\sigma_F}$

If the values for the safety factors are not acceptable adjust the parameters in the initial design proposal and repeat the process.

Example 10.1

Determine the bending and contact stresses and associated safety factors for a straight bevel gear set comprising a 25-tooth bevel and a 35-tooth gear with a module of 3 mm. The pinion speed is 3000 rpm, and the transmission power is 8.5 kW. The pinion and gear are manufactured from grade 1 steel through-hardened to 280 HB and 240 HB respectively with a quality number of 8 and the teeth are crowned. The gear set is driven by an electric motor and the driven machinery has a uniform driving characteristic. The gear is straddle mounted, and the pinion is cantilevered.

Solution

The initial values for the design are as follows:

$N_P = 25$; $N_G = 25$; $m = 3$ mm; $n_P = 3000$ rpm, power = 3 kW, pinion material grade 1 through-hardened steel, $\text{HB}_1 = 280$ HB, $\text{HB}_2 = 240$ HB.

The torque, T_q, is given by

$$T_q = \frac{\text{Power}}{\omega} = \frac{8500}{(2\pi/60)3000} = 27.06 \text{ N m}$$

The pinion and gear pitch diameters are

$$d_P = mN_P = 3 \times 25 = 75 \text{ mm}$$

$$d_G = mN_G = 3 \times 35 = 105 \text{ mm}$$

The gear and pinion pitch angles are

$$\alpha_G = \tan^{-1}\left(\frac{N_G}{N_P}\right) = \tan^{-1}\left(\frac{35}{25}\right) = 54.46^\circ$$

$$\alpha_P = 90 - \alpha_G = 35.54^\circ$$

The back cone distance, L, is given by

$$L = \frac{d_P}{2 \sin \alpha_P} = \frac{75}{2 \sin 35.54} = 64.52 \text{ mm}$$

The face width, F, is normally $L/3$, so

$$F = \frac{64.52}{3} = 21.51 \text{ mm}$$

From Figure 10.8, the pitting resistance geometry factor for a 20° pressure angle straight bevel gears, for a pinion with 25 teeth and a gear with 35 teeth is $Z_I = 0.075$.

From Table 10.1, for smooth or uniform driver and driven characteristics, the overload factor, K_A, is $K_A = 1$.

The pitch line velocity is

$$V_{et} = \omega \frac{d}{2} = \left(\frac{2\pi}{60} \times 3000\right) \frac{75 \times 10^{-3}}{2} = 11.78 \text{ m/s}$$

$$B = 0.25(12 - Q_v)^{0.667} = 0.25(12 - 8)^{0.667} = 0.6303$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.6303) = 70.71$$

The dynamic factor is given by

$$K_v = \left(\frac{A}{A + \sqrt{200 v_{et}}}\right)^{-B} = \left(\frac{70.71}{70.71 + \sqrt{200 \times 11.78}}\right)^{-0.6303} = 1.39$$

The maximum recommended pitch line velocity is

$$v_{et \text{ max}} = \frac{[A + (Q_v - 3)]^2}{200} = \frac{[70.71 + (8 - 3)]^2}{200} = 28.66 \text{ m/s}$$

As $v_{et} < v_{et \text{ max}}$, the pitch line velocity is acceptable.

As one of the gears is straddle mounted, the load distribution modifier, $K_{mb} = 1.1$.

The load distribution factor, $K_{H\beta}$, is given by

$$K_{H\beta} = K_{mb} + 5.6 \times 10^{-6} F^2 = 1.1 + 5.6 \times 10^{-6} (21.51)^2 = 1.103$$

The size factor for pitting resistance, Z_x, is given by

$$Z_x = 0.00492F + 0.4375 = (0.00492 \times 21.51) + 0.4375 = 0.5433$$

Table 10.5: Approximate values for the elastic coefficient Z_E ($\sqrt{\text{MPa}}$).

Pinion material	E_{pinion} (GPa)	Gear material					
		Steel	Malleable iron	Nodular iron	Cast iron	Aluminum bronze	Tin bronze
Steel	200	191	181	179	174	162	158
Malleable iron	170	181	174	172	168	158	154
Nodular iron	170	179	172	170	166	156	152
Cast iron	150	174	168	166	163	154	149
Aluminum bronze	120	162	158	156	154	145	141
Tin bronze	110	158	154	152	149	141	137

Source: Data sourced from ANSI/AGMA 218.01

For properly crowned teeth, $Z_{xc} = 1$.

For a steel pinion and a steel gear, from Table 10.5, $Z_E = 191\sqrt{\text{MPa}}$

From Eqn (10.9), the AGMA contact stress for pitting resistance, for the pinion, is given by

$$\sigma_H = Z_E\sqrt{\frac{2000T_{q1}}{Fd_{e1}^2 Z_I}K_A K_V K_{H\beta} Z_x Z_{xc}}$$

$$= 191\sqrt{\frac{2000 \times 27.06}{21.51 \times 75^2 \times 0.076} \times 1 \times 1.39 \times 1.103 \times 0.5433 \times 1}$$

$$\sigma_H = 422.9 \text{ MPa}$$

From Eqn (10.9), the AGMA contact stress for pitting resistance, for the gear, is given by

$$\sigma_H = Z_E\sqrt{\frac{2000T_{q1}}{Fd_{e1}^2 Z_I}K_A K_V K_{H\beta} Z_x Z_{xc}}$$

$$= 191\sqrt{\frac{2000 \times 27.06}{21.51 \times 105^2 \times 0.076} \times 1 \times 1.39 \times 1.103 \times 0.5433 \times 1}$$

$$\sigma_H = 302.0 \text{ MPa}$$

If the number of cycles for the gear set is taken as 1×10^9, the stress cycle factor, Z_{NT}, is given by

$$Z_{NT} = 3.4822n_L^{-0.0602} = 3.4822\left(1 \times 10^9\right)^{-0.0602} = 1.000$$

$$B_1 = 0.00898\left(\frac{H_{B1}}{H_{B2}}\right) - 0.00829 = 0.00898\left(\frac{280}{240}\right) - 0.00829 = 0.002187$$

The hardness ratio factor, Z_W, is given by

$$Z_W = 1 + B_1\left(\frac{N_G}{N_P} - 1\right) = 1 + 0.002187\left(\frac{35}{25} - 1\right) = 1.001$$

The temperature factor can be taken as $K_\theta = 1$.

The reliability factor for 99% reliability can be taken as $Z_Z = 1$.

The allowable contact stress for grade 1 through-hardened steel can be estimated from Eqn (10.40).

For the pinion, $\sigma_{H\ \mathrm{lim}} = 2.22H_B + 200 = (2.22 \times 280) + 200 = 821.6$ MPa

For the gear, $\sigma_{H\ \mathrm{lim}} = 2.22H_B + 200 = (2.22 \times 240) + 200 = 732.8$ MPa

Setting the contact safety factor, $S_H = 1$, to enable an estimate for the permissible AGMA contact stress to be calculated, then, for the pinion,

$$\sigma_{HP} = \frac{\sigma_{H\ \mathrm{lim}} Z_{NT} Z_W}{S_H K_\theta Z_Z} = \frac{821.6 \times 1 \times 1.001}{1 \times 1 \times 1} = 822.4 \text{ MPa}$$

For the gear,

$$\sigma_{HP} = \frac{\sigma_{H\ \mathrm{lim}} Z_{NT} Z_W}{S_H K_\theta Z_Z} = \frac{732.8 \times 1 \times 1.001}{1 \times 1 \times 1} = 733.5 \text{ MPa}$$

The ratio of the contact stress to the permissible contact stress for the pinion is

$$N_{sH} = \frac{\sigma_{HP}}{\sigma_H} = \frac{822.4}{422.9} = 1.9$$

The ratio of the contact stress to the permissible contact stress for the gear is

$$N_{sH} = \frac{\sigma_{HP}}{\sigma_H} = \frac{733.5}{302.0} = 2.4$$

The relevant factors to enable the AGMA bending stress to be evaluated can now be established.

From Figure 10.9, the bending strength geometry factor for the pinion is $Y_J = 0.245$.

From Figure 10.9, the bending strength geometry factor for the gear is $Y_J = 0.213$.

The size factor Y_x is given by

$$Y_x = 0.4867 + 0.008399 m_{et} = 0.4867 + (0.008399 \times 3) = 0.5119$$

The tooth lengthwise curvature factor, Y_β, can be taken as $Y_\beta = 1$.

The AGMA bending stress for the pinion is given by

$$\sigma_F = \frac{2000T_{q1}}{Fd_{e1}}\frac{K_A K_v}{m_{et}}\frac{Y_x K_{H\beta}}{Y_\beta Y_J} = \frac{2000 \times 27.06}{21.51 \times 75} \times \frac{1 \times 1.39}{3} \times \frac{0.5119 \times 1.103}{1 \times 0.245} = 35.81 \text{ MPa}$$

The AGMA bending stress for the gear is given by

$$\sigma_F = \frac{2000T_{q1}}{Fd_{e1}}\frac{K_A K_v}{m_{et}}\frac{Y_x K_{H\beta}}{Y_\beta Y_J} = \frac{2000 \times 27.06}{21.51 \times 105} \times \frac{1 \times 1.39}{3} \times \frac{0.5119 \times 1.103}{1 \times 0.213} = 29.42 \text{ MPa}$$

The allowable bending stress for grade 1 through-hardened steel can be estimated from Eqn (10.35).

For the pinion,

$$\sigma_{F\,\text{lim}} = 0.533H_B + 88.3 = (0.533 \times 280) + 88.3 = 237.5 \text{ MPa}$$

For the gear,

$$\sigma_{F\,\text{lim}} = 0.533H_B + 88.3 = (0.533 \times 240) + 88.3 = 216.2 \text{ MPa}$$

The stress cycle factor for bending strength, Y_{NT}, can be calculated from

$$Y_{NT} = 1.6831n_L^{-0.0323} = 1.6831\left(1 \times 10^9\right)^{-0.0323} = 0.8618$$

The reliability factor for 99% reliability can be taken as $Y_Z = 1$.

Setting the contact safety factor, $S_F = 1$, to enable an estimate for the permissible AGMA bending stress to be calculated, then, for the pinion,

$$\sigma_{FP} = \frac{\sigma_{F\,\text{lim}} Y_{NT}}{S_F K_\theta Y_z} = \frac{237.5 \times 0.8618}{1 \times 1 \times 1} = 204.7 \text{ MPa}$$

For the gear

$$\sigma_{FP} = \frac{\sigma_{F\,\text{lim}} Y_{NT}}{S_F K_\theta Y_z} = \frac{216.2 \times 0.8618}{1 \times 1 \times 1} = 186.3 \text{ MPa}$$

The ratio of the bending stress to the permissible bending stress for the pinion is

$$N_{sF} = \frac{\sigma_{FP}}{\sigma_F} = \frac{204.7}{35.81} = 5.7$$

The ratio of the bending stress to the permissible bending stress for the gear is

$$N_{sF} = \frac{\sigma_{FP}}{\sigma_F} = \frac{186.3}{29.42} = 6.3$$

Comment: The safety factors for bending are significantly higher than those for contact resistance, let alone comparing N_{sF} to N_{sH}^2 or N_{sH}^3. This suggests the design proposal is overly conservative in bending and that alternative initial values could result in a cheaper design.

Example 10.2

A bevel gear set has been proposed for a right angle drive from an electrical motor running at 14,000 rpm. The power being transmitted is 900 W from an electric motor, and the driven machine is expected to experience medium shocks. The design proposed has a spiral bevel pinion with 15 teeth and the gear has 32 teeth with a 20° pressure angle and a 35° spiral angle. The module selected is $m = 1.5$ mm. A grade 1 steel through-hardened with a hardness of 280 HB for the pinion and 180 HB for the gear is proposed. The quality number for manufacture is 8 and the teeth are crowned. Determine the bending and contact stresses and associated safety factors and comment on the suitability of the proposed gear set.

Solution

The initial values for the design are as follows:

$N_P = 15$; $N_G = 25$; $m = 1.5$ mm; $n_P = 14{,}000$ rpm, power = 0.9 kW, pinion material grade 1 through-hardened steel, $\text{HB}_1 = 280$ HB, $\text{HB}_2 = 180$ HB.

The torque, T_q, is given by

$$T_q = \frac{\text{Power}}{\omega} = \frac{900}{(2\pi/60)14{,}000} = 0.6139 \text{ N m}$$

The pinion and gear pitch diameters are

$$d_P = mN_P = 1.5 \times 15 = 22.5 \text{ mm}$$

$$d_G = mN_G = 1.5 \times 32 = 48 \text{ mm}$$

The gear and pinion pitch angles are

$$\alpha_G = \tan^{-1}\left(\frac{N_G}{N_P}\right) = \tan^{-1}\left(\frac{32}{15}\right) = 64.89^\circ$$

$$\alpha_P = 90 - \alpha_G = 25.11^\circ$$

The back cone distance, L, is given by

$$L = \frac{d_P}{2 \sin \alpha_P} = \frac{22.5}{2 \sin 25.11} = 26.51 \text{ mm}$$

The face width, F, is normally $L/3$, so

$$F = \frac{26.51}{3} = 8.835 \text{ mm}$$

From Figure 10.10, the pitting resistance geometry factor for a 20° pressure angle and 35° spiral angle and 90° shaft angle spiral bevel gears, for a pinion with 15 teeth and a gear with 32 teeth is $Z_I = 0.097$.

From Table 10.1, for smooth or uniform driver and medium shock driven characteristics, the overload factor, K_A, is $K_A = 1.5$.

The pitch line velocity is

$$V_{et} = \omega \frac{d}{2} = \left(\frac{2\pi}{60} \times 14{,}000\right) \frac{22.5 \times 10^{-3}}{2} = 16.49 \text{ m/s}$$

$$B = 0.25(12 - Q_v)^{0.667} = 0.25(12 - 8)^{0.667} = 0.6303$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.6303) = 70.71$$

The dynamic factor is given by

$$K_v = \left(\frac{A}{A + \sqrt{200 v_{et}}}\right)^{-B} = \left(\frac{70.71}{70.71 + \sqrt{200 \times 16.49}}\right)^{-0.6303} = 1.455$$

The maximum recommended pitch line velocity is

$$v_{et \text{ max}} = \frac{[A + (Q_v - 3)]^2}{200} = \frac{[70.71 + (8 - 3)]^2}{200} = 28.66 \text{ m/s}$$

As $v_{et} < v_{et \text{ max}}$, the pitch line velocity is acceptable.

As one of the gears is straddle mounted, the load distribution modifier, $K_{mb} = 1.1$.

The load distribution factor, $K_{H\beta}$, is given by

$$K_{H\beta} = K_{mb} + 5.6 \times 10^{-6} F^2 = 1.1 + 5.6 \times 10^{-6} (8.835)^2 = 1.100$$

The size factor for pitting resistance, Z_x, is given by

$$Z_x = 0.00492F + 0.4375 = (0.00492 \times 8.835) + 0.4375 = 0.4810$$

For properly crowned teeth, $Z_{xc} = 1$.

For a steel pinion and a steel gear, from Table 10.5, $Z_E = 191\sqrt{\text{MPa}}$

From Eqn (10.9), the AGMA contact stress for pitting resistance, for the pinion, is given by

$$\sigma_H = Z_E\sqrt{\frac{2000T_{q1}}{Fd_{e1}^2 Z_I}K_A K_V K_{H\beta} Z_x Z_{xc}}$$

$$= 191\sqrt{\frac{2000 \times 0.6139}{8.835 \times 22.5^2 \times 0.097} \times 1.5 \times 1.455 \times 1.1 \times 0.481 \times 1}$$

$$\sigma_H = 345.3 \text{ MPa}$$

From Eqn (10.9), the AGMA contact stress for pitting resistance, for the gear, is given by

$$\sigma_H = Z_E\sqrt{\frac{2000T_{q1}}{Fd_{e1}^2 Z_I}K_A K_V K_{H\beta} Z_x Z_{xc}}$$

$$= 191\sqrt{\frac{2000 \times 0.6139}{8.835 \times 48^2 \times 0.097} \times 1.5 \times 1.455 \times 1.1 \times 0.481 \times 1}$$

$$\sigma_H = 161.8 \text{ MPa}$$

If the number of cycles for the gear set is taken as 1×10^9, the stress cycle factor, Z_{NT}, is given by

$$Z_{NT} = 3.4822n_L^{-0.0602} = 3.4822\left(1 \times 10^9\right)^{-0.0602} = 1.000$$

$$B_1 = 0.00898\left(\frac{H_{B1}}{H_{B2}}\right) - 0.00829 = 0.00898\left(\frac{280}{180}\right) - 0.00829 = 0.005679$$

The hardness ratio factor, Z_W, is given by

$$Z_W = 1 + B_1\left(\frac{N_G}{N_P} - 1\right) = 1 + 0.005679\left(\frac{32}{15} - 1\right) = 1.006$$

The temperature factor can be taken as $K_\theta = 1$.

The reliability factor for 99% reliability can be taken as $Z_Z = 1$.

The allowable contact stress for grade 1 through-hardened steel can be estimated from Eqn (10.40).

For the pinion, $\sigma_{H\,\mathrm{lim}} = 2.22H_B + 200 = (2.22 \times 280) + 200 = 821.6$ MPa

For the gear, $\sigma_{H\,\mathrm{lim}} = 2.22H_B + 200 = (2.22 \times 180) + 200 = 599.6$ MPa

Setting the contact safety factor, $S_H = 1$, to enable an estimate for the permissible AGMA contact stress to be calculated, then, for the pinion,

$$\sigma_{HP} = \frac{\sigma_{H\,\mathrm{lim}} Z_{NT} Z_W}{S_H K_\theta Z_Z} = \frac{821.6 \times 1 \times 1.006}{1 \times 1 \times 1} = 827.0 \text{ MPa}$$

For the gear,

$$\sigma_{HP} = \frac{\sigma_{H\,\mathrm{lim}} Z_{NT} Z_W}{S_H K_\theta Z_Z} = \frac{599.6 \times 1 \times 1.006}{1 \times 1 \times 1} = 603.5 \text{ MPa}$$

The ratio of the contact stress to the permissible contact stress for the pinion is

$$N_{sH} = \frac{\sigma_{HP}}{\sigma_H} = \frac{827.0}{345.3} = 2.4$$

The ratio of the contact stress to the permissible contact stress for the gear is

$$N_{sH} = \frac{\sigma_{HP}}{\sigma_H} = \frac{603.5}{161.8} = 3.7$$

The relevant factors to enable the AGMA bending stress to be evaluated can now be established.

From Figure 10.11, the bending strength geometry factor for the pinion is $Y_J = 0.21$.

From Figure 10.11, the bending strength geometry factor for the gear is $Y_J = 0.205$.

The size factor Y_x is given by

$$Y_x = 0.4867 + 0.008399 m_{et} = 0.4867 + (0.008399 \times 1.5) = 0.4993$$

The tooth lengthwise curvature factor, Y_β, can be taken as $Y_\beta = 1$.

The AGMA bending stress for the pinion is given by

$$\sigma_F = \frac{2000 T_{q1}}{F d_{e1}} \frac{K_A K_v}{m_{et}} \frac{Y_x K_{H\beta}}{Y_\beta Y_J} = \frac{2000 \times 0.6139}{8.835 \times 22.5} \times \frac{1.5 \times 1.455}{1.5} \times \frac{0.4993 \times 1.1}{1 \times 0.21} = 23.50 \text{ MPa}$$

The AGMA bending stress for the gear is given by

$$\sigma_F = \frac{2000T_{q1}}{Fd_{e1}}\frac{K_A K_v}{m_{et}}\frac{Y_x K_{H\beta}}{Y_\beta Y_J} = \frac{2000 \times 0.6139}{8.835 \times 48} \times \frac{1.5 \times 1.455}{1.5} \times \frac{0.4993 \times 1.1}{1 \times 0.205} = 11.29 \text{ MPa}$$

The allowable bending stress for grade 1 through-hardened steel can be estimated from Eqn (10.35).

For the pinion,

$$\sigma_{F\,\text{lim}} = 0.533H_B + 88.3 = (0.533 \times 280) + 88.3 = 237.5 \text{ MPa}$$

For the gear,

$$\sigma_{F\,\text{lim}} = 0.533H_B + 88.3 = (0.533 \times 180) + 88.3 = 184.2 \text{ MPa}$$

The stress cycle factor for bending strength, Y_{NT}, can be calculated from

$$Y_{NT} = 1.6831n_L^{-0.0323} = 1.6831\left(1 \times 10^9\right)^{-0.0323} = 0.8618$$

The reliability factor for 99% reliability can be taken as $Y_Z = 1$.

Setting the contact safety factor, $S_F = 1$, to enable an estimate for the permissible AGMA bending stress to be calculated, then, for the pinion,

$$\sigma_{FP} = \frac{\sigma_{F\,\text{lim}} Y_{NT}}{S_F K_\theta Y_z} = \frac{237.5 \times 0.8618}{1 \times 1 \times 1} = 204.7 \text{ MPa}$$

For the gear

$$\sigma_{FP} = \frac{\sigma_{F\,\text{lim}} Y_{NT}}{S_F K_\theta Y_z} = \frac{184.2 \times 0.8618}{1 \times 1 \times 1} = 158.8 \text{ MPa}$$

The ratio of the bending stress to the permissible bending stress for the pinion is

$$N_{sF} = \frac{\sigma_{FP}}{\sigma_F} = \frac{204.7}{23.5} = 8.7$$

The ratio of the bending stress to the permissible bending stress for the gear is

$$N_{sF} = \frac{\sigma_{FP}}{\sigma_F} = \frac{158.8}{11.29} = 14$$

Comment: The safety factors for bending are significantly higher than those for contact resistance, let alone comparing N_{sF} to N_{sH}^2 or N_{sH}^3. This suggests the design proposal is overly conservative in bending and that alternative initial values could result in a cheaper design.

10.5 Conclusions

Bevel gears can be used to transmit power through a 90° angle. The efficiency of transmission can be high, and this form of gearing can be found in a wide range of applications from machine tools to differential drives. The AGMA equations presented provide a conservative means for assessing the bending and contact stresses.

References

ANSI/AGMA 2003-B97. American National Standard, 2003. Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth. AGMA.

American Gear Manufacturers Association. ANSI/AGMA 218.01.

American Gear Manufacturers Association. ANSI/AGMA 2001-D04. Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth.

Further Reading

Shigley, J.E., Mischke, C.R., 1986. Gearing. McGraw Hill.

Web Sites

www.agma.org/
www.bga.org.uk/
www.motionco.co.uk/?gclid=CMDP3p_bzaYCFUhO4QodpCr7iw
www.arrowgear.com/products/spiral_bevel_gears.htm
www.qtcgears.com/rfq/bevelgears.htm
www.spiralbevel.com/
www.gears-manufacturers.com/spiral-bevel-gears.html
www.amarillogear.com/spiralbevel.html
www.bostongear.com/products/open/miter.html
www.gearshub.com/bevel-gears.html
www.roymech.co.uk/Useful_Tables/Drive/Bevel_Gears.html
www.derangear.com/spiral_gear.htm
www.geartechnology.com/issues/

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

d	pitch diameter (mm)
d_G	pitch diameter for the pinion (mm)
d_P	pitch diameter for the gear (mm)
d_{e1}	pinion outer pitch diameter (mm)
E_1	Young's modulus for the pinion (MPa)
E_2	Young's modulus for the gear (MPa)

F	net face width (mm)
F'_1	effective face width of the pinion (mm)
F'_2	effective face width of the gear (mm)
g_c	length of line contact (mm)
H_{B1}	minimum Brinell hardness for the pinion (HB, MPa)
H_{B2}	minimum Brinell hardness for the gear (HB, MPa)
K_θ	temperature factor
K_A	overload factor
$K_{H\beta}$	load distribution factor
K_{mb}	load distribution modifier
K_v	dynamic factor
L	orthogonal length from the gear back cone to the axis (mm)
m	module
m_{et}	metric outer traverse module (mm)
m_{et}	outer traverse module (mm)
m_G	gear ratio
m_{mt}	mean traverse module (mm)
n_1	pinion speed (rpm)
n_P	pinion speed (rpm)
n_G	gear speed (rpm)
N	number of teeth
N_G	number of teeth in the gear
N_P	number of teeth in the pinion
P	power (kW)
P_{ay}	allowable transmitted power (kW)
P_{az}	allowable transmitted power (kW)
Q_v	transmission accuracy number
R_{a1}	pinion surface roughness (μm)
r_{av}	pitch radius at the midpoint of the tooth for the gear under consideration (m)
r_{c0}	cutter radius (mm)
R_m	mean cone distance (mm)
r_{mpt1}	mean transverse pitch radius for the pinion (mm)
r_{mpt2}	mean transverse pitch radius for the gear (mm)
r_{myo1}	mean transverse radius to the point of load application for the pinion (mm)
r_{myo2}	mean transverse radius to the point of load application for the gear (mm)
S_F	bending safety factor
S_H	contact safety factor
T	peak operating gear temperature (K)
T_q	torque (N m)
T_{q1}	operating pinion torque (N m)
v_{et}	pitch line velocity at the outside pitch diameter (m/s)
W_a	axial force (N)
W_r	radial force (N)
W_t	transmitted load (N)
Y_β	tooth lengthwise curvature factor

Y_i inertia ratio for gears with low contact ratio
Y_J bending strength geometry factor
Y_{K1} tooth form factor for the pinion
Y_{K2} tooth form factor for the gear
Y_{NT} stress cycle factor
Y_x size factor
Y_Z reliability factor
Z_E elastic coefficient $(\sqrt{\text{MPa}})$
Z_i inertia factor for gears with a low contact ratio
Z_I pitting resistance geometry factor
Z_{NT} stress cycle factor
Z_W hardness ratio factor
Z_x size factor
Z_{xc} crowning factor
Z_Z reliability factor
α pitch angle (°)
α_G pitch angle for the gear (°)
α_n normal pressure angle (°)
α_P pitch angle for the pinion (°)
β_m mean spiral angle (°)
ε_{NI} load sharing ratio
ε_{NJ} load sharing ratio
ϕ pressure angle (°)
ρ_{y0} relative radius of profile curvature (mm)
ν_1 Poisson's ratio for the pinion
ν_2 Poisson's ratio for the gear
$\sigma_{F\,\text{lim}}$ allowable bending stress (N/mm^2, MPa)
σ_F bending stress at the root of the tooth (N/mm^2, MPa)
σ_{FP} permissible bending stress (N/mm^2, MPa)
$\sigma_{H\,\text{lim}}$ allowable contact stress (N/mm^2, MPa)
σ_H contact stress (N/mm^2, MPa)
σ_{HP} permissible contact stress (N/mm^2, MPa)
ω angular velocity (rad/s)
ψ spiral angle (°)

CHAPTER 11

Worm Gears

Chapter Outline

11.1 Introduction 439
11.2 Force Analysis 446
11.3 AGMA Equations 449
11.4 Design Procedure 453
11.5 Conclusions 455
References 456
Further Reading 456
Nomenclature 457

Abstract

Worm and wheel gears are widely used for nonparallel, nonintersecting, right angle gear drive system applications where a high transmission gearing ratio is required. In comparison to other gear, belt, and chain transmission elements, worm and wheel gear sets tend to offer a more compact solution. In certain configurations, a worm and wheel gear set can provide sufficiently high friction to be self-locking. This can be a desirable feature if a defined position is required for a gear train, if it is not braked or unpowered. This chapter provides an overview of worms and wheels and outlines a selection procedure.

11.1 Introduction

A worm gear is a cylindrical helical gear with one or more threads and resembles a screw thread. A worm wheel or worm gear is a cylindrical gear with flanks cut in such a way as to ensure contact with the flanks of the worm gear. The worm wheel is analogous to a nut that fits on the screw thread of the worm. If the worm is restrained axially within its housing and if the worm is rotated, the worm gear will also rotate. Typical forms for worms and worm gears are shown in Figure 11.1.

In a worm and wheel gear set rotary power can be transmitted between nonparallel and nonintersecting shafts. A worm and wheel gear set is typically used when the speed ratio of the two shafts is high, say three or more.

Worm and wheel gear sets are used for steering gear, winch blocks, e.g. see Figure 11.2, low-speed gearboxes, rotary tables, and remote valve control. Worm and wheel gear sets are

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00011-3
Copyright © 2014 Elsevier Ltd. All rights reserved.

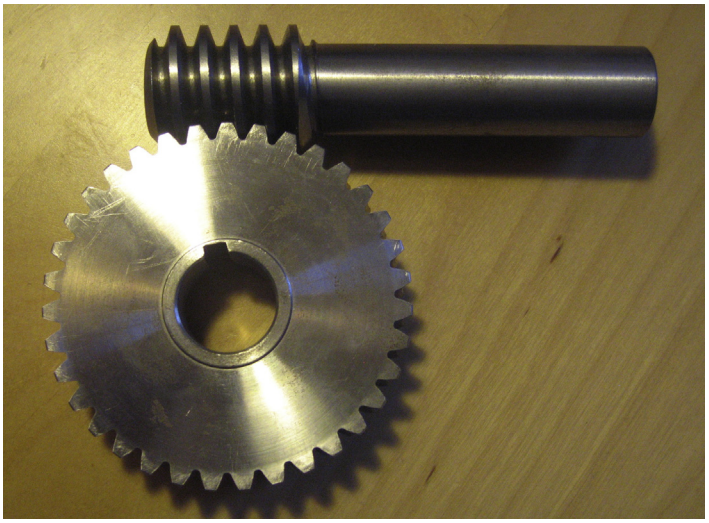


Figure 11.1
Worm and wheel gear sets.

capable of high-speed reduction and high load applications where nonparallel, non-intersecting shafts are used. The 90° configuration is most common, although other angles are possible. Frictional heat generation is high in worm gears because of the high sliding velocities, so continuous lubrication and provision for heat dissipation are required.

The direction of rotation of the worm wheel depends on the direction of rotation of the worm and on whether the worm teeth have a right-hand thread or a left-hand thread. The direction of rotation for a worm and wheel gear sets is illustrated is Figure 11.3.

Worms usually have just one tooth and can, therefore, produce gearing ratios as high as the number of teeth on the gear wheel. Herein lies the principal merit of worm and wheel gear sets. In comparison to other gear sets, which are typically limited to a gear ratio of up to 10:1, worm

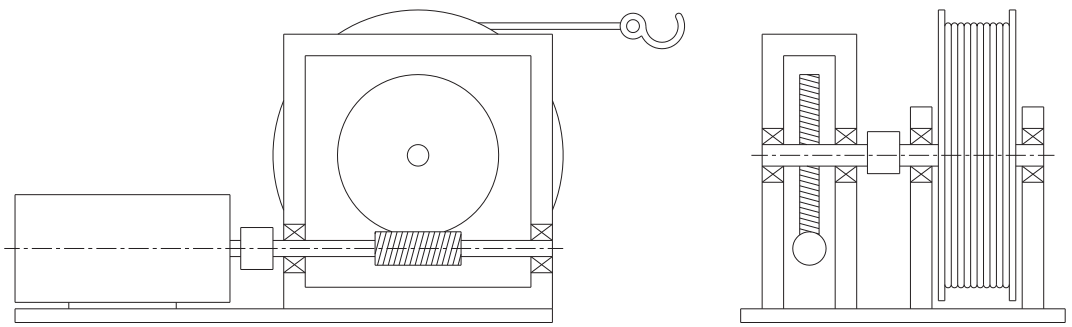


Figure 11.2
Possible outline winch configuration incorporating a worm and wheel gear set.

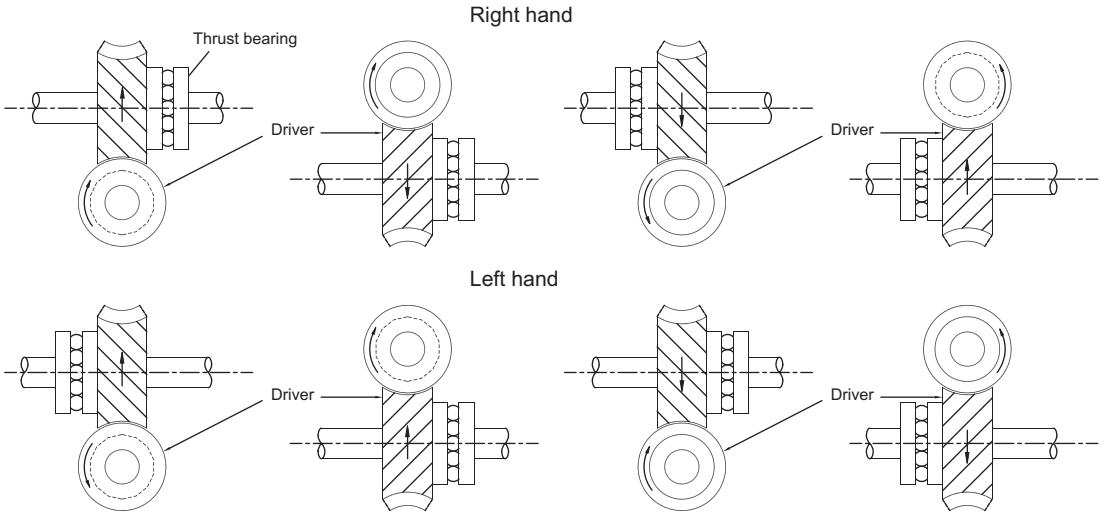


Figure 11.3
Rotation and hand relations for worm and wheel gear sets. *After Boston Gear Division.*

and wheel gear sets can achieve gear ratios of up to 360:1, although most manufacturers quote ranges between 3:1 and 100:1. Ratios above 30:1 generally have one thread on the worm, while ratios below 30:1 tend to have a worm with multiple threads (sometimes referred to as starts).

The gear ratio for a worm and wheel gear set is given by

$$m_G = \frac{N_G}{N_W} \tag{11.1}$$

where

m_G = gear ratio,
N_G = number of teeth in the worm gear, and
N_W = number of threads in the worm.

A particular merit of worm and wheel gear sets is their ability to self-lock. If a worm set is self-locking it will not back drive and any torque applied to the worm gear will not rotate the worm. A self-locking worm and wheel gear set can only be driven forward by rotation of the worm. Such a gear set can, therefore, be used to hold a load. This is commonly exploited in a car jack. Whether a worm and wheel gear set will be self-locking depends on frictional contact between the worm and the worm wheel flanks.

There are two types of worm and wheel gear sets, depending on whether the teeth of one or both wrap around each other.

- Single enveloping worm and wheel gear sets, see Figure 11.4, and
- Double enveloping worm and wheel gear sets, see Figure 11.5.

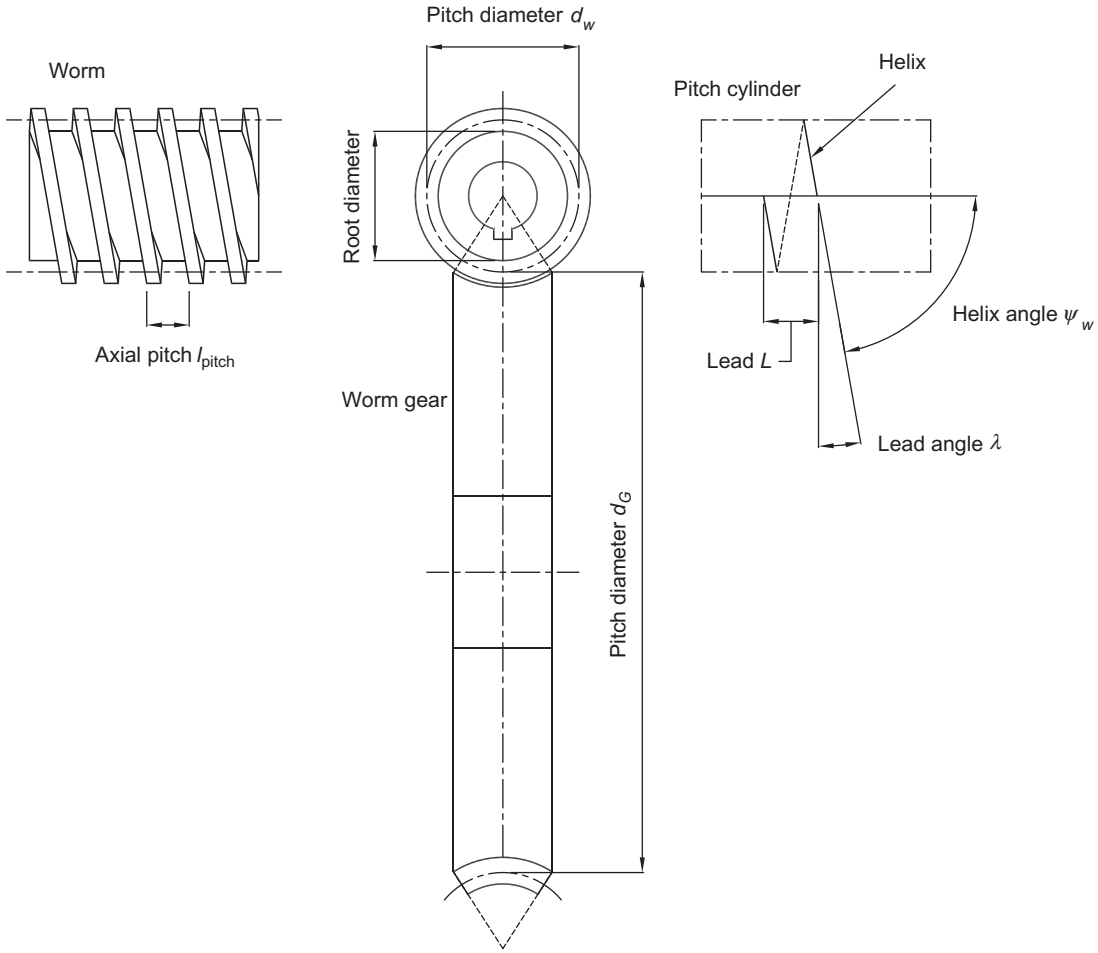


Figure 11.4
Nomenclature for a single enveloping worm and wheel gear set.

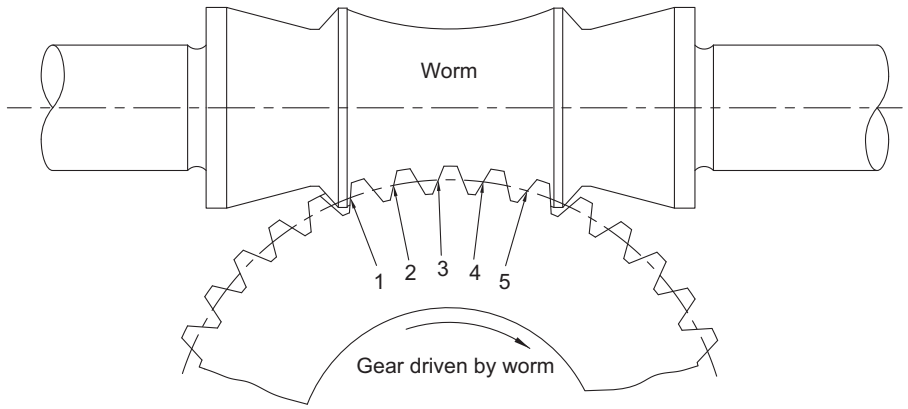


Figure 11.5
Double enveloping worm and wheel gear set.

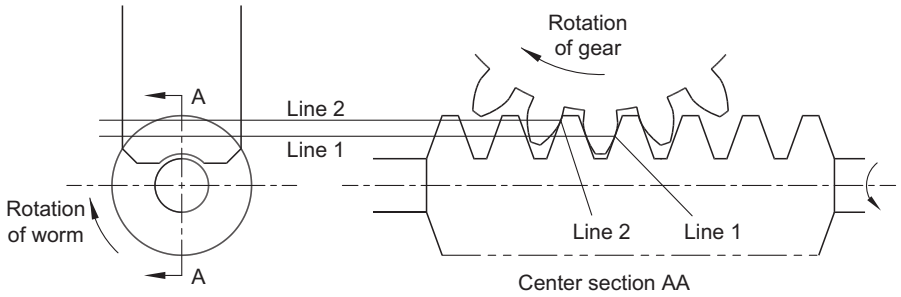


Figure 11.6
Lines of contact for a worm and wheel gear set.

As the worm rotates through the worm gear, lines of contact either roll or progress from the tip to the root of the worm gear teeth. At any instant in time, there may be two or three teeth in contact and transmitting power as illustrated in Figure 11.6.

Some of the key geometric features and dimensions for a worm gear are illustrated in Figure 11.7.

The helix angle on a worm is usually high, and the helix angle on the worm wheel is low. Normal convention is to define a lead angle, λ, on the worm and a helix angle, ψ_G, on the

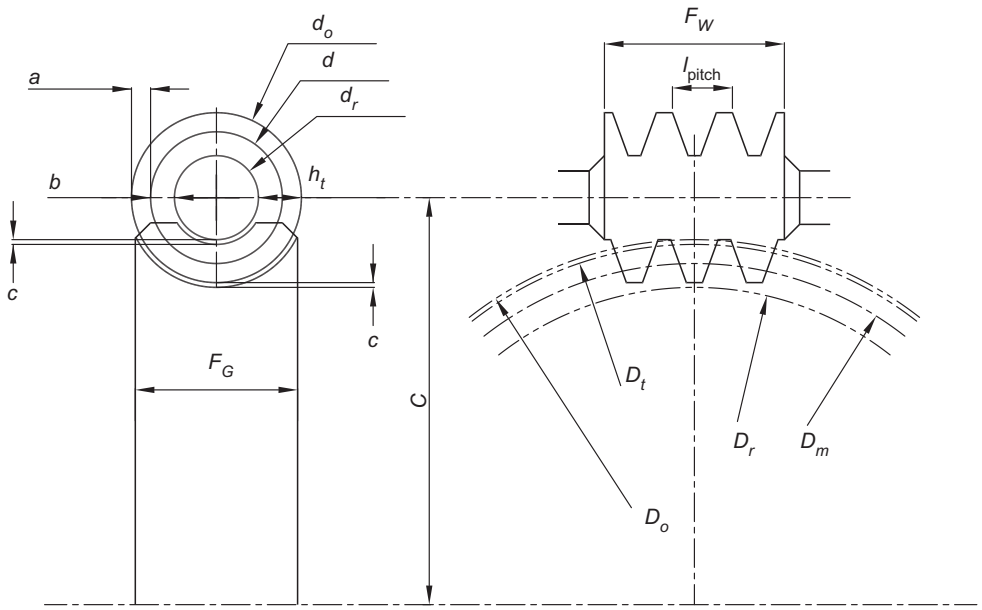


Figure 11.7
Worm gear dimensions.

worm gear. For a 90° configuration, $\lambda = \psi_G$. The distance that a point on the mating worm gear moves axially in one revolution of the worm is called the lead, L.

The following relationships apply to the lead, L, and lead angle, λ:

$$L = l_{\text{pitch}} N_W = \frac{\pi d_G N_W}{N_G} \tag{11.2}$$

$$\tan \lambda = \frac{L}{\pi d_W} \tag{11.3}$$

where

L = lead (mm),
l_{pitch} = worm axial pitch (mm),
N_W = number of teeth on the worm,
d_G = pitch diameter of the worm gear (mm),
N_G = number of teeth on the worm gear,
λ = lead angle (°), and
d_W = pitch diameter of the worm (mm).

The worm lead angle and the worm helix angle, ψ_W, are related by $\lambda = 90° - \psi_W$.

The lead angle will vary from the root to the outside diameter of the worm, as indicated in Figure 11.8. Generally, self-locking occurs for lead angles below 6°. However, care is necessary in relying solely on self-locking to brake or sustain a load, as vibration has been

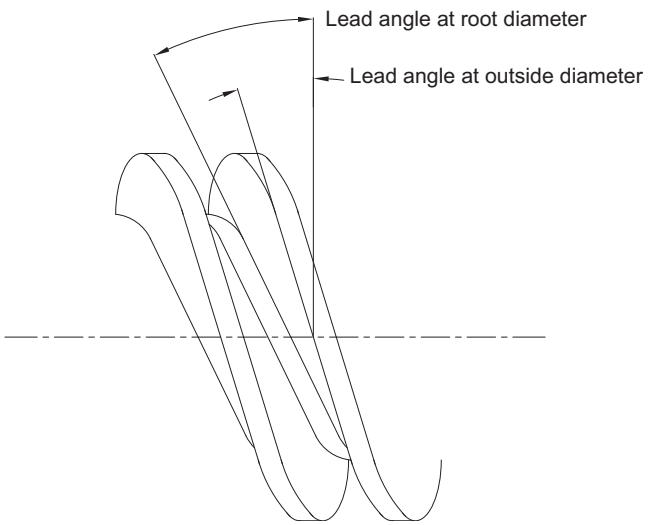


Figure 11.8
Variation of the lead angle on a worm gear. *Figure after AGMA.*

known to result in a reduction of the frictional contact between the worm and wheel for lead angles below 6° and resulting movement or failure of the device.

The axial pitch of the worm and the transverse circular pitch of the wheel will be equal for a 90° set configuration,

$$d = mN \tag{11.4}$$

The worm can have any pitch diameter, as this is not related to the number of teeth. General guidance for optimum power capacity indicates that the pitch diameter, d, of the worm should fall in the following ranges (AGMA 6022-C93):

$$\frac{C^{0.875}}{1.6} \leq d_{\max} \leq \frac{C^{0.875}}{1.07} \tag{11.5}$$

$$\frac{C^{0.875}}{3} \leq d_{\min} \leq \frac{C^{0.875}}{2} \tag{11.6}$$

where:

C = center distance (mm),
d = worm pitch diameter (mm),
$d_{\max}$ = maximum worm pitch diameter (mm), and
$d_{\min}$ = minimum worm pitch diameter (mm).

Dudley (1984) using Imperial units, see also Radzevich (2012), recommends

$$d \approx \frac{C^{0.875}}{2.2} \tag{11.7}$$

The pitch diameter of the worm gear, d_G, is related to the center distance C and the pitch diameter of the worm, by

$$d_G = 2C - d \tag{11.8}$$

The addendum, a, and dedendum, b, are given by

$$a = 0.3183 l_{\text{pitch}} \tag{11.9}$$

$$b = 0.3683 l_{\text{pitch}} \tag{11.10}$$

The face width of a worm gear, Figure 11.7, is limited by the worm diameter. The ANSI/AGMA 6034-B92 recommendation for the minimum face width, for a pitch exceeding 4.06 mm, is given by

$$F_G = 0.67d \tag{11.11}$$

The tooth forms for worm and wheel gear sets are not involutes. They are manufactured as matched sets. The worm is subject to high stresses and is normally made using a hardened steel such as AISI 1020, 1117, 8620, 4320 hardened to HRC 58−62, or a medium carbon steel such as AISI 4140 or 4150 induction or flame hardened to a case of HRC 58−62 (Norton, 2006). They are typically ground or polished to a roughness of $R_a = 0.4$ μm. The worm gear needs to be of softer material that is compliant enough to run-in and conform to the worm under the high sliding running conditions. Sand cast or forged bronze is commonly used. Cast iron and polymers are sometimes used for lightly loaded, low-speed applications.

An analysis of the forces associated with a worm and wheel gear set can be undertaken readily, and this is outlined in Section 11.2. Such information is critical in order to enable suitable bearings to be selected for both shafts. Worm and wheel gear sets tend to fail due to pitting and wear (see Maitra (1994) and Dudley (1984)). The American Gear Manufacturer Association power ratings, based on wear and pitting resistance, are presented in Section 11.3 and an associated design procedure in Section 11.4.

11.2 Force Analysis

The force exerted on a worm by a gear is illustrated in Figure 11.9 where, for the time being, friction has been neglected. The resultant force, W, will have three components:

$$W_x = W \cos \phi_n \sin \lambda \tag{11.12}$$

$$W_y = W \sin \phi_n \tag{11.13}$$

$$W_z = W \cos \phi_n \cos \lambda \tag{11.14}$$

where

W_x = force component in the tangential direction on the worm (N),
W_y = force component in the radial direction on the worm (N),
W_z = force component in the axial direction on the worm (N),
λ = lead angle (°), and
ϕ_n = normal pressure angle of the worm thread at the mean diameter (°).

Standard pressure angles for worm and wheel gear sets are 14.5, 17.5, 20, 22.5, 25, 27.5, and 30°. The higher the pressure, the higher the tooth strength; albeit at the expense of higher friction, bearing loads, and bending stresses in the worm.

The minimum number of worm gear teeth, $N_{\min}$, as a function of the pressure angle is listed in Table 11.1.

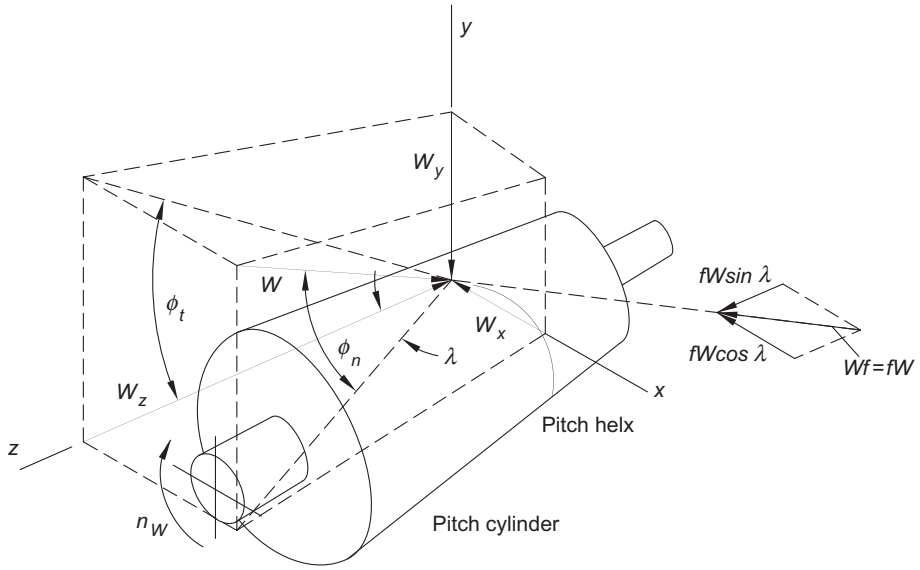


Figure 11.9
Pitch cylinder of the worm, showing the forces exerted on the worm by the worm gear.
Image after Shigley (1986).

As the forces on the worm and worm gear are equal and opposite, the tangential, radial, and axial forces are given by

$$W_{Wt} = -W_{Ga} = W_x \tag{11.15}$$

$$W_{Wr} = -W_{Gr} = W_y \tag{11.16}$$

$$W_{Wa} = -W_{Gt} = W_z \tag{11.17}$$

where

W_{Wt} = tangential force component acting against the worm (N),
W_{Wr} = radial force component acting against the worm (N),

Table 11.1: Suggested minimum number of teeth for the worm (AGMA).

Pressure angle (°)	N_{min}
14.5	40
17.5	27
20	21
22.5	17
25	14
27.5	12
30	10

W_{Wa} = axial force component acting against the worm (N),
W_{Gt} = tangential force component acting against the gear (N),
W_{Gr} = radial force component acting against the gear (N), and
W_{Wa} = axial force component acting against the gear (N).

Introducing a coefficient of friction, f, to account for the sliding motion experienced in the motion between a worm thread and the wheel teeth surfaces,

$$W_x = W(\cos\phi_n \sin\lambda + f\cos\lambda) \tag{11.18}$$

$$W_y = W\sin\phi_n \tag{11.19}$$

$$W_z = W(\cos\phi_n \cos\lambda - f\sin\lambda) \tag{11.20}$$

From Eqns (11.15−11.18)

$$W_f = fW = \frac{fW_{Gt}}{f\sin\lambda - \cos\phi_n\cos\lambda} \tag{11.21}$$

$$W_{Wt} = W_{Gt}\frac{\cos\phi_n \sin\lambda + f\cos\lambda}{f\sin\lambda - \cos\phi_n\cos\lambda} \tag{11.22}$$

The efficiency of a worm and wheel gear set can be defined by

$$\eta = \frac{W_{Wt,\text{ without friction}}}{W_{Wt,\text{ with friction}}} \tag{11.23}$$

or

$$\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} \tag{11.24}$$

A typical value for the coefficient of friction for worm gears is $f \approx 0.05$. The variation of efficiency with helix angle is given in Table 11.2.

Experiments have shown that efficiency for a worm and wheel gear set is a function of the sliding velocity. Taking V_G as the pitch line velocity of the gear and V_W as the pitch line velocity of the worm, the sliding velocity, V_s, is given by vector addition:

$$V_W = V_G + V_s \tag{11.25}$$

or

$$V_s = \frac{V_W}{\cos\lambda} \tag{11.26}$$

Table 11.2: Worm and wheel efficiency (taking $f = 0.05$).

ψ	λ	F	ϕ_n	η
1	1	0.05	20	24.7
2.5	2.5	0.05	20	45.0
5	5	0.05	20	61.9
7.5	7.5	0.05	20	70.7
10	10	0.05	20	76.1
15	15	0.05	20	82.2
20	20	0.05	20	85.6
25	25	0.05	20	87.5
30	30	0.05	20	88.7

where

V_G = pitch line velocity of the gear (m/s),
V_W = pitch line velocity of the worm (m/s), and
V_S = sliding velocity (m/s).

11.3 AGMA Equations

Worm sets are generally rated by their capacity to handle a particular level of input power, output power, or allowable torque at a particular speed for the input or output shaft. The AGMA power rating is based on pitting and wear resistance, as this is the usual failure mode for worm sets. The AGMA rating (ANSI/AGMA 6034-B92) is based on 10 h of continuous operation under a uniform load.

The input-power rating, P_{input}, is given by

$$P_{\text{input}} = P_{\text{output}} + P_{\text{loss}} \tag{11.27}$$

where P_{loss} is the power lost due to friction in the mesh (kW).

The output power is given by

$$P_{\text{output}} = \frac{nW_{tg}d_g}{1.91 \times 10^7 m_G} \tag{11.28}$$

where

n = rotational speed of the worm (rpm),
W_{tg} = worm gear tangential force (N),
P_{output} = output power (kW),
m_G = gear ratio, and
d_g = mean gear diameter (mm).

The power lost is given by

$$P_{\text{loss}} = \frac{V_t W_f}{1000} \tag{11.29}$$

where

P_{loss} = lost power (kW),
V_t = sliding velocity at the mean worm diameter (m/s), and
W_f = friction force (N).

The AGMA tangential load on a worm gear is given by

$$W_t = \frac{C_s C_m C_v d_g^{0.8} F}{75.948} \tag{11.30}$$

where

C_s = materials factor,
d_g = mean diameter of the gear (mm),
F = effective face width (mm),
C_m = ratio correction factor, and
C_v = velocity factor.

The friction force can be determined by

$$W_f = \frac{f W_t}{\cos \lambda \cos \phi_n} \tag{11.31}$$

where

f = coefficient of friction,
W_t = tangential load on the worm gear tooth (N),
λ = lead angle (°), and
ϕ_n = normal pressure angle of the worm thread at the mean diameter (°).

The sliding velocity at the mean worm diameter can be determined by

$$V_t = \frac{n d_m}{19{,}098 \cos \lambda} \tag{11.32}$$

where

n = rotational speed of the worm (rpm), and
d_m = mean worm diameter (mm).

Values for the ratio correction factor, the velocity factor, and materials factors can be found from tables provided in the ANSI/AGMA 6034-B92 standard. The following equations for

the ratio correction factor, the velocity factor, and materials factors provide approximations to the values given in the tables.

The ratio correction factor C_m is a function of the gear ratio, m_G.

For $3 \leq m_G \leq 20$

$$C_m = 0.02\left(-m_G^2 + 40m_G - 76\right)^{0.5} + 0.46 \tag{11.33}$$

For $20 \leq m_G \leq 76$

$$C_m = 0.0107\left(-m_G^2 + 56m_G + 5145\right)^{0.5} \tag{11.34}$$

For $m_G > 76$

$$C_m = 1.1483 - 0.00658m_G \tag{11.35}$$

For $0 \leq V_t \leq 3.556$ m/s

$$C_v = 0.659e^{-0.2165V_t} \tag{11.36}$$

For $3.556 \leq V_t \leq 15.24$ m/s

$$C_v = 0.652V_t^{-0.571} \tag{11.37}$$

For $V_t > 15.24$ m/s

$$C_v = 1.098V_t^{-0.774} \tag{11.38}$$

For $V_t = 0$, take $f = 0.15$.

For $0 \leq V_t \leq 0.0508$ m/s

$$f = 0.124e^{\left(-2.233V_t^{0.645}\right)} \tag{11.39}$$

For $V_t > 0.0508$ m/s

$$f = 0.103e^{\left(-1.185V_t^{0.45}\right)} \tag{11.40}$$

The materials factor C_s depends on the method of casting.

For $C \leq 76.2$ mm, an initial estimate for the materials factor can be obtained from

$$C_s = 720 + 0.000633C^3 \tag{11.41}$$

This value can be compared to the values obtained for the relevant means of casting, as indicated by the following relationships and the smaller value used.

For sand cast gears then:

for $d_m < 63.5$ mm, $C_s = 1000$
for $d_m > 63.5$ mm,

$$C_s = 1859.104 - 476.5454 \log_{10} d_m \tag{11.42}$$

For chill cast bronze gears then:

for $d_m < 203.2$ mm, $C_s = 1000$
for $d_m > 203.2$ mm

$$C_s = 2052.011 - 455.8259 \log_{10} d_m \tag{11.43}$$

For centrifugally cast gears then:

for $d_m < 635$ mm, $C_s = 1000$
for $d_m > 635$ mm,

$$C_s = 1503.811 - 179.7503 \log_{10} d_m \tag{11.44}$$

The efficiency, in percent, for worm gearing is given by

$$\eta = \frac{P_{\text{output}}}{P_{\text{input}}} \times 100 \tag{11.45}$$

Substituting for the output power

$$\eta = \frac{nW_t\, d_m}{1.91 \times 10^7 m_G P_{\text{input}}} \times 100 \tag{11.46}$$

where

P_{output} = rated output power (kW),
P_{input} = rated input power (kW),
n = rotational speed of the worm (rpm),
W_t = tangential load on the worm gear (N),
d_m = mean diameter of the gear (mm),
m_G = gear ratio.

11.4 Design Procedure

An outline design procedure for a worm and wheel gear set using the AGMA equations is listed next.

1. Define the number of starts.
2. Define the center distance, C.
3. Determine a suitable worm gear diameter.
4. Determine the lead.
5. Determine the lead angle.
6. Determine the maximum recommended face width, F.
7. Determine the material factor, C_s.
8. Determine the ratio correction factor, C_m.
9. Determine the tangential velocity, V_t.
10. Determine the velocity factor, C_v.
11. Determine the tangential load, W_t.
12. Determine the coefficient of friction, f.
13. Determine the friction force, W_f.
14. Determine the output power, P_{output}.
15. Determine the power lost in mesh, P_{loss}.
16. Determine the rated input power, P_{input}.
17. Estimate the efficiency of the gear set, η.
18. Determine the output torque, T_q.
19. Establish whether the power rating and output torque are sufficient for the application.
20. If not, alter the number of starts, worm diameter, center distance, etc. to provide suitable power rating and output torque.

Example 11.1

Develop a design for a worm and wheel gear set. The prime mover is an electric motor running at 1470 rpm. A reduction ratio of 50:1 is required. The peak torque required is 850 N m. Assume sand cast gears.

Solution

Input speed = 1470 rpm. Ratio 50:1.

Output = 29.4 rpm.

Sand cast gears.

If a single start worm is specified, then a 50 tooth worm gear will be needed to give a ratio of 50:1.

Fifty teeth are above the minimum recommended (see Table 11.1).

An estimate for the center distance needs to be made. If the center distance between the worm and wheel is taken as 140 mm, then from Eqns (11.5) and (11.6), the pitch diameter range is found to be between 20.72 and 70.55 mm. A midvalue of 50 mm is selected here.

From Eqn (11.8),

$$d_G = 2C - d = (2 \times 140) - 50 = 230.0 \text{ mm}$$

The lead, from Eqn (11.2), is

$$L = \frac{\pi d_G N_W}{N_G} = \pi \times 230.0 \times \frac{1}{50} = 14.45 \text{ mm}$$

The lead angle, from Eqn (11.3),

$$\lambda = \tan^{-1}\left(\frac{L}{\pi d_w}\right) = \tan^{-1}\left(\frac{14.45}{\pi \times 50}\right) = 5.256^\circ$$

Generally, self-locking occurs for lead angles below 6°. This is less than 6° so the worm set will be self-locking.

The face width can be determined from Eqn (11.11)

$$F_G = 0.67d = 0.67 \times 50 = 33.5 \text{ mm}$$

The materials factor for sand cast gears is given by Eqn (11.42), if the mean worm diameter is greater than 63.5 mm. Here, the mean worm diameter is 50 mm so $C_s = 1000$

$$m_G = 50.$$

From Eqn (11.34),

$$C_m = 0.7896$$

The tangential velocity at the mean worm diameter can be determined from Eqn (11.32),

$$V_t = \frac{nd}{19,098 \cos \lambda} = \frac{1470 \times 50.0}{19,098 \cos 5.256} = 3.865 \text{ m/s}$$

The velocity factor, from Eqn (11.37), is

$$C_v = 0.3013$$

From Eqn (11.30), the tangential load is given by

$$W_t = \frac{1000 \times 0.7896 \times 0.3013 \times 230^{0.8} \times 33.50}{75.948} = 8134 \text{ N}$$

The coefficient of friction is given by Eqn (11.40)

$$\mu = 0.103e^{\left(-1.185V_t^{0.45}\right)} = 0.103e^{\left(-1.185\times 3.865^{0.45}\right)} = 0.01167$$

The friction force is given by

$$W_f = \frac{fW_t}{\cos\lambda\cos\phi} = \frac{0.01167 \times 8134}{\cos 5.256 \cos 20} = 101.5 \text{ N}$$

The rated output power, Eqn (11.28), is

$$P_{\text{output}} = \frac{nW_{tg}d_g}{1.91 \times 10^7 m_G} = \frac{1470 \times 8134 \times 230}{1.91 \times 10^7 \times 50} = 2.880 \text{ kW}$$

The lost power, Eqn (11.29), is

$$P_{\text{loss}} = \frac{V_t W_f}{1000} = \frac{3.865 \times 101.5}{1000} = 0.392 \text{ kW}$$

The input-power rating is $2.880 + 0.392 = 3.272$ kW.

The efficiency of the gear set is $2.880/3.272 = 0.881 = 88.1\%$.

The output torque is given by

$$T_q = W_t \frac{d}{2} = 8134 \times \frac{0.230}{2} = 935.4 \text{ N m}$$

This torque exceeds the requirement, suggesting that the design is suitable. If the value was below that required, then an alternative center distance could be explored and, if necessary, the number of starts could be increased.

11.5 Conclusions

Worm and wheel gears are usually used for nonparallel nonintersecting right angle gear drive system applications, where a high gear ratio is required. They can also be used for medium-speed reductions. The worm is generally the driving member. The high transmission ratio leads to a compact solution for many applications in comparison to other types of gearing. For certain arrangements, self-locking is possible, and this can provide an attribute for some applications where a set position is desirable if the drive train is not braked or powered.

References

Books and Papers

Dudley, D.W., 1984. Handbook of Practical Gear Design. McGraw Hill.
Maitra, G.M., 1994. Handbook of Gear Design, second ed. Tata McGraw Hill.
Norton, R.L., 2006. Machine Design, third ed. Pearson.
Radzevich, S.P., 2012. Dudley's gear handbook of practical gear design and manufacture, 2nd ed. CRC Press.
Shigley, J.E., 1986. Mechanical Engineering Design, first metric ed. McGraw Hill.

Further Reading

Shigley, J.E., Mischke, C.R., Budynas, R.G., 2004. Mechanical Engineering Design, seventh ed. McGraw Hill.
Townsend, D.P., 1992. Dudley's Gear Handbook, second ed. McGraw Hill.

Standards

AGMA Design manual for cylindrical wormgearing. ANSI/AGMA Standard 6022-C93. Reaffirmed 2008.
AGMA Practice for enclosed cylindrical wormgear speed reducers and gearmotors. ANSI/AGMA Standard 6034-B92. Reaffirmed 2005.
BS 721-1:1963. Specification for worm gearing. Imperial units.
BS 721-2:1983. Specification for worm gearing. Metric units.
BS ISO TR 10828:1997. Worm gears. Geometry of worm profiles.
PD ISO/TR 14521:2010. Gears. Calculation of load capacity of wormgears.

Web Sites

At the time of going to press the world-wide-web contained useful information relating to this chapter at the following sites:

www.ashokaengineering.com/
www.bandhgears.co.uk
www.bellgears.co.uk
www.bostongear.com/products/open/worms.html
www.brentwingearcompany.co.uk
www.davall.co.uk
www.delroyd.com
www.gearcutting.com
www.gearmanufacturer.net/
www.girard-transmissions.com
www.hewitt-topham.co.uk/
www.hopwoodgear.com
www.hpcgears.com
www.huco.com
www.mmestrygears.com
www.muffettgears.co.uk/
www.qtcgears.com/
www.rarodriguez.co.uk
www.traceygear.com
www.wmberg.com

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

a	addendum (mm)
b	dedendum (mm)
c	clearance (mm)
C	center distance (mm)
C_m	ratio correction factor
C_s	materials factor
C_v	velocity factor
d	worm pitch diameter (mm)
d_g	mean diameter of the gear (mm)
d_G	pitch diameter of the worm gear (mm)
d_m	mean diameter of the worm gear (mm)
d_{max}	maximum pitch diameter of the worm (mm)
d_{min}	minimum pitch diameter of the worm (mm)
d_o	worm outer diameter (mm)
d_r	worm root diameter (mm)
d_W	pitch diameter of the worm (mm)
D_m	mean gear diameter (mm)
D_o	worm gear outer diameter (mm)
D_r	worm gear root diameter (mm)
D_t	worm gear throat diameter (mm)
f	coefficient of friction
F	effective face width (mm)
F_G	worm gear face width (mm)
F_W	worm face width (mm)
h_t	full depth of worm thread (mm)
l_{pitch}	worm axial pitch (mm)
L	lead (mm)
m_G	gear ratio
n	rotational speed of the worm (rpm)
N_G	number of teeth on the worm gear
N_{min}	minimum number of worm gear teeth
N_W	number of threads on the worm
P_{input}	rated input power (kW)
P_{loss}	lost power (kW)
P_{output}	rated output power (kW)

R_a	roughness (μm)
T_q	torque (N m)
V_G	pitch line velocity of the gear (m/s)
V_s	sliding velocity (m/s)
V_t	sliding velocity at the mean worm diameter (m/s)
V_W	pitch line velocity of the worm (m/s)
W	resultant force (N)
W_f	friction force (N)
W_{Gr}	radial force component acting against the gear (N)
W_{Gt}	tangential force component acting against the gear (N)
W_t	tangential load on the worm gear (N)
W_{tg}	worm gear tangential force (N)
W_{Wa}	axial force component acting against the gear (N)
W_{Wa}	axial force component acting against the worm (N)
W_{Wr}	radial force component acting against the worm (N)
W_{Wt}	tangential force component acting against the worm (N)
W_x	force component in the tangential direction on the worm (N)
W_y	force component in the radial direction on the worm (N)
W_z	force component in the axial direction on the worm (N)
ϕ_n	normal pressure angle of the worm thread at the mean diameter (°)
η	efficiency
λ	lead angle (°)
ψ	helix angle (°)
ψ_G	helix angle (°)
ψ_W	worm helix angle (°)

CHAPTER 12

Belt and Chain Drives

Chapter Outline

12.1 Introduction 459
12.2 Belt Drives 461
12.2.1 Belt Selection 463
12.2.2 Wedge Belt Selection 464
12.2.3 Synchronous Belts 473
12.2.4 Flat Belt Drives 492
12.3 Chain Drives 495
12.3.1 Roller Chain Selection 501
12.4 Conclusions 510
References 510
Further Reading 510
Nomenclature 511

Abstract

Belt and chain drives are used to transmit power from one rotational drive to another. A belt is a flexible power transmission element that runs tightly on a set of pulleys. A chain drive consists of a series of pin-connected links that run on a set of sprockets. This chapter introduces various types of belt and chain drives and presents selection procedures for wedge, synchronous, and flat belts and also for roller chains.

12.1 Introduction

Belt and chain drives consist of flexible elements running on either pulleys or sprockets, as illustrated in Figure 12.1. The purpose of a belt or chain drive is to transmit power from one rotating shaft to another. The speed ratio between the driving and driven shaft is dependent on the ratio of the pulley or sprocket diameters, as is given by

$$V_{\text{pitch line}} = \omega_1 R_1 = \omega_2 R_2 \tag{12.1}$$

$$\text{Angular velocity ratio} = \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} \tag{12.2}$$

where $V_{\text{pitch line}}$ is the pitch line velocity (m/s), ω_1 is the angular velocity of the driving pulley or sprocket (rad/s), ω_2 is the angular velocity of the driven pulley or sprocket (rad/s), R_1 is the

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00012-5
Copyright © 2014 Elsevier Ltd. All rights reserved.

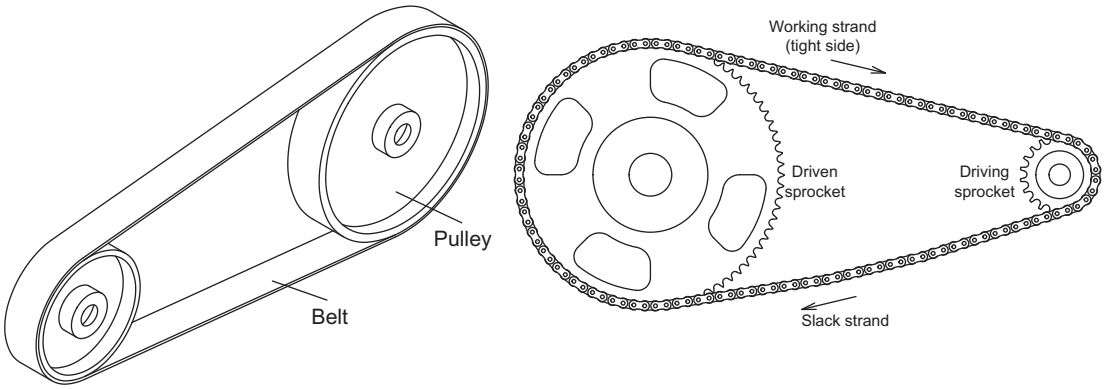


Figure 12.1
Belt drive and chain drive.

radius of the driving pulley or sprocket (m), R_2 is the radius of the driven pulley or sprocket (m).

A belt drive transmits power between shafts by using a belt to connect pulleys on the shafts by means of frictional contact or mechanical interference. A chain consists of a series of links connected by pins. The chain is designed to mesh with corresponding teeth on sprockets located on both the driving and the driven shafts.

Power transmission between shafts can be achieved by a variety of means, including belt, chain, and gear drives, and their use should be compared for suitability and optimization for any given application. In addition to power transmission, the constant speed ratio or synchronization of the angular position of the driving and driven shaft may be critical to operation. This can be achieved by means of gears, chains, or special toothed belts called synchronous or timing belts.

Both belt and chain drives can transmit power between shafts that are widely separated, giving designers greater scope for control over machine layout. In comparison to gears, they do not require as much precision in the location of center distances. Both belt and chain drives are often cheaper than the equivalent gear drive. Belts and chains are generally complementary, covering a range of operational requirements.

In general, belt drives are used when rotational speeds are of the order of 10−60 m/s. At lower speeds, the tension in the belt becomes too high for typical belt sections. At higher speeds, centrifugal forces throw the belts off the pulleys reducing the torque capacity and dynamic phenomena, and thus reducing the effectiveness and life of the belt drive. Chain drives are typically used at lower speeds and consequently higher torques than belts. Recall that for a rotating machine, torque is proportional to power/ω. For a given power, as the angular velocity decreases, the torque increases.

Belt drives have numerous advantages over gear and chain drives, including easy installation, low maintenance, high reliability, adaptability to nonparallel drive, and high transmission speeds. The principal disadvantages of belt drives are their limited power transmission capacity and limited speed ratio capability. Belt drives are less compact than either gear or chain drives, and they are susceptible to changes in environmental conditions such as contamination with lubricants. In addition, vibration and shock loading can damage belts.

Chains are usually more compact than belt drives for a given speed ratio and power capacity. Chain drives are generally more economical than the equivalent gear drive and are usually competitive with belt drives. Chains are inherently stronger than belt drives due to the use of steels in their manufacture, and they can therefore support higher tension and transmit greater power. The disadvantages of chain drives are limited speed ratios and power transmission capability, as well as safety issues. Chain drives can break and get thrown off the sprockets with strong forces and high speeds. Guards should be provided for chain drives and some belt drives to prevent damage caused by a broken chain or belt and also to prevent careless access to the chain or belt drive.

12.2 Belt Drives

As mentioned previously, a belt drive consists of a flexible element that runs on a set of pulleys mounted on the shafts. Belt drives can be used simply to transmit power from one shaft to another with the speed of the driving and driven shaft being equal. In this case, the pulley diameters would be equal. Alternatively, the driven shaft velocity can be decreased by using a bigger diameter pulley on the driven shaft than on the driving shaft. It is also possible to use belt drives to step up or increase the speed of a driven shaft, but this is a less common application.

There are various types of belt drive configurations, including flat, round, V, wedge, and synchronous belt drives, each with their individual merits. The cross sections of various belts are illustrated in Figure 12.2. Most belts are manufactured from rubber or polymer-based materials.

Power is transmitted by means of friction in the case of flat, round, wedge, and V belts and by a combination of friction and positive mechanical interference in the case of synchronous belt drives. When the driven shaft rotates, friction between the pulley and the belt causes the belt to grip the pulley, increasing the tension in the side near to the point of first rotational contact. The tensile force exerts a tangential force, and associated torque, on the driven pulley. The opposite side of the belt is also under tension but to a lesser extent, and it is referred to as the slack side.

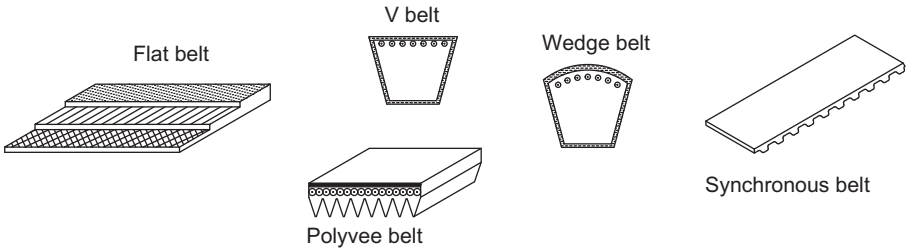


Figure 12.2
Various belt cross sections.

A frequent application of belt drives is to reduce the speed output from electric motors, which typically run at specific synchronous speeds that are high in comparison to the desired application drive speed. Because of their good "twistability," belt drives are well suited to applications where the rotating shafts are in different planes. Some of the standard layouts are shown in Figure 12.3. Belts are installed by moving the shafts closer together, slipping the belt over the pulleys, and then moving the shafts back into their operating locations.

Flat belts have high strength, can be used for large speed ratios (>8:1), have a low pulley cost, produce low noise levels, and are good at absorbing torsional vibration. The belts are typically made from multiple plies, with each layer serving a special purpose. A typical three-ply belt consists of a friction ply made from synthetic rubber, polyurethane, or chrome leather, a tension ply made from polyamide strips or polyester cord, and an outer skin made from

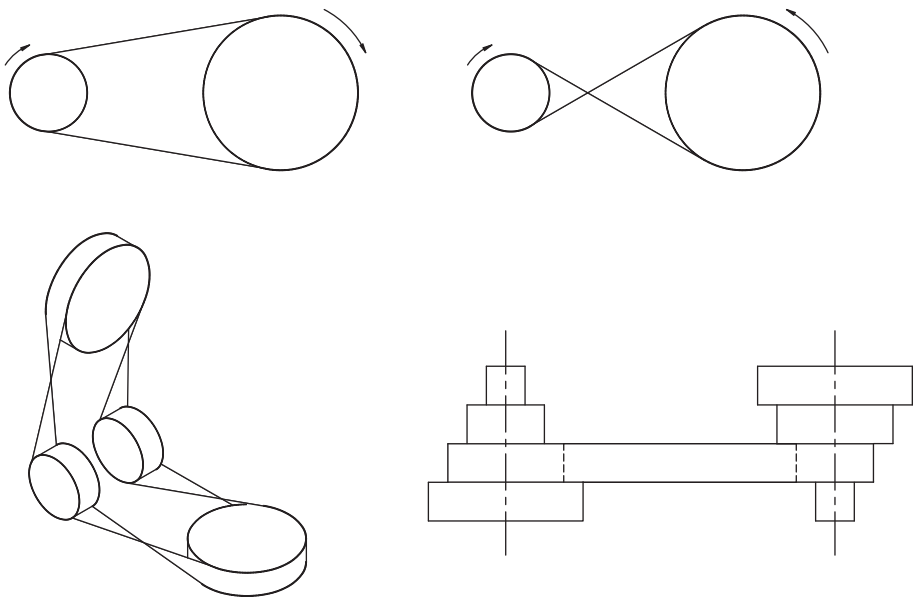


Figure 12.3
Example of belt drive configurations.

polyamide fabric, chrome leather, or an elastomer. The corresponding pulleys are made from cast iron or polymer materials and are relatively smooth to limit wear. The driving force is limited by the friction between the belt and the pulley. Applications include manufacturing tools, saw mills, textile machinery, food processing machines, multiple spindle drives, pumps, and compressors.

The most widely used type of belt in industrial and automotive applications is the V belt or wedge belt. It is familiar from its automotive application, where it is used to connect the crankshaft to accessory drives such as the alternator, water pump, and cooling fan. It is also used for general engineering purposes from domestic appliances to heavy-duty rolling machines. The V or wedge shape causes the belt to wedge into the corresponding groove in the pulley, increasing friction and torque capacity. Multiple belts are commonly used so that a cheaper small cross-sectional area belt can be used to transmit more power. Note that long center distances are not recommended for V or wedge belts.

Synchronous belts, also called timing belts, have teeth that mesh with corresponding teeth on the pulleys (see Perneder and Osbourne, 2012). This mechanical interference or positive contact between the pulleys and the belt provides angular synchronization between the driving and the driven shafts and ensures a constant speed ratio. Synchronous belts combine the advantages of normal friction belt drives with the capability of synchronous drive. The meshing of the belt and pulleys is critical to their effective operation. The teeth of the advancing belt must mesh correctly with the corresponding grooves on the pulley wheels and remain in mesh throughout the arc of contact. To achieve this, the pitch of the belts and the pulleys must correspond exactly. A disadvantage of synchronous belts can be the noise generated by the compression of air between the teeth, especially at high speeds.

Table 12.1 lists the comparative merits of various belt drives. Figure 12.4 provides a guide for the selection of belt type.

12.2.1 Belt Selection

Two approaches are presented here for the selection of a belt drive. Use can be made of the design procedures and accompanying charts provided by most belt drive manufacturers.

Table 12.1: Comparison of belt performance.

Parameter	Flat belt drive	V belt drive	Wedge belt drive	Synchronous belt drive
Optimum efficiency	98%	80%	86%	98%
Maximum speed (m/s)	70	30	40	50
Minimum pulley diameter (mm)	40	67	60	16
Maximum speed ratio	20	7	8	9
Optimum tension ratio	2.5	5	5	—

Source: Reproduced from Hamilton (1994).

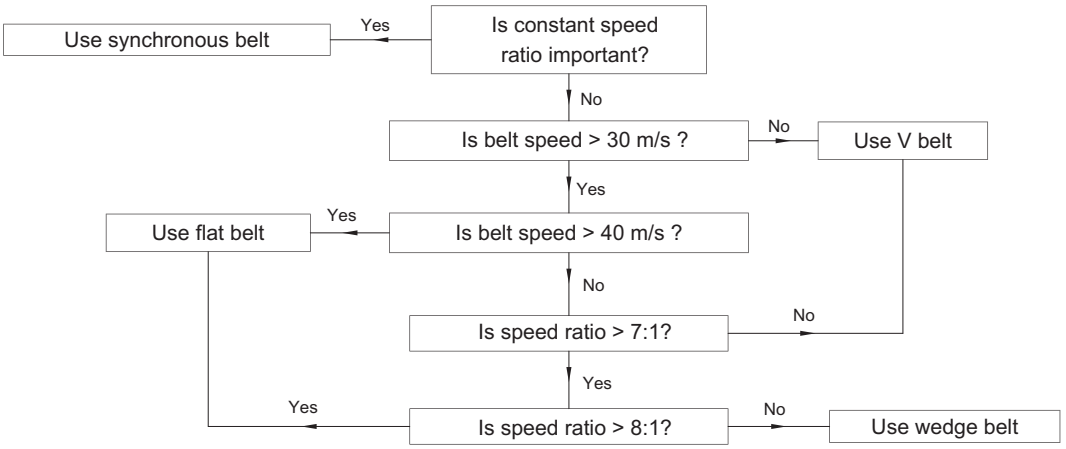


Figure 12.4
Procedure for the selection of belt type. *After Hamilton (1994).*

The use of charts for the belt selection is illustrated by considering a wedge belt drive in Section 12.2.2, and for synchronous belts in Section 12.2.3. Alternatively, use can be made of fundamental relationships for the belt tensions and torque transmission and a belt selected based on its maximum permissible tensile stress. These equations are illustrated by an example using a flat belt drive in Section 12.2.4.

12.2.2 Wedge Belt Selection

The typical design and selection procedure for use in conjunction with power-speed rating charts supplied by commercial belt companies is outlined below for a wedge belt.

1. Define the operating conditions.	These include the nominal power to be transmitted, the rotational speeds of the shafts, and any space, layout, or other constraints, such as environmental conditions.
2. Determine the service factor.	Service factors are used to down-rate the power transmission capability listed by belt suppliers to account for the differences between practical applications and test conditions. Table 12.2 lists typical values for service factors.
3. Calculate the design power.	This is the product of the nominal power and the service factor.
4. Select the belt type.	To assist in the selection of which type of belt drive to use, the procedure given in Figure 12.4 can be used as a guideline.
5. Select a belt.	Using a manufacturer's rating chart, select a specific belt for the design power and speed.

6. Select the pulley diameters.	Pulleys are normally available in standard sizes. Choose the smallest sizes available such that the speed ratio is acceptable.
7. Set the center distance.	This is dependent on the application. As a general guideline, the center distance should be greater than the diameter of the larger pulley.
8. Determine the belt length.	Note that belts are usually manufactured in standard lengths, so some iteration around the design parameters may be necessary to arrive at a satisfactory compromise.
9. Apply power-correction factors.	These are used to compensate for speed ratio and the belt geometry and are provided within belt manufacturer's design guides.
10. Determine the allowable power per belt (or per belt width for flat belts).	This is a function of the belt dimensions and is available from the manufacturer's design guides.
11. Determine the number of belts.	The number of belts is given by dividing the design power by the allowable power per belt and rounding up to the nearest integer.

Figure 12.5 shows a design chart for power rating versus speed, and Tables 12.3−12.6 list values for the minimum pulley diameter, center distance, power ratings, and additional power increment for wedge belts, and additional power increments per SPB belt. The use of these in conjunction with the procedure given is illustrated by the following examples (Examples 12.1 and 12.2).

Example 12.1

Select a wedge belt and determine the pulley diameters for a reciprocating compressor driven by a 28 kW two-cylinder diesel engine. The engine speed is 1500 rpm and the compressor speed is 950 rpm. The proposed distance between the engine and compressor shaft centers is approximately 1.5 m. The system is expected to be used for less than 10 h per day.

Solution

The speed ratio is $\dfrac{1500}{950} = 1.58$.

From Table 12.2 for "heavy starts," "heavy duty," and less than 10 h operation per day, the service factor is 1.4.

The design power $= 28 \times 1.4 = 39.2$ kW.

From Figure 12.5, the combination of power equal to 39.2 kW and the speed equal to 1500 rpm is found to be within the envelope curve and range defined as suitable for SPB belt drives. The combination is also with the envelope range suitable for SPA belts. As the

Table 12.2: Service factors.

	Type of prime mover					
	Soft starts. Electric motors: AC—star delta start, DC—shunt wound. IC engines with four or more cylinders. Prime movers fitted with centrifugal clutches, dry or fluid couplings, or electronic soft start drives.			Heavy starts. Electric motors: AC—star delta start, DC—shunt wound. IC engines with fewer than four cylinders. Prime movers not fitted with soft start drives.		
	Duty (hours per day)					
Types of driven machine	**<10**	**10—16**	**>16**	**<10**	**10—16**	**>16**
Light duty, e.g. agitators (uniform density), blowers, exhausters, and fans (up to 7.5 kW), centrifugal compressors, rotodynamic pumps, uniformly loaded belt conveyors.	1.0	1.1	1.2	1.1	1.2	1.3
Medium duty, e.g. agitators (variable density), blowers, exhausters, and fans (over 7.5 kW), rotary compressors and pumps (other than centrifugal), nonuniformly loaded conveyors, generators, machine tools, printing machinery, and sawmill machinery.	1.1	1.2	1.3	1.2	1.3	1.4
Heavy duty, e.g. brick machinery, bucket elevators, reciprocating compressors and pumps, heavy-duty conveyors, hoists, pulverizers, punches, presses, quarry plant, and textile machinery.	1.2	1.3	1.4	1.4	1.5	1.6
Extra heavy duty, e.g. crushers.	1.3	1.4	1.5	1.5	1.6	1.8

Source: After Fenner Power Transmission UK.

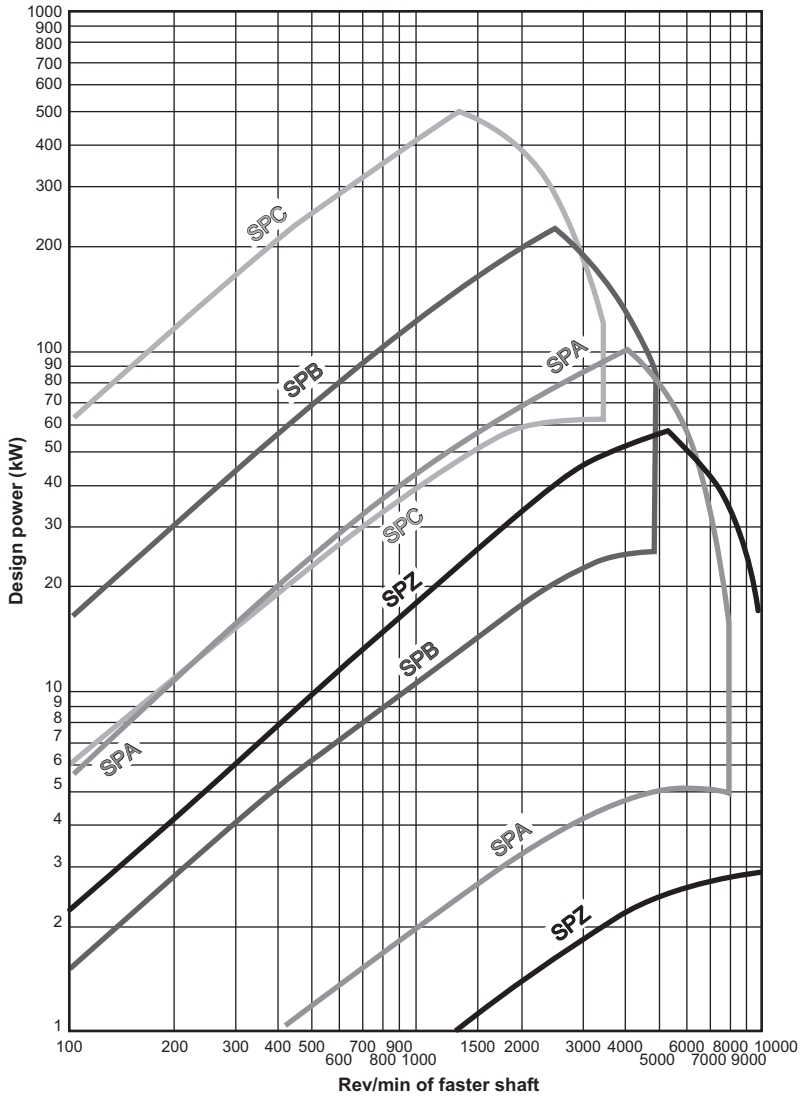


Figure 12.5
Selection chart for wedge belts. *Courtesy of Fenner Drives UK.*

power is near the upper range of the SPA curve, an SPB belt is selected here, although the choice is marginal.

From Table 12.3 for a design power of 39.2 kW and a driving shaft speed of 1500 rpm, an approximate minimum pulley diameter of 160 mm is suitable.

From Table 12.4, the actual pulley diameters can be selected by tracing down the left-hand side to the speed ratio and reading across for the pitch diameters of the driving and driven

Table 12.3: Minimum recommended pulley diameters for wedge belt drives.

	Minimum pulley diameters (mm) Design power (kW)																			
Speed of faster shaft (rpm)	**<1**	**3.0**	**4.0**	**5.0**	**7.5**	**10.0**	**15.0**	**20.0**	**25**	**30**	**40**	**50**	**60**	**75**	**90**	**110**	**130**	**150**	**200**	**250**
500	56	90	100	112	125	140	180	200	212	236	250	280	280	315	375	400	450	475	500	560
600	56	85	90	100	112	125	140	180	200	212	224	250	265	280	300	335	375	400	475	500
720	56	80	85	90	100	106	132	150	160	170	200	236	250	265	280	300	335	375	450	500
960	56	75	80	85	95	100	112	132	150	180	180	200	224	250	280	280	300	335	400	450
1200	56	71	80	80	95	95	106	118	132	150	160	180	200	236	236	250	265	300	335	355
1440	56	63	75	80	85	85	100	112	125	140	160	170	190	212	236	236	250	280	315	335
1800	56	63	71	75	80	85	95	106	112	125	150	160	170	190	212	224	236	265	300	335
2880	56	60	67	67	80	80	85	90	100	112	125	140	160	170	180	212	224	236	—	—

Source: Courtesy of Fenner Power Transmission UK.

Table 12.4: Center distances for selected SPB wedge belts. N.B. this is only a partial selection from a typical catalogue relevant to the worked examples.

Combined arc and belt length correction factor					0.85		0.90		0.95		1.00			1.05			1.10		1.15	
	Pitch diameter of pulleys		Power per belt (kW)		Belt Length for SPB belt type															
Speed ratio	Driver	Driven	1440 rpm	960 rpm	1250	1400	1800	2000	2240	2500	2800	3150	3550	4000	4500	5000	5600	6300	7100	8000
1.27	315	400	24.56	17.91	–	–	–	436	557	687	837	1013	1213	1438	1688	1938	2238	2588	2988	3438
1.27	118	150	6.37	4.58	414	489	689	789	–	–	–	–	–	–	–	–	–	–	–	–
1.28	125	160	7.24	5.18	401	476	676	776	–	–	–	–	–	–	–	–	–	–	–	–
1.29	140	180	7.69	5.54	373	448	648	748	869	998	1148	1342	1524	1749	1999	2249	2549	2899	3299	3749
1.29	132	170	8.10	5.78	387	462	663	763	–	–	–	–	–	–	–	–	–	–	–	–
1.53	118	180	6.51	4.67	390	465	665	765	–	–	–	–	–	–	–	–	–	–	–	–
1.56	180	280	12.03	8.61	259	335	536	637	757	887	1038	1213	1413	1638	1888	2138	2438	2788	3188	3638
1.56	160	250	9.95	7.13	300	375	576	676	797	927	1077	1252	1452	1677	1927	2178	2478	2828	3228	3678
1.57	150	236	8.89	6.37	319	394	595	696	816	946	1096	1271	1471	1696	1946	2196	2496	2847	3247	3697
1.57	200	315	14.05	10.05	–	290	492	593	713	844	994	1169	1369	1594	1845	2095	2395	2745	3145	3595
1.58	224	355	16.54	11.86	–	–	440	541	662	793	943	1118	1319	1544	1794	2044	2344	2694	3095	3545
1.59	315	500	24.81	18.09	–	–	–	–	471	603	754	930	1131	1357	1607	1858	2158	2508	2908	3359
1.60	125	200	7.49	5.35	368	443	644	744	–	–	–	–	–	–	–	–	–	–	–	–
1.60	140	224	7.95	5.71	336	412	613	713	833	963	1113	1288	1489	1714	1964	2214	2514	2864	3264	3714
1.60	250	400	19.02	13.68	–	–	382	484	605	736	886	1062	1262	1488	1738	1988	2288	2368	3039	3489
1.87	190	355	13.17	9.42	–	–	465	566	687	818	968	1144	1344	1570	1820	2070	2371	2721	3121	3571
1.89	212	400	15.37	11.00	–	–	409	511	633	764	915	1090	1291	1516	1767	2017	2317	2668	3068	3518
1.89	125	236	7.49	5.35	337	413	614	714	–	–	–	–	–	–	–	–	–	–	–	–
1.89	112	212	5.88	4.23	367	443	644	744	–	–	–	–	–	–	–	–	–	–	–	–
1.89	132	250	8.36	5.95	320	396	597	697	–	–	–	–	–	–	–	–	–	–	–	–
2.09	170	355	11.21	8.02	–	–	479	580	702	833	983	1159	1360	1585	1835	2086	2386	2736	3136	3586
2.10	150	315	9.11	6.53	–	324	528	629	750	881	1031	1207	1407	1633	1883	2133	2433	2784	3184	3634
2.11	190	400	13.26	9.48	–	–	424	526	648	780	931	1107	1307	1533	1784	2034	2334	2685	3085	3535
2.11	112	236	5.97	4.30	346	422	624	724	–	–	–	–	–	–	–	–	–	–	–	–
2.12	118	250	6.72	4.81	429	406	607	708	–	–	–	–	–	–	–	–	–	–	–	–
2.23	224	500	16.64	11.92	–	–	–	408	534	667	820	997	1198	1425	1676	1926	2227	2578	2978	3429
2.24	125	280	7.59	5.42	297	374	577	677	–	–	–	–	–	–	–	–	–	–	–	–
2.25	140	315	8.04	5.78	252	331	535	637	758	888	1039	1214	1415	1640	1891	2141	2441	2791	3191	3642
2.25	280	630	21.87	15.81	–	–	–	–	364	505	662	842	1046	1273	1525	1777	2078	2429	2830	3281
2.35	170	400	11.21	8.02	–	–	437	540	663	794	945	1121	1322	1548	1799	2049	2350	2700	3100	3550
2.54	315	800	24.81	18.15	–	–	–	–	–	–	–	654	865	1097	1353	1606	1909	2261	2663	3115
2.63	190	500	13.26	9.48	–	–	–	430	557	691	844	1021	1223	1450	1701	1952	2253	2603	3004	3455
2.67	150	400	9.11	6.53	–	–	451	554	677	808	960	1136	1337	1563	1814	2064	2365	2715	3116	3566
2.67	118	315	6.72	4.81	267	346	551	652	–	–	–	–	–	–	–	–	–	–	–	–
2.67	236	630	17.79	12.77	–	–	–	–	390	533	692	873	1077	1305	1557	1809	2111	2462	2863	3314
3.39	236	800	17.86	12.82	–	–	–	–	–	–	–	705	918	1152	1408	1662	1966	2319	2722	3174
3.50	180	630	12.31	8.80	–	–	–	–	425	569	729	911	1116	1345	1598	1850	2152	2504	2905	3356
3.57	112	400	6.04	4.34	–	–	476	580	–	–	–	–	–	–	–	–	–	–	–	–
3.57	140	500	8.11	5.82	–	–	351	462	590	725	879	1057	1259	1486	1738	1989	2290	2641	3042	3493
3.57	224	800	16.71	11.97	–	–	–	–	–	–	–	713	926	1160	1416	1671	1975	2328	2731	3183
3.57	280	1000	21.94	15.85	–	–	–	–	–	–	–	–	673	925	1190	1450	1758	2114	2519	2973
3.71	170	630	11.28	8.06	–	–	–	–	431	576	736	918	1123	1352	1605	1857	2159	2511	2913	3364
3.77	212	800	15.53	11.11	–	–	–	–	–	–	–	720	934	1168	1425	1679	1983	2337	2739	3192
3.79	132	500	8.52	6.06	–	–	356	467	–	–	–	–	–	–	–	–	–	–	–	–
3.94	160	630	10.23	7.32	–	–	–	–	437	582	742	925	1130	1359	1612	1865	2167	2519	2920	3371
4.72	212	1000	15.53	11.11	–	–	–	–	–	–	–	–	714	968	1235	1496	1805	2162	2568	3022
5.00	160	800	10.23	7.32	–	–	–	–	–	–	554	753	968	1203	1461	1716	2021	2374	2778	3230
5.00	200	1000	14.34	10.25	–	–	–	–	–	–	–	–	722	976	1243	1504	1813	2171	2576	3031
5.26	190	1000	13.33	9.53	–	–	–	–	–	–	–	–	728	982	1250	1511	1820	2178	2584	3038
5.33	150	800	9.18	6.57	–	–	–	–	–	–	–	759	975	1210	1468	1723	2028	2382	2785	3238

Source: Courtesy of Fenner Power Transmission UK.

Table 12.5: Power ratings for SPB wedge belts.

	Rated power (kW) per belt for small pulley pitch diameter (mm)												
rpm of faster shaft	**140**	**150**	**160**	**170**	**180**	**190**	**200**	**212**	**224**	**236**	**250**	**280**	**315**
100	0.73	0.82	0.92	1.01	1.10	1.20	1.29	1.40	1.51	1.62	1.74	2.01	2.33
200	1.33	1.51	1.69	1.87	2.05	2.22	2.40	2.61	2.82	3.02	3.26	3.78	4.37
300	1.89	2.15	2.41	2.67	2.93	3.18	3.44	3.74	4.04	4.35	4.70	5.44	6.30
400	2.42	2.76	3.09	3.43	3.77	4.10	4.43	4.83	5.22	5.61	6.07	7.04	8.15
500	2.92	3.33	3.75	4.16	4.57	4.98	5.39	5.87	6.36	6.84	7.39	8.58	9.94
600	3.40	3.89	4.38	4.87	5.35	5.83	6.31	6.89	7.45	8.02	8.67	10.06	11.66
700	3.86	4.43	4.99	5.55	6.11	6.66	7.21	7.87	8.52	9.17	9.92	11.50	13.32
720	3.95	4.53	5.11	5.69	6.26	6.82	7.39	8.06	8.73	9.39	10.16	11.79	13.65
800	4.31	4.95	5.59	6.22	6.84	7.47	8.08	8.82	9.55	10.28	11.12	12.90	14.93
900	4.75	5.46	6.16	6.86	7.56	8.25	8.93	9.75	10.56	11.36	12.29	14.25	16.47
960	5.00	5.75	6.50	7.24	7.98	8.71	9.43	10.29	11.15	11.99	12.97	15.03	17.37
1000	5.17	5.95	6.72	7.49	8.25	9.01	9.76	10.65	11.53	12.41	13.42	15.55	17.96
1100	5.58	6.42	7.27	8.10	8.93	9.75	10.56	11.52	12.48	13.43	14.52	16.80	19.39
1200	5.97	6.89	7.79	8.69	9.58	10.46	11.34	12.37	13.40	14.41	15.57	18.01	20.75
1300	6.36	7.34	8.31	9.27	10.22	11.16	12.09	13.19	14.28	15.36	16.59	19.17	22.05
1400	6.73	7.77	8.81	9.83	10.84	11.84	12.82	13.99	15.14	16.27	17.57	20.28	23.28
1440	6.88	7.95	9.00	10.05	11.08	12.10	13.11	14.30	15.47	16.63	17.96	20.70	23.75
1500	7.09	8.20	9.29	10.37	11.44	12.49	13.53	14.76	15.97	17.15	18.51	21.33	24.43
1600	7.44	8.61	9.76	10.90	12.02	13.12	14.21	15.50	16.76	18.00	19.41	22.33	25.51
1700	7.78	9.01	10.21	11.40	12.58	13.73	14.87	16.21	17.52	18.81	20.27	23.27	26.51
1800	8.11	9.39	10.65	11.90	13.12	14.32	15.50	16.89	18.25	19.58	21.08	24.15	27.43
1900	8.43	9.76	11.08	12.37	13.64	14.88	16.11	17.54	18.94	20.31	21.85	24.97	28.27
2000	8.73	10.12	11.48	12.82	14.14	15.43	16.69	18.16	19.60	20.99	22.57	25.72	29.01
2100	9.02	10.46	11.88	13.26	14.62	15.94	17.24	18.75	20.22	21.64	23.23	26.41	29.67
2200	9.31	10.79	12.25	13.68	15.07	16.44	17.76	19.31	20.80	22.24	23.85	27.03	30.22
2300	9.57	11.11	12.61	14.08	15.51	16.90	18.26	19.83	21.35	22.80	24.42	27.57	30.68
2400	9.83	11.41	12.95	14.46	15.92	17.34	18.72	20.32	21.85	23.31	24.93	28.05	31.04
2500	10.08	11.70	13.28	14.82	16.31	17.76	19.16	20.77	22.31	23.78	25.38	28.44	—
2600	10.31	11.97	13.59	15.16	16.68	18.14	19.56	21.19	22.73	24.19	25.78	28.76	—
2700	10.53	12.23	13.88	15.47	17.02	18.50	19.93	21.56	23.11	24.56	26.12	28.99	—
2800	10.73	12.47	14.15	15.77	17.33	18.83	20.27	21.90	23.44	24.87	26.40	—	—
2880	10.89	12.65	14.35	15.99	17.57	19.07	20.51	22.14	23.67	25.08	26.57	—	—
2900	10.93	12.69	14.40	16.04	17.62	19.13	20.57	22.20	23.72	25.12	26.61	—	—
3000	11.10	12.90	14.63	16.30	17.89	19.40	20.84	22.46	23.96	25.33	26.76	—	—

Source: Courtesy of Fenner Power Transmission UK.

Table 12.6: Additional power increment per SPB belt.

Speed of faster shaft (rpm)	Additional power (kW) per belt for speed ratio									
	1.00 to 1.01	1.02 to 1.05	1.06 to 1.11	1.12 to 1.18	1.19 to 1.26	1.27 to 1.38	1.39 to 1.57	1.58 to 1.94	1.95 to 3.38	3.39 and over
100	0.00	0.01	0.02	0.04	0.04	0.06	0.07	0.07	0.08	0.08
200	0.00	0.01	0.04	0.07	0.09	0.11	0.13	0.15	0.16	0.17
300	0.00	0.02	0.06	0.10	0.14	0.17	0.20	0.22	0.24	0.25
400	0.00	0.03	0.07	0.13	0.19	0.22	0.26	0.29	0.32	0.34
500	0.00	0.04	0.09	0.17	0.23	0.28	0.33	0.37	0.40	0.43
600	0.00	0.04	0.12	0.20	0.28	0.34	0.40	0.45	0.48	0.51
700	0.00	0.05	0.13	0.24	0.33	0.39	0.46	0.52	0.57	0.59
720	0.00	0.05	0.14	0.25	0.33	0.41	0.48	0.54	0.59	0.62
800	0.00	0.06	0.16	0.28	0.37	0.45	0.53	0.60	0.65	0.69
900	0.00	0.07	0.18	0.31	0.42	0.51	0.60	0.66	0.72	0.77
960	0.00	0.07	0.19	0.32	0.44	0.54	0.62	0.70	0.77	0.81
1000	0.00	0.07	0.19	0.34	0.46	0.56	0.66	0.74	0.81	0.86
1100	0.00	0.08	0.22	0.37	0.51	0.62	0.72	0.81	0.89	0.94
1200	0.00	0.09	0.23	0.41	0.56	0.68	0.79	0.89	0.97	1.03
1300	0.00	0.09	0.25	0.44	0.60	0.73	0.86	0.96	1.05	1.11
1400	0.00	0.10	0.28	0.48	0.65	0.79	0.93	1.04	1.13	1.20
1440	0.00	0.10	0.28	0.48	0.66	0.79	0.94	1.06	1.15	1.21
1500	0.00	0.10	0.29	0.51	0.69	0.84	0.99	1.11	1.21	1.28
1600	0.00	0.11	0.31	0.54	0.75	0.90	1.05	1.19	1.29	1.37
1700	0.00	0.12	0.34	0.58	0.79	0.95	1.12	1.26	1.37	1.45
1800	0.00	0.13	0.35	0.61	0.84	1.01	1.19	1.34	1.45	1.54
1900	0.00	0.13	0.37	0.65	0.88	1.07	1.25	1.41	1.54	1.63
2000	0.00	0.14	0.39	0.68	0.93	1.13	1.32	1.48	1.62	1.71
2100	0.00	0.15	0.41	0.72	0.98	1.18	1.39	1.56	1.69	1.79
2200	0.00	0.16	0.43	0.75	1.02	1.24	1.45	1.63	1.78	1.88
2300	0.00	0.16	0.45	0.78	1.07	1.29	1.51	1.71	1.86	1.97
2400	0.00	0.17	0.47	0.82	1.11	1.35	1.58	1.78	1.94	2.05
2500	0.00	0.18	0.49	0.85	1.16	1.41	1.65	1.86	2.02	2.14
2600	0.00	0.19	0.51	0.89	1.21	1.46	1.72	1.92	2.10	2.22
2700	0.00	0.19	0.53	0.92	1.25	1.52	1.78	1.99	2.18	2.31
2800	0.00	0.20	0.54	0.95	1.29	1.57	1.84	2.07	2.26	2.39
2880	0.00	0.20	0.56	0.97	1.32	1.60	1.88	2.11	2.31	2.44
2900	0.00	0.21	0.57	0.99	1.34	1.63	1.91	2.15	2.34	2.48
3000	0.00	0.22	0.59	1.02	1.39	1.69	1.98	2.23	2.42	2.57

Source: Courtesy of Fenner Power Transmission UK.

pulleys. The minimum diameter from Table 12.3 can be used as a guideline when there is a choice.

From Table 12.4 for a speed ratio of 1.58, suitable pitch diameters are $D_1 = 224$ mm and $D_2 = 355$ mm. The nearest center distance listed at the desired 1.5 m is 1.544 m; tracing the column upward defines the belt length as 4000 mm, and by tracing the shading upward the arc-length correction factor is 1.05.

From Table 12.5, the rated power per belt for $n_1 = 1500$ rpm and $D_1 = 224$ mm is 15.97 kW/belt.

From Table 12.6 the additional power per belt accounting for the speed ratio is 1.11 kW.

The corrected value for the power per belt is $(15.97 + 1.11) \times 1.05 = 17.93$ kW/belt.

The total number of belts is therefore $\dfrac{39.2}{17.93} = 2.19$.

Because a partial belt cannot be used, this number should be rounded up to the nearest integer. Thus three SPB4000 belts should be used running on sprockets of 224 and 315 mm pitch diameter with a center distance of 1.544 m.

Example 12.2

A wedge belt drive is required to transmit 18.5 kW from an electric motor running at 1455 rpm to a uniformly loaded conveyor running at 400 rpm. The desired center distance is 1.4 m and expected use is 15 h a day. Select a suitable belt, or belts, and determine the pulley diameters.

Solution

18.5 kW, $n_1 = 1455$ rpm, $n_2 = 400$ rpm, $C \approx 1.4$ m.

The speed ratio is $1455/400 = 3.6375$.

From Table 12.2, the service factor is 1.1.

Design power $= 18.5 \times 1.1 = 20.35$ kW.

From Figure 12.5, an SPB belt is suitable.

From Table 12.3, the minimum pulley diameter is approximately 112 mm.

From Table 12.4, selecting the closest ratio as 3.57, $D_1 = 224$ mm, $D_2 = 800$ mm, and $C = 1.416$ m (SPB4500). The combined arc and belt length correction factor is 1.0.

From Table 12.5, the rated power per belt is approximately 15.47 kW. From Table 12.6, the additional power per belt is 1.21 kW.

The corrected power per belt is $(15.47 + 1.21) \times 1.0 = 16.68$ kW.

$$\text{Number of belts required} = \frac{20.35}{16.68} = 1.22.$$

Rounding up, use two SPB4500 belts with pulley diameters of 224 and 800 mm and center distance 1.416 m.

12.2.3 Synchronous Belts

Synchronous belt drives have teeth on the belt and corresponding grooves on the pulley wheels. The meshing teeth provide positive angular location and hence there is normally no relative motion or slipping between the two elements in mesh giving a constant speed ratio between the driving and driven shafts. Synchronous belts can therefore be used for applications such as automatic machinery where a definite motion sequence or indexing is necessary.

The principal dimensions of a synchronous belt drive are the number of grooves, the pitch, and the width. The belt pitch is the distance between two adjacent tooth centers measured on the pitch line of the belt. The belt pitch length is the total length of the belt measured along the pitch line.

The selection procedure for synchronous belts is specific to the belt manufacturer concerned, although there are similarities. Here a procedure based on TDP3 belt drives provided by Fenner is presented as outlined in the following box, Figure 12.6 and Tables 12-7—12.19.

1.	Define the principal requirements.	These include the nominal power to be transmitted; the rotational speeds of the shafts; the power capability and starting characteristics of the prime mover and power absorbed by the driven machine; any space, layout, or other constraints such as the center distance and shaft diameters; environmental conditions.
2.	Determine the service factor.	Service factors are used to down-rate the power transmission capability listed by belt suppliers to account for the differences between practical applications and test conditions. Table 12.13 lists typical values for service factors.
3.	Calculate the design power.	This is the product of the nominal power and the service factor.
4.	Belt pitch.	Use the belt pitch selection chart, Figure 12.6, to identify a suitable pitch for the drive. The chart defines envelopes of suitability based on the intersection between the smaller pulley speed and design power to identify the belt pitch.
5.	Speed ratio.	This is the ratio of rotational speed of the faster shaft over the rotational speed of the slower shaft.
6.	Pulley selection.	The charts given in Tables 12.14—12.19 can be used for the particular pitch concerned to identify the number of grooves for the driving and driven pulleys.

7.	Belt length and center distance.	Using the charts given in Tables 12.14–12.19, read across the row for the selected number of grooves for the driving and driven pulleys and select the center distance that is closest to that required.
8.	Power rating and belt width.	For the chosen belt pitch, select the appropriate power rating table, Table 12.7 or Table 12.10. Locate the smaller pulley groove number speed combination and note the power rating. Multiply this rating by the belt length factor, Table 12.8 or Table 12.11. Divide the design power by the length corrected power rating to give the required belt width factor. Refer to Tables 12.9 and 12.12 and select the belt width with a factor equal to or greater than that required.

Example 12.3

A synchronous belt is required for a gear pump that is driven by a 1470 rpm AC electric motor. The pump rotational speed required is 720 rpm within ±15 rpm. The motor has been matched to the pump power requirements and has a rating of 50 kW. The center distance required between the motor axis and the gear pump drive axis is 700–750 mm. The pump shaft diameter is 70 mm. The pump operation required is continuous.

Solution

Assuming continuous operation and medium duty characteristics, from Table 12.13, the service factor is 1.7.

The design power is given by $1.7 \times 50 = 85$ kW.

The belt pitch selection chart, Figure 12.6, for a power of 85 kW and a small pulley speed of 1470 rpm, indicates that a TDP3 14 mm pitch belt drive would be suitable.

The speed ratio is $1470/720 = 2.042$.

From Table 12.18, in the corresponding chart for a 14 mm pitch belt with a speed ratio of 2.05, which is the nearest listed to the desired speed ratio of 2.05, the corresponding number of grooves on the driving and driven pulleys are 44 and 90, respectively.

A speed ratio of 2.05 gives an output speed of 717 rpm. This is within the ±15 rpm stated requirement.

From Table 12.18, for the 2.05 ratio, 44 and 90 groove pulley combination, a belt length of 2450 mm gives a center distance of 749 mm. This is within the center distance range requirement.

Table 12.10 gives a power rating of 69.77 for a pulley with 44 grooves running at 1450 rpm for a 40 mm belt.

Table 12.7: Power ratings (kW) for 20-mm-wide 8MXP belts.

Rev/min of small pulley	Number of grooves														
	22	24	26	28	30	32	34	36	38	40	44	48	56	64	72
100	0.43	0.50	0.57	0.65	0.73	0.81	0.89	0.97	1.06	1.14	1.30	1.45	1.71	1.88	1.94
200	0.83	0.97	1.11	1.26	1.41	1.57	1.72	1.88	2.04	2.19	2.50	2.79	3.27	3.59	3.71
300	1.23	1.43	1.64	1.85	2.07	2.30	2.53	2.76	2.99	3.21	3.66	4.07	4.76	5.22	5.37
400	1.61	1.88	2.15	2.43	2.72	3.01	3.31	3.61	3.91	4.20	4.78	5.31	6.21	6.78	6.97
500	1.99	2.32	2.65	3.00	3.35	3.71	4.08	4.44	4.81	5.17	5.87	6.52	7.60	8.29	8.51
600	2.37	2.75	3.14	3.55	3.97	4.40	4.83	5.26	5.69	6.11	6.93	7.69	8.96	9.76	9.99
720	2.82	3.27	3.73	4.22	4.71	5.21	5.73	6.23	6.74	7.24	8.20	9.09	10.57	11.49	11.74
800	3.10	3.60	4.11	4.64	5.19	5.74	6.29	6.85	7.40	7.95	8.99	9.96	11.56	12.54	12.79
960	3.65	4.23	4.83	5.45	6.08	6.72	7.37	8.01	8.65	9.28	10.50	11.62	13.44	14.54	14.80
1000	3.82	4.42	5.05	5.70	6.36	7.03	7.71	8.38	9.05	9.71	10.97	12.13	14.02	15.16	15.41
1200	4.52	5.23	5.97	6.73	7.50	8.29	9.08	9.86	10.64	11.40	12.86	14.20	16.36	17.63	17.85
1450	5.38	6.21	7.08	7.98	8.89	9.81	10.73	11.64	12.55	13.43	15.12	16.66	19.11	20.51	20.67
1600	5.57	6.43	7.33	8.25	9.18	10.12	11.07	12.00	12.93	13.83	15.55	17.12	19.58	20.95	21.07
1800	6.54	7.55	8.59	9.66	10.75	11.84	12.94	14.02	15.09	16.13	18.10	19.89	22.67	24.17	24.21
2000	7.18	8.29	9.43	10.59	11.77	12.96	14.15	15.32	16.47	17.59	19.71	21.62	24.56	26.08	26.02
2500	8.74	10.07	11.43	12.82	14.22	15.62	17.01	18.39	19.73	21.02	23.46	25.60	28.82	30.30	29.91
2850	9.79	11.26	12.76	14.29	15.83	17.37	18.89	20.38	21.83	23.24	25.84	28.11	31.42	32.79	32.10
3000	10.23	11.76	13.32	14.91	16.50	18.09	19.66	21.20	22.70	24.14	26.80	29.12	32.44	33.74	32.91
3500	11.65	13.36	15.10	16.87	18.63	20.38	22.10	23.78	25.40	26.94	29.77	32.18	35.46	36.43	35.06
4000	13.00	14.88	16.78	18.70	20.61	22.49	24.34	26.12	27.83	29.45	32.37	34.79	37.88	38.39	36.37
4500	14.28	16.31	18.36	20.41	22.44	24.44	26.37	28.23	30.00	31.66	34.61	36.98	39.71	39.63	36.86
5000	15.50	17.66	19.83	22.00	24.13	26.21	28.21	30.12	31.92	33.59	36.49	38.73	40.98	40.16	36.55
5500	16.65	18.93	21.21	23.47	25.68	27.81	29.86	31.79	33.58	35.23	38.02	40.06	41.68	40.00	35.44
6000	17.74	20.12	22.49	24.82	27.08	29.26	31.31	33.24	35.00	36.60	39.21	40.98	41.82	39.16	33.56

Source: Table reproduced courtesy of Fenner.

Table 12.8: Belt length correction factors for 20-mm-wide 8MXP belts.

Belt length (mm)	384–600	640–880	960–1200	1280–1760	1800–4400
Length factor	0.8	0.9	1.0	1.1	1.2

Source: Table reproduced courtesy of Fenner.

Table 12.9: Belt width factors for 20-mm-wide 8MXP belts.

Belt width (mm)	20	30	50	85
Width factor	1.00	1.58	2.73	4.76

Source: Table reproduced courtesy of Fenner.

Table 12.10: Power ratings (kW) for 40-mm-wide 14MXP belts.

	28	29	30	32	34	36	38	40	44	48	56	64
10	0.44	0.47	0.50	0.55	0.60	0.65	0.69	0.74	0.88	0.92	1.10	1.29
20	0.85	0.90	0.96	1.06	1.15	1.24	1.33	1.42	1.69	1.78	2.13	2.48
50	1.99	2.12	2.24	2.48	2.71	2.92	3.14	3.35	3.98	4.20	5.03	5.87
100	3.77	4.03	4.27	4.73	5.16	5.58	5.99	6.40	7.60	8.00	9.59	11.18
200	7.09	7.58	8.04	8.91	9.72	10.52	11.29	12.06	14.30	15.06	18.02	20.95
300	10.19	10.91	11.57	12.82	14.00	15.13	16.25	17.34	20.55	21.61	25.78	29.88
400	13.13	14.06	14.92	16.52	18.04	19.50	20.92	22.31	26.40	27.73	32.98	38.10
500	15.93	17.06	18.11	20.05	21.88	23.64	25.34	27.02	31.89	33.48	39.68	45.67
600	18.61	19.93	21.16	23.42	25.54	27.58	29.55	31.48	37.06	38.87	45.91	52.63
720	21.78	23.34	24.77	27.40	29.87	32.22	34.50	36.72	43.13	45.20	53.17	60.71
800	23.65	25.33	26.88	29.73	32.38	34.90	37.34	39.71	46.51	48.69	57.04	64.84
960	27.20	29.20	30.90	34.20	37.20	40.00	42.70	45.33	52.90	55.30	64.30	72.60
1000	28.30	30.30	32.14	35.50	38.60	41.55	44.38	47.11	54.64	57.29	66.51	74.85
1200	32.59	34.88	36.97	40.78	44.28	47.57	50.70	53.71	62.11	64.73	74.38	82.77
1450	37.49	40.10	42.46	46.74	50.62	54.24	57.65	60.90	69.77	72.47	82.07	89.83
1600	40.20	42.97	45.47	49.98	54.04	57.81	61.34	64.66	73.62	76.30	85.57	92.58
1800	43.54	46.50	49.72	53.93	58.18	62.08	65.69	69.07	77.83	80.50	88.95	94.56
2000	46.59	49.71	52.51	57.46	61.84	65.80	69.43	72.78	81.29	83.66	90.90	94.63
2200	49.35	52.61	55.51	60.59	65.02	68.99	72.57	75.81	83.74	85.81	91.44	92.82
2500	52.95	56.35	59.33	64.49	68.89	72.72	76.09	79.04	85.59	87.06	89.47	86.40
2850	56.46	59.94	62.96	68.05	72.25	75.77	78.71	81.12	85.46	85.99	83.65	
3000	57.70	61.20	64.20	69.21	73.26	76.58	79.26	81.37	84.54	84.58	79.82	
3500	60.83	64.23	67.06	71.55	74.87	77.27	78.85	79.68	77.95	76.02		
4000	62.41	65.54	68.03	71.64	73.86	74.94	75.01	74.12	66.07			

Source: Table reproduced courtesy of Fenner.

Table 12.11: Belt length correction factors for 40-mm-wide 14MXP belts.

Belt length (mm)	966–1190	1400–1610	1778–1890	2100–2450	2590–3360	3500–6860
Length factor	0.80	0.90	0.95	1.00	1.05	1.10

Source: Table reproduced courtesy of Fenner.

Table 12.12: Belt width factors for 40-mm-wide 14MXP belts.

Belt width (mm)	40	55	85	115	170
Width factor	1.00	1.44	2.31	3.18	4.78

Source: Table reproduced courtesy of Fenner.

Table 12.13: Synchronous belt drive service factors.

Types of driven machine	Types of prime mover					
Special cases For speed increasing drives of: 1.00—1.24: no additional factor 1.25—1.74: add 0.1 1.75—2.49: add 0.2 2.50—3.49: add 0.3 3.50 and greater: add 0.4 Seasonal/intermittent use: subtract 0.2 Idler pulley used on drive: add 0.2	"Soft" starts AC electric motors — Star/delta start — Synchronous — Split wound — Inverter control DC electric motors — Shunt wound — Stepper motors I/C engines with four or more cylinders. Prime movers with centrifugal clutches or fluid couplings.			"Heavy" starts AC electric motors — DOL start — Single phase — Slip ring DC electric motors — Series wound — Compound — Servo motors I/C engines with <4 cylinders		
	Hours per day duty			Hours per day duty		
	<10	16 ≥ Hours > 10	>16	<10	16 ≥ Hours > 10	>16
Light duty Agitators (uniform density), Bakery machinery: Dough mixers, Blowers except positive displacement. Centrifugal pumps and compressors. Belt conveyors (uniformly loaded). Exhausters. Fans up to 7.5 kW. Paper machinery: Agitators, calenders, dryers, printing machinery: Linotype machines, cutters, folders. Screens: Drum, conical. Woodworking machinery: Lathes, band saws.	1.2	1.4	1.6	1.6	1.8	2.0
Medium duty Agitators and mixers (variable density), belt conveyors (not uniformly loaded), brick and clay machinery, augers, mixers,	1.3	1.5	1.7	1.7	1.9	2.1

Continued

Table 12.13: Synchronous belt drive service factors.—cont'd

Types of driven machine	Types of prime mover					
granulators. Fans over 7.5 kW. Generators, line shafts. Laundry machinery. Punches, presses, shears. Printing machinery: Presses, newspaper, rotary embossing, flat bed magazine. Pumps: Positive displacement, rotary. Screens, vibrating. Machine tools.						
Heavy duty Blowers, positive displacement. Bucket elevators, centrifuges. Conveyors: Drag, pan, screw. Paper machinery: Beaters, jordans, mash pumps, pulpers. Pumps, piston. Pulverizers. Woodworking machinery. Textile machinery. Exiters.	1.5	1.7	1.9	1.9	2.1	2.3
Extra heavy duty Brick machinery, pug mills. Compressors, piston. Crushers: Gyratory, jaw roll. Hoists. Mills: Ball, rod, tube, rubber. Rubber machinery: Calenders, extruders, mills.	1.7	1.9	2.1	2.1	2.3	2.5

Source: Table courtesy of Fender.

Table 12.14: Fenner torque drive plus 8MXP and 8M drives. Center distance (mm).

Speed ratio	Number of grooves on: Driving pulley	Driven pulley	Belt pitch length (mm) 480	560	600	640	720	800	880	960	1040	1120	1200	1280	1440	1600	1760	1800	2000	2400	2600	2800	Speed ratio
			60 teeth	70 teeth	75 teeth	80 teeth	90 teeth	100 teeth	110 teeth	120 teeth	130 teeth	140 teeth	150 teeth	160 teeth	180 teeth	200 teeth	220 teeth	225 teeth	250 teeth	300 teeth	325 teeth	350 teeth	
1.00	24	24	144	184	204	224	264	304	344	384	424	464	504	544	624	704	784	804	904	1104	1204	1304	1.00
1.00	26	26	136	176	196	216	256	296	336	376	416	456	496	536	616	696	776	796	896	1096	1196	1296	1.00
1.00	28	28	128	168	188	208	248	288	328	368	408	448	488	528	608	688	768	788	888	1088	1188	1288	1.00
1.00	30	30	120	160	180	200	240	280	320	360	400	440	480	520	600	680	760	780	880	1080	1180	1280	1.00
1.00	32	32	112	152	172	192	232	272	312	352	392	432	472	512	592	672	752	772	872	1072	1172	1272	1.00
1.00	34	34	104	144	164	184	224	264	304	344	384	424	464	504	584	664	744	764	864	1064	1164	1264	1.00
1.00	36	36	—	136	156	176	216	256	296	336	376	416	456	496	576	656	736	756	856	1056	1156	1256	1.00
1.00	38	38	—	128	148	168	208	248	288	328	368	408	448	488	568	648	728	748	848	1048	1148	1248	1.00
1.00	40	40	—	120	140	160	200	240	280	320	360	400	440	480	560	640	720	740	840	1040	1140	1240	1.00
1.00	44	44	—	—	—	144	184	224	264	304	344	384	424	464	544	624	704	724	824	1024	1124	1224	1.00
1.00	48	48	—	—	—	—	168	208	248	288	328	368	408	448	528	608	688	708	808	1008	1108	1208	1.00
1.00	56	56	—	—	—	—	—	176	216	256	296	336	376	416	496	576	656	676	776	976	1076	1176	1.00
1.00	64	64	—	—	—	—	—	—	184	224	264	304	344	384	464	544	624	644	744	944	1044	1144	1.00
1.00	72	72	—	—	—	—	—	—	—	—	232	272	312	352	432	512	592	612	712	912	1012	1112	1.00
1.00	80	80	—	—	—	—	—	—	—	—	—	240	280	320	400	480	560	580	680	880	980	1080	1.00
1.05	38	40	—	124	144	164	204	244	284	324	364	404	444	484	564	644	724	744	844	1044	1144	1244	1.05
1.06	36	38	—	132	152	172	212	252	292	332	372	412	452	492	572	652	732	752	852	1052	1152	1252	1.06
1.06	34	36	—	140	160	180	220	260	300	340	380	420	460	500	580	660	740	760	860	1060	1160	1260	1.06
1.06	32	34	108	148	168	188	228	268	308	348	388	428	468	508	588	668	748	768	868	1068	1168	1268	1.06
1.07	30	32	116	156	176	196	236	276	316	356	396	436	476	516	596	676	756	776	876	1076	1176	1276	1.07
1.07	28	30	124	164	184	204	244	284	324	364	404	444	484	524	604	684	764	784	884	1084	1184	1284	1.07
1.08	26	28	132	172	192	212	252	292	332	372	412	452	492	532	612	692	772	792	892	1092	1192	1292	1.08
1.08	24	26	140	180	200	220	260	300	340	380	420	460	500	540	620	700	780	800	900	1100	1200	1300	1.08
1.09	44	48	—	—	—	136	176	216	256	296	336	376	416	456	536	616	696	716	816	1016	1116	1216	1.09
1.10	40	44	—	—	132	152	192	232	272	312	352	392	432	472	552	632	712	732	832	1032	1132	1232	1.10
1.11	36	40	—	128	148	168	208	248	288	328	368	408	448	488	568	648	728	748	848	1048	1148	1248	1.11
1.11	72	80	—	—	—	—	—	—	—	—	216	256	296	336	416	496	576	596	696	896	996	1096	1.11
1.12	34	38	—	136	156	176	216	256	296	336	376	416	456	496	576	656	736	756	856	1056	1156	1256	1.12
1.13	32	36	104	144	164	184	224	264	304	344	384	424	464	504	584	664	744	764	864	1064	1164	1264	1.13
1.13	64	72	—	—	—	—	—	—	—	208	248	288	328	368	448	528	608	628	728	928	1028	1128	1.13
1.13	80	90	—	—	—	—	—	—	—	—	—	—	260	300	380	460	540	560	660	860	960	1060	1.13
1.13	30	34	112	152	172	192	232	272	312	352	392	432	472	512	592	672	752	772	872	1072	1172	1272	1.13
1.14	28	32	120	160	180	200	240	280	320	360	400	440	480	520	600	680	760	780	880	1080	1180	1280	1.14

Continued

Table 12.14: Fenner torque drive plus 8MXP and 8M drives. Center distance (mm).—cont'd

	Number of grooves on:		Belt pitch length (mm)																				
			480	560	600	640	720	800	880	960	1040	1120	1200	1280	1440	1600	1760	1800	2000	2400	2600	2800	
Speed ratio	Driving pulley	Driven pulley	60 teeth	70 teeth	75 teeth	80 teeth	90 teeth	100 teeth	110 teeth	120 teeth	130 teeth	140 teeth	150 teeth	160 teeth	180 teeth	200 teeth	220 teeth	225 teeth	250 teeth	300 teeth	325 teeth	350 teeth	Speed ratio
1.14	56	64	—	—	—	—	—	—	200	240	280	320	360	400	480	560	640	660	760	960	1060	1160	1.14
1.15	26	30	128	168	188	208	248	288	328	368	408	448	488	528	608	688	768	788	888	1088	1188	1288	1.15
1.16	38	44	—	—	136	156	196	236	276	316	356	396	436	476	556	636	716	736	836	1036	1136	1236	1.16
1.17	24	28	136	176	196	216	256	296	336	376	416	456	496	536	616	696	776	796	896	1096	1196	1296	1.17
1.17	48	56	—	—	—	—	152	192	232	272	312	352	392	432	512	592	672	692	792	992	1092	1192	1.17
1.18	34	40	—	132	152	172	212	252	292	332	372	412	452	492	572	652	732	752	852	1052	1152	1252	1.18
1.19	32	38	—	140	160	180	220	260	300	340	380	420	460	500	580	660	740	760	860	1060	1160	1260	1.19
1.20	30	36	108	148	168	188	228	268	308	348	388	428	468	508	588	668	748	768	868	1068	1168	1268	1.20
1.20	40	48	—	—	—	144	184	224	264	304	344	384	424	464	544	624	704	724	824	1024	1124	1224	1.20
1.21	28	34	116	156	176	196	236	276	316	356	396	436	476	516	596	676	756	776	876	1076	1176	1276	1.21
1.22	36	44	—	120	140	160	200	240	280	320	360	400	440	480	560	640	720	740	840	1040	1140	1240	1.22
1.23	26	32	124	164	184	204	244	284	324	364	404	444	484	524	604	684	764	784	884	1084	1184	1284	1.23
1.25	24	30	132	172	192	212	252	292	332	372	412	452	492	532	612	692	772	792	892	1092	1192	1292	1.25
1.25	32	40	—	136	156	176	216	256	296	336	376	416	456	496	576	656	736	756	856	1056	1156	1256	1.25
1.25	64	80	—	—	—	—	—	—	—	—	231	271	311	351	432	512	592	612	712	912	1012	1112	1.25
1.25	72	90	—	—	—	—	—	—	—	—	—	235	275	315	395	475	556	576	676	876	976	1076	1.25
1.26	38	48	—	—	127	147	188	228	268	308	348	388	428	468	548	628	708	728	828	1028	1128	1228	1.26
1.27	30	38	103	144	164	184	224	264	304	344	384	424	464	504	584	664	744	764	864	1064	1164	1264	1.27
1.27	44	56	—	—	—	—	159	199	240	280	320	360	400	440	520	600	680	700	800	1000	1100	1200	1.27
1.29	28	36	112	152	172	192	232	272	312	352	392	432	472	512	592	672	752	772	872	1072	1172	1272	1.29
1.29	56	72	—	—	—	—	—	—	183	223	263	303	343	383	464	544	624	644	744	944	1044	1144	1.29
1.29	34	44	—	123	143	164	204	244	284	324	364	404	444	484	564	644	724	744	844	1044	1144	1244	1.29
1.31	26	34	120	160	180	200	240	280	320	360	400	440	480	520	600	680	760	780	880	1080	1180	1280	1.31
1.33	24	32	128	168	188	208	248	288	328	368	408	448	488	528	608	688	768	788	888	1088	1188	1288	1.33
1.33	30	40	—	139	159	180	220	260	300	340	380	420	460	500	580	660	740	760	860	1060	1160	1260	1.33
1.33	36	48	—	—	131	151	191	231	272	312	352	392	432	472	552	632	712	732	832	1032	1132	1232	1.33
1.33	48	64	—	—	—	—	—	175	215	255	295	335	375	416	496	576	656	676	776	976	1076	1176	1.33
1.36	28	38	107	147	168	188	228	268	308	348	388	428	468	508	588	668	748	768	868	1068	1168	1268	1.36
1.38	32	44	—	127	147	167	207	248	288	328	368	408	448	488	568	648	728	748	848	1048	1148	1248	1.38
1.38	26	36	115	155	176	196	236	276	316	356	396	436	476	516	596	676	756	776	876	1076	1176	1276	1.38
1.40	40	56	—	—	—	—	167	207	247	287	327	367	407	448	528	608	688	708	808	1008	1108	1208	1.40
1.40	80	112	—	—	—	—	—	—	—	—	—	—	—	—	334	414	494	514	615	815	915	1015	1.40

Source: Table courtesy of Fenner.

Table 12.15: Fenner torque drive plus 8MXP and 8M drives. Center distance (mm).

	Number of grooves on:		Belt pitch length (mm)																				
			480	560	600	640	720	800	880	960	1040	1120	1200	1280	1440	1600	1760	1800	2000	2400	2600	2800	
Speed ratio	Driving pulley	Driven pulley	60 teeth	70 teeth	75 teeth	80 teeth	90 teeth	100 teeth	110 teeth	120 teeth	130 teeth	140 teeth	150 teeth	160 teeth	180 teeth	200 teeth	220 teeth	225 teeth	250 teeth	300 teeth	325 teeth	350 teeth	Speed ratio
1.41	64	90	—	—	—	—	—	—	—	—	—	250	290	330	411	491	571	591	691	891	991	1091	1.41
1.41	34	48	—	—	135	155	195	235	275	315	356	396	436	476	556	636	716	736	836	1036	1136	1236	1.41
1.42	24	34	123	164	184	204	244	284	324	364	404	444	484	524	604	684	764	784	884	1084	1184	1284	1.42
1.43	28	40	103	143	163	183	223	264	304	344	384	424	464	504	584	664	744	764	864	1064	1164	1264	1.43
1.43	56	80	—	—	—	—	—	—	—	206	246	286	327	367	447	527	607	627	727	927	1028	1128	1.43
1.45	44	64	—	—	—	—	—	182	223	263	303	343	383	423	503	583	664	684	784	984	1084	1184	1.45
1.46	26	38	111	151	171	191	231	272	312	352	392	432	472	512	592	672	752	772	872	1072	1172	1272	1.46
1.47	30	44	—	131	151	171	211	251	291	332	372	412	452	492	572	652	732	752	852	1052	1152	1252	1.47
1.47	38	56	—	—	—	—	170	211	251	291	331	371	411	451	532	612	692	712	812	1012	1112	1212	1.47
1.50	24	36	119	159	179	199	240	280	320	360	400	440	480	520	600	680	760	780	880	1080	1180	1280	1.50
1.50	32	48	—	118	139	159	199	239	279	319	359	399	440	480	560	640	720	740	840	1040	1140	1240	1.50
1.50	48	72	—	—	—	—	—	—	198	238	278	319	359	399	479	559	639	659	759	960	1060	1160	1.50
1.54	26	40	107	147	167	187	227	267	307	348	388	428	468	508	588	668	748	768	868	1068	1168	1268	1.54
1.56	36	56	—	—	—	134	174	214	255	295	335	375	415	455	535	615	696	716	816	1016	1116	1216	1.56
1.56	72	112	—	—	—	—	—	—	—	—	—	—	—	267	348	429	509	530	630	830	931	1031	1.56
1.57	28	44	—	134	155	175	215	255	295	335	375	416	456	496	576	656	736	756	856	1956	1156	1256	1.57
1.58	24	38	115	155	175	195	235	275	315	356	396	436	476	516	596	676	756	776	876	1076	1176	1276	1.58
1.60	30	48	—	122	142	162	203	243	283	323	363	403	443	483	564	644	724	744	844	1044	1144	1244	1.60
1.60	40	64	—	—	—	—	149	190	230	270	310	351	391	431	511	591	671	691	791	992	1092	1192	1.60
1.61	56	90	—	—	—	—	—	—	—	—	224	264	305	345	426	506	586	606	707	907	1007	1107	1.61
1.64	44	72	—	—	—	—	—	164	205	245	286	326	366	406	487	567	647	667	767	967	1067	1167	1.64
1.65	34	56	—	—	—	137	178	218	258	299	339	379	419	459	539	619	699	719	820	1020	1120	1220	1.65
1.67	24	40	110	151	171	191	231	271	311	351	391	432	472	512	592	672	752	772	872	1072	1172	1272	1.67
1.67	48	80	—	—	—	—	—	—	179	220	261	301	342	382	462	542	623	643	743	943	1043	1143	1.67
1.68	38	64	—	—	—	—	152	193	234	274	314	354	395	435	515	595	675	695	795	995	1095	1196	1.68
1.69	26	44	—	138	158	179	219	259	299	339	379	419	459	499	580	660	740	760	860	1060	1160	1260	1.69
1.71	28	48	—	125	146	166	206	247	287	327	367	407	447	487	567	647	728	748	848	1048	1148	1248	1.71
1.75	32	56	—	—	—	141	181	222	262	302	343	383	423	463	543	623	703	723	823	1024	1124	1224	1.75
1.75	64	112	—	—	—	—	—	—	—	—	—	—	240	281	363	444	524	545	645	846	946	1046	1.75
1.78	36	64	—	—	—	—	156	197	237	278	318	358	398	439	519	599	679	699	799	999	1099	1199	1.78
1.80	40	72	—	—	—	—	—	171	212	253	293	334	374	414	494	575	655	675	775	975	1075	1175	1.80
1.80	80	144	—	—	—	—	—	—	—	—	—	—	—	—	—	342	424	445	546	748	848	948	1.80
1.82	44	80	—	—	—	—	—	—	186	227	268	309	349	389	470	550	630	650	751	951	1051	1151	1.82
1.83	24	44	—	142	162	182	223	263	303	343	383	423	463	503	583	664	744	764	864	1064	1164	1264	1.83

Continued

Table 12.15: Fenner torque drive plus 8MXP and 8M drives. Center distance (mm).—cont'd

Speed ratio	Number of grooves on: Driving pulley	Driven pulley	Belt pitch length (mm) 480	560	600	640	720	800	880	960	1040	1120	1200	1280	1440	1600	1760	1800	2000	2400	2600	2800	Speed ratio
			60 teeth	70 teeth	75 teeth	80 teeth	90 teeth	100 teeth	110 teeth	120 teeth	130 teeth	140 teeth	150 teeth	160 teeth	180 teeth	200 teeth	220 teeth	225 teeth	250 teeth	300 teeth	325 teeth	350 teeth	
1.85	26	48	—	129	149	170	210	250	291	331	371	411	451	491	571	651	731	751	852	1052	1152	1252	1.85
1.87	30	56	—	—	—	144	185	226	266	306	346	387	427	467	547	627	707	727	827	1027	1128	1228	1.87
1.88	48	90	—	—	—	—	—	—	—	197	238	279	320	360	441	521	602	622	722	922	1023	1123	1.88
1.88	34	64	—	—	—	—	159	200	241	281	322	362	402	442	523	603	683	703	803	1003	1103	1203	1.88
1.89	38	72	—	—	—	—	—	175	216	256	297	337	378	418	498	578	659	679	779	979	1079	1179	1.89
2.00	24	48	—	132	153	173	214	254	294	335	375	415	455	495	575	655	735	755	855	1056	1156	1256	2.00
2.00	28	56	—	—	127	148	189	229	270	310	350	390	431	471	551	631	711	731	831	1031	1131	1231	2.00
2.00	32	64	—	—	—	—	163	204	245	285	325	366	406	446	526	607	687	707	807	1007	1107	1207	2.00
2.00	36	72	—	—	—	—	—	178	219	260	301	341	381	422	502	582	662	682	783	983	1083	1183	2.00
2.00	40	80	—	—	—	—	—	—	193	234	275	316	356	397	477	558	638	658	758	959	1059	1159	2.00
2.00	56	112	—	—	—	—	—	—	—	—	—	—	254	295	377	458	539	559	660	861	961	1062	2.00
2.00	72	144	—	—	—	—	—	—	—	—	—	—	—	—	—	356	438	459	561	762	863	964	2.00
2.05	44	90	—	—	—	—	—	—	—	204	245	286	327	367	448	529	609	629	730	930	1030	1130	2.05
2.10	80	168	—	—	—	—	—	—	—	—	—	—	—	—	—	—	367	388	491	695	796	897	2.10
2.11	38	80	—	—	—	—	—	—	197	238	279	320	360	400	481	561	642	662	762	963	1063	1163	2.11
2.12	34	72	—	—	—	—	—	182	223	264	304	345	385	425	506	586	666	686	787	987	1087	1187	2.12
2.13	30	64	—	—	—	—	166	207	248	289	329	369	410	450	530	610	691	711	811	1011	1111	1211	2.13
2.15	26	56	—	—	130	151	192	233	273	314	354	394	434	474	555	635	715	735	835	1035	1135	1235	2.15
2.22	36	80	—	—	—	—	—	—	200	242	282	323	364	404	485	565	646	666	766	966	1067	1167	2.22
2.25	32	72	—	—	—	—	—	185	226	267	308	348	389	429	509	590	670	690	790	991	1091	1191	2.25
2.25	40	90	—	—	—	—	—	—	—	210	252	293	334	375	456	536	617	637	737	938	1038	1138	2.25
2.25	64	144	—	—	—	—	—	—	—	—	—	—	—	—	286	370	453	473	575	777	878	979	2.25
2.29	28	64	—	—	—	—	170	211	252	292	333	373	413	454	534	614	694	715	815	1015	1115	1215	2.29
2.33	24	56	—	—	134	155	196	236	277	317	358	398	438	478	559	639	719	739	839	1039	1139	1239	2.33
2.33	48	112	—	—	—	—	—	—	—	—	—	225	268	309	392	473	554	574	675	876	977	1077	2.33
2.33	72	168	—	—	—	—	—	—	—	—	—	—	—	—	—	—	380	401	505	709	811	912	2.33
2.35	34	80	—	—	—	—	—	161	204	245	286	327	367	408	488	569	649	669	770	970	1070	1171	2.35
2.37	38	90	—	—	—	—	—	—	—	214	255	297	338	378	459	540	620	641	741	942	1042	1142	2.37
2.40	30	72	—	—	—	—	146	188	230	271	311	352	392	433	513	594	674	694	794	995	1095	1195	2.40
2.40	80	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	432	640	742	844	2.40
2.46	26	64	—	—	—	131	173	215	255	296	337	377	417	457	538	618	698	718	819	1019	1119	1219	2.46

Source: Table courtesy of Fenner.

Table 12.16: Fenner torque drive plus 8MXP and 8M drives. Center distance (mm).

Speed ratio	Number of grooves on: Driving pulley	Number of grooves on: Driven pulley	Belt pitch length (mm): 480 60 teeth	560 70 teeth	600 75 teeth	640 80 teeth	720 90 teeth	800 100 teeth	880 110 teeth	960 120 teeth	1040 130 teeth	1120 140 teeth	1200 150 teeth	1280 160 teeth	1440 180 teeth	1600 200 teeth	1760 220 teeth	1800 225 teeth	2000 250 teeth	2400 300 teeth	2600 325 teeth	2800 350 teeth	Speed ratio
2.50	32	80	—	—	—	—	—	165	207	248	290	330	371	411	492	573	653	673	774	974	1074	1174	2.50
2.50	36	90	—	—	—	—	—	—	—	217	259	300	341	382	463	544	624	644	745	946	1046	1146	2.50
2.55	44	112	—	—	—	—	—	—	—	—	—	232	274	316	399	480	561	582	683	884	984	1085	2.55
2.57	28	72	—	—	—	—	150	192	233	274	315	356	396	436	517	597	678	698	798	998	1099	1199	2.57
2.57	56	144	—	—	—	—	—	—	—	—	—	—	—	—	299	384	467	487	589	792	893	994	2.57
2.63	64	168	—	—	—	—	—	—	—	—	—	—	—	—	—	—	394	415	519	724	825	927	2.63
2.65	34	90	—	—	—	—	—	—	178	220	262	304	345	385	467	547	628	648	749	949	1050	1150	2.65
2.67	24	64	—	—	—	134	177	218	259	300	340	381	421	461	542	622	702	722	822	1023	1123	1223	2.65
2.67	30	80	—	—	—	—	—	168	210	252	293	334	375	415	496	576	657	677	777	978	1078	1178	2.67
2.67	72	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	446	654	757	858	2.67
2.77	26	72	—	—	—	—	153	195	237	278	319	359	400	440	521	601	681	702	802	1002	1102	1203	2.77
2.80	40	112	—	—	—	—	—	—	—	—	—	238	281	323	406	487	569	589	690	891	992	1092	2.80
2.81	32	90	—	—	—	—	—	—	181	224	266	307	348	389	470	551	632	652	752	953	1053	1154	2.81
2.86	28	80	—	—	—	—	—	171	214	255	297	338	378	419	500	580	661	681	781	982	1082	1182	2.86
2.95	38	112	—	—	—	—	—	—	—	—	—	242	284	326	409	491	572	593	694	895	996	1096	2.95
3.00	24	72	—	—	—	—	156	199	240	281	322	363	403	444	524	605	685	705	806	1006	1106	1206	3.00
3.00	30	90	—	—	—	—	—	—	184	227	269	311	352	393	474	555	635	656	756	957	1057	1157	3.00
3.00	48	144	—	—	—	—	—	—	—	—	—	—	—	—	312	397	480	501	604	807	908	1009	3.00
3.00	56	168	—	—	—	—	—	—	—	—	—	—	—	—	—	320	407	428	533	738	840	941	3.00
3.00	64	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	350	459	668	771	873	3.00
3.08	26	80	—	—	—	—	—	174	217	259	300	341	382	422	503	584	664	685	785	986	1086	1186	3.08
3.11	36	112	—	—	—	—	—	—	—	—	—	245	288	330	413	495	576	596	697	899	999	1100	3.11
3.21	28	90	—	—	—	—	—	—	187	230	273	314	355	396	477	558	639	659	760	961	1061	1161	3.21
3.27	44	144	—	—	—	—	—	—	—	—	—	—	—	—	319	404	487	508	611	814	915	1016	3.27
3.29	34	112	—	—	—	—	—	—	—	—	204	248	291	333	416	498	579	600	701	903	1003	1104	3.29
3.33	24	80	—	—	—	—	—	178	220	262	304	345	385	426	507	588	668	688	789	989	1090	1190	3.33
3.43	56	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	340	363	472	682	785	887	3.43
3.46	26	90	—	—	—	—	—	—	191	234	276	318	359	400	481	562	643	663	764	985	1065	1165	3.46
3.50	32	112	—	—	—	—	—	—	—	—	207	251	294	337	420	502	583	603	705	906	1007	1107	3.50
3.50	48	168	—	—	—	—	—	—	—	—	—	—	—	—	—	333	420	442	547	752	854	956	3.50
3.60	40	144	—	—	—	—	—	—	—	—	—	—	—	—	325	411	494	515	618	821	922	1023	3.60
3.73	30	112	—	—	—	—	—	—	—	—	210	255	298	340	423	505	587	607	708	910	1011	1111	3.73

Continued

Table 12.16: Fenner torque drive plus 8MXP and 8M drives. Center distance (mm).—cont'd

	Number of grooves on:		Belt pitch length (mm)																			
			480	560	600	640	720	800	880	960	1040	1120	1200	1280	1440	1600	1760	1800	2000	2400	2600	2800	
Speed ratio	Driving pulley	Driven pulley	60 teeth	70 teeth	75 teeth	80 teeth	90 teeth	100 teeth	110 teeth	120 teeth	130 teeth	140 teeth	150 teeth	160 teeth	180 teeth	200 teeth	220 teeth	225 teeth	250 teeth	300 teeth	325 teeth	350 teeth	Speed ratio
3.75	24	90	—	—	—	—	—	—	194	237	279	321	362	403	485	566	647	667	767	968	1069	1169	3.75
3.79	38	144	—	—	—	—	—	—	—	—	—	—	—	—	328	414	498	518	621	825	926	1027	3.79
3.82	44	168	—	—	—	—	—	—	—	—	—	—	—	—	—	339	427	448	553	760	862	963	3.82
4.00	28	112	—	—	—	—	—	—	—	—	213	258	301	343	427	509	590	611	712	914	1014	1115	4.00
4.00	36	144	—	—	—	—	—	—	—	—	—	—	—	—	331	417	501	522	625	829	930	1031	4.00
4.00	48	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	352	375	485	696	799	901	4.00
4.20	40	168	—	—	—	—	—	—	—	—	—	—	—	—	—	346	433	455	560	767	869	970	4.20
4.24	34	144	—	—	—	—	—	—	—	—	—	—	—	244	335	421	505	525	628	832	933	1035	4.24
4.31	26	112	—	—	—	—	—	—	—	—	216	261	304	347	430	512	594	614	716	917	1018	1119	4.31
4.36	44	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	358	381	492	703	806	908	4.36
4.42	38	168	—	—	—	—	—	—	—	—	—	—	—	—	—	349	437	459	564	770	872	974	4.42
4.50	32	144	—	—	—	—	—	—	—	—	—	—	—	247	338	424	508	529	632	836	937	1038	4.50
4.67	24	112	—	—	—	—	—	—	—	—	219	264	308	350	434	516	597	618	719	921	1022	1122	4.67
4.67	36	168	—	—	—	—	—	—	—	—	—	—	—	—	—	352	440	461	567	774	876	978	4.67
4.80	30	144	—	—	—	—	—	—	—	—	—	—	—	250	341	427	511	532	635	839	941	1042	4.80
4.80	40	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	365	388	498	710	813	916	4.90
4.94	34	168	—	—	—	—	—	—	—	—	—	—	—	—	—	355	443	465	570	777	879	981	4.94
5.05	38	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	368	391	502	713	816	919	5.05
5.14	28	144	—	—	—	—	—	—	—	—	—	—	—	253	344	431	515	536	639	843	944	1046	5.14
5.25	32	168	—	—	—	—	—	—	—	—	—	—	—	—	—	358	446	468	574	781	883	995	5.25
5.33	36	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	371	394	505	716	820	923	5.33
5.54	26	144	—	—	—	—	—	—	—	—	—	—	—	256	348	434	518	539	642	847	948	1049	5.54
5.60	30	168	—	—	—	—	—	—	—	—	—	—	—	—	—	361	450	471	577	784	887	988	5.60
5.65	34	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	374	397	508	720	823	926	5.65
6.00	24	144	—	—	—	—	—	—	—	—	—	—	—	259	351	437	522	542	646	850	952	1053	6.00
6.00	28	168	—	—	—	—	—	—	—	—	—	—	—	—	269	364	453	475	581	785	890	992	6.00
6.00	32	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	377	400	511	723	827	930	6.00
6.40	30	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	380	403	515	727	830	933	6.40
6.46	26	168	—	—	—	—	—	—	—	—	—	—	—	—	272	368	478	456	584	791	894	996	6.46
6.86	28	192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	383	406	519	730	834	937	6.86

Source: Table courtesy of Fenner.

Table 12.17: Fenner torque drive plus 14MXP and 14M drives. Center distance (mm).

	Number of grooves on:		Belt pitch length (mm)																
			966	1190	1400	1610	1778	1890	2100	2310	2450	2590	2800	3150	3500	3850	4326	4578	
Speed ratio	Driving pulley	Driven pulley	69 teeth	85 teeth	100 teeth	115 teeth	127 teeth	135 teeth	150 teeth	165 teeth	175 teeth	185 teeth	200 teeth	225 teeth	250 teeth	275 teeth	309 teeth	327 teeth	Speed ratio
1.00	28	28	287	399	504	609	693	749	854	959	1029	1099	1204	1379	1554	1729	1967	2093	1.00
1.00	29	29	280	392	497	602	686	742	847	952	1022	1092	1197	1372	1547	1722	1960	2086	1.00
1.00	30	30	273	385	490	595	679	735	840	945	1015	1085	1190	1365	1540	1715	1953	2079	1.00
1.00	32	32	259	371	476	581	665	721	826	931	1001	1071	1176	1351	1526	1701	1939	2065	1.00
1.00	34	34	245	357	462	567	651	707	812	917	987	1057	1162	1337	1512	1687	1925	2051	1.00
1.00	36	36	231	343	448	553	637	693	798	903	973	1043	1148	1323	1498	1673	1911	2037	1.00
1.00	38	38	217	329	434	539	623	679	784	889	959	1029	1134	1309	1484	1659	1897	2023	1.00
1.00	40	40	203	315	420	525	609	665	770	875	945	1015	1120	1295	1470	1645	1883	2009	1.00
1.00	44	44	—	287	392	497	581	637	742	847	917	987	1092	1267	1442	1617	1855	1981	1.00
1.00	48	48	—	259	364	469	553	609	714	819	889	959	1064	1239	1414	1589	1827	1953	1.00
1.00	56	56	—	—	308	413	497	553	658	763	833	903	1008	1183	1358	1533	1771	1897	1.00
1.03	29	30	276	388	493	598	682	738	843	948	1018	1088	1193	1368	1543	1718	1956	2082	1.00
1.04	28	29	283	395	500	605	689	745	850	955	1025	1095	1200	1375	1550	1725	1963	2089	1.04
1.05	38	40	210	322	427	532	616	672	777	882	952	1022	1127	1302	1477	1652	1890	2016	1.05
1.06	36	38	224	336	441	546	630	686	791	896	966	1036	1141	1316	1491	1666	1904	2030	1.06
1.06	34	36	238	350	455	560	644	700	805	910	980	1050	1155	1330	1505	1680	1918	2044	1.06
1.06	32	34	252	364	469	574	658	714	819	924	994	1064	1169	1344	1519	1694	1932	2058	1.06
1.07	30	32	266	378	483	588	672	728	833	938	1008	1078	1183	1358	1533	1708	1946	2072	1.07
1.07	28	30	280	392	497	602	686	742	847	952	1022	1092	1197	1372	1547	1722	1960	2086	1.07
1.09	44	48	—	273	378	483	567	623	728	833	903	973	1078	1253	1428	1603	1841	1967	1.09
1.10	40	44	—	301	406	511	595	651	756	861	931	1001	1106	1281	1456	1631	1869	1995	1.10
1.10	29	32	269	381	486	591	675	731	836	941	1011	1081	1186	1361	1536	1711	1949	2075	1.10
1.11	36	40	217	329	434	539	623	679	784	889	959	1029	1134	1309	1484	1659	1897	2023	1.11
1.12	34	38	231	343	448	553	637	693	798	903	973	1043	1148	1323	1498	1673	1911	2037	1.12
1.13	32	36	245	357	462	567	651	707	812	917	987	1057	1162	1337	1512	1687	1925	2051	1.13
1.13	30	34	259	371	476	581	665	721	826	931	1001	1071	1176	1351	1526	1701	1939	2065	1.13
1.14	28	32	273	385	490	595	679	735	840	945	1015	1085	1190	1365	1540	1715	1953	2079	1.14
1.14	56	64	—	—	—	385	469	525	630	735	805	875	980	1155	1330	1505	1743	1869	1.14
1.16	38	44	—	308	413	518	602	658	763	868	938	1008	1113	1288	1463	1638	1876	2002	1.16
1.17	48	56	—	—	336	441	525	581	686	791	861	931	1036	1211	1386	1561	1799	1925	1.17

Continued

Table 12.17: Fenner torque drive plus 14MXP and 14M drives. Center distance (mm).—cont'd

	Number of grooves on:		Belt pitch length (mm)																
			966	1190	1400	1610	1778	1890	2100	2310	2450	2590	2800	3150	3500	3850	4326	4578	
Speed ratio	Driving pulley	Driven pulley	69 teeth	85 teeth	100 teeth	115 teeth	127 teeth	135 teeth	150 teeth	165 teeth	175 teeth	185 teeth	200 teeth	225 teeth	250 teeth	275 teeth	309 teeth	327 teeth	Speed ratio
1.17	29	34	262	374	479	584	668	724	829	934	1004	1074	1179	1354	1529	1704	1942	2068	1.17
1.18	34	40	224	336	441	546	630	686	791	896	966	1036	1141	1316	1491	1666	1904	2030	1.18
1.19	32	38	238	350	455	560	644	700	805	910	980	1050	1155	1330	1505	1680	1918	2044	1.19
1.20	30	36	252	364	469	574	658	714	819	924	994	1064	1169	1344	1519	1694	1932	2058	1.20
1.20	40	48	—	286	392	497	581	637	742	847	917	987	1092	1267	1442	1617	1855	1981	1.20
1.21	28	34	266	378	483	588	672	728	833	938	1008	1078	1183	1358	1533	1708	1946	2072	1.21
1.22	36	44	202	314	420	525	609	665	770	875	945	1015	1120	1295	1470	1645	1883	2009	1.22
1.24	29	36	255	367	472	577	661	717	822	927	997	1067	1172	1347	1522	1697	1935	2061	1.24
1.25	32	40	230	343	448	553	637	693	798	903	973	1043	1148	1323	1498	1673	1911	2037	1.25
1.26	38	48	—	293	398	504	588	644	749	854	924	994	1099	1274	1449	1624	1862	1988	1.26
1.27	30	38	244	357	462	567	651	707	812	917	987	1057	1162	1337	1512	1687	1925	2051	1.27
1.27	44	56	—	244	349	454	538	594	699	805	875	945	1050	1225	1400	1575	1813	1939	1.27
1.29	28	36	258	371	476	581	665	721	826	931	1001	1071	1176	1351	1526	1701	1939	2065	1.29
1.29	56	72	—	—	—	355	440	496	601	706	776	846	951	1126	1302	1477	1715	1841	1.29
1.29	34	44	209	321	426	532	616	672	777	882	952	1022	1127	1302	1477	1652	1890	2016	1.29
1.31	29	38	248	360	465	570	654	710	815	920	990	1060	1165	1340	1515	1690	1928	2054	1.31
1.33	30	40	237	349	454	560	644	700	805	910	980	1050	1155	1330	1505	1680	1918	2044	1.33
1.33	36	48	—	300	405	510	594	650	756	861	931	1001	1106	1281	1456	1631	1869	1995	1.33
1.33	48	64	—	—	306	411	496	552	657	762	832	902	1007	1182	1358	1533	1771	1897	1.33
1.36	28	38	251	363	468	574	658	714	819	924	994	1064	1169	1344	1519	1694	1932	2058	1.36
1.38	32	44	215	328	433	538	622	678	784	889	959	1029	1134	1309	1484	1659	1897	2023	1.38
1.38	29	40	240	353	458	563	647	703	808	913	983	1053	1158	1333	1508	1683	1921	2047	1.38
1.40	40	56	—	257	362	468	552	608	713	818	888	958	1063	1238	1414	1589	1827	1953	1.40
1.41	34	48	—	306	412	517	601	657	762	867	937	1008	1113	1288	1463	1638	1876	2002	1.41
1.43	28	40	244	356	461	566	650	706	812	917	987	1057	1162	1337	1512	1687	1925	2051	1.43

Source: Table courtesy of Fenner.

Table 12.18: Fenner torque drive plus 14MXP and 14M drives. Center distance (mm).

	Number of grooves on:		Belt pitch length (mm)																
			966	1190	1400	1610	1778	1890	2100	2310	2450	2590	2800	3150	3500	3850	4326	4578	
Speed ratio	Driving pulley	Driven pulley	69 teeth	85 teeth	100 teeth	115 teeth	127 teeth	135 teeth	150 teeth	165 teeth	175 teeth	185 teeth	200 teeth	225 teeth	250 teeth	275 teeth	309 teeth	327 teeth	Speed ratio
1.43	56	80	—	—	—	325	410	466	571	677	747	817	922	1098	1273	1448	1686	1812	1.43
1.45	44	64	—	—	319	425	509	565	671	776	846	916	1021	1196	1371	1546	1784	1910	1.45
1.47	30	44	222	335	440	545	629	685	790	895	965	1036	1141	1316	1491	1666	1904	2030	1.47
1.47	38	56	—	263	369	474	559	615	720	825	895	965	1070	1245	1420	1595	1834	1960	1.47
1.50	32	48	200	313	418	524	608	664	769	874	944	1014	1119	1295	1470	1645	1883	2009	1.50
1.50	48	72	—	—	—	381	466	522	628	733	803	873	979	1154	1329	1504	1742	1868	1.50
1.52	29	44	225	338	443	548	633	689	794	899	969	1039	1144	1319	1494	1669	1907	2033	1.52
1.56	36	56	—	269	375	481	565	621	727	832	902	972	1077	1252	1427	1602	1840	1966	1.56
1.57	28	44	228	341	447	552	636	692	797	902	972	1042	1147	1323	1498	1673	1911	2037	1.57
1.60	30	48	206	319	425	530	615	671	776	881	951	1021	1126	1301	1476	1652	1890	2016	1.60
1.60	40	64	—	—	332	438	522	579	684	789	859	929	1035	1210	1385	1560	1798	1924	1.60
1.61	56	90	—	—	—	—	370	427	534	640	710	780	886	1061	1237	1412	1650	1776	1.61
1.64	44	72	—	—	287	394	479	535	641	746	817	887	992	1167	1343	1518	1756	1882	1.64
1.65	34	56	—	276	382	488	572	628	733	839	909	979	1084	1259	1434	1609	1847	1973	1.65
1.66	29	48	209	323	428	534	618	674	779	884	955	1025	1130	1305	1480	1655	1893	2019	1.66
1.67	48	80	—	—	—	350	435	492	598	703	774	844	949	1125	1300	1475	1714	1840	1.67
1.68	38	64	—	—	338	444	529	585	691	796	866	936	1041	1217	1392	1567	1805	1931	1.68
1.71	28	48	212	326	432	537	621	678	783	888	958	1028	1133	1308	1483	1658	1896	2023	1.71
1.75	32	56	—	282	388	494	579	635	740	845	915	986	1091	1266	1441	1616	1854	1980	1.75
1.78	36	64	—	—	344	451	535	592	697	803	873	943	1048	1223	1399	1574	1812	1938	1.78
1.80	40	72	—	—	300	407	492	548	654	760	830	900	1005	1181	1356	1531	1770	1896	1.80
1.82	44	80	—	—	—	362	448	505	611	717	787	857	963	1138	1314	1489	1727	1853	1.82
1.87	30	56	—	288	395	501	585	641	747	852	922	992	1097	1273	1448	1623	1861	1987	1.87
1.88	48	90	—	—	—	—	395	452	559	665	736	807	912	1088	1264	1439	1677	1804	1.88
1.88	34	64	—	243	351	457	542	598	704	809	879	950	1055	1230	1405	1581	1819	1945	1.88
1.89	38	72	—	—	306	413	498	555	661	766	837	907	1012	1188	1356	1538	1776	1902	1.89
1.93	29	56	—	291	389	504	588	645	750	855	926	996	1101	1276	1451	1626	1865	1991	1.93
2.00	28	56	—	294	401	507	592	648	753	859	929	999	1104	1279	1455	1630	1868	1994	2.00
2.00	32	64	—	249	357	464	548	605	710	816	886	956	1062	1237	1412	1587	1826	1952	2.00
2.00	36	72	—	—	312	419	505	561	667	773	843	913	1019	1194	1370	1545	1783	1909	2.00

Continued

Table 12.18: Fenner torque drive plus 14MXP and 14M drives. Center distance (mm).—cont'd

	Number of grooves on:		Belt pitch length (mm)																
			966	1190	1400	1610	1778	1890	2100	2310	2450	2590	2800	3150	3500	3850	4326	4578	
Speed ratio	Driving pulley	Driven pulley	69 teeth	85 teeth	100 teeth	115 teeth	127 teeth	135 teeth	150 teeth	165 teeth	175 teeth	185 teeth	200 teeth	225 teeth	250 teeth	275 teeth	309 teeth	327 teeth	Speed ratio
2.00	40	80	—	—	—	374	460	517	624	730	800	870	976	1152	1327	1502	1741	1867	2.00
2.00	56	112	—	—	—	—	—	—	444	553	625	696	802	979	1155	1331	1570	1696	2.00
2.05	44	90	—	—	—	320	407	465	572	678	749	820	925	1101	1277	1452	1691	1817	2.05
2.11	38	80	—	—	—	380	467	524	630	736	807	877	983	1158	1334	1509	1747	1874	2.11
2.12	34	72	—	—	318	426	511	568	674	779	850	920	1026	1201	1376	1552	1790	1916	2.12
2.13	30	64	—	255	363	470	555	611	717	823	893	963	1068	1244	1419	1594	1832	1959	2.13
2.21	29	64	—	258	366	473	558	615	720	826	896	966	1072	1247	1422	1598	1836	1962	2.21
2.22	36	80	—	—	—	387	473	530	636	743	813	884	989	1165	1340	1516	1754	1880	2.22
2.25	32	72	—	—	324	432	517	574	680	786	856	927	1032	1208	1383	1558	1797	1923	2.25
2.25	40	90	—	—	—	331	419	477	584	691	762	833	938	1114	1290	1466	1704	1831	2.25
2.29	28	64	—	261	369	476	561	618	724	829	899	970	1075	1250	1426	1601	1839	1965	2.29
2.33	48	112	—	—	—	—	—	—	468	577	649	721	828	1005	1181	1358	1597	1723	2.33
2.35	34	80	—	—	282	393	479	536	643	749	820	890	996	1172	1347	1523	1761	1887	2.35
2.37	38	90	—	—	—	337	425	483	591	697	768	839	945	1121	1297	1472	1711	1837	2.37
2.40	30	72	—	—	330	438	524	580	687	792	863	933	1039	1214	1390	1565	1804	1930	2.40
2.48	29	72	—	—	333	441	527	584	690	796	866	937	1042	1218	1393	1569	1807	1933	2.48
2.50	32	80	—	—	288	399	485	542	649	755	826	897	1002	1178	1354	1529	1758	1894	2.50
2.50	36	90	—	—	—	343	431	489	597	704	775	845	951	1128	1303	1479	1718	1844	2.50
2.55	44	112	—	—	—	—	—	368	480	590	662	733	840	1018	1194	1371	1610	1736	2.55
2.57	28	72	—	—	336	444	530	587	693	799	869	940	1045	1221	1397	1572	1810	1937	2.57
2.57	56	144	—	—	—	—	—	—	—	—	485	561	671	852	1031	1209	1450	1577	2.57
2.65	34	90	—	—	—	349	437	495	603	710	781	852	958	1134	1310	1486	1724	1851	2.65
2.67	30	80	—	—	294	405	491	549	656	762	833	903	1009	1185	1360	1536	1775	1901	2.67
2.76	29	80	—	—	297	408	494	552	659	765	836	906	1012	1188	1364	1539	1778	1904	2.76
2.80	40	112	—	—	—	—	—	379	492	602	674	746	853	1031	1207	1384	1623	1750	2.80

Source: Table courtesy of Fenner.

Table 12.19: Fenner torque drive plus 14MXP and 14M drives. Center distance (mm).

	Number of grooves on:		Belt pitch length (mm)																
			966	1190	1400	1610	1778	1890	2100	2310	2450	2590	2800	3150	3500	3850	4326	4578	
Speed ratio	Driving pulley	Driven pulley	69 teeth	85 teeth	100 teeth	115 teeth	127 teeth	135 teeth	150 teeth	165 teeth	175 teeth	185 teeth	200 teeth	225 teeth	250 teeth	275 teeth	309 teeth	327 teeth	Speed ratio
2.81	32	90	—	—	—	354	443	501	609	716	787	858	964	1141	1317	1492	1731	1858	2.81
2.86	28	80	—	—	300	411	498	555	662	768	839	910	1015	1191	1367	1543	1781	1907	2.86
2.95	38	112	—	—	—	—	—	385	498	608	680	752	859	1037	1214	1390	1630	1756	2.95
3.00	30	90	—	—	—	360	449	507	615	723	794	865	971	1147	1323	1499	1738	1864	3.00
3.00	48	144	—	—	—	—	—	—	—	—	508	584	695	877	1056	1234	1475	1603	3.00
3.00	56	168	—	—	—	—	—	—	—	—	—	—	560	749	933	1113	1356	1484	3.00
3.10	29	90	—	—	—	363	452	510	619	726	797	868	974	1150	1327	1502	1741	1868	3.10
3.11	36	112	—	—	—	—	—	390	504	614	686	758	865	1043	1220	1397	1636	1763	3.11
3.21	28	90	—	—	—	366	455	513	622	729	800	871	977	1154	1330	1506	1745	1871	3.21
3.27	44	144	—	—	—	—	—	—	—	441	519	595	707	889	1069	1247	1488	1616	3.27
3.29	34	112	—	—	—	—	—	396	509	620	692	764	872	1050	1227	1403	1643	1769	3.29
3.43	56	192	—	—	—	—	—	—	—	—	—	—	—	635	826	1012	1259	1388	3.43
3.50	32	112	—	—	—	—	—	401	515	626	698	770	878	1056	1233	1410	1649	1776	3.50
3.50	48	168	—	—	—	—	—	—	—	—	—	—	583	773	957	1138	1381	1509	3.50
3.60	40	144	—	—	—	—	—	—	—	452	530	607	719	901	1081	1260	1501	1629	3.60
3.73	30	112	—	—	—	—	343	407	521	632	704	777	884	1062	1240	1416	1656	1783	3.73
3.79	38	144	—	—	—	—	—	—	—	457	536	612	725	907	1087	1266	1507	1635	3.79
3.82	44	168	—	—	—	—	—	—	—	—	—	—	594	784	969	1150	1394	1522	3.82
3.86	29	112	—	—	—	—	346	410	524	635	707	780	887	1065	1243	1419	1659	1786	3.86
4.00	28	112	—	—	—	—	349	413	527	638	710	783	890	1069	1246	1423	1662	1789	4.00
4.00	36	144	—	—	—	—	—	—	—	462	542	618	730	913	1094	1272	1514	1641	4.00
4.00	48	192	—	—	—	—	—	—	—	—	—	—	—	654	849	1035	1283	1413	4.00
4.20	40	168	—	—	—	—	—	—	—	—	—	—	605	796	981	1162	1406	1534	4.20
4.24	34	144	—	—	—	—	—	—	—	488	547	624	736	919	1100	1279	1520	1648	4.24
4.36	44	192	—	—	—	—	—	—	—	—	—	—	—	668	861	1047	1295	1425	4.36
4.42	38	168	—	—	—	—	—	—	—	—	—	488	610	802	986	1168	1412	1541	4.42
4.50	32	144	—	—	—	—	—	—	—	473	553	630	742	925	1106	1285	1527	1654	4.50
4.67	36	168	—	—	—	—	—	—	—	—	—	493	616	807	992	1174	1419	1547	4.67
4.80	30	144	—	—	—	—	—	—	—	479	558	635	748	931	1112	1291	1533	1661	4.80
4.80	40	192	—	—	—	—	—	—	—	—	—	—	—	678	872	1059	1307	1437	4.80

Continued

Table 12.19: Fenner torque drive plus 14MXP and 14M drives. Center distance (mm).—cont'd

	Number of grooves on:		Belt pitch length (mm)															
			966	1190	1400	1610	1778	1890	2100	2310	2450	2590	2800	3150	3500	3850	4326	4578	
Speed ratio	Driving pulley	Driven pulley	69 teeth	85 teeth	100 teeth	115 teeth	127 teeth	135 teeth	150 teeth	165 teeth	175 teeth	185 teeth	200 teeth	225 teeth	250 teeth	275 teeth	309 teeth	327 teeth	Speed ratio
4.94	34	168	—	—	—	—	—	—	—	—	—	499	621	813	998	1180	1425	1553	4.94
4.97	29	144	—	—	—	—	—	—	—	481	561	638	751	934	1115	1294	1536	1664	4.97
5.05	38	192	—	—	—	—	—	—	—	—	—	—	—	684	878	1065	1313	1443	5.05
5.14	28	144	—	—	—	—	—	—	—	484	564	641	754	937	1118	1297	1539	1667	5.14
5.25	32	168	—	—	—	—	—	—	—	—	—	504	627	819	1004	1186	1431	1560	5.25
5.33	36	192	—	—	—	—	—	—	—	—	—	—	—	689	884	1071	1319	1449	5.33
5.60	30	168	—	—	—	—	—	—	—	—	—	509	632	825	1010	1192	1437	1566	5.60
5.65	34	192	—	—	—	—	—	—	—	—	—	—	—	695	889	1076	1325	1455	5.65
5.79	29	168	—	—	—	—	—	—	—	—	—	512	635	828	1013	1195	1440	1569	5.79
6.00	28	168	—	—	—	—	—	—	—	—	—	514	638	830	1016	1198	1443	1572	6.00
6.00	32	192	—	—	—	—	—	—	—	—	—	—	—	700	895	1082	1331	1462	6.00
6.40	30	192	—	—	—	—	—	—	—	—	—	—	—	706	901	1088	1337	1468	6.40
6.62	29	192	—	—	—	—	—	—	—	—	—	—	—	708	904	1091	1340	1471	6.62
6.86	28	192	—	—	—	—	—	—	—	—	—	—	—	711	906	1094	1343	1474	6.86

Source: Table courtesy of Fenner.

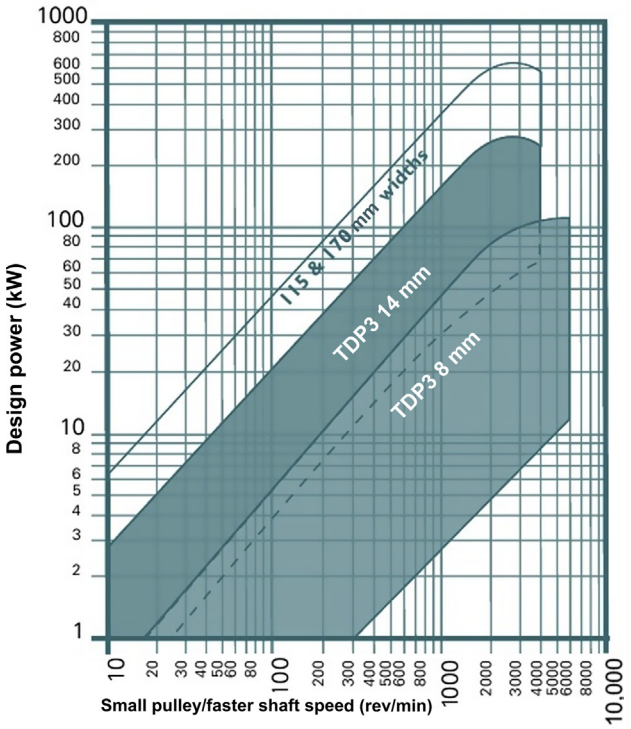


Figure 12.6
Belt pitch selection guide. *Chart courtesy of Fenner.*

From Table 12.11 the belt length factor is 1.00.

The ratio of the design power to the power rating is 85/69.77 = 1.22. From Table 12.12 the next larger standard width factor is 1.44, with a corresponding belt width of 55 mm. A belt width of 55 mm should therefore be used.

The pulley specification is therefore as follows.

Motor pulley: 44-14M-55 HTD pulley, corresponding to a 44 groove pulley with a 55 mm belt width.

Pump pulley: 90-14M-55 HTD pulley, corresponding to a 90 groove pulley with a 55 mm belt width.

Belt: 14MXP-2450-55 torque drive plus 3 belt, corresponding to a 2450-mm-long 55 mm wide belt.

It must also be confirmed that the bores for the pulleys are compatible with the motor and pump shaft diameters.

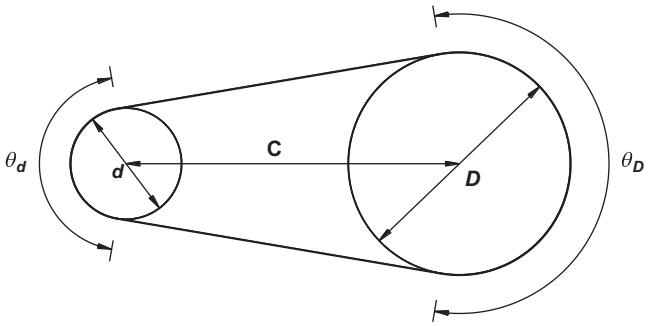


Figure 12.7
Belt drive geometry definition.

12.2.4 Flat Belt Drives

For the simple belt drive configuration shown in Figure 12.7, the angles of contact between the belt and the pulleys are given by

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C} \tag{12.3}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C} \tag{12.4}$$

where

d is the diameter of the small pulley (m),
D is the diameter of the large pulley (m),
C is the distance between the pulley centers (m),
θ_d is the angle of contact between the belt and the small pulley (rad),
θ_D is the angle of contact between the belt and the large pulley (rad).

The length of the belt can be obtained by summing the arc lengths of contact and the spanned distances and is given by

$$L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\theta_D + d\theta_d) \tag{12.5}$$

The power transmitted by a belt drive is given by

$$\text{Power} = (F_1 - F_2)V \tag{12.6}$$

where

F_1 is the belt tension in the tight side (N),
F_2 is the belt tension in the slack side (N),
V is the belt speed (per ms).

The torque is given by

$$\text{Torque} = (F_1 - F_2)r \tag{12.7}$$

Assuming that the friction is uniform throughout the arc of contact and ignoring centrifugal effects, the ratio of the tensions in the belts can be modeled by Eytlewein's formula

$$\frac{F_1}{F_2} = e^{\mu\theta} \tag{12.8}$$

where

μ is the coefficient of friction,
θ is the angle of contact (rad), usually taken as the angle for the smaller pulley.

The centrifugal forces acting on the belt along the arcs of contact reduce the surface pressure. The centrifugal force is given by

$$F_c = \rho V^2 A = mV^2 \tag{12.9}$$

where

ρ is the density of the belt material (kg/m^3),
A is the cross-sectional area of the belt (m^2),
m is the mass per unit length of the belt (kg/m).

The centrifugal force acts on both the tight and the slack sides of the belt, and Eytlewein's formula can be modified to model the effect:

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta} \tag{12.10}$$

The maximum allowable tension, $F_{1,\max}$, in the tight side of a belt depends in the allowable stress of the belt material, $\sigma_{\max}$. Typical values for the maximum permissible stress are given in Table 12.20.

$$F_{1,\max} = \sigma_{\max} A \tag{12.11}$$

The required cross-sectional area for a belt drive can be found from

$$A = \frac{F_1 - F_2}{\sigma_1 - \sigma_2} \tag{12.12}$$

Example 12.4

A fan is belt-driven by an electric motor running at 1500 rpm. The pulley diameters for the fan and motor are 500 and 355 mm, respectively. A flat belt has been selected with a width

Table 12.20: Guideline values for the maximum permissible stress for high-performance flat belts.

Multiply structure			Maximum permissible stress (MN/m^2)
Friction surface coating	Core	Top surface	
Elastomer	Polyamide sheet	Polyamide fabric	8.3–19.3
Elastomer	Polyamide sheet	Elastomer	6.6–13.7
Chrome leather	Polyamide sheet	None	6.3–11.4
Chrome leather	Polyamide sheet	Polyamide fabric	5.7–14.7
Chrome leather	Polyamide sheet	Chrome leather	4–8
Elastomer	Polyester cord	Elastomer	Up to 21.8
Chrome leather	Polyester cord	Polyamide fabric	5.2–12
Chrome leather	Polyester cord	Chrome leather	3.1–8

of 100 mm, a thickness of 3.5 mm, a coefficient of friction of 0.8, a density of 1100 kg/m^3, and permissible stress of 11 MN/m^2. The center distance is 1500 mm. Determine the power capacity of the belt.

Solution

The arcs of contact for the driving and driven pulleys are:

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C} = \pi - 2\sin^{-1}\frac{500-355}{2\times 1500} = 3.045 \text{ rad},$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C} = \pi + 2\sin^{-1}\frac{500-355}{2\times 1500} = 3.238 \text{ rad}.$$

The maximum tension in the tight side (see Table 12.20) is given as a function of the maximum permissible stress in the belt by:

$$F_1 = \sigma_{\max} A = 11\times 10^6\left(3.5\times 10^{-3}\times 100\times 10^{-3}\right) = 3850 \text{ N}.$$

The belt velocity, $V = 1500\times\frac{2\pi}{60}\times\frac{0.355}{2} = 27.88$ m/s.

The mass per unit length, $m = \rho A = 1100\times 0.1\times 0.0035 = 0.385$ kg/m.

The centrifugal load is given by $F_c = 0.385(27.88)^2 = 299.3$ N.

Using Eqn (12.10):

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta} = \frac{3850 - 299.3}{F_2 - 299.3} = e^{0.8\times 3.045}$$

$$\Rightarrow F_2 = 610.0 \text{ N}$$

The power capacity is given by $(F_1 - F_2)V = (3850 - 610)27.88 = 90.3$ kW.

Example 12.5

A flat belt is required to transmit 22 kW from a 250-mm-diameter pulley running at 1450 rpm to a 355-mm-diameter pulley. The coefficient of friction can be taken as 0.7, the density of the belt is 1100 kg/m^3, and the maximum permissible stress is 7 MPa. The distance between the shaft centers is 1.8 m. The proposed belt is 3.5 mm thick. Calculate the width required.

Solution

22 kW, $D_1 = 250$ mm, $D_2 = 355$ mm, $n_1 = 1450$ rpm, $\mu = 0.7$, $\rho = 1100$ kg/m^3, $\sigma_{\max} = 7$ MPa, $C = 1.8$ m, $t = 3.5$ mm.

$$\theta_d = \pi - 2\sin^{-1}\frac{355 - 250}{2 \times 1800} = 3.083.$$

$$F_1 = \sigma_{\max} A = \sigma_{\max} tw = 7 \times 10^6 \times 3.5 \times 10^{-3} w = 24500w.$$

$$F_c = \rho A V^2 = \rho tw \times \left(1450 \times \frac{2\pi}{60} \times \frac{0.25}{2}\right)^2$$

$$= w \times 1100 \times 3.5 \times 10^{-3} \times \left(1450 \times \frac{2\pi}{60} \times \frac{0.25}{2}\right)^2 = 1387w$$

$$\frac{F_1 - 1387w}{F_2 - 1387w} = e^{2.1581} = 8.655$$

$$F_1 - 1387w = 8.655(F_2 - 1387w)$$

$$F_1 - F_2 = 22{,}000/18.98 = 1159.$$

$$F_2 = F_1 - 1159 = 24{,}500w - 1159.$$

$$\frac{24,500w - 1387w}{8.655} = 24,500w - 1159 - 1387w$$

$w = 0.0567$ m, $w = 56.7$ mm. Use $w = 60$ mm.

12.3 Chain Drives

A chain is a power transmission device consisting of a series of pin-connected links as illustrated in Figure 12.8. The chain transmits power between two rotating shafts by meshing

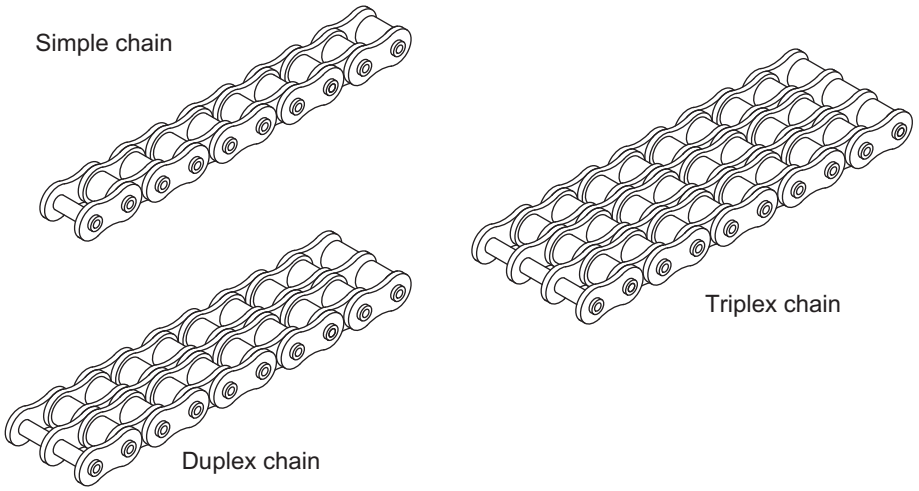


Figure 12.8
Roller chain.

with toothed sprockets as shown in Figure 12.9. A cross section of a sprocket for the case of a triplex chain is illustrated in Figure 12.10.

Chain drives are usually manufactured using high-strength steel, and for this reason they are capable of transmitting high torque. Chain drives are complementary and competitive with belt drives serving the function of transmitting a wide range of powers for shaft speeds up to about 6000 rpm. At higher speeds, the cyclic impact between the chain links and the sprocket teeth, the high noise, and the difficulties in providing lubrication limit the application of chain drives. Table 12.21 shows a comparison of chain, belt, and gear attributes. Chain drives are principally used for power transmission, conveyors, and for supporting or lifting loads (see American Chain Association, 2005). Applications range from motorcycle and bicycle transmissions, automotive camshaft drives (although synchronous

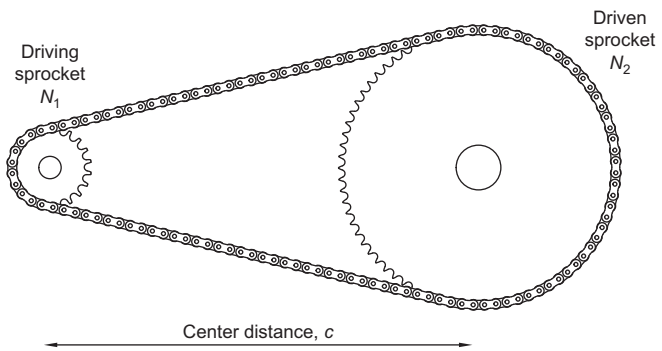


Figure 12.9
Simple chain drive.

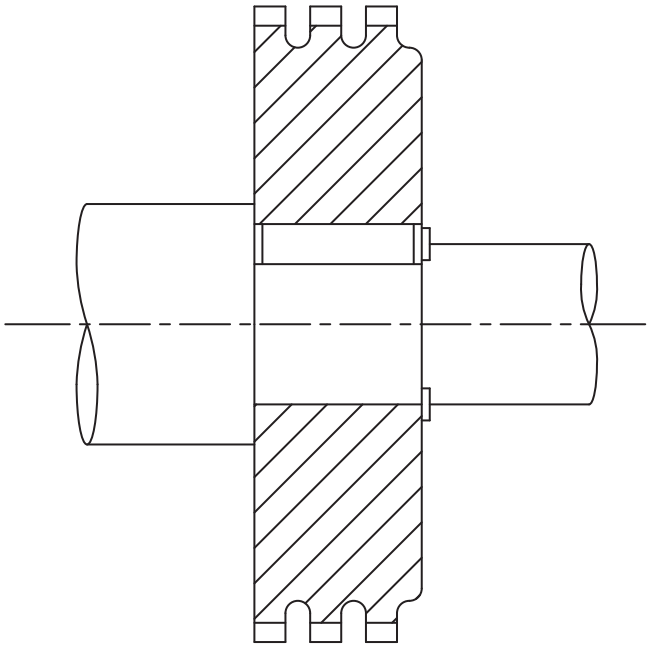


Figure 12.10
Triplex sprocket.

belts have replaced chains in many modern automotive applications), machine tools and aerospace drives such as the thruster nozzles for the Harrier, to conveyors and packaging machinery. Efficiencies of up to 98.9%, ratios of up to 9:1 (where necessary), and power transmission of several 100 kW can be achieved. Chain drives are typically used with

Table 12.21: Comparison of chain, belt, and gear performance.

Feature	Chain drive	Belt drive	Gear drive
Efficiency	A	A	A
Positive drive	A	A*	A
Large center distance	A	A	C
Wear resistance	A	B	A
Multiple drives	A	A	C
Heat resistance	A	C	B
Chemical resistance	A	C	B
Oil resistance	A	C	A
Power range	A	B	A
Speed range	C	A	A
Ease of maintenance	A	B	C
Environment	A	C	A

A = excellent, B = good, C = poor.
* Using a synchronous belt drive.

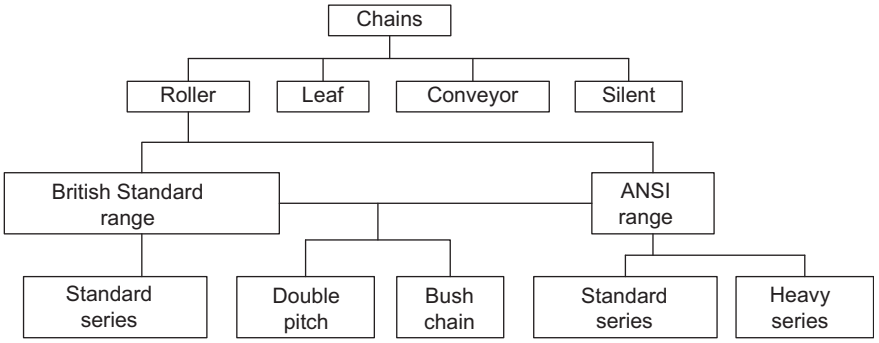


Figure 12.11
Chain types.

reduction ratios of up to 3:1 giving high efficiency and ratios of up to 5 giving reasonable efficiency.

The range of chain drives is extensive, as illustrated in Figure 12.11. The most common type is a roller chain, which is used for high-power transmission and conveyor applications. The selection and specification of this type of chain is introduced in Section 12.3.1. A roller chain is made up of a series of links. Each chain consists of side plates, pins, bushes, and rollers, as shown in Figure 12.12. The chain runs on toothed sprockets, and as the teeth of the sprockets engage with the rollers, rolling motion occurs and the chain articulates onto the sprocket. A roller chain is classified by its pitch. This is the distance between the corresponding points of adjacent links. Roller chains are available in stainless steel and nylon (for hygienic applications) as well as a series of other materials for specialist applications.

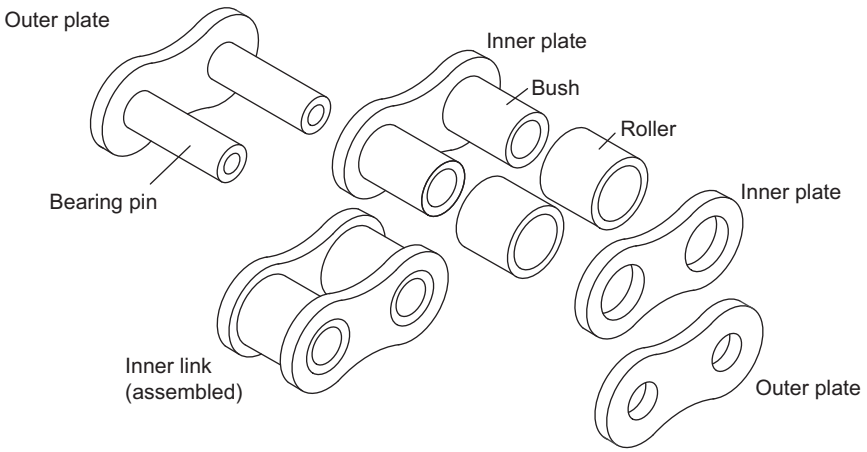


Figure 12.12
Roller chain components.

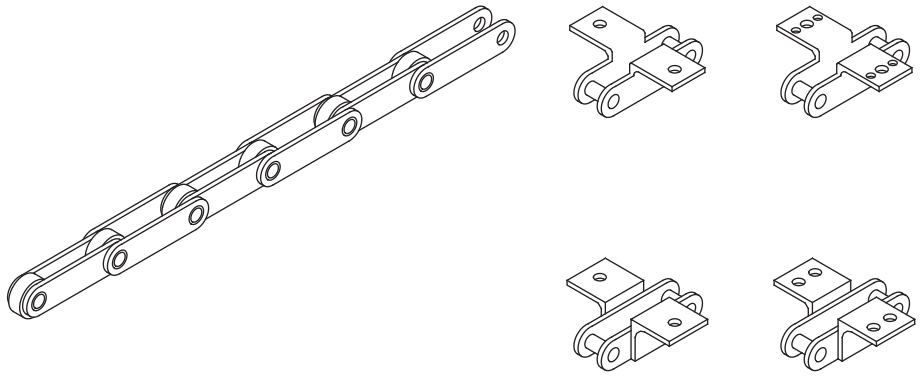


Figure 12.13
Conveyor chain.

A conveyor chain is specially designed for use in materials handling and conveyor equipment and is characterized by its long pitch, large roller diameters, and high tensile strength. The applications of a conveyor chain demand that it is capable of pulling significant loads, usually along a straight line at relatively low speeds. The sides of conveyor chains often incorporate special features to aid connection to conveyor components, as illustrated in Figure 12.13.

A leaf chain consists of a series of pin-connected side plates, as illustrated in Figure 12.14. It is generally used for load balancing applications and is essentially a special form of transmission chain.

A silent chain (also called an inverted tooth chain) has teeth formed in the link plates, as shown in Figure 12.15. The chain consists of alternately mounted links so that the chain can

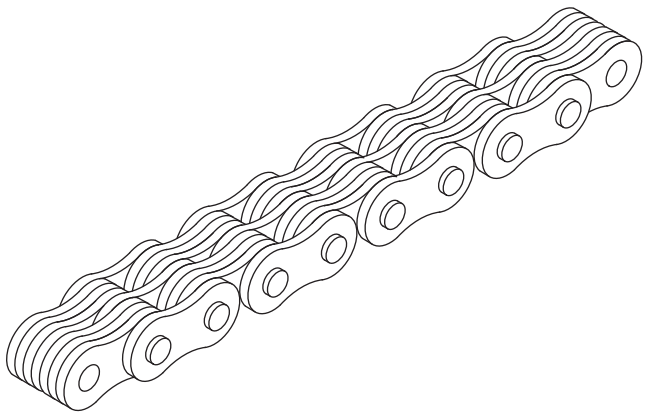


Figure 12.14
Leaf chain.

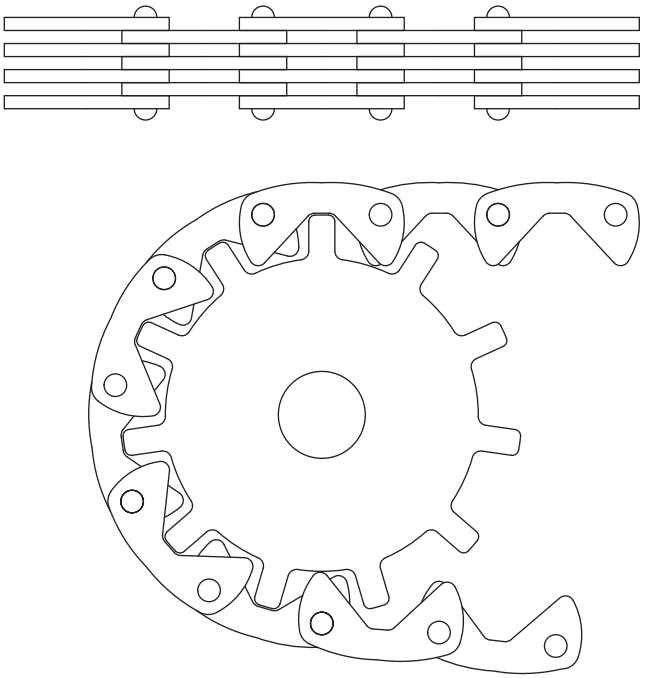


Figure 12.15
Silent chain.

articulate onto the mating sprocket teeth. Silent chains can operate at higher speeds than comparatively sized roller chains and, as the name suggests, more quietly.

The direction of rotation relative to the chain layout is important to the effective operation of a chain drive. Figure 12.16 shows the recommended and acceptable practice for chain drive layouts. The idler sprockets shown are used to engage the chain in the slack region for chain drives with large center distances or when the drive is vertical.

Chain design is based on ensuring that the power transmission capacity is within limits for three modes of failure: fatigue, impact, and galling. Chains are designed so that the maximum tensile stress is below the fatigue endurance limit for the finite life of the material. Failure would nevertheless eventually occur but it would be due to wear, not fatigue. In service, failures due to wear can be eliminated by inspection and replacement intervals. When the chain rollers mesh with the sprocket teeth, an impact occurs and a Hertz contact stress occurs, similar to that found for gear meshing. The power rating charts for chain drives limit the selection of the drive so that these modes of failure should not occur, assuming proper installation, operation, and lubrication.

Details of the selection procedure for standard roller chains are outlined in Section 12.3.1. For details of chain geometry, the reader is referred to the standards listed in the

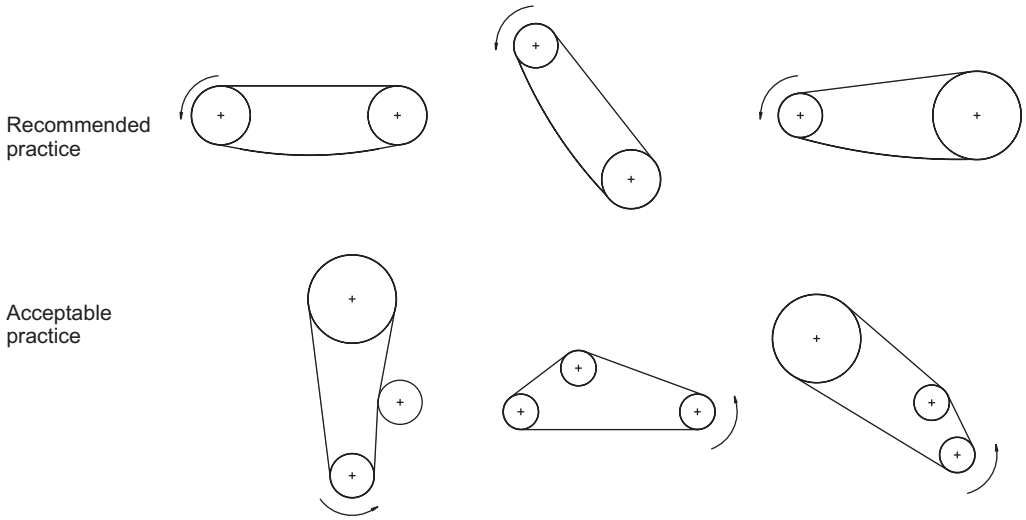


Figure 12.16
Typical chain drive layouts.

references, and for details of selection and design procedures for other chain types, the reader is referred to manufacturers' catalogues.

12.3.1 Roller Chain Selection

Once the use of a chain drive has been shown to be preferable to other forms of drive, the type of chain to be used can be selected from the range available, as illustrated in Figure 12.11. The next step is the design of the chain drive layout and selection of the standard components available from chain manufacturers. The method outlined here is for roller chains. Procedures for the selection of other chain types can be found in manufacturers' catalogues.

The method is based upon the use of power rating charts for the chain drive, which ensure 15,000 h operation assuming proper installation, operation, and lubrication. The steps for the method are listed below.

1. Determine the power to be transmitted.
2. Determine the speeds of the driving and driven shafts.
3. Determine the characteristics of the driving and driven shaft, e.g. type of running, whether smooth or shock loadings, etc.
4. Set the approximate center distance. This should normally be in the range of 30−50 times the chain pitch.
5. Select the speed ratio. This is dependent on the standard sizes available as listed in Table 12.22. Ideally sprockets should have a minimum of 19 teeth. For high-speed drives

Table 12.22: Chain reduction ratios as a function of the standard sprockets available.

Number of teeth in the driven sprocket N_2	Number of teeth in the drive sprocket N_1					
	15	17	19	21	23	25
25	—	—	—	—	—	1.00
38	2.53	2.23	2.00	1.80	1.65	1.52
57	3.80	3.35	3.00	2.71	2.48	2.28
76	5.07	4.47	4.00	3.62	3.30	3.04
95	6.33	5.59	5.00	4.52	4.13	3.80
114	7.60	6.70	6.00	5.43	4.96	4.56

Source: Reproduced from Renold (1996).

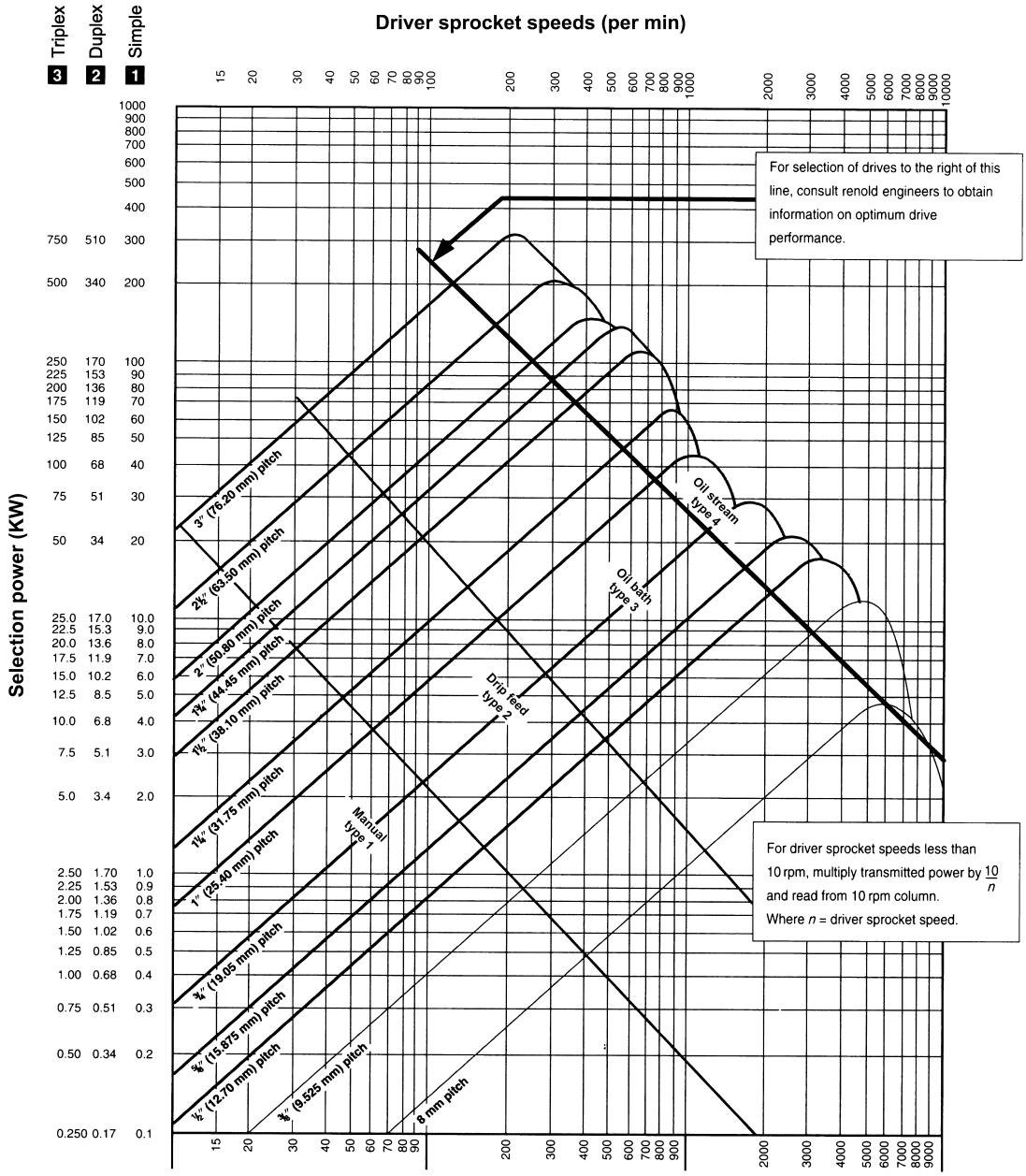
subjected to transient loads, the minimum number of teeth rises to 25. Note that the maximum number of teeth should not exceed 114.

6. Establish the application and tooth factors: Table 12.23 gives values for the application factor f_1. The tooth factor is given by $f_2 = 19/N_1$ assuming that the selection rating charts are based on a 19-tooth sprocket.
7. Calculate the selection power. Selection power = Power $\times f_1 \times f_2$.
8. Select the chain drive pitch. Use power-speed rating charts as supplied by chain manufacturers (see Figure 12.17). The smallest pitch of a simple chain should be used as this normally gives the most economical drive. If the power requirement at a given speed is beyond the capacity of a single strand of chain, then the use of a multistrand chain, such as duplex (two strands), triplex (three strands), and up to decuplex (10 strands) for the ANSI range, permits higher power to be transmitted and can be considered.

Table 12.23: Application factor.

Driven machine characteristics	Characteristics of driver		
	Smooth running, e.g. electric motors, IC engines with hydraulic coupling	Slight shocks, e.g. IC engines with more than six cylinders, electric motors with frequent starts	Heavy shocks, e.g. IC engines with fewer than six cylinders
Smooth running, e.g. fans, pumps, compressors, printing machines, uniformly loaded conveyors.	1	1.1	1.3
Moderate shocks, e.g. concrete mixing machines, nonuniformly loaded conveyors, mixers	1.4	1.5	1.7
Heavy shocks, e.g. planars, presses, drilling rigs.	1.8	1.9	2.1

Source: Reproduced from Renold (1996).



1 Kilowatt = 1.34 hp.

Figure 12.17
British Standard chain drives rating chart using a 19-tooth drive sprocket.
Courtesy of Renold Chain.

9. Calculate the chain length. Eqn (12.13) gives the chain length as a function of the number of pitches. Note that the value for the length should be rounded up to the nearest even integer.
10. Calculate the exact center distance. This can be calculated using Eqn (12.14).
11. Specify the lubrication method.

The chain length, in pitches, is given by

$$L = \frac{N_1 + N_2}{2} + \frac{2C}{p} + \left(\frac{N_2 - N_1}{2\pi}\right)^2 \frac{p}{C} \tag{12.13}$$

where

L is the number of pitches,
N_1 is the number of teeth in the driving sprocket,
N_2 is the number of teeth in the driven sprocket,
C is the center distance (m),
p is the chain pitch (m).
The exact center distance is given by

$$C = \frac{p}{8}\left[2L - N_2 - N_1 + \sqrt{(2L - N_2 - N_1)^2 - \frac{\pi}{3.88}(N_2 - N_1)^2}\right] \tag{12.14}$$

Chain drives should be protected against dirt and moisture (tell this to a mountain biker!). Lubrication should be provided using a nondetergent mineral-based oil. For the majority of applications, multigrade SAE20/50 is suitable. There are five principal types of lubrication: manual application, drip feed, bath, stream (see Figure 12.18), and dry lubrication. Grease lubrication is not recommended but can be used for chain speeds of less than 4 m/s^{-1}. In order to ensure the grease penetrates the working parts of the chain, it should be heated until it becomes liquid and the chain dipped into the grease until the air has been displaced. This process should be repeated at regular service intervals. For dry lubrication, solid lubricant is contained in a volatile carrier fluid. When applied to a chain, the carrier transports the lubricant into the chain and then evaporates, leaving the chain lubricated but dry to touch. Applications for dry lubrication include food processing, dusty environments, and fabric handling.

The pitch diameters for the driving and driven sprockets are given by

$$D_1 = \frac{N_1 p}{\pi} \quad D_2 = \frac{N_2 p}{\pi} \tag{12.15}$$

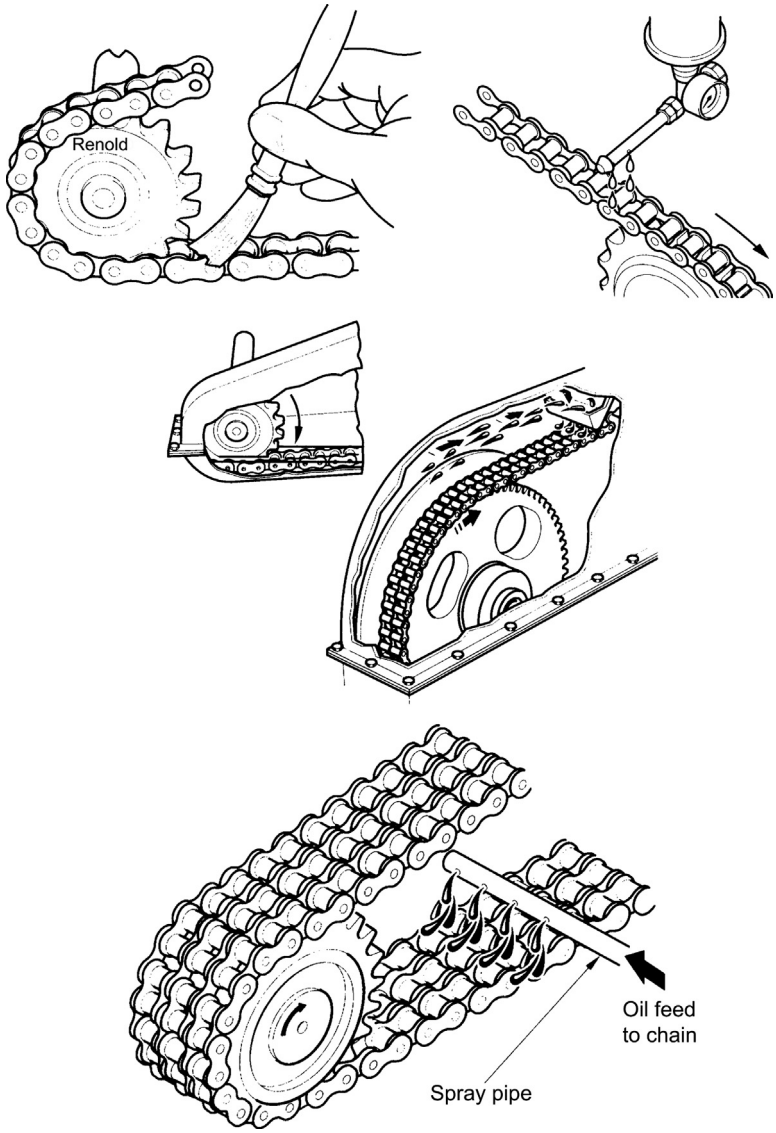


Figure 12.18
Chain lubrication methods. *Courtesy of Renold Chain.*

and the angle of contact (in radians) between the chain and the sprockets by

$$\theta_1 = \pi - 2\sin^{-1}\frac{p(N_2 - N_1)}{2\pi C} \tag{12.16a}$$

$$\theta_2 = \pi + 2\sin^{-1}\frac{p(N_2 - N_1)}{2\pi C} \tag{12.16b}$$

Note that the minimum angle of wrap recommended for the small sprocket is 120°.

The chain tension is given by

$$\text{Chain tension} = \frac{\text{Power}}{N_1 \omega_1 p/2\pi} \tag{12.17}$$

Standard sprockets can be purchased. The choice of sprocket material depends on the number of teeth and the operating conditions as shown in Table 12.24.

Example 12.6

A chain drive is required for a gear pump operating at 400 rpm driven by a 5.5 kW electric motor running at 1440 rpm. The center distance between the motor and pump shafts is approximately 470 mm.

Solution

The desired reduction ratio is $\frac{1440}{400} = 3.6$.

The nearest ratio available (see Table 12.22) using standard sized sprockets is 3.62. This requires a driving sprocket of 21 teeth and a driven sprocket of 76 teeth.

The application factor from Table 12.23 is $f_1 = 1.0$.

The tooth factor $f_2 = \frac{19}{N_1} = \frac{19}{21} = 0.905$.

The selection power $= 5.5 \times 1.0 \times 0.905 = 4.98$ kW.

Using the BS/ISO selection chart, Figure 12.17, a 12.7 mm pitch simple BS chain drive is suitable.

The chain length is given by

Table 12.24: Selection of sprocket materials.

Sprocket	Smooth running	Moderate shocks	Heavy shocks
Up to 29 teeth	080M40 or 070M55	080M40 or 070M55 hardened and tempered or case-hardened mild steel	080M40 or 070M55 hardened and tempered or case-hardened mild steel
Over 30 teeth	Cast iron	Mild steel	080M40 or 070M55 hardened and tempered or case-hardened mild steel

Source: Adapted from Renold (1996).

$$L = \frac{N_1 + N_2}{2} + \frac{2C}{p} + \left(\frac{N_2 - N_1}{2\pi}\right)^2 \frac{p}{C}$$

$$= \frac{21 + 76}{2} + \frac{2 \times 470}{12.7} + \left(\frac{76 - 21}{2\pi}\right)^2 \frac{12.7}{470} = 124.6 \text{ pitches.}$$

Rounding up to the nearest even integer gives $L = 126$ pitches.

The exact center distance can now be calculated using

$$C = \frac{p}{8}\left[2L - N_2 - N_1 + \sqrt{(2L - N_2 - N_1)^2 - \frac{\pi}{3.88}(N_2 - N_1)^2}\right].$$

$$C = \frac{12.7}{8}\left[2 \times 126 - 76 - 21 + \sqrt{(2 \times 126 - 76 - 21)^2 - \frac{\pi}{3.88}(76 - 21)^2}\right] = 479.2 \text{ mm.}$$

From the rating chart, Figure 12.17, the required lubrication type is an oil bath. Use of a SAE20/50 multigrade lubricant would likely suffice in the absence of more detailed knowledge concerning the operating conditions.

Example 12.7

Specify a suitable drive for a gear pump operating at 400 rpm driven by a 30 kW electric motor running at 728 rpm. The center distance between the motor and pump shafts is approximately 1 m.

Solution

The desired reduction ratio is $\frac{728}{400} = 1.82$.

From Table 12.22, the nearest ratio using standard sized sprockets is 1.8. $N_1 = 21$, $N_2 = 38$.

From Table 12.23, $f_1 = 1.0$.

The tooth factor $f_2 = \frac{19}{N_1} = \frac{19}{21} = 0.905$.

The selection power $= 30 \times 1.0 \times 0.905 = 27.15$ kW.

Using the BS/ISO selection chart, Figure 12.17, a 25.4 mm pitch simple BS chain drive is suitable with oil bath lubrication.

$$L = \frac{21 + 38}{2} + \frac{2 \times 1000}{25.4} + \left(\frac{38 - 21}{2\pi}\right)^2 \frac{25.4}{1000} = 108.4 \text{ pitches.}$$

Rounding up to the nearest even integer gives $L = 110$ pitches.

The exact center distance is

$$C = \frac{25.4}{8}\left[2 \times 110 - 38 - 21 + \sqrt{(2 \times 110 - 38 - 21)^2 - \frac{\pi}{3.88}(38 - 21)^2}\right] = 1020 \text{ mm}.$$

Example 12.8

Specify a suitable chain drive for a packaging machine operating at 75 rpm driven by a 2.2 kW electric motor running at 710 rpm. The maximum permissible center distance between the motor and packaging machine shafts is approximately 1 m.

Solution

There is no unique solution to this question. What follows is one possible solution.

The speed ratio 710/75 = 9.47 is too high for a single reduction and is outside the range listed in Table 12.22. A two-stage reduction might be feasible.

Examination of Table 12.22 for a combination of reductions shows that choosing ratios of 3.8 and 2.48 might be suitable.

As $3.8 \times 2.48 = 9.424$ is close to the target reduction, this combination is likely to be acceptable and would give an output speed of 75.3 rpm.

Using the larger reduction on the higher speed drive, for a ratio of 3.8 from Table 12.22, $N_1 = 25$, $N_2 = 95$.

The application factor from Table 12.23 assuming moderate shocks is 1.4.

The tooth factor $f_2 = \dfrac{19}{N_1} = \dfrac{19}{25} = 0.76$.

The selection power $= 2.2 \times 1.4 \times 0.76 = 2.34$ kW.

Using the BS/ISO selection chart, Figure 12.17, the following drives would be suitable:

12.7 mm pitch in oil bath (simple).
9.525 mm drip feed (duplex).

Selecting the simple chain, $p = 12.7$ mm.

Take the distance between the sprocket centers as the minimum recommended, i.e. $30 \times p = 30 \times 12.7 = 381$ mm to ensure the design fits in the space available. This constraint can be relaxed at a later stage if appropriate.

The chain length is given by

$$L = \frac{25+95}{2} + \frac{2 \times 381}{12.7} + \left(\frac{95-25}{2\pi}\right)^2 \frac{12.7}{381} = 124.1 \text{ pitches.}$$

Rounding up to the nearest even integer gives $L = 126$ pitches.

The exact center distance is given by

$$C = \frac{12.7}{8}\left[2 \times 126 - 95 - 25 + \sqrt{(132)^2 - \frac{\pi}{3.88}(70)^2}\right] = 393.7 \text{ mm.}$$

For the second chain with a ratio of 2.48, $N_1 = 23$, $N_2 = 57$.

The application factor from Table 12.23 is 1.4.

The tooth factor $f_2 = \dfrac{19}{N_1} = \dfrac{19}{23} = 0.8261$.

The selection power $= 2.2 \times 1.4 \times 0.8261 = 2.544$ kW.

Using the BS/ISO selection chart, Figure 12.17, the following drives would be suitable:

12.7 mm pitch in oil bath (simple).
9.525 mm drip feed (duplex).

Selecting the simple chain, $p = 12.7$ mm.

Again the distance between the sprocket centers can be taken as the minimum recommended, i.e. $30 \times p = 30 \times 12.7 = 381$ mm.

The chain length is given by

$$L = \frac{23+57}{2} + \frac{2 \times 381}{12.7} + \left(\frac{57-23}{2\pi}\right)^2 \frac{12.7}{381} = 101.0 \text{ pitches.}$$

Rounding up to the nearest even integer gives $L = 102$ pitches.

The exact center distance is given by

$$C = \frac{12.7}{8}\left[2 \times 102 - 57 - 23 + \sqrt{(124)^2 - \frac{\pi}{3.88}(34)^2}\right] = 387.6 \text{ mm.}$$

The combination of the two center distances fits in the space available.

An alternative might be to arrange the chain sprockets vertically or at an angle to the horizontal in order to accommodate the center distance restriction.

12.4 Conclusions

Belt and chain drives can be used for transmission of mechanical power between two rotating shafts. Belt drives are often cheaper than the equivalent gears and useful for transmitting power between shafts that are widely separated or nonparallel drives. Chain drives are usually more compact than the equivalent belt drive and can be used in oily environments where the equivalent belt would be prone to slipping. There is a wide range of belt and chain drives, and this chapter has served to introduce the technology and the selection and specification of wedge and flat belts and roller chain drives.

References

Books and Papers

American Chain Association, 2005. Standard Handbook of Chains: Chains for Power Transmission and Material Handling, second ed. CRC Press.
Fenner Power Transmission UK. Belt drives design manual.
Hamilton, P., 1994. Belt drives. In: Hurst, K. (Ed.), Rotary Power Transmission Design. McGraw Hill.
Perneder, R., Osbourne, I., 2012. Handbook Timing Belts: Principles, Calculations, Applications. Springer.
Renold Power Transmission Ltd, 1996. The Designer Guide, second ed.

Further Reading

Standards

BS 2969: 1980. Specification for high tensile steel chains (round link) for chain conveyors and coal ploughs.
BS 2969:1980, ISO 610-1979. Specification for high-tensile steel chains (round link) for chain conveyors and coal ploughs.
BS 3790:2006. Specification for belt drives. Endless wedge belts, endless V-belts, banded wedge belts, banded V-belts and their corresponding pulleys.
BS 4116-4:1992. Conveyor chains, their attachments and associated chain wheels. Specification for chains and attachments (British series).
BS 5801:1979, ISO 4348-1978. Specification for flat-top chains and associated chain wheels for conveyors.
BS ISO 10190:2008. Motor cycle chains. Characteristics and test methods.
BS ISO 10823:1996. Guidance on the selection of roller chain drives.
BS ISO 1275:2006. Double-pitch precision roller chains, attachments and associated chain sprockets for transmission and conveyors.
BS ISO 155:1998. Belt drives. Pulleys. Limiting values for adjustment of centres.
BS ISO 1977:2000. Conveyor chains, attachments and sprockets.
BS ISO 2790:2004. Belt drives. V-belts for the automotive industry and corresponding pulleys. Dimensions.
BS ISO 487:1998. Steel roller chains, types S and C, attachments and sprockets.
BS ISO 5287:2003. Belt drives. Narrow V-belts for the automotive industry. Fatigue test.
BS ISO 5294:2012. Synchronous belt drives. Pulleys.
BS ISO 5296:2012. Synchronous belt drives. Belts with pitch codes MXL, XXL, XL, L, H, XH and XXH. Metric and inch dimensions.
BS ISO 9633:2001. Cycle chains. Characteristics and test methods.

Web Sites

At the time of going to press, the world-wide-web contained useful information relating to this chapter at the following sites:

www.beijingthc.com
www.beltingindustries.com
www.chiorino.co.uk
www.chjc-chain.com
www.daidocorp.com
www.fennerdrives.com
www.fptgroup.com/fenner.asp
www.gates.com
www.goodyearep.com/
www.habasitiakoka.com
www.hmcross.com
www.hutchinsontransmission.com
www.jonsonrubber.com
www.kettenwulf.com/de/
www.kilangrantai.com
www.kkrubber.com
www.martinsprocket.com
www.mpta.org/
www.ngfeurope.com
www.omfarubbers.com
www.powertransmission.com
www.renold.com
www.samplabelting-sa.com
www.siegling.com
www.sprocketco.com
www.tsubakimoto.com
www.visusa.com
www.vmchain.com
www.websterchain.com

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

C	center distance (m)
D	pitch diameter (m)
f_1	application factor
f_2	tooth factor
L	number of pitches
N_1	number of teeth in the driving sprocket
N_2	number of teeth in the driven sprocket
p	chain pitch (m)

R_1	radius of driving pulley or sprocket (m)
R_2	radius of driven pulley or sprocket (m)
$V_{\text{pitch line}}$	pitch line velocity (m/s)
θ	angle of contact (rad)
ω_1	angular velocity of driving pulley or sprocket (rad/s)
ω_2	angular velocity of driven pulley or sprocket (rad/s).

CHAPTER 13

Clutches and Brakes

Chapter Outline

13.1 Introduction 513
13.2 Clutches 516
13.2.1 Design of Disc Clutches 521
13.3 Brakes 531
13.3.1 Disc Brakes 535
13.3.2 Drum Brakes 540
13.3.3 Short-Shoe External Drum Brakes 540
13.3.4 Long-Shoe External Drum Brakes 543
13.3.5 Long-Shoe Internal Drum Brakes 554
13.3.6 Band Brakes 557
13.4 Conclusions 562
References 562
Further Reading 562
Nomenclature 563

Abstract

A clutch is a device that permits the smooth, gradual connection of two shafts rotating at different speeds. A brake enables the controlled dissipation of energy to slow down, stop, or control the speed of a system. This chapter describes the basic principles of frictional clutches and brakes and outlines design and selection procedures for disc clutches as well as disc and drum brakes.

13.1 Introduction

When a rotating machine is started it must be accelerated from rest to the desired speed. A clutch is a device used to connect or disconnect a driven component from a prime mover such as an engine or motor. A familiar application is the use of a clutch between an engine's crankshaft and the gearbox in automotive settings. The need for the clutch arises from the relatively high torque requirement to get a vehicle moving and the low torque output from an internal combustion engine at low levels of rotational speed. The disconnection of the engine from the drive enables the engine to speed up unloaded to about 1000 rpm, where it generates sufficient torque to drive the transmission. The clutch can then be engaged, allowing power to be transmitted to the gearbox, transmission shafts, and wheels. A brake is a

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00013-7
Copyright © 2014 Elsevier Ltd. All rights reserved.

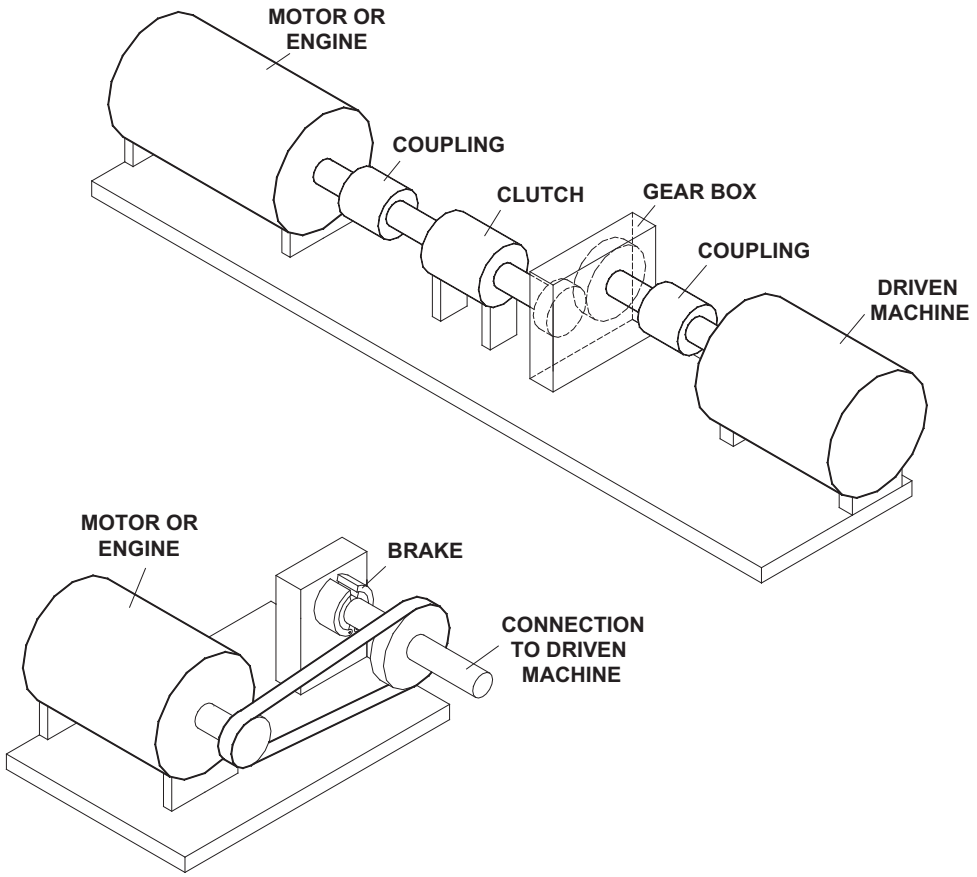


Figure 13.1
Typical applications of clutches and brakes.

device used to reduce or control the speed of a system or bring it to rest. Typical applications of a clutch and brake are illustrated in Figure 13.1. Clutches and brakes are similar devices providing frictional, magnetic, or mechanical connection between two components. If one component rotates and the other is fixed to a nonrotating plane of reference the device will function as a brake, and if both rotate it will function as a clutch.

Whenever the speed or direction of motion of a body is changed there is force exerted on the body. If the body is rotating, a torque must be applied to the system to speed it up or slow it down. If the speed changes, so does the energy, either by addition or absorption.

The acceleration, α, of a rotating machine is given by

$$\alpha = \frac{T}{I} \tag{13.1}$$

where T is the torque (N m) and I is the mass moment of inertia (kg m^2).

The mass moment of inertia can often be approximated by considering an assembly to be made up of a series of cylinders and discs and summing the individual values for the disc and cylinder mass moments of inertia. The mass moments of inertia for a cylinder and a disc are given by Eqns (13.2) and (13.3), respectively.

$$I_{\text{cylinder}} = \frac{1}{2}\rho\pi L\left(r_o^4 - r_i^4\right) \tag{13.2}$$

$$I_{\text{disc}} = \frac{1}{2}\rho\pi L\left(r_o^4\right) \tag{13.3}$$

The desired level of acceleration will depend on the application; once the acceleration has been set and I determined, the approximate value of torque required to accelerate or brake the load can be estimated. This can be used as the principal starting point for the design or selection of the clutch or brake geometry.

Torque is equal to the ratio of power and angular velocity. In other words, torque is inversely proportional to angular velocity. This implies that it is usually advisable to locate the clutch or brake on the highest speed shaft in the system so that the required torque is a minimum. Size, cost, and response time are lower when the torque is lower. The disadvantage is that the speed differential between the components can result in increased slipping and associated frictional heating, potentially causing overheating problems.

Friction type clutches and brakes are the most common. Two or more surfaces are pushed together with a normal force to generate a friction torque (see Figure 13.2). Normally, at least one of the surfaces is metal and the other a high friction material referred to as the lining. The frictional contact can occur radially, as for a cylindrical arrangement, or axially as in a disc arrangement. The function of a frictional clutch or brake surface material is to develop a substantial friction force when a normal force is applied. Ideally, a material with a high

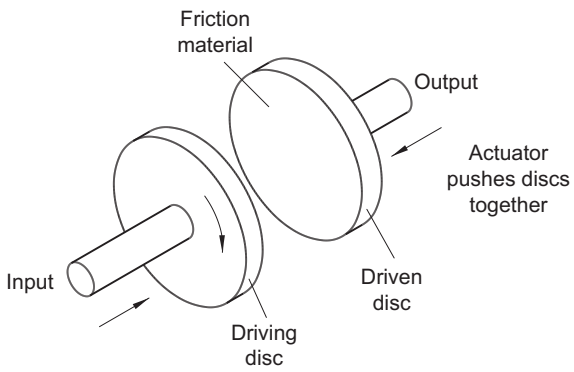


Figure 13.2
Idealized friction disc clutch or brake.

coefficient of friction, constant properties, good resistance to wear, and chemical compatibility is required. Clutches and brakes transfer or dissipate significant quantities of energy, and their design must enable the absorption and transfer of this heat without damage to the component parts of the surroundings.

With the exception of high volume automotive clutches and brakes, engineers rarely need to design a clutch or a brake from scratch. Clutch and brake assemblies can be purchased from specialist suppliers, and the engineer's task is to specify the torque and speed requirements, the loading characteristics, and the system inertias and to select an appropriately sized clutch or brake and the lining materials.

13.2 Clutches

The function of a clutch is to permit the connection and disconnection of two shafts, either when both are stationary or when there is a difference in the relative rotational speeds of the shafts. Clutch connection can be achieved by a number of techniques from direct mechanical friction, electromagnetic coupling, hydraulic or pneumatic means, or by some combination. There are various types of clutches, as outlined in Figure 13.3. The devices considered here are of the friction type.

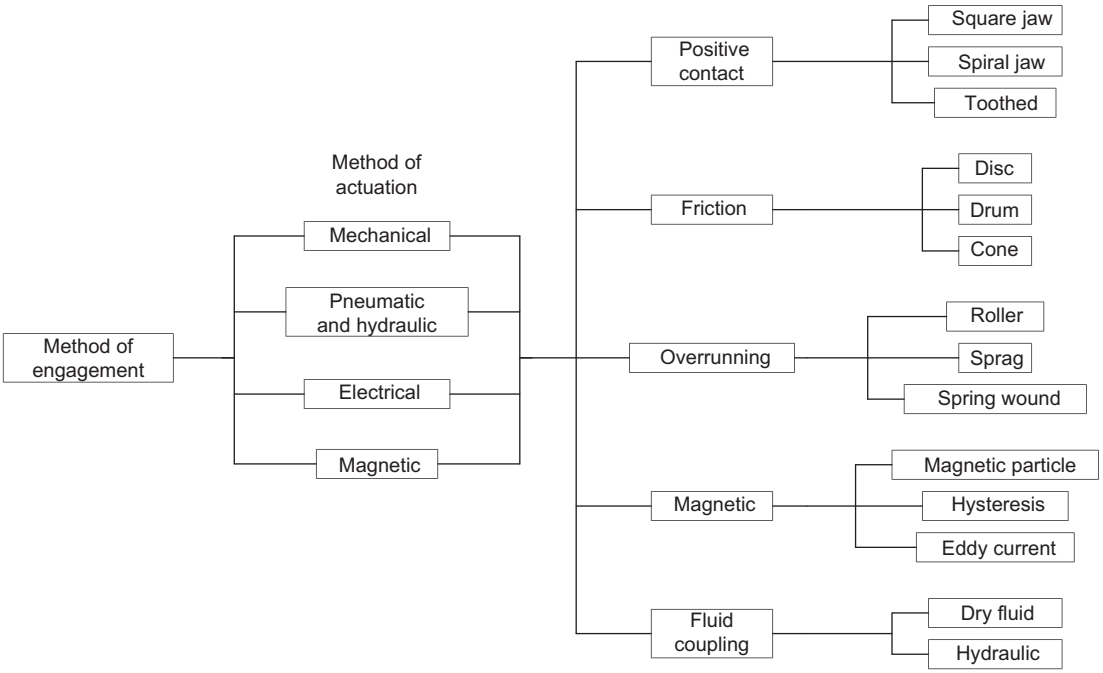


Figure 13.3
Clutch classification. *Reproduced with adaptations from Hindhede et al. (1983).*

Clutches must be designed principally to satisfy four requirements:

1. The necessary actuation force should not be excessive;
2. The coefficient of friction should be constant;
3. The energy converted to heat must be dissipated; and
4. Wear must be limited to provide reasonable clutch life.

Alternatively the objective in clutch design can be stated as maximization of a maintainable friction coefficient and minimization of wear. Correct clutch design and selection is critical because a clutch that is too small for an application will slip and overheat, and a clutch that is too large will have a high inertia and may overload the drive.

Example 13.1

Calculate the torque a clutch must transmit to accelerate a pulley with a moment of inertia of 0.25 kg m^2 to

1. 500 rpm in 2.5 s,
2. 1000 rpm in 2 s.

Solution

$T = I\alpha$,

$\alpha = \Delta n/t$,

$I = 0.25$ kg m^2.

1. $T = 0.25\left(\frac{500(2\pi/60)}{2.5}\right) = 5.236$ N m.

2. $T = 0.25\left(\frac{1000(2\pi/60)}{2}\right) = 13.10$ N m.

Positive contact clutches have teeth or serrations that provide mechanical interference between mating components. Figure 13.4 shows a square jaw clutch, and Figure 13.5 shows a multiple serration positive contact clutch. This principle can be combined with frictional surfaces as in an automotive synchromesh clutch. As helical gears cannot be shifted in and out of mesh easily, the pragmatic approach is to keep the gears engaged in mesh and allow the gear to rotate freely on the shaft when no power is required but provide positive location of the gear on the shaft when necessary (see also Chapter 8, Section 8.3.1).

Friction clutches consist of two surfaces, or two sets of surfaces, that can be forced into frictional contact. A frictional clutch allows gradual engagement between two shafts. The frictional surfaces can be forced together by springs or hydraulic pistons, or magnetically. Various forms exist such as disc, cone, and radial clutches. The design of the primary geometry for disc clutches is described in Section 13.2.1.

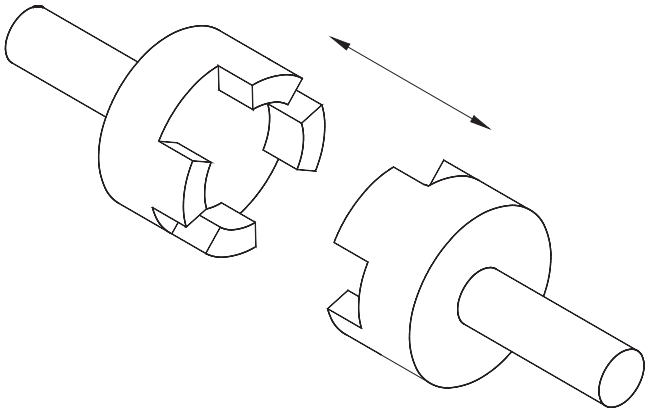


Figure 13.4
Square jaw clutch.

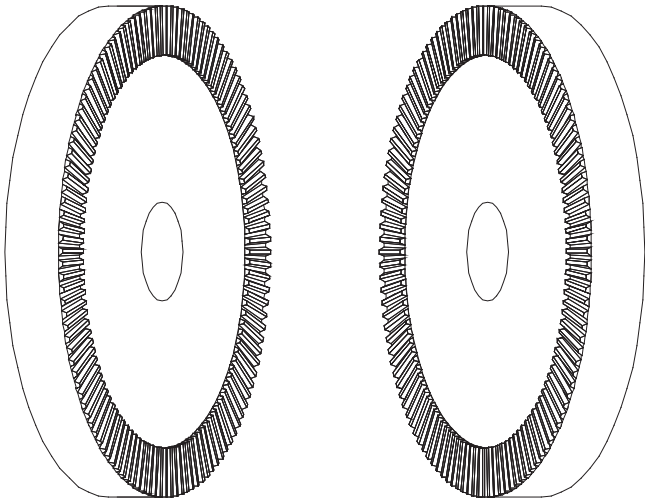


Figure 13.5
Multiple serration clutch.

Over-running clutches operate automatically based on the relative velocity of the mating components. They allow relative motion in one direction only. If the rotation attempts to reverse the constituent, components of the clutch grab the shaft and lock up. Applications include backstops, indexing, and freewheeling, as on a bicycle when the wheel speed is greater than the drive sprocket. The range of overrunning clutches comprise the simple ratchet and pawl, Figure 13.6, roller, sprag, and spring wound clutches.

Roller clutches consist of balls or cylindrical rollers located in wedge-shaped chambers between inner and outer races, Figure 13.7. Rotation is possible in one direction, but when

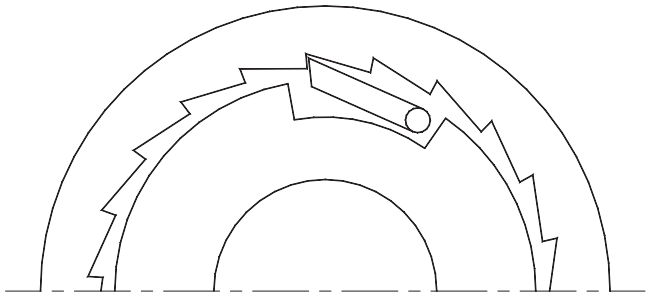


Figure 13.6
Ratchet and pawl. *Reproduced from Neale (1995).*

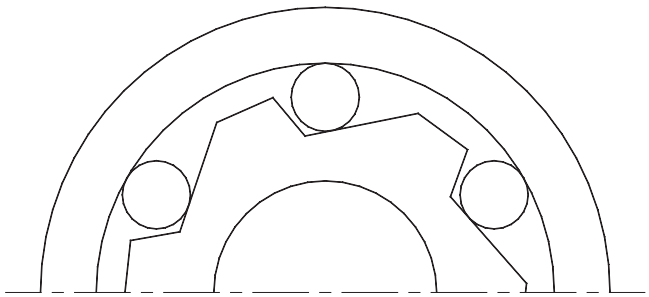


Figure 13.7
Roller clutch. *Reproduced from Neale (1995).*

this is reversed the rollers wedge themselves between the races, jamming the inner and outer races together. Figure 13.8 illustrates the principal components of a sprag clutch, which consists of an inner and outer race, like a bearing, but instead of balls the space between the races contains specially shaped elements called "sprags," which permit rotation in one direction only. If the clutch motion is reversed, the sprags jam and lock the clutch. A spring wound clutch is shown in Figure 13.9. Friction between hubs and the coil spring causes the

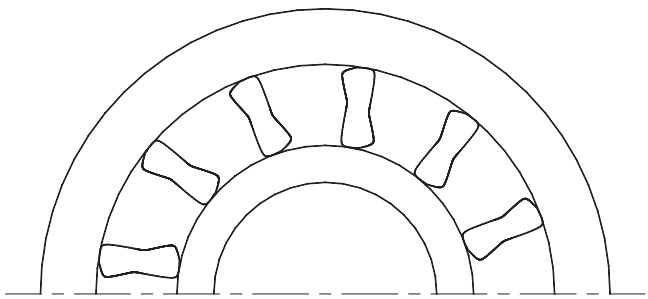


Figure 13.8
Sprag clutch. *Reproduced from Neale (1995).*

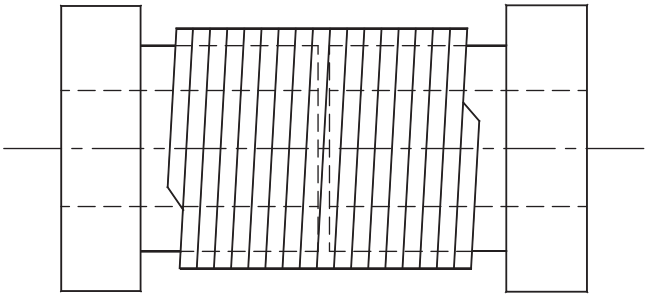


Figure 13.9
Spring wound clutch. *Reproduced from Neale (1995).*

spring to tighten onto hubs and drive in one direction. In the opposite direction the spring unwraps and slips easily on the hubs.

Magnetic clutches use a magnetic field to couple the rotating components together. Magnetic clutches are smooth, quiet, and have a long life, as there is no direct mechanical contact and hence no wear except at the bearings.

Fluid couplings transmit torque through a fluid such as oil. A fluid coupling always transmits some torque because of the swirling nature of the fluid contained within the device. For this reason it is necessary to maintain some braking force in automatic transmission automobiles that utilize fluid coupling torque converters in the automatic gearbox.

Centrifugal clutches engage automatically when the shaft speed exceeds some critical value. Friction elements are forced radially outward and engage against the inner radius of a mating cylindrical drum, Figure 13.10. Common applications of centrifugal clutches include chainsaws, overload-releases, and go-karts.

The selection of clutch type and configuration depends on the application. Table 13.1 can be used as the first step in determining the type of clutch to be used. Clutches are rarely designed from scratch. Either an existing design is available and is modified for a new

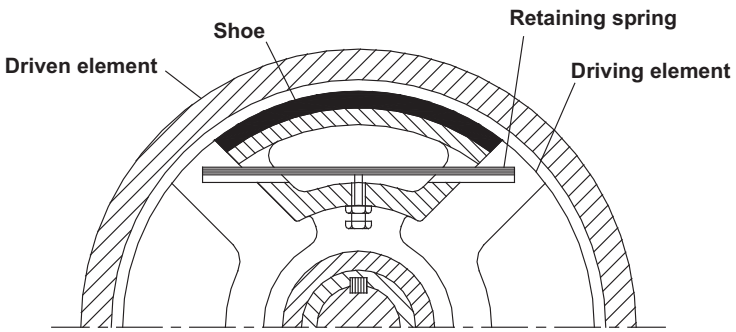


Figure 13.10
Centrifugal clutch. *Reproduced from Neale (1995).*

Table 13.1: Clutch selection criteria.

Type of clutch	Characteristics	Typical applications
Sprag	One-way clutch; profiled elements jam against the outer edge to provide drive; high torque capacity.	One-way operation, e.g. backstop for hoists.
Roller	One-way clutch; rollers ride up ramps and drive by wedging into place.	One-way operation.
Cone clutch	Embodies the mechanical principle of the wedge which reduces the axial force required to transmit a given torque.	Contractor's plant; feed drives for machine tools.
Single disc clutch	Used when diameter is not restricted; simple construction.	Automobile drives.
Multiple disc clutch	The power transmitted can be increased by using more plates allowing a reduction in diameter.	Machine tool head stocks; motorcycles.
Centrifugal clutch	Automatic engagement at a critical speed.	Electric motor drives; industrial diesel drives.
Magnetic	Compact; low wear.	Machine tool gearboxes; numerical control machine tools.

Source: Reproduced with adaptations from Neale (1994).

application, or a clutch can be bought in from a specialist manufacturer. In the latter case the type, size, and materials for the clutch lining must be specified. This requires determination of the system characteristics such as speed, torque, loading characteristic (e.g. shock loads), and operating temperatures. Many of these factors have been lumped into a multiplier called a service factor. A lining material is typically tested under steady conditions using an electric motor drive. The torque capacity obtained from this test is then derated by the service factor according to the particular application to take account of vibrations and loading conditions. Table 13.2 gives an indication of the typical values for service factors.

13.2.1 Design of Disc Clutches

Disc clutches can consist of single or multiple discs, as illustrated in Figures 13.11, 13.12 and 13.13. Generally, multiple disc clutches enable greater torque capacity but are harder to cool. Frictional clutches can be run dry or wet using oil. Typical coefficients of friction are 0.07 for a wet clutch and 0.45 for a dry clutch. While running a clutch wet in oil reduces the coefficient of friction, it enhances heat transfer and the potential for cooling of the components. The expedient solution to the reduction of the friction coefficient is to use more discs and hence the use of multiple disc clutches.

Table 13.2: Service factors.

Description of general system	Typical driven system	Type of driver			
		Small electric motors, turbine	IC engines (4–6 cylinders). Medium to large electric motors	IC engines (2 or 3 cylinders)	Single cylinder engine
Steady power source, steady load, no shock or overload	Belt drive, small generators, centrifugal pumps, fans, machine tools	1.5	1.7	1.9	2.2
	Light machinery for wood, metal and textiles, conveyor belts	1.8	2.0	2.4	2.7
Steady power source with some irregularity of load up to 1.5 times nominal power	Larger conveyor belts, larger machines, reciprocating pumps	2.0	2.2	2.4	2.7
	Presses, punches, piston pumps, cranes, hoists	2.5	2.7	2.9	3.2
Frequent start-stops, overloads, cycling, high inertia starts, high power, pulsating power source	Stone crushers, roll mills, heavy mixers, single cylinder compressors	3.0	3.2	3.4	3.7

Source: Vedamuttu, 1994.

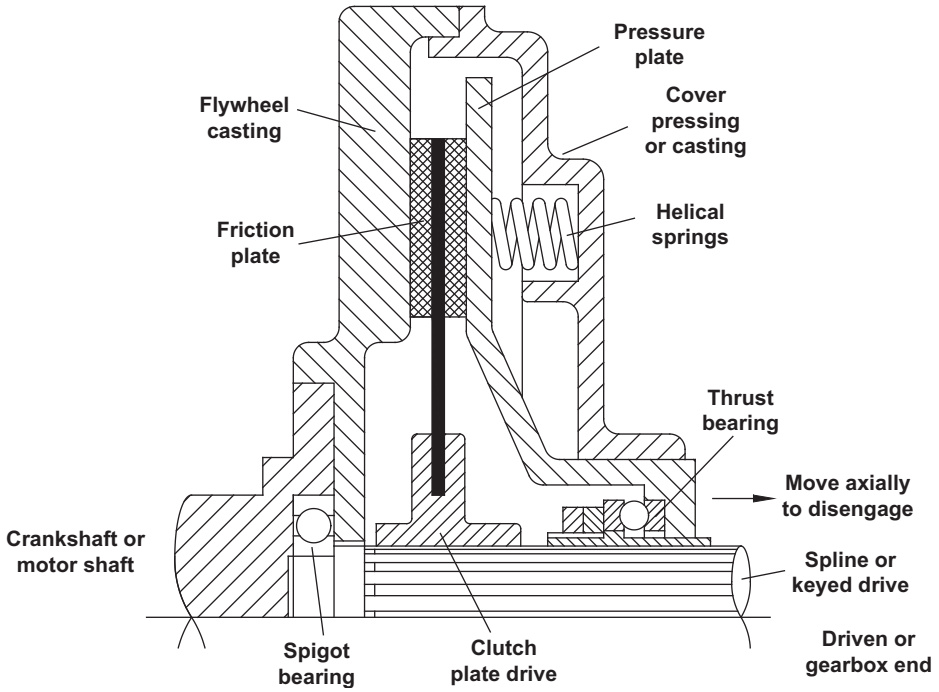


Figure 13.11
Disc clutch. *Reproduced with adaptations from Neale (1994).*

Two basic assumptions are used in the development of procedures for disc clutch design based upon a uniform rate of wear at the mating surfaces or a uniform pressure distribution between the mating surfaces. The equations for both of these methods are outlined in this section.

The assumption of a uniform pressure distribution at the interface between mating surfaces is valid for an unworn accurately manufactured clutch with rigid outer discs.

The area of an elemental annular ring on a disc clutch, Figure 13.14, is $\delta A = 2\pi r \delta r$. Now $F = pA$, where p is the assumed uniform interface pressure, so $\delta F = 2\pi r p \delta r$. For the disc, the normal force acting on the entire face is

$$F = \int_{r_i}^{r_o} 2\pi r p \mathrm{d}r = 2\pi p \left[\frac{r^2}{2}\right]_{r_i}^{r_o} \tag{13.4}$$

$$F = \pi p\left(r_o^2 - r_i^2\right) \tag{13.5}$$

Note that F is also the necessary force required to clamp the clutch discs together.

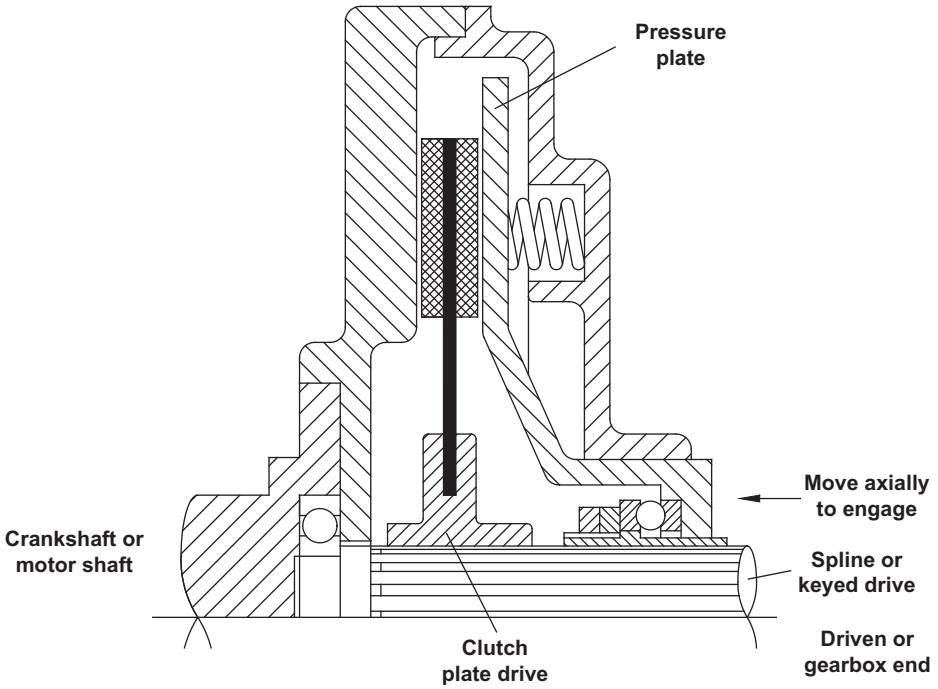


Figure 13.12
Disc clutch shown disengaged. Little or no torque is transferred from the crankshaft to the driven load. *Reproduced with adaptations from Neale (1994).*

The friction torque δT that can be developed on an elemental ring is the product of the elemental normal force, given by $\mu\delta F$ and the radius:

$$\delta T = r\mu\delta F = 2\mu\pi r^2 p\delta r \tag{13.6}$$

where μ is the coefficient of friction that models the less than ideal frictional contact that occurs between two surfaces.

The total torque is given by integration of Eqn (13.6) between the limits of the annular ring, r_i and r_o:

$$T = \int_{r_i}^{r_o} 2\mu\pi r^2 p\mathrm{d}r = 2\pi p\mu\left[\frac{r^3}{3}\right]_{r_i}^{r_o} = \frac{2}{3}\pi p\mu\left(r_o^3 - r_i^3\right) \tag{13.7}$$

This equation represents the torque capacity of a clutch with a single frictional interface. In practice, clutches use an even number of frictional surfaces as illustrated in Figures 13.11 and 13.13.

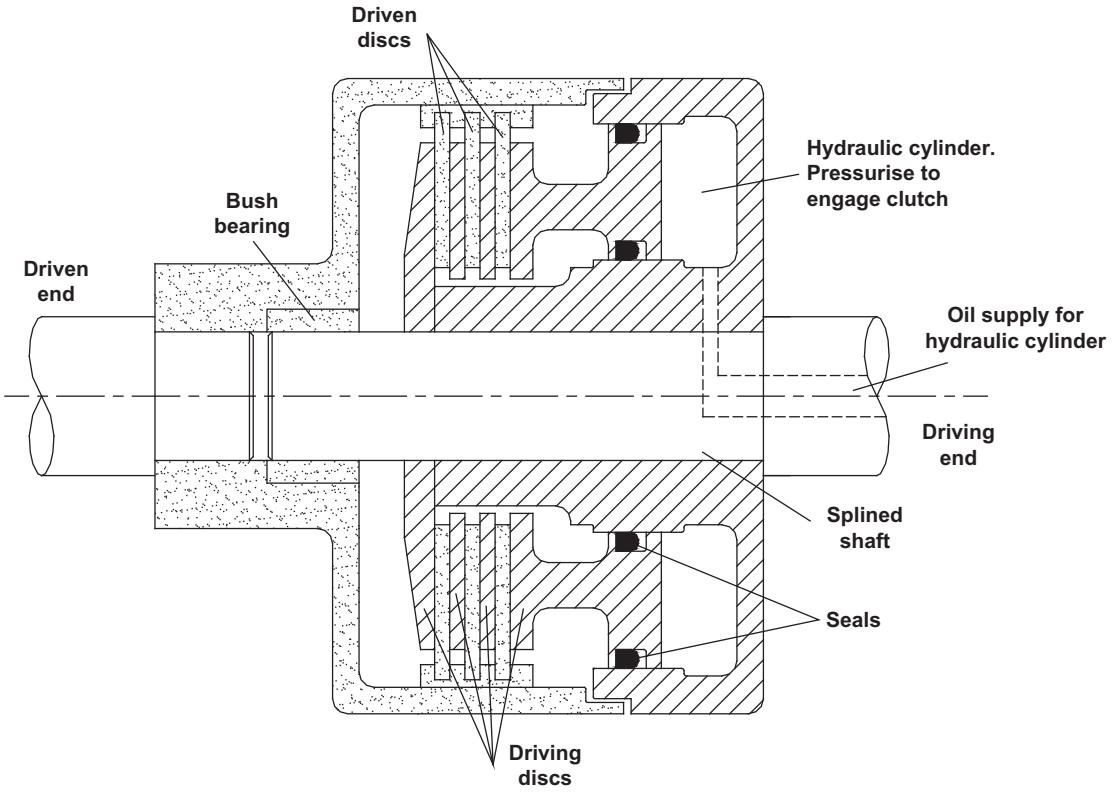


Figure 13.13
Multiple disc clutch. *Reproduced with adaptations from Juvinall and Marshek (1991).*

For a clutch with N faces the torque capacity is given by

$$T = \frac{2}{3}\pi p \mu N\left(r_o^3 - r_i^3\right) \tag{13.8}$$

Using Eqns (13.5) and (13.8) and substituting for the pressure, p, gives an equation for the torque capacity as a function of the axial clamping force.

$$T = \frac{2}{3}\mu F N\left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right) \tag{13.9}$$

The equations assuming uniform wear are developed below. The wear rate is assumed to be proportional to the product of the pressure and velocity. So

$$pr\omega = \text{constant} \tag{13.10}$$

where ω is the angular velocity (rad/s).

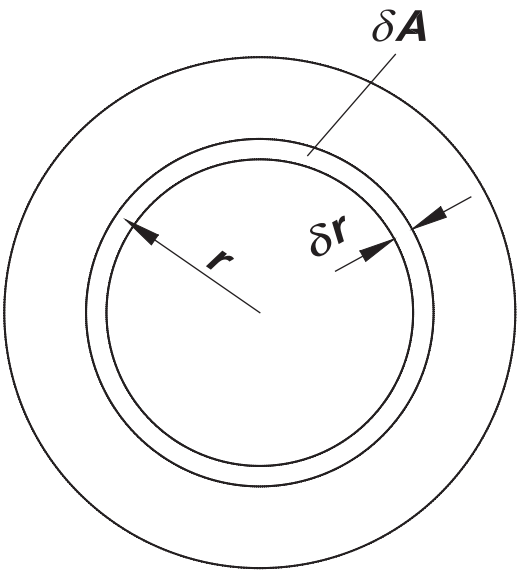


Figure 13.14
Area of an elemental ring on a disc clutch.

For a constant angular velocity, the maximum pressure will occur at the smallest radius.

$$p_{\max} r_i \omega = \text{constant}. \tag{13.11}$$

Eliminating the angular velocity and constant from the above two equations gives a relationship for the pressure as a function of the radius:

$$p = p_{\max} \frac{r_i}{r} \tag{13.12}$$

The value for the maximum permissible pressure is dependent on the clutch lining material; typical values are listed in Table 13.3.

Table 13.3: Typical values for dynamic friction coefficients, permissible contact pressures, and temperature limits.

Material	μ_{dry}	μ_{oil}	p_{max} (MN/m^2)	T (°C)
Molded compounds	0.25–0.45	0.06–0.10	1.035–2.07	200–260
Woven materials	0.25–0.45	0.08–0.10	0.345–0.69	200–260
Sintered metal	0.15–0.45	0.05–0.08	1.035–2.07	230–680
Cork	0.30–0.50	0.15–0.25	0.055–0.1	80
Wood	0.20–0.45	0.12–0.16	0.345–0.62	90
Cast iron	0.15–0.25	0.03–0.06	0.69–1.725	260
Paper based	–	0.10–0.17		
Graphite/resin	–	0.10–0.14		

The elemental axial force on an elemental annular ring is given by

$$\delta F = 2\pi p r \delta r \tag{13.13}$$

Integrating to give the total axial force:

$$F = \int_{r_i}^{r_o} 2\pi p r \mathrm{d}r = \int_{r_i}^{r_o} 2\pi p_{\max} \frac{r_i}{r} r \mathrm{d}r = 2\pi p_{\max} r_i (r_o - r_i) \tag{13.14}$$

The elemental torque is given by

$$\delta T = \mu r \delta F \tag{13.15}$$

So

$$T = \int_{r_i}^{r_o} 2\mu\pi p_{\max} r_i r \mathrm{d}r = p_{\max} \mu \pi r_i \left(r_o^2 - r_i^2\right) \tag{13.16}$$

Rearranging Eqn (13.16) gives

$$p_{\max} = \frac{F}{2\pi r_i (r_o - r_i)} \tag{13.17}$$

Substituting Eqn (13.17) into Eqn (13.16) gives

$$T = \frac{\mu F}{2} \left(\frac{r_o^2 - r_i^2}{r_o - r_i} \right) = \frac{\mu F}{2} (r_o + r_i) \tag{13.18}$$

For N frictional surfaces

$$T = \int_{r_i}^{r_o} 2\mu\pi N p_{\max} r_i r \mathrm{d}r = p_{\max} \mu \pi N r_i \left(r_o^2 - r_i^2\right) \tag{13.19}$$

gives

$$T = \frac{\mu N F}{2} (r_o + r_i) \tag{13.20}$$

By differentiating Eqn (13.16) with respect to r_i and equating the result to zero, the maximum torque for any outer radius r_o is found to occur when

$$r_i = \sqrt{1/3}\, r_o \tag{13.21}$$

This useful formula can be used to set the inner radius if the outer radius is constrained to a particular value.

Note that Eqn (13.20) indicates a lower torque capacity than the uniform pressure assumption. This is because the higher initial wear at the outer diameter shifts the center of pressure toward the inner radius.

Clutches are usually designed based on uniform wear. The uniform wear assumption gives a lower torque capacity clutch than the uniform pressure assumption. The preliminary design procedure for disc clutch design requires the determination of the torque and speed, specification of space limitations, selection of materials, i.e. the coefficient of friction and the maximum permissible pressure, and the selection of principal radii, r_o and r_i. Common practice is to set the value of r_i between $0.45r_o$ and $0.8r_o$. This procedure for determining the initial geometry is itemized below:

1. Determine the service factor;
2. Determine the required torque capacity, $T = \text{power}/\omega$;
3. Determine the coefficient of friction μ;
4. Determine the outer radius r_o;
5. Find the inner radius r_i; and
6. Find the axial actuation force required.

The material used for clutch plates is typically gray cast iron or steel. The friction surface will consist of a lined material that may be molded, woven, sintered, or solid. Molded linings consist of a polymeric resin used to bind powdered fibrous material and brass and zinc chips. Table 13.3 lists typical values for the performance of friction linings.

Example 13.2

A clutch is required for transmission of power between a four-cylinder internal combustion engine and a small machine. Determine the radial dimensions for a single face dry disc clutch with a molded lining which should transmit 5 kW at 1800 rpm, Figure 13.15. Base the design on the uniform wear assumption.

Solution

From Table 13.2, a service factor of 2 should be used. The design will therefore be undertaken using a power of 2×5 kW $= 10$ kW.

The torque is given by

$$T = \frac{\text{Power}}{\omega} = \frac{10{,}000}{1800 \times (2\pi/60)} = 53 \text{ N m}.$$

From Table 13.3, taking midrange values for the coefficient friction and the maximum permissible pressure for molded linings gives $\mu = 0.35$ and $p_{\max} = 1.55$ MN/m^2.

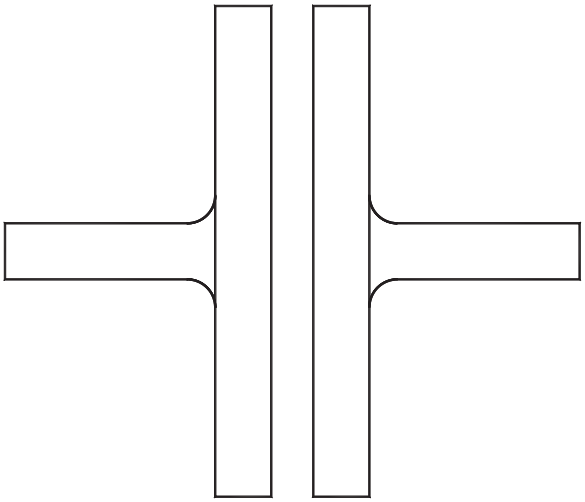


Figure 13.15
Single face clutch.

Taking $r_i = \sqrt{1/3}r_o$ and substituting for r_i in Eqn (13.16) gives

$$T = \pi\mu\sqrt{1/3}r_o p_{\max}\left(r_o^2 - \left(\sqrt{1/3}r_o\right)^2\right) = \pi\mu\sqrt{1/3}p_{\max}\left(r_o^3 - \frac{1}{3}r_o^3\right) = \pi\mu\sqrt{4/27}p_{\max}r_o^3,$$

$$r_o = \left(\frac{T}{\pi\mu p_{\max}\sqrt{4/27}}\right)^{1/3} = \left(\frac{53.05}{\pi 0.35 \times 1.55 \times 10^6\sqrt{4/27}}\right)^{1/3} = 0.04324 \text{ m},$$

$$r_i = \sqrt{1/3}r_o = 0.02497 \text{ m},$$

$$F = 2\pi r_i p_{\max}(r_o - r_i) = 2\pi \times 0.02497 \times 1.55 \times 10^6(0.04324 - 0.02497) = 4443 \text{ N}.$$

So the clutch consists of a disc of inner and outer radius 25 and 43 mm, respectively, with a molded lining having a coefficient of friction value of 0.35 and a maximum permissible contact pressure of 1.55 MPa and an actuating force of 4.4 kN.

Example 13.3

A disc clutch, running in oil, is required for a vehicle with a four-cylinder engine. The design power for initial estimation of the clutch specification is 90 kW at 4500 rpm. Determine the radial dimensions and actuating force required. Base the design on the uniform wear assumption.

Solution

From Table 13.2, a service factor of 2.7 should be used to account for starts and stops and the four-cylinder engine. The design will therefore be undertaken using a power of $2.7 \times 90 = 243$ kW.

The torque is given by

$$T = \frac{\text{Power}}{\omega} = \frac{243{,}000}{4500 \times (2\pi/60)} = 515.7 \text{ N m}.$$

From Table 13.3, taking midrange values for the coefficient friction and the maximum permissible pressure for molded linings gives $\mu = 0.35$ and $p_{\max} = 1.55$ MN/m^2.

Taking $r_i = \sqrt{1/3}r_o$ and substituting for r_i in Eqn (13.19) gives

$$r_o = \left(\frac{T}{\pi\mu N p_{\max}\sqrt{4/27}}\right)^{1/3} = \left(\frac{515.7}{\pi 0.35 \times 2 \times 1.55 \times 10^6\sqrt{4/27}}\right)^{1/3} = 0.07325 \text{ m},$$

$$r_i = \sqrt{1/3}r_o = 0.04229 \text{ m},$$

$$F = 2\pi N r_i p_{\max}(r_o - r_i) = 2\pi \times 2 \times 0.04229 \times 1.55 \times 10^6(0.07325 - 0.04229)$$
$$= 25500 \text{ N}.$$

So the clutch consists of a disc of inner and outer radius 42.3 and 73.3 mm, respectively, with a molded lining having a coefficient of friction value of 0.35 and a maximum permissible contact pressure of 1.55 MPa and an actuating force of 25.5 kN.

Example 13.4

A multiple disc clutch, running in oil, is required for a motorcycle with a three-cylinder engine. The power demand is 75 kW at 8500 rpm. The preliminary design layout indicates that the maximum diameter of the clutch discs should not exceed 100 mm. In addition, previous designs have indicated that a molded lining with a coefficient of friction of 0.068 in oil and a maximum permissible pressure of 1.2 MPa is reliable. Within these specifications, determine the radii for the discs, the number of discs required and the clamping force.

Solution

The torque is given by

$$T = \frac{\text{Service factor} \times \text{Power}}{\omega} = \frac{3.4 \times 75{,}000}{8500 \times (2\pi/60)} = 286.5 \text{ N m}.$$

Select the outer radius to be the largest possible, i.e. $r_o = 50$ mm. Using $r_i = \sqrt{1/3}r_o$, $r_i = 28.87$ mm. From Eqn (13.19), the number of frictional surfaces, N, can be determined.

$$N = \frac{T}{\pi p_{\max} r_i \mu \left(r_o^2 - r_i^2\right)} = \frac{286.5}{\pi 1.2 \times 10^6 \times 0.02887 \times 0.068(0.05^2 - 0.02887^2)} = 23.23$$

This must be an even number, so the number of frictional surfaces is taken as $N = 24$. This requires thirteen driving discs and twelve driven discs to implement.

Using Eqn (13.14) the clamping force can be calculated:

$$F = \frac{2T}{\mu N(r_o + r_i)} = \frac{2 \times 286.5}{0.068 \times 24(0.05 + 0.02887)} = 4452 \text{ N}.$$

As well as the disc or discs, detailed design of a disc clutch such as the automotive variant illustrated in Figure 13.16 requires specification and consideration of all the associated components such as the shaft, splined hub, damping springs in the disc, Figure 13.17, actuating springs, cover plate, spline, bearings, flywheel, crankshaft, gearbox connections, and release mechanism. The choice of whether to use coil springs or a diaphragm spring, Figure 13.18, depends on the clamping force required and cost considerations. Diaphragm springs have been developed from Belleville springs and are made from a steel disc. The inner portion of the disc has a number of radial slots to form actuating fingers. Stress relieving holes are included at the outer end of the fingers to prevent cracking. Multicoil spring units are used in heavy commercial vehicles, where it is difficult to provide the required clamping force from a single diaphragm.

13.3 Brakes

The basic function of a brake is to absorb kinetic energy and dissipate it in the form of heat. An idea of the magnitude of energy that must be dissipated can be obtained from considering the familiar example of a car undergoing an emergency stop in 7 s from 60 mph (96 km/h). If the car's mass is 1400 kg and assuming that 65% of the car's weight is loaded onto the front axles during rapid braking, then the load on the front axle is

$$1400 \times 9.81 \times 0.65 = 8927 \text{ N}$$

This will be shared between two brakes, so the energy that must be absorbed by one brake is

$$E = \frac{1}{2} m \left(V_i^2 - V_f^2\right) \tag{13.22}$$

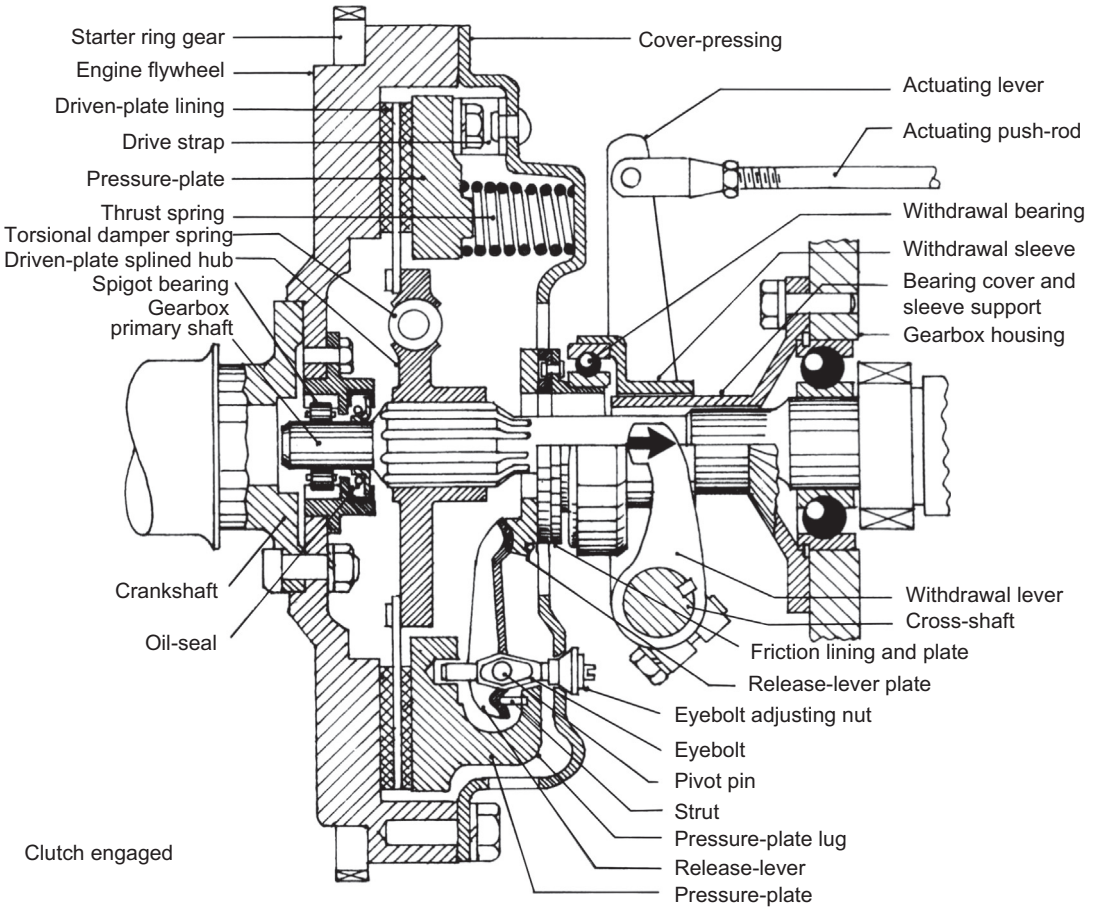


Figure 13.16
Automotive clutch. *Reproduced from Heisler (1999).*

where $m =$ mass (kg), $V_i =$ initial velocity (m/s), and $V_f =$ final velocity (m/s).

$$E = \frac{1}{2}m\left(V_i^2 - V_f^2\right) = \frac{1}{2} \times \left(\frac{8927}{9.81} \times 0.5\right) \times \left(\left(\frac{96 \times 10^3}{3600}\right)^2 - 0^2\right) = 161.8 \text{ kJ}$$

If the car brakes uniformly in 7 s, the heat that must be dissipated is $161.8 \times 10^3/7 = 23.1$ kW. From your experience of heat transfer from, say, 1-kW domestic heaters, you will recognize that this is a significant quantity of heat to transfer away from the relatively compact components that make up brake assemblies.

Convective heat transfer can be modeled by Fourier's equation:

$$Q = hA\Delta T = hA\left(T_s - T_f\right) \tag{13.23}$$

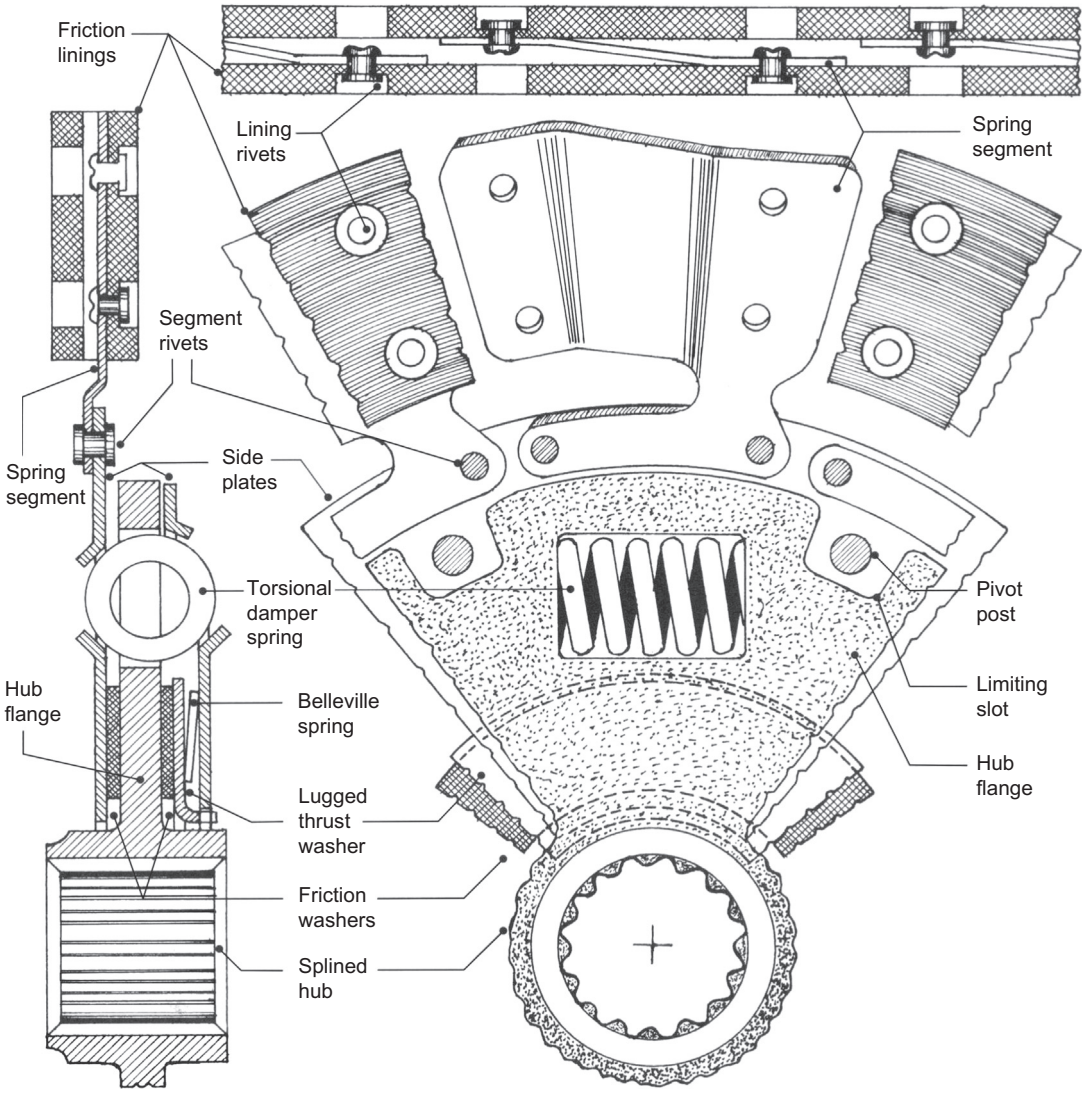


Figure 13.17
Automotive disc illustrating the use of torsional damping springs. *Reproduced from Heisler (1999).*

where Q = heat transfer rate (W), h = the heat transfer coefficient (W/m^{2}K), T_s = temperature of the surface (K or °C), T_f = temperature of the surrounding fluid (K or °C), and A = surface area (m^2).

This equation indicates that the ability of a brake to dissipate the heat generated increases as the surface area increases or as the heat transfer coefficient rises. For air, the heat transfer coefficient is usually dependent on the local flow velocity and on the geometry. A method often used for disc brakes to increase both the surface area and the local flow is

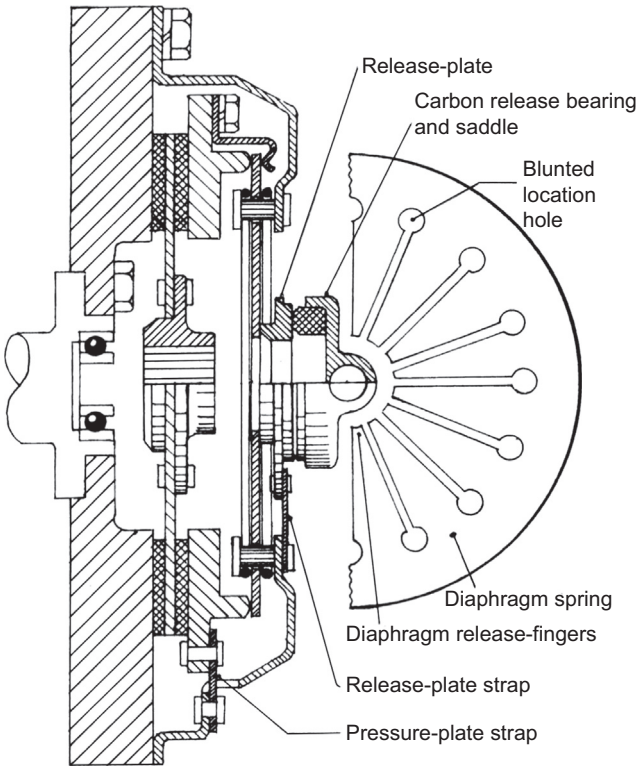


Figure 13.18
Diaphragm spring. *Reproduced from Heisler (1999).*

to machine multiple axial or radial holes in the disc (this also reduces the mass and inertia).

Example 13.5

Calculate the energy that must be absorbed in stopping a 100-ton Airbus airliner traveling at 250 km/h in an aborted takeoff, stopping in 40 s.

Solution

$$E = \frac{1}{2}m\left(V_i^2 - V_f^2\right) = \frac{1}{2} \times 100 \times 10^3 \left(\left(\frac{250 \times 10^3}{3600}\right)^2 - 0\right) = 241.1 \text{ MJ}.$$

Assuming that the aircraft brakes uniformly, the power that must be dissipated is

$$\frac{241.1 \times 10^6}{40} = 6.028 \text{ MW}.$$

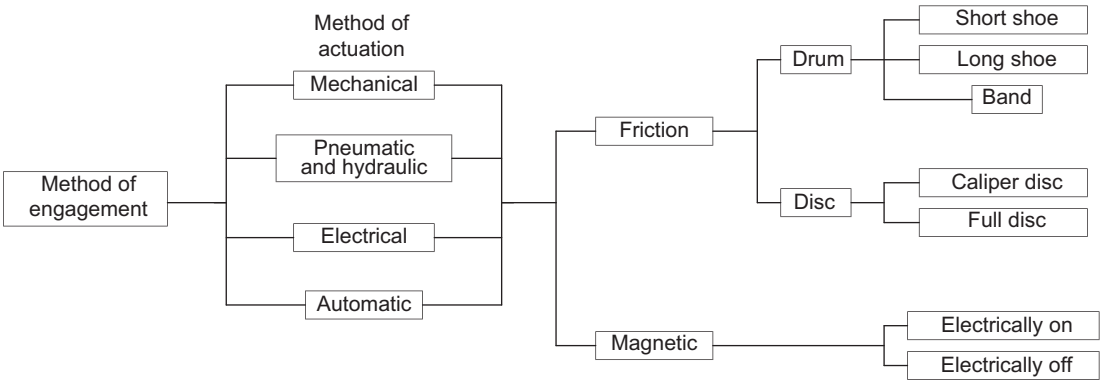


Figure 13.19
Brake classification.

This is a significant quantity of power, equivalent to six thousand 1-kW electric heaters and evenly divided between multiple brakes, gives an indication why aborted takeoffs can result in burnt-out brakes.

Note that the effects of any thrust reversal and aerodynamic drag has been ignored. Both of these effects would reduce the heat dissipated within the brakes.

There are numerous brake types, as shown in Figure 13.19. The selection and configuration of a brake depends on the requirements. Table 13.4 gives an indication of brake operation against various criteria. The brake factor listed in Table 13.4 is the ratio of frictional braking force generated to the actuating force applied.

Brakes can be designed so that once engaged, the actuating force applied is assisted by the braking torque. This kind of brake is called a self-energizing brake and is useful for braking large loads. Great care must be exercised in brake design. It is possible and sometimes desirable to design a brake, which once engaged, will grab and lock up (called self-locking action).

A critical aspect of all brakes is the material used for frictional contact. Normally one component will comprise a steel or cast iron disc or drum and this is brought into frictional contact against a geometrically similar component with a brake lining made up of one of the materials listed in Table 13.3.

Section 13.3.1 gives details about the configuration design of disc brakes and Section 13.3.2 introduces the design of drum brakes.

13.3.1 Disc Brakes

Disc brakes are familiar from automotive applications where they are used extensively for car and motorcycle wheels. These typically consist of a cast iron disc, bolted to the wheel hub. This is sandwiched between two pads actuated by pistons supported in a caliper mounted

Table 13.4: Comparative table of brake performance.

Type of brake	Maximum operating temperature	Brake factor	Stability	Dryness	Dust and dirt	Typical applications
Differential band brake	Low	High	Low	Unstable but still effective	Good	Winches, hoist, excavators, tractors
External drum brake (leading trailing edge)	Low	Medium	Medium	Unstable if humid, poor if wet	Good	Mills, elevators, winders
Internal drum brake (leading trailing edge)	Higher than external brake	Medium	Medium	Unstable if humid, ineffective if wet	Good if sealed	Vehicles (rear axles on passenger cars)
Internal drum brake (two leading shoes)	Higher than external brake	High	Low	Unstable if humid, ineffective if wet	Good if sealed	Vehicles (rear axles on passenger cars)
Internal drum brake (duo-servo)	Low	High	Low	Unstable if humid, ineffective if wet	Good if sealed	Vehicles (rear axles on passenger cars)
Caliper disc brake	High	Low	High	Good	Poor	Vehicles and industrial machinery
Full disc brake	High	Low	High	Good	Poor	Machine tools and other industrial machinery

Source: Reproduced from Neale (1994).

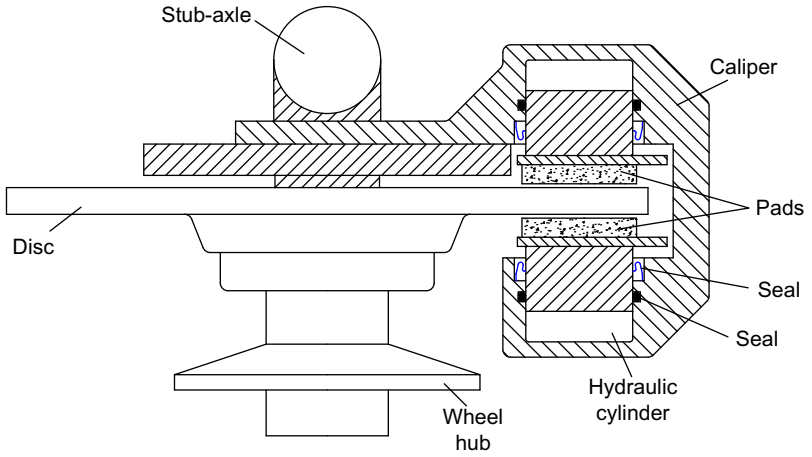


Figure 13.20
Automotive disc brake.

on the stub shaft (see Figure 13.20). When the brake pedal is pressed, hydraulically pressurized fluid is forced into the cylinders, pushing the opposing pistons and brake pads into frictional contact with the disc. The advantages of this form of braking are steady braking, easy ventilation, balancing thrust loads, and design simplicity. There is no self-energizing action, so the braking action is proportional to the applied force. The use of a discrete pad allows the disc to cool as it rotates, enabling heat transfer between the cooler disc and the hot brake pad. As the pads on either side of the disc are pushed to the disc with equal forces, the net thrust load on the disc cancels.

With reference to Figure 13.21, the torque capacity per pad is given by

$$T = \mu F r_e \tag{13.24}$$

where r_e is an effective radius.

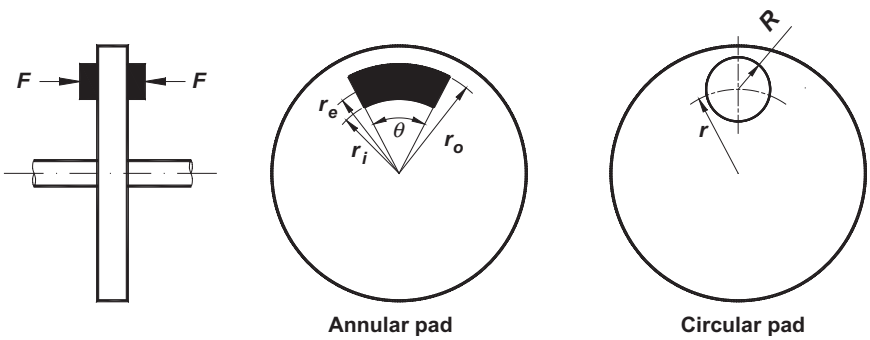


Figure 13.21
Caliper disc brake.

The actuating force assuming constant pressure is given by

$$F = p_{av}\theta\frac{r_o^2 - r_i^2}{2} \tag{13.25}$$

or assuming uniform wear by

$$F = p_{\max}\theta r_i(r_o - r_i) \tag{13.26}$$

where θ (in radians) is the included angle of the pad, r_i is the inner radius of the pad, and r_o is the outer radius of the pad.

The relationship between the average and the maximum pressure for the uniform wear assumption is given by

$$\frac{p_{av}}{p_{\max}} = \frac{2r_i/r_o}{1 + r_i/r_o} \tag{13.27}$$

For an annular disc brake, the effective radius is given by Eqn (13.28), assuming constant pressure and Eqn (13.29) assuming uniform wear.

$$r_e = \frac{2\left(r_o^3 - r_i^3\right)}{3\left(r_o^2 - r_i^2\right)} \tag{13.28}$$

$$r_e = \frac{r_i + r_o}{2} \tag{13.29}$$

For circular pads the effective radius is given by $r_e = r\delta$, where values for δ are given in Table 13.5 as a function of the ratio of the pad radius and the radial location, R/r. The actuating force for circular pads can be calculated using

$$F = \pi R^2 p_{av} \tag{13.30}$$

Table 13.5: Circular pad disk brake design values.

R/r	$\delta = r_e/r$	$p_{\max}/p_{av}$
0	1.000	1.000
0.1	0.983	1.093
0.2	0.969	1.212
0.3	0.957	1.367
0.4	0.947	1.578
0.5	0.938	1.875

Source: Fazekas (1972).

Example 13.6

A caliper brake is required for the front wheels of a sport's car with a braking capacity of 820 N m for each brake. Preliminary design estimates have set the brake geometry as $r_i = 100$ mm, $r_o = 160$ mm, and $\theta = 45°$. A pad with a coefficient of friction of 0.35 has been selected. Determine the required actuating force and the average and maximum contact pressures.

Solution

The torque capacity per pad $= 820/2 = 410$ N m.

The effective radius is $r_e = \dfrac{0.1 + 0.16}{2} = 0.13$ m.

The actuating force is given by $F = \dfrac{T}{\mu r_e} = \dfrac{410}{0.35 \times 0.13} = 9.011$ kN.

The maximum contact pressure is given by

$$p_{\max} = \frac{F}{\theta r_i (r_o - r_i)} = \frac{9.011 \times 10^3}{45 \times (2\pi/360) \times 0.1 \times (0.16 - 0.1)} = 1.912 \text{ MN/m}^2.$$

The average pressure is given by

$$p_{av} = p_{\max} \frac{2r_i/r_o}{1 + r_i/r_o} = 1.471 \text{ MN/m}^2.$$

Example 13.7

A caliper brake is required for the front wheels of a passenger car with a braking capacity of 320 N m for each brake. Preliminary design estimates have set the brake geometry as $r_i = 100$ mm, $r_o = 140$ mm, and $\theta = 40°$. Pads with a coefficient of friction of 0.35 have been selected. Each pad is actuated by means of a hydraulic cylinder of nominal diameter 25.4 mm. Determine the required actuating force, the average and the maximum contact pressures, and the required hydraulic pressure for brake actuation.

Solution

The torque capacity per pad is $320/2 = 160$ N m

The effective radius is given by $r_e = \dfrac{0.1 + 0.14}{2} = 0.12$ m

The actuation force required is $F = \dfrac{T}{\mu r_e} = \dfrac{160}{0.35 \times 0.12} = 3810$ N

The maximum pressure is

$$p_{\max} = \frac{F}{\theta r_i (r_o - r_i)} = \frac{3810}{40(2\pi/360) \times 0.1(0.14 - 0.1)} = 1.364 \times 10^6 \text{ N/m}^2$$

The average pressure is

$$p_{av} = \frac{2r_i/r_o}{1 + r_i/r_o} p_{\max} = \frac{2 \times 0.1/0.14}{1 + 0.1/0.14} 1.36 \times 10^6 = 1.137 \times 10^6 \text{ N/m}^2$$

The area of one of the hydraulic cylinders is $\pi 0.0127^2 = 5.067 \times 10^{-4}$ m^2

The hydraulic pressure required is given by

$$p_{\text{hydraulic}} = \frac{F}{A_{\text{cylinder}}} = \frac{3810}{5.067 \times 10^{-4}} = 7.519 \times 10^6 \text{ N/m}^2$$

i.e. $p_{\text{hydraulic}} \approx 75$ bar.

Full disc brakes, consisting of a complete annular ring pad, are principally used for industrial machinery. The disc clutch equations developed in Section 13.2.1 are applicable to their design. The disc configuration can be designed to function as either a clutch or a brake (a clutch–brake combination) to transmit a load or control its speed.

13.3.2 Drum Brakes

Drum brakes apply friction to the external or internal circumference of a cylinder. A drum brake consists of the brake shoe, which has the friction material bonded to it, and the brake drum. For braking, the shoe is forced against the drum developing the friction torque. Drum brakes can be divided into two groups depending on whether the brake shoe is external or internal to the drum. A further classification can be made in terms of the length of the brake shoe: short, long, or complete band.

Short-shoe internal brakes are used for centrifugal brakes that engage at a particular critical speed. Long-shoe internal drum brakes are used principally in automotive applications. Drum brakes (or clutches) can be designed to be self-energizing. Once engaged, the friction force increases the normal force nonlinearly, increasing the friction torque as in a positive feedback loop. This can be advantageous in braking large loads, but it makes control much more difficult. One problem associated with some drum brakes is stability. If the brake has been designed so that the braking torque is not sensitive to small changes in the coefficient of friction, which would occur if the brake is worn or wet, the brake is said to be stable. If a small change in the coefficient of friction causes a significant change to the braking torque, the brake is unstable and will tend to grab if the friction coefficient rises or the braking torque will drop noticeably if the friction coefficient reduces.

13.3.3 Short-Shoe External Drum Brakes

The schematic of a short-shoe external drum brake is given in Figure 13.22. If the included angle of contact between the brake shoe and the brake drum is less than 45°, the force

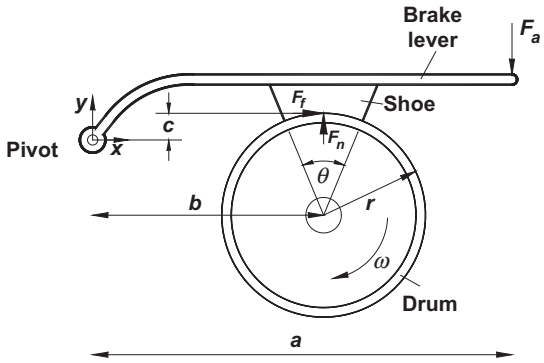


Figure 13.22
Short-shoe external drum brake.

between the shoe and the drum is relatively uniform and can be modeled by a single concentrated load F_n at the center of the contact area. If the maximum permissible pressure is $p_{\max}$, the force F_n can be estimated by

$$F_n = p_{\max} r \theta w \tag{13.31}$$

where $w =$ width of the brake shoe (m) and $\theta =$ angle of contact between the brake shoe and the lining (rad).

The frictional force, F_f, is given by

$$F_f = \mu F_n \tag{13.32}$$

where μ is the coefficient of friction.

The torque on the brake drum is

$$T = F_f r = \mu F_n r \tag{13.33}$$

Summing moments, for the shoe arm, about the pivot gives

$$\sum M_{\text{pivot}} = aF_a - bF_n + cF_f = 0$$

$$F_a = \frac{bF_n - cF_f}{a} = F_n \frac{b - \mu c}{a} \tag{13.34}$$

Resolving forces gives the reactions at the pivot:

$$R_x = -F_f \tag{13.35}$$

$$R_y = F_a - F_n \tag{13.36}$$

Note that for the configuration and direction of rotation shown in Figure 13.22, the friction moment $\mu F_n c$ adds or combines with the actuating moment aF_a. Once the actuating force is applied, the friction generated at the shoe acts to increase the braking torque. This kind of braking action is called self-energizing. If the brake direction is reversed, the friction moment term $\mu F_n c$ becomes negative and the applied load F_a must be maintained to generate braking torque. This combination is called self de-energizing. From Eqn (13.34), note that if the brake is self-energizing and if $\mu c > b$, then the force required to actuate the brake is zero or negative and the brake action is called self-locking. If the shoe touches the drum it will grab and lock. This is usually undesirable, with exceptions being hoist stops or over-running clutch type applications.

Example 13.8

A double short-shoe external brake is illustrated in Figure 13.23. The actuating force required to limit the drum rotation to 100 rpm is 2.4 kN. The coefficient of friction for the brake lining is 0.35. Determine the braking torque and the rate of heat generation.

Solution

For the top shoe:

$$F \times 0.56 - F_n 0.3 + \mu F_n 0.05 = 0,$$

$$F_n = \frac{2400 \times 0.56}{0.3 - 0.35 \times 0.05} = 4758 \text{ N}.$$

For the bottom shoe:

$$-F \times 0.56 + F_n 0.3 + \mu F_n 0.05 = 0,$$

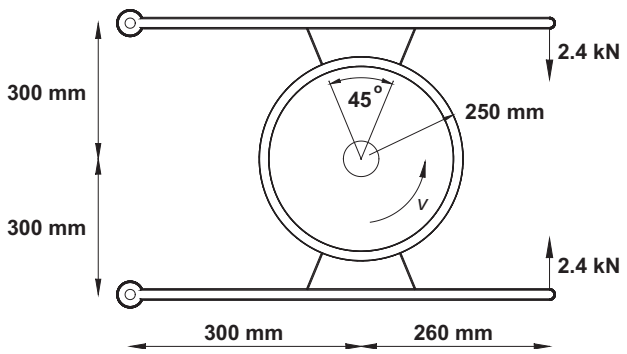


Figure 13.23
Double short-shoe external brake.

$$F_n = \frac{2400 \times 0.56}{0.3 + 0.35 \times 0.05} = 4233 \text{ N}.$$

$$\text{Torque} = \mu(F_{n\ \text{total}}) \times 0.125 = 0.35(4758 + 4233) \times 0.125 = 393.4 \text{ N m}.$$

$$\text{Heat generation} = \omega \times T = 100 \times \frac{2\pi}{60} \times 393.4 = 4119 \text{ W}.$$

13.3.4 Long-Shoe External Drum Brakes

If the included angle of contact between the brake shoe and the drum is greater than 45°, the pressure between the shoe and the brake lining cannot be regarded as uniform and the approximations made for the short-shoe brake analysis are inadequate. Most drum brakes use contact angles greater than 90°. Brake shoes are not rigid, and the local deflection of the shoe affects the pressure distribution. Detailed shoe analysis is possible using finite element software. For initial synthesis/specification of brake geometry, a simpler analysis suffices.

For a single block brake (see Figure 13.24) the force exerted on the drum by the brake shoe must be supported by the bearings. To balance this load and provide a compact braking arrangement, two opposing brake shoes are usually used in a caliper arrangement as shown in Figure 13.25.

The following equations can be used (with reference to Figure 13.24) to determine the performance of a long-shoe brake.

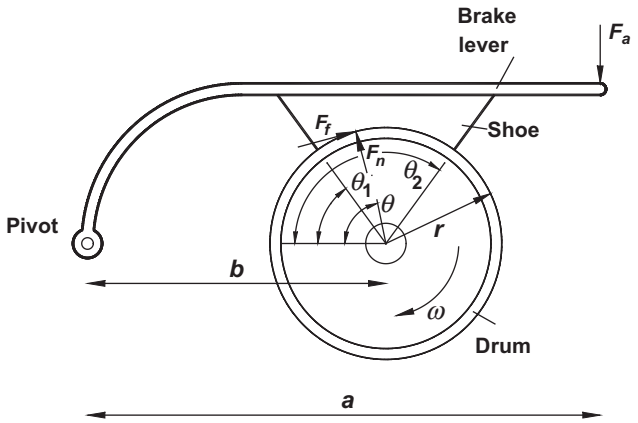


Figure 13.24
Long-shoe external drum brake.

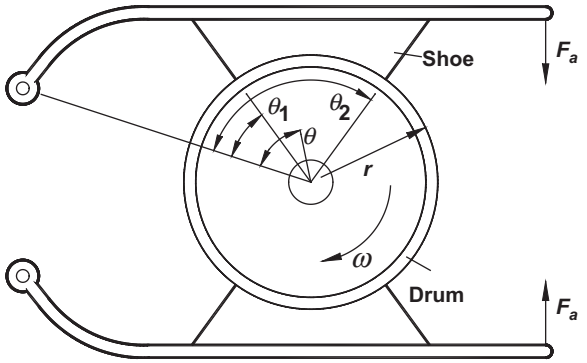


Figure 13.25
Double long-shoe external drum brake.

The braking torque T is given by

$$T = \mu w r^2 \frac{p_{\max}}{(\sin\theta)_{\max}} (\cos\theta_1 - \cos\theta_2) \tag{13.37}$$

where μ = coefficient of friction, w = width of the brake shoe (m), r = radius of drum (m), $p_{\max}$ = maximum allowable pressure for the lining material (N/m^2), θ = angular location (rad), $(\sin\theta)_{\max}$ = maximum value of $\sin\theta$, θ_1 = center angle from the shoe pivot to the heel of the lining (rad), θ_2 = center angle from the shoe pivot to the toe of the lining (rad).

This is based on the assumption that the local pressure p at an angular location θ is related to the maximum pressure, $p_{\max}$, by

$$p = \frac{p_{\max} \sin\theta}{(\sin\theta)_{\max}} \tag{13.38}$$

The local pressure p will be a maximum when $\theta = 90^\circ$. If $\theta_2 < 90^\circ$, the pressure will be a maximum at the toe, θ_2. The relationship given in Eqn (13.38) assumes that there is no deflection at the shoe or the drum, no wear on the drum, and that the shoe wear is proportional to the frictional work and hence the local pressure. Note that if $\theta = 0$, the pressure is zero. This indicates that frictional material near the pivot or heel of the brake does not contribute significantly to the braking action. For this reason, common practice is to leave out the frictional material near the heel and start it at an angle typically between $\theta_1 = 10^\circ$ and $\theta_1 = 30^\circ$.

With the direction of rotation shown in Figure 13.24 (i.e. the brake is self-energizing), the magnitude of the actuation force is given by

$$F_a = \frac{M_n - M_f}{a} \tag{13.39}$$

where M_n = the moment of the normal forces (N m), M_f = the moment due to the frictional forces (N m), and a = the orthogonal distance between the brake pivot and the line of action of the applied force (m).

The normal and frictional moments can be determined using Eqns (13.40) and (13.41), respectively. If the geometry and materials are selected such that $M_f = M_n$, the actuation force becomes zero. Such a brake would be self-locking. The slightest contact between the shoe and drum would bring the two surfaces into contact and the brake would snatch giving rapid braking. Alternatively, values for the brake geometry and materials can be selected to give different levels of self-energization depending on the relative magnitudes of M_n and M_f.

$$M_n = \frac{wrbp_{\max}}{(\sin\theta)_{\max}}\left[\frac{1}{2}(\theta_2 - \theta_1) - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right] \tag{13.40}$$

$$M_f = \frac{\mu wrp_{\max}}{(\sin\theta)_{\max}}\left[r(\cos\theta_1 - \cos\theta_2) + \frac{b}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right] \tag{13.41}$$

If the direction of rotation for the drum shown in Figure 13.24 is reversed, the brake becomes self de-energizing and the actuation force is given by

$$F_a = \frac{M_n + M_f}{a} \tag{13.42}$$

The pivot reactions can be determined by resolving the horizontal and vertical forces. For a self-energizing brake they are given by

$$R_x = \frac{wrp_{\max}}{(\sin\theta)_{\max}}\left[\left(0.5\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right) - \mu\left(\frac{\theta_2}{2} - \frac{\theta_1}{2} - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right)\right] - F_x \tag{13.43}$$

$$R_y = \frac{wrp_{\max}}{(\sin\theta)_{\max}}\left[\left(0.5\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right) + \mu\left(\frac{\theta_2}{2} - \frac{\theta_1}{2} - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right)\right] + F_y \tag{13.44}$$

and for a self de-energizing brake by

$$R_x = \frac{wrp_{\max}}{(\sin\theta)_{\max}}\left[\left(0.5\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right) + \mu\left(\frac{\theta_2}{2} - \frac{\theta_1}{2} - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right)\right] - F_x \tag{13.45}$$

$$R_y = \frac{wrp_{\max}}{(\sin\theta)_{\max}}\left[\left(0.5\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right) - \mu\left(\frac{\theta_2}{2} - \frac{\theta_1}{2} - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right)\right] + F_y \tag{13.46}$$

Example 13.9

Design a long-shoe drum brake to produce a friction torque of 75 N m to stop a drum rotating at 140 rpm. Initial design calculations have indicated that a shoe lining with $\mu = 0.25$ and using a value of $p_{max} = 0.5 \times 10^6$ N/m^2 in the design will give suitable life.

Solution

First propose trial values for the brake geometry, say $r = 0.1$ m, $b = 0.2$ m, $a = 0.3$ m, $\theta_1 = 30^\circ$, $\theta_2 = 150^\circ$. Using Eqn (13.37) and solving for the width of the shoe,

$$w = \frac{T(\sin\theta)_{max}}{\mu r^2 p_{max}(\cos\theta_1 - \cos\theta_2)} = \frac{75 \sin 90}{0.25 \times 0.1^2 \times 0.5 \times 10^6(\cos 30 - \cos 150)} = 0.0346 \text{ m}$$

Select the width to be 35 mm, as this is a standard size. The actual maximum pressure experienced will be

$$p_{max} = 0.5 \times 10^6 \frac{0.0346}{0.035} = 494{,}900 \text{ N/m}^2.$$

From Eqn (13.40) the moment of the normal force with respect to the shoe pivot is

$$M_n = \frac{0.035 \times 0.1 \times 0.2 \times 0.4949 \times 10^6}{\sin 90}\left[\frac{1}{2}\left(120 \times \frac{2\pi}{360}\right) - \frac{1}{4}(\sin 300 - \sin 60)\right]$$
$$= 512.8 \text{ N m}.$$

From Eqn (13.41) the moment of the frictional forces with respect to the shoe pivot is

$$M_f = \frac{0.25 \times 0.035 \times 0.1 \times 0.4949 \times 10^6}{\sin 90}\left[0.1(\cos 30 - \cos 150) + \frac{0.2}{4}(\cos 300 - \cos 60)\right]$$
$$= 75 \text{ N m}.$$

From Eqn (13.39) the actuation force is

$$F_a = \frac{M_n - M_f}{a} = \frac{512.8 - 75}{0.3} = 1459 \text{ N}.$$

For the double long-shoe external drum brake illustrated in Figure 13.25, the left hand shoe is self-energizing, and the frictional moment reduces the actuation load. The right hand shoe, however, is self de-energizing, and its frictional moment acts to reduce the maximum pressure that occurs on the right hand brake shoe. The normal and frictional moments for a self-energizing and self de-energizing brake are related by

$$M'_n = \frac{M_n p'_{\max}}{p_{\max}} \tag{13.47}$$

$$M'_f = \frac{M_f p'_{\max}}{p_{\max}} \tag{13.48}$$

where M'_n = the moment of the normal forces for the self de-energizing brake (N m), M'_f = the moment due to the frictional forces for the self de-energizing brake (N m), and $p'_{\max}$ = the maximum pressure on the self de-energizing brake (N/m^2).

Example 13.10

For the double long shoe external drum brake illustrated in Figure 13.26 determine the limiting force on the lever such that the maximum pressure on the brake lining does not exceed 1.4 MPa and determine the torque capacity of the brake. The face width of the shoes is 30 mm and the coefficient of friction between the shoes and the drum can be taken as 0.28.

Solution

First it is necessary to calculate values for θ_1 and θ_2 as these are not indicated directly on the diagram.

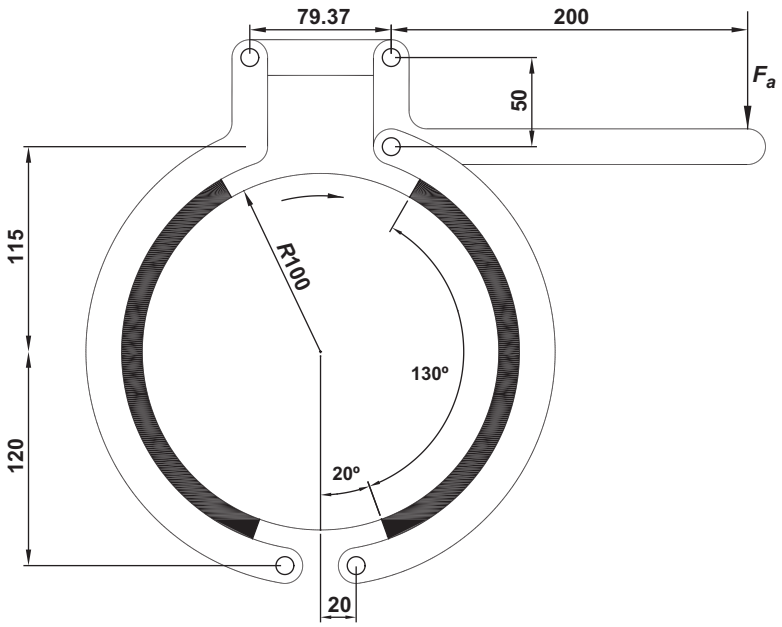


Figure 13.26
Double long-shoe external drum brake.

$$\theta_1 = 20^\circ - \tan^{-1}\left(\frac{20}{120}\right) = 10.54^\circ$$

$$\theta_2 = 20^\circ + 130^\circ - \tan^{-1}\left(\frac{20}{120}\right) = 140.5^\circ$$

The maximum value of sin θ would be sin $90 = 1$.

The distance between the pivot and the drum center, $b = \sqrt{0.02^2 + 0.12^2} = 0.1217$ m.

The normal moment is given by

$$\begin{aligned} M_n &= \frac{wrbp_{\max}}{(\sin\theta)_{\max}}\left[\frac{1}{2}(\theta_2 - \theta_1) - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right] \\ &= \frac{0.03 \times 0.1 \times 0.1217 \times 1.4 \times 10^6}{\sin 90} \\ &\quad \times \left[\frac{1}{2}\left((140.5 - 10.54) \times \frac{2\pi}{360}\right) - \frac{1}{4}(\sin 281 - \sin 21.08)\right] \\ &= 751.1 \text{ N m.} \end{aligned}$$

$$\begin{aligned} M_f &= \frac{\mu wrp_{\max}}{(\sin\theta)_{\max}}\left[r(\cos\theta_1 - \cos\theta_2) + \frac{b}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right] \\ &= \frac{0.28 \times 0.03 \times 0.1 \times 1.4 \times 10^6}{\sin 90} \\ &\quad \times \left[0.1(\cos 10.54 - \cos 140.5) + \frac{0.1217}{4}(\cos 281 - \cos 21.08)\right] \\ &= 179.8 \text{ N m.} \end{aligned}$$

The orthogonal distance between the actuation force and the pivot, $a = 0.12 + 0.115 + 0.05 = 0.285$ m.

The actuation load on the left hand shoe is given by

$$F_{a \text{ left shoe}} = \frac{M_n - M_f}{a} = \frac{751.1 - 179.8}{0.285} = 2004 \text{ N.}$$

The torque contribution from the left hand shoe is given by

$$\begin{aligned} T_{\text{left shoe}} &= \mu wr^2 \frac{p_{\max}}{(\sin\theta)_{\max}}(\cos\theta_1 - \cos\theta_2) \\ &= 0.28 \times 0.03 \times 0.1^2 \times 1.4 \times 10^6(\cos 10.54 - \cos 140.5) = 206.4 \text{ N m.} \end{aligned}$$

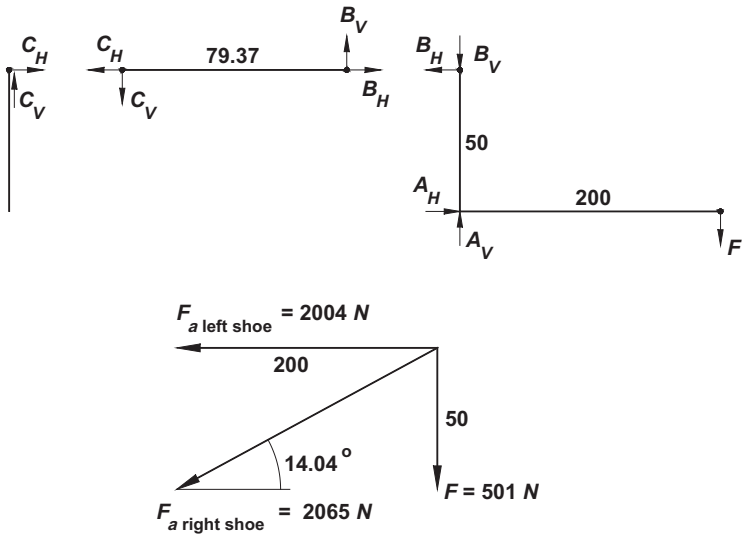


Figure 13.27
Free body diagrams.

The actuation force on the right hand shoe can be determined by considering each member of the lever mechanism as a free body (see Figure 13.27).

$$F - A_V + B_V = 0.$$

$$A_H = B_H.$$

$$B_H = C_H.$$

$$A_H = C_H.$$

$$0.2F = 0.05B_H, \quad F = B_H/4, \quad B_H = 2004\ \text{N}, \quad F = 2004/4 = 501\ \text{N}.$$

So the limiting lever force is $F = 501$ N.

$$C_V = 0, \quad B_V = 0.$$

The actuating force for the right hand lever is the resultant of F and B_H. The resultant angle is given by $\tan^{-1}(0.05/0.2) = 14.04°$.

$$F_{a\ \text{right shoe}} = \frac{2004}{\cos 14.04} = 2065\ \text{N}$$

The orthogonal distance between the actuation force vector and the pivot, Figure 13.28, is given by

$$a = (0.235 - 0.01969 \tan 14.04) \times \cos 14.04 = 0.2232\ \text{m}$$

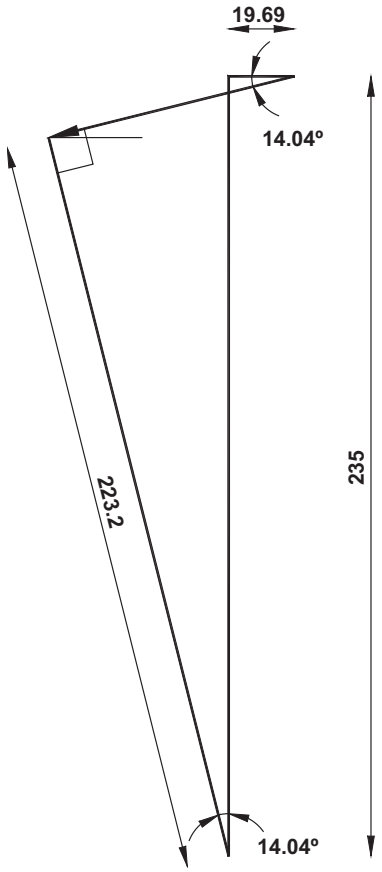


Figure 13.28
Orthogonal distance.

The normal and frictional moments for the right hand shoe can be determined using Eqns (13.47) and (13.48).

$$M'_n = \frac{M_n p'_{\max}}{p_{\max}} = \frac{751.1 p'_{\max}}{1.4 \times 10^6}$$

$$M'_f = \frac{M_f p'_{\max}}{p_{\max}} = \frac{179.8 p'_{\max}}{1.4 \times 10^6}$$

For the right hand shoe the maximum pressure can be determined from

$$F_{a \text{ right shoe}} = \frac{M'_n + M'_f}{a} = 2065 = \frac{751.1 p'_{\max} - 179.8 p'_{\max}}{1.4 \times 10^6 \times 0.2232}$$

$$p'_{\max} = 1.130 \times 10^6 \text{ N/m}^2.$$

The torque contribution from the right hand shoe is

$$T_{\text{right shoe}} = \mu w r^2 \frac{p'_{\max}}{(\sin\theta)_{\max}} (\cos\theta_1 - \cos\theta_2)$$
$$= 0.28 \times 0.03 \times 0.1^2 \times 1.13 \times 10^6 (\cos 10.54 - \cos 140.5) = 166.6 \text{ N m}.$$

The total torque is given by

$$T_{\text{total}} = T_{\text{left shoe}} + T_{\text{right shoe}} = 206.4 + 166.6 = 373 \text{ N m}$$

Example 13.11

A double external long-shoe drum brake is illustrated in Figure 13.29. The face width of the shoes is 50 mm, and the maximum permissible lining pressure is 1 MPa. If the coefficient of friction is 0.32, determine the limiting actuating force and the torque capacity.

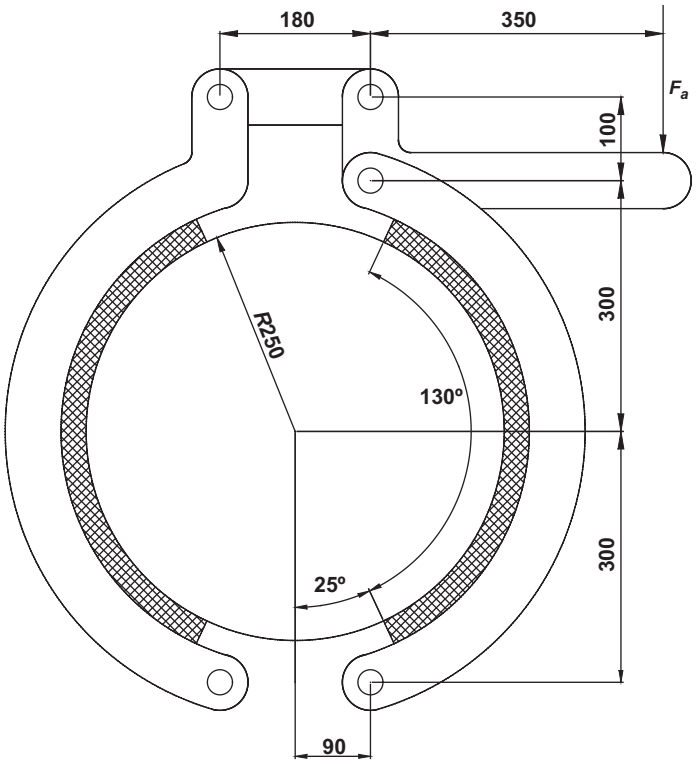


Figure 13.29
Double external long-shoe drum brake.

Solution

First it is necessary to calculate values for θ_1 and θ_2 as these are not indicated directly on the diagram.

$$\theta_1 = 25^\circ - \tan^{-1}\left(\frac{90}{300}\right) = 8.301^\circ.$$

$$\theta_2 = 25^\circ + 130^\circ - \tan^{-1}\left(\frac{90}{300}\right) = 138.3^\circ.$$

The maximum value of sin θ would be sin $90 = 1$.

The distance between the pivot and the drum center,

$$b = \sqrt{0.09^2 + 0.3^2} = 0.3132 \text{ m}.$$

The normal moment is given by

$$\begin{aligned} M_n &= \frac{wrbp_{\max}}{(\sin\theta)_{\max}}\left[\frac{1}{2}(\theta_2 - \theta_1) - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right] \\ &= \frac{0.05 \times 0.25 \times 0.3132 \times 1 \times 10^6}{\sin 90} \\ &\quad \times \left[\frac{1}{2}\left((138.3 - 8.301) \times \frac{2\pi}{360}\right) - \frac{1}{4}(\sin 276.6 - \sin 16.6)\right] \\ &= 5693 \text{ N m}. \end{aligned}$$

$$\begin{aligned} M_f &= \frac{\mu wrp_{\max}}{(\sin\theta)_{\max}}\left[r(\cos\theta_1 - \cos\theta_2) + \frac{b}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right] \\ &= \frac{0.32 \times 0.05 \times 0.25 \times 1 \times 10^6}{\sin 90} \\ &\quad \times \left[0.25(\cos 8.301 - \cos 138.3) + \frac{0.3132}{4}(\cos 276.6 - \cos 16.6)\right] \\ &= 1472 \text{ N m}. \end{aligned}$$

The orthogonal distance between the actuation force and the pivot, $a = 0.7$ m.

The actuation load on the left hand shoe is given by

$$F_{a \text{ left shoe}} = \frac{M_n - M_f}{a} = \frac{5693 - 1472}{0.7} = 6031 \text{ N}.$$

The torque contribution from the left hand shoe is given by

$$T_{\text{left shoe}} = \mu w r^2 \frac{p_{\max}}{(\sin\theta)_{\max}}(\cos\theta_1 - \cos\theta_2)$$
$$= 0.32 \times 0.05 \times 0.25^2 \times 1 \times 10^6(\cos 8.301 - \cos 138.3) = 1736 \text{ N m}.$$

The actuation force on the right hand shoe can be determined by considering each member of the lever mechanism as a free body.

$$F - A_V + B_V = 0.$$

$$A_H = B_H.$$

$$B_H = C_H.$$

$$A_H = C_H.$$

$$0.35F = 0.1B_H,$$

$$F = 0.1B_H/0.35.$$

$$B_H = 6031.$$

$$F = 6031 \times 0.1/0.35 = 1723 \text{ N}.$$

So the limiting lever force is $F = 1723$ N.

$$C_V = 0,$$

$$B_V = 0.$$

The actuating force for the right hand lever is the resultant of F and B_H.

The resultant angle is given by $\tan^{-1}(0.1/0.35) = 15.95^\circ$.

$$F_{a \text{ right shoe}} = \frac{1723}{\cos 15.95} = 6272 \text{ N}.$$

The perpendicular distance between the actuation force vector and the pivot is given by

$$a = 0.6 \times \cos 15.95 = 0.5769 \text{ m}.$$

The normal and frictional moments for the right hand shoe can be determined using Eqns (10.47) and (10.48).

$$M_n' = \frac{M_n p_{\max}'}{p_{\max}} = \frac{5693 p_{\max}'}{1 \times 10^6}.$$

$$M'_f = \frac{M_f p'_{\max}}{p_{\max}} = \frac{1472 p'_{\max}}{1 \times 10^6}.$$

For the right hand shoe the maximum pressure can be determined from

$$F_{a \text{ right shoe}} = \frac{M'_n + M'_f}{a} = 6272 = \frac{5693 p'_{\max} - 1472 p'_{\max}}{1 \times 10^6 \times 0.5769}$$

$$p'_{\max} = 0.8571 \times 10^6 \text{ N/m}^2.$$

The torque contribution from the right hand shoe is

$$T_{\text{right shoe}} = \mu w r^2 \frac{p'_{\max}}{(\sin\theta)_{\max}} (\cos\theta_1 - \cos\theta_2)$$

$$= 0.32 \times 0.05 \times 0.25^2 \times 0.8571 \times 10^6 (\cos 8.301 - \cos 138.3) = 1488 \text{ N m}.$$

The total torque is given by

$$T_{\text{total}} = T_{\text{left shoe}} + T_{\text{right shoe}} = 1736 + 1488 = 3224 \text{ N m}$$

13.3.5 Long-Shoe Internal Drum Brakes

Most drum brakes use internal shoes that expand against the inner radius of the drum. Long-shoe internal drum brakes are principally used in automotive applications. An automotive drum brake typically comprises two brake shoes and linings supported on a back plate bolted to the axle casing. The shoes are pivoted at one end on anchor pins or abutments fixed onto the back plate (see Figure 13.30). The brake can be actuated by a double hydraulic piston expander that forces the free ends of the brake apart so the nonrotating shoes come into frictional contact with the rotating brake drum. A leading and trailing shoe layout consists of a pair of shoes pivoted at a common anchor point as shown in Figure 13.30. The leading shoe is identified as the shoe whose expander piston moves in the direction of rotation of the drum. The frictional drag between the shoe and the drum will tend to assist the expander piston in forcing the shoe against the drum; this action is referred to as self-energizing or the self-servo action of the shoe. The trailing shoe is the one whose expander piston moves in the direction opposed to the rotation of the drum. The frictional force opposes the expander, hence a trailing brake shoe provides less braking torque than an equivalent leading shoe actuated by the same force. The equations developed for external long-shoe drum brakes are also valid for internal long-shoe drum brakes.

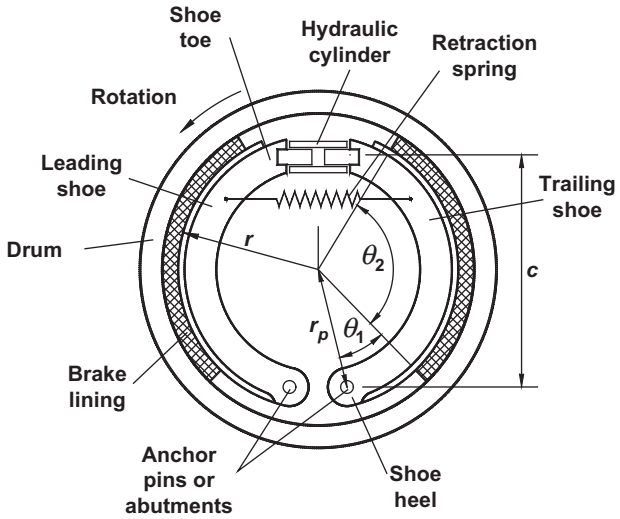


Figure 13.30
Double long-shoe internal drum brake.

Example 13.12

Determine the actuating force and the braking capacity for the double internal long-shoe brake illustrated in Figure 13.31. The lining is sintered metal with a coefficient of friction of 0.32, and the maximum lining pressure is 1.2 MPa. The drum radius is 68 mm and the shoe width is 25 mm.

Solution

$$b = \sqrt{0.015^2 + 0.055^2} = 0.05701 \text{ m}$$

As the brake lining angles relative to the pivot, brake axis line, are not explicitly shown on the diagram, they must be calculated.

$$\theta_1 = 4.745^\circ, \quad \theta_2 = 124.7^\circ$$

As $\theta_2 > 90^\circ$, the maximum value of sin θ is sin $90 = 1 = (\sin \theta)_{\max}$.

For this brake with the direction of rotation as shown, the right hand shoe is self-energizing.

For the right hand shoe:

$$M_n = \frac{0.025 \times 0.068 \times 0.05701 \times 1.2 \times 10^6}{1} \times \left[\frac{1}{2}\left((124.7 - 4.745) \times \frac{2\pi}{360}\right) - \frac{1}{4}(\sin 249.4 - \sin 9.49)\right] = 153.8 \text{ N m}.$$

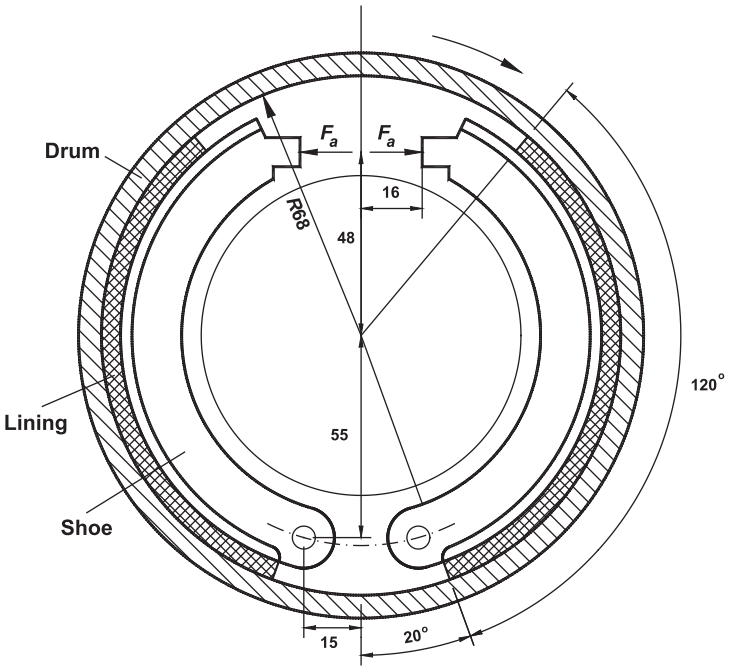


Figure 13.31
Double long-shoe internal drum brake.

$$M_f = \frac{0.32 \times 0.025 \times 0.068 \times 1.2 \times 10^6}{1} \times \left[0.068(\cos 4.745 - \cos 124.7) + \frac{0.05701}{4}(\cos 249.4 - \cos 9.49)\right] = 57.1 \text{ N m}.$$

$$a = 0.055 + 0.048 = 0.103 \text{ m}.$$

$$F_a = \frac{M_n - M_f}{a} = \frac{153.8 - 57.1}{0.103} = 938.9 \text{ N}.$$

The actuating force is 938.9 N.

The torque applied by the right hand shoe is given by

$$T_{\text{right shoe}} = \frac{\mu w r^2 p_{\max}}{(\sin\theta)_{\max}}(\cos\theta_1 - \cos\theta_2) = \frac{0.32 \times 0.025 \times 0.068^2 \times 1.2 \times 10^6}{1}(\cos 4.745 - \cos 124.7) = 69.54 \text{ N m}.$$

The torque applied by the left hand shoe cannot be determined until the maximum operating pressure $p'_{\max}$ for the left hand shoe has been calculated.

As the left hand shoe is self de-energizing, the normal and frictional moments can be determined using Eqns (13.47) and (13.48).

$$M'_n = \frac{M_n p'_{\max}}{p_{\max}} = \frac{153.8 p'_{\max}}{1.2 \times 10^6}$$

$$M'_f = \frac{M_f p'_{\max}}{p_{\max}} = \frac{57.1 p'_{\max}}{1.2 \times 10^6}$$

The left hand shoe is self de-energizing, so

$$F_a = \frac{M_n + M_f}{a}.$$

$F_a = 938.9$ N as calculated earlier.

$$938.9 = \frac{153.8 p'_{\max} + 57.1 p'_{\max}}{1.2 \times 10^6 \times 0.103}$$

$$p'_{\max} = 0.5502 \times 10^6 \text{ N/m}^2.$$

The torque applied by the left hand shoe is given by

$$T_{\text{left shoe}} = \frac{\mu w r^2 p'_{\max}}{(\sin\theta)_{\max}} (\cos\theta_1 - \cos\theta_2)$$

$$= \frac{0.32 \times 0.025 \times 0.068^2 \times 0.5502 \times 10^6}{1} (\cos 4.745 - \cos 124.7) = 31.89 \text{ N m}.$$

The total torque applied by both shoes is

$$T_{\text{total}} = T_{\text{right shoe}} + T_{\text{left shoe}} = 69.54 + 31.89 = 101.4 \text{ N m}.$$

From this example, the advantage in torque capacity of using self-energizing brakes is apparent. Both the left hand and the right hand shoes could be made self-energizing by inverting the left hand shoe, having the pivot at the top. This would be advantageous if rotation occurred in just one direction. If, however, drum rotation is possible in either direction, it may be more suitable to have one brake self-energizing for forward motion and one self-energizing for reverse motion.

13.3.6 Band Brakes

One of the simplest types of braking device is the band brake. This consists of a flexible metal band lined with a frictional material wrapped partly around a drum. The brake is

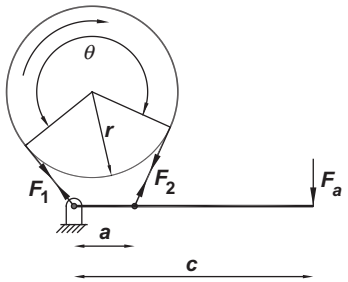


Figure 13.32
Band brake.

actuated by pulling the band against the drum, as illustrated in Figure 13.32. For the clockwise rotation shown in Figure 13.32, the friction forces increase F_1 relative to F_2. The relationship between the tight and slack sides of the band is given by

$$\frac{F_1}{F_2} = e^{\mu\theta} \tag{13.49}$$

where F_1 = tension in the tight side of the band (N), F_2 = tension in the slack side of the band (N), μ = coefficient of friction, and θ = angle of wrap (rad).

The point of maximum contact pressure for the friction material occurs at the tight end and is given by

$$p_{\max} = \frac{F_1}{rw} \tag{13.50}$$

where w is the width of the band (m).

The torque braking capacity is given by

$$T = (F_1 - F_2)r \tag{13.51}$$

The relationship, for the simple band brake shown in Figure 13.32, between the applied lever force F_a and F_2 can be found by taking moments about the pivot point.

$$F_a c - F_2 a = 0$$

$$F_a = F_2 \frac{a}{c} \tag{13.52}$$

The brake configuration shown in Figure 13.32 is self-energizing for clockwise rotation. The level of self-energization can be enhanced by using the differential band brake configuration shown in Figure 13.33. Summation of the moments about the pivot gives

$$F_a c - F_2 a + F_1 b = 0 \tag{13.53}$$

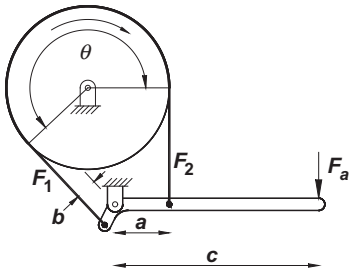


Figure 13.33
Self-energizing band brake.

So the relationship between the applied load F_a and the band brake tensions is given by

$$F_a = \frac{F_2 a - F_1 b}{c} \tag{13.54}$$

Note that the value of b must be less than a so that applying the lever tightens F_2 more than it loosens F_1. Substituting for F_1 in Eqn (13.54) gives

$$F_a = \frac{F_2\left(a - be^{\mu\theta}\right)}{c} \tag{13.55}$$

The brake can be made self-locking if $a < be^{\mu\theta}$, and the slightest touch on the lever would cause the brake to grab or lock abruptly. This principle can be used to permit rotation in one direction only, as in hoist and conveyor applications.

Example 13.13

Design a band brake to exert a braking torque of 85 N m. Assume the coefficient of friction for the lining material is 0.25 and the maximum permissible pressure is 0.345 MPa.

Solution

Propose a trial geometry, say $r = 150$ mm, $\theta = 225°$, and $w = 50$ mm.

$$F_1 = p_{\max} rw = 0.345 \times 10^6 \times 0.15 \times 0.05 = 2587 \text{ N}.$$

$$F_2 = \frac{F_1}{e^{\mu\theta}} = \frac{2587.5}{e^{0.25(225\times 2\pi/360)}} = 969 \text{ N}.$$

$$T = (F_1 - F_2)r = (2587.5 - 969)0.15 = 242.7 \text{ N m}.$$

This torque is much greater than the 80 N m desired, so try a different combination of r, θ, and w until a satisfactory design is achieved.

Try $r = 0.1$ m, $\theta = 225°$, and $w = 50$ mm.

$$F_1 = p_{\max} rw = 0.345 \times 10^6 \times 0.1 \times 0.05 = 1725 \text{ N}.$$

$$F_2 = \frac{F_1}{e^{\mu\theta}} = \frac{1725}{e^{0.25(225\times 2\pi/360)}} = 646.3 \text{ N}.$$

$$T = (F_1 - F_2)r = (1725 - 646.3)0.1 = 107.9 \text{ N m}.$$

Try $r = 0.09$ m, $\theta = 225°$, and $w = 50$ mm.

$$F_1 = p_{\max} rw = 0.345 \times 10^6 \times 0.09 \times 0.05 = 1552.5 \text{ N}.$$

$$F_2 = \frac{F_1}{e^{\mu\theta}} = \frac{1552.5}{e^{0.25(225\times 2\pi/360)}} = 581.7 \text{ N}.$$

$$T = (F_1 - F_2)r = (1552.5 - 581.7)0.09 = 87.4 \text{ N m}.$$

The actuating force is given by $F_a = F_2 a/c$. If $a = 0.08$ m and $c = 0.15$ m then,

$$F_a = 581.7 \times \frac{0.08}{0.15} = 310.2 \text{ N}.$$

Practical braking systems comprise an energy supplying device, a control device, a transmission device for controlling the braking force, and the brakes themselves. For example, in the case of a braking system for passenger cars, the braking system could control the brake pedal, a vacuum booster, master hydraulic fluid cylinder, brake fluid reservoir, a device to warn the driver of a brake circuit failure, a sensor to warn of low brake fluid level, valves, hydraulic cylinders, springs, pads, and discs. For heavy vehicles, say two to three tons and over, the force required for braking can become greater than a person can exert. In such cases some assistance needs to be given to the driver. This can be achieved by using a servo mechanism that adds to the driver's effort, for example a vacuum assisted brake servo unit, or by using power operation in which case the driver's effort is simply for control purposes and is not transmitted directly to the brakes.

The performance of braking systems in, say, automotive applications has been significantly enhanced over the purely mechanical variants described so far by the use of sensors, actuators and sophisticated control systems. An example is antilock braking systems (ABS), which are closed-loop control devices within the braking system. The objective of ABS is to prevent wheel lockup during braking, and as a result retain greater steering control and vehicle stability. The principal components of an ABS system are a hydraulic modulator, wheel based sensors, and the engine control unit (ECU). The ECU processes the signals and controls and triggers the hydraulics. The control loop for an antilock braking system is shown in Figure 13.34. ABS systems are considered in more detail in the Bosch Automotive Handbook (2000) and Garrett et al. (2001).

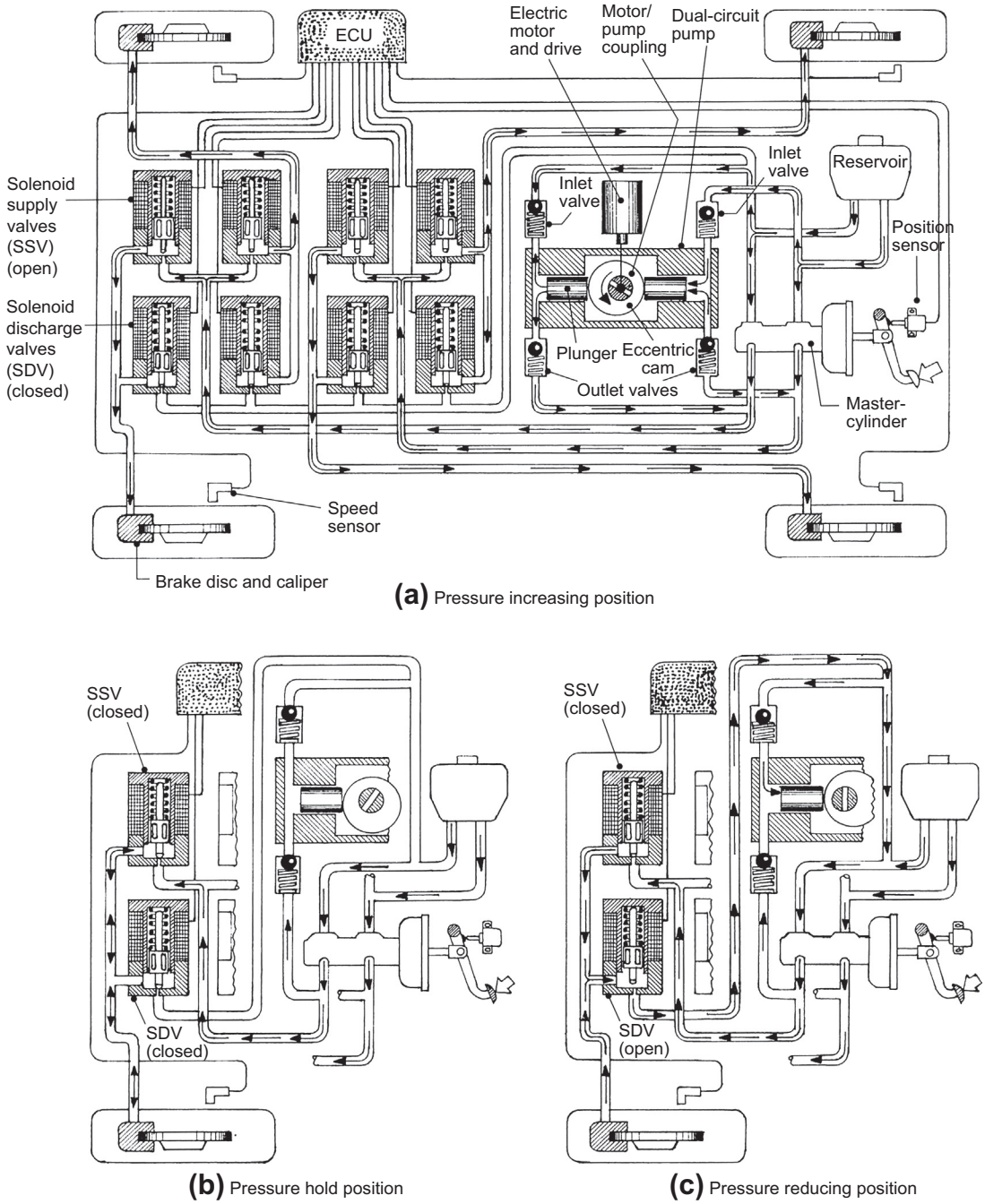


Figure 13.34

Control loop for an antilock braking system (ABS). (a) Pressure increasing position; (b) Pressure hold position; (c) Pressure reducing position. *Reproduced from Heisler (1999).*

It should be noted that the design of braking systems in the case of automotive applications is subject to extensive regulation; requirements are detailed in documents such as StVZO, EC Directive 71,320, and ECE Directive 13.

13.4 Conclusions

Clutches are designed to permit the smooth, gradual engagement or disengagement of a prime mover from a driven load. Brakes are designed to decelerate a system. Clutches and brakes are similar devices providing frictional, magnetic, or direct positive connection between two components. In principle, one device could function as either a clutch or a brake. If one component rotates and the other is fixed to a nonrotating plane of reference, the device will function as a brake, and if both rotate the device will function as a clutch.

This chapter has concentrated on rotating clutches and brakes and specifically on the design of friction based devices. In the case of brakes, control systems enable significant advantages over purely mechanical devices, enabling the prevention of wheel lockup during braking and as a result retaining greater driveability. The detailed design of a clutch or braking system involves integration of a wide range of skills such as bearings, shafts, splines, teeth, flywheels, casings, frictional surfaces, hydraulics, sensors, and control algorithms. Both brakes and clutches can be purchased from specialist suppliers or alternatively key components such as brake pads or clutch discs can be specified and bought in from specialist suppliers and integrated into a fit for purpose machine design.

References

Books and Papers

Bosch, 2000. Automotive Handbook, fifth ed. Bentley Publishers.
Fazekas, G.A., 1972. On circular spot brakes. Trans. ASME, J. Eng. Ind., 859–863.
Garrett, T.K., Newton, K., Steeds, W., 2001. The Motor Vehicle, thirteenth ed. Butterworth Heinemann.
Heisler, H., 1999. Vehicle and Engine Technology, second ed. Butterworth Heinemann.
Hindhede, U., Zimmerman, J.R., Hopkins, R.B., Erisman, R.J., Hull, W.C., Lang, J.D., 1983. Machine Design Fundamentals. A Practical Approach. Wiley.
Juvinall, R.C., Marshek, K.M., 1991. Fundamentals of Machine Component Design. Wiley.
Neale, M.J., 1994. Drives and Seals: A Tribology Handbook. Butterworth Heinemann.
Neale, M.J. (Ed.), 1995. The Tribology Handbook. Butterworth Heinemann.
Vedamuttu, P., 1994. Clutches. In: Hurst, K. (Ed.), Rotary Power Transmission Design. McGraw Hill.

Further Reading

Baker, A.K., 1992. Industrial Brake and Clutch Design. Pentech Press.
Limpert, R., 1999. Brake Design and Safety, second ed. SAE.
Proctor, J., 1961. Selecting clutches for mechanical drives. Product Eng., 43–58.

Standards

British Standards Institution. BS AU 180: Part 4: 1982. Brake Linings. Method for Determining Effects of Heat on Dimensions and Form of Disc Brake Pads.
British Standards Institution. BS 4639: 1987. Specification for Brakes and Braking Systems for Towed Agricultural Vehicles.
British Standards Institution. BS AU 249:1993, ISO 7649:1991. Specification for Clutch Housings for Reciprocating Internal Combustions Engines. Nominal Dimensions and Tolerances.

Web Sites

At the time of going to press the world-wide-web contained useful information relating to this chapter at the following sites.

www.boschusa.com/AutoParts/BrakePads/
www.clutchfacing.com
www.cross-morse.co.uk
www.dba.com.au
www.frendisa.com
www.iatcoinc.com
www.jbrakes.com
www.kfabsc.com
www.luk.de
www.mondelengineering.com/mondProductsMain.htm
www.nascoaircraft.com
www.onassisauto.com
www.trwauto.com
www.warnernet.com
www.winnard.co.uk
www.xyd-autoparts.com

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

a	distance (m)
b	distance (m)
c	distance (m)
A	area (m^2)
ABS	antilock braking system
ECU	engine control unit
F	force (N)
F_f	frictional force (N)
F_n	normal force (N)
F_1	tension in the tight side of the band (N)
F_2	tension in the slack side of the band (N)
h	heat transfer coefficient (W/m^2 K)

I	mass moment of inertia (kg m^2)
IC	internal combustion
L	length or thickness (m)
m	mass (kg)
M	moment (N m)
M_f	frictional moment (N m)
M'_f	moment due to the frictional forces for a self de-energizing brake (N m)
M_n	normal moment (N m)
M'_n	moment of the normal forces for a self de-energizing brake (N m)
N	number of disc faces
p	pressure (N/m^2)
p_{av}	average pressure (N/m^2)
$p_{\max}$	maximum permissible pressure (N/m^2)
$p'_{\max}$	maximum pressure on the self de-energizing brake (N/m^2)
Q	heat transfer rate (W)
r	radius (m)
r_e	effective radius (m)
r_i	inner radius (m)
r_o	outer radius (m)
R	pad radius, reaction (N)
T	torque (N m), temperature (°C)
T_f	temperature of the surrounding fluid (K or °C)
T_s	temperature of the surface (K or °C)
V_i	initial velocity (m/s)
V_f	final velocity (m/s)
w	width of the brake shoe (m)
α	acceleration (m/s^2)
δ	ratio of effective radius to radius
δF	elemental axial force (N)
δr	elemental radial distance (m)
δT	elemental torque (N m)
μ	coefficient of friction
θ	angle (rad)
ρ	density (kg/m^3)
ω	angular velocity (rad/s)
ΔT	temperature difference (°C)

CHAPTER 14

Seals

Chapter Outline

14.1 Introduction to Seals 565
14.2 Static Seals 567
- 14.2.1 Elastomeric Seal Rings 567
- 14.2.2 Gaskets 574
- 14.2.3 Foodstuffs Containers 575

14.3 Dynamic Seals 577
- 14.3.1 Seals for Rotating Machinery 577

14.4 Labyrinth Seals 581
- 14.4.1 Martin's Equation 593
- 14.4.2 Orifice Flow Approach 595
- 14.4.3 Zimmermann and Wolff's Model 596
- 14.4.4 Stepped Labyrinth Seals 608
- 14.4.5 Staggered Labyrinths 611
- 14.4.6 Radial Labyrinths 612

14.5 Axial and Bush Seals 612
14.6 Seals for Reciprocating Components 616
14.7 Conclusions 619
References 619
Further Reading 622
Nomenclature 623

Abstract

Seals are devices used to prevent or limit leakage of fluids or particulates. Seals are widely used for applications where components do not move significantly relative to each other and also for applications where there is significant movement between components as a result of rotation or reciprocation. The aims of this chapter are to introduce the variety of seal configurations, and to provide guidelines for the selection of seals and introduce calculation methods for the quantification of leakage rates for labyrinth, axial, and bush seals.

14.1 Introduction to Seals

The purpose of a seal is to prevent or limit flow between components. Seals are an important aspect of machine design where pressurized fluids must be contained within an area of a

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00014-9

Copyright © 2014 Elsevier Ltd. All rights reserved.

machine such as a hydraulic cylinder, contaminants excluded, or lubricants retained. Seals fall into two general categories:

1. Static seals, where sealing takes place between two surfaces that do not move relative to each other; and
2. Dynamic seals, where sealing takes place between two surfaces that move relative to each other by, for example, rotary or reciprocating motion.

Any clearance between two components will permit the passage of fluid molecules in either direction, the direction depending on the pressures and momentum associated with the fluid. The basic sealing problem is illustrated in Figure 14.1, where either boundary shown may be stationary or moving. Fluid can move between different regions of space by means of diffusion, free convection, or forced convection. The size of a typical gas or vapor molecule is of the order of 10^{-9} m in diameter. They can therefore diffuse through very small gaps such as pores in a machine casing or seal component. Convection involves the mixing of one portion of fluid with another by means of gross movements of the mass of fluid. The fluid motion may be caused by external mechanical means, such as by a fan or pump in which case the process is called forced convection. Alternatively, if the fluid motion is caused by density differences, due to for instance temperature differences, the process is called natural or free convection.

Some of the typical considerations in selecting the type of seal include:

1. the nature of the fluid to be contained or excluded, whether liquid, gaseous, two phase, multiphase, multicomponent flow, or particulate laden;
2. static pressure levels on either side of the seal;
3. the nature of any relative motion between the seal and mating components, e.g. rotating, reciprocating, contacting or noncontacting, high speed or low speed;
4. the level of sealing required, for example whether some leakage can be tolerated or a contacting seal can be used;

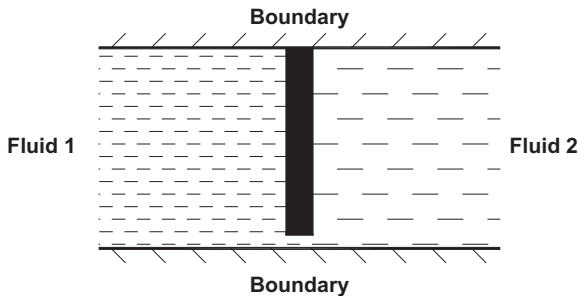


Figure 14.1
The basic sealing geometry.

5. operating temperatures;
6. life expectancy;
7. serviceability; and
8. total cost considering components and lifetime service and replacement costs.

The variety of seals and sealing systems is extensive. Figure 14.2 shows a general classification diagram for seals detailing the principal types. Several of these will be introduced in Sections 14.2 and 14.3. It should be noted, however, that once a general sealing requirement and possible solution have been identified, the best source of specific information is usually the seal manufacturers, or for sealing bearings, the bearing manufacturers. Figure 14.3 provides a general guide to the selection of seal type.

The subject of sealing for fluid applications is also introduced by Flitney (2007), Muller and Nau (1998), SAE (1998), Steinetz and Hendricks (1997), Neale (1994), and Zuk (1976) and for the specific case of labyrinth seals by ESDU (2009).

14.2 Static Seals

Static seals aim to provide a complete physical barrier to leakage flow. To achieve this, the seal material must be resilient enough to flow into and fill any irregularities in the surfaces being sealed and at the same time remain rigid enough to resist extrusion into clearances. Elastomeric seals and gaskets fulfill these criteria and are described in Sections 14.2.1 and 14.2.2, respectively. The particular demands of sealing of foodstuffs are introduced in Section 14.2.3.

14.2.1 Elastomeric Seal Rings

The "O" ring, Figure 14.4, is a simple and versatile type of seal with a wide range of applications for both static and dynamic sealing. An O ring seal is a molded elastomeric ring "nipped" in a cavity in which the seal is located. The principle of operation for an O ring sealing against a fluid at various pressures is illustrated in Figure 14.5.

Elastomeric seal rings require the seal material to have an interference fit with one of the mating parts of the assembly. O rings are available in a wide range of sizes, with internal diameters from 3.1 to 249.1 mm and section diameters of 1.6, 2.4, 3.0, 4.1, 5.7, and 8.4 mm as defined in British Standard BS 4518. Table 14.1 shows the dimensions for a small number of the seal sizes available. A more extensive table is normally available in the form of a sales catalog from manufacturers or in BS 4518. Figure 14.6 shows the groove dimensions, which must be specified to house the O ring seal and ensure the seal is nipped or compressed sufficiently to enable effective sealing. The groove dimensions and corresponding data for static face sealing are given in Figure 14.7 and Table 14.2 respectively.

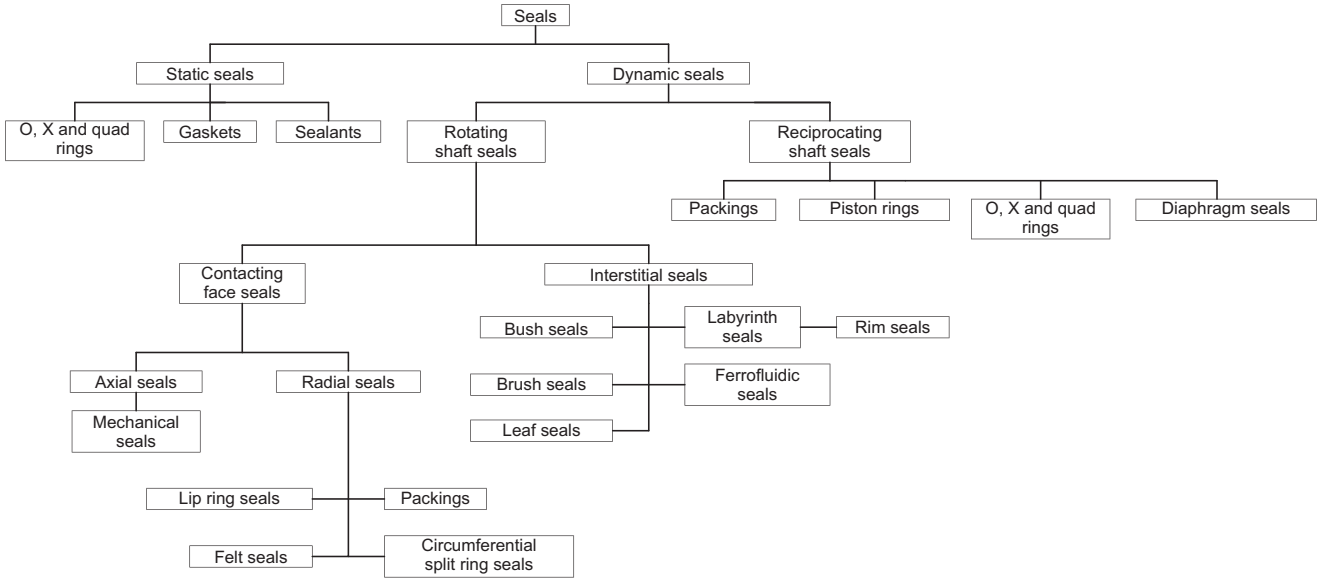


Figure 14.2
General seal classification chart. *ESDU (2009)*.

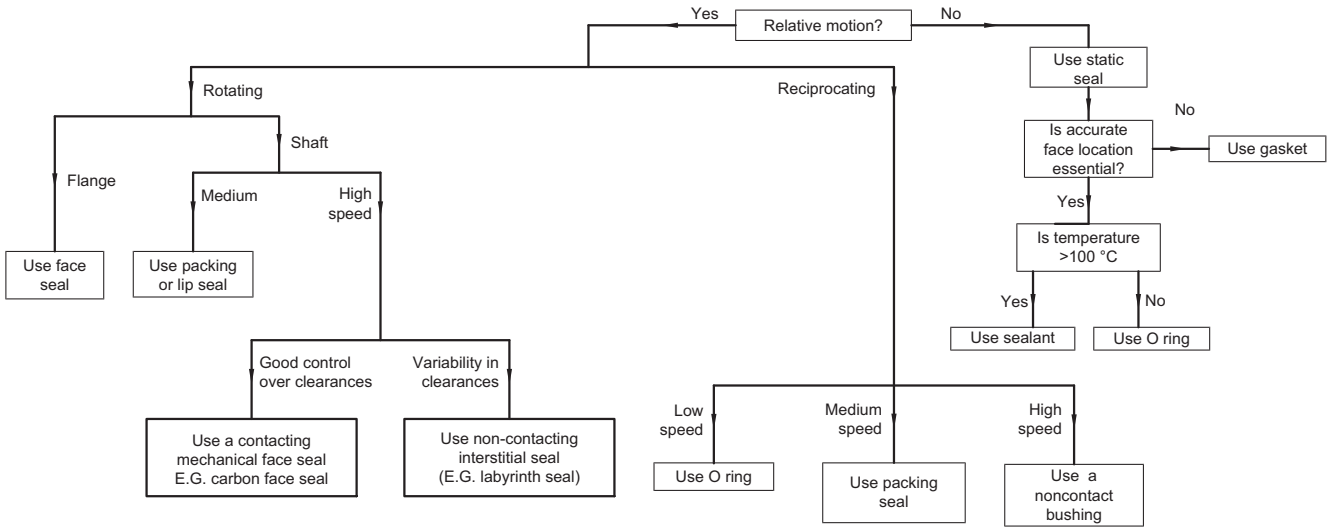


Figure 14.3
Seal selection procedure. *Adapted from Hamilton (1994).*

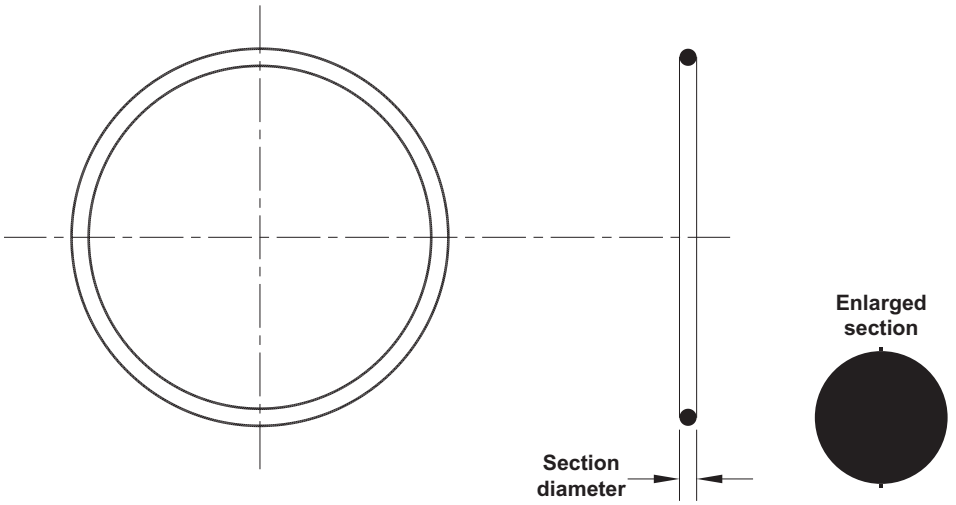


Figure 14.4
General O ring geometry.

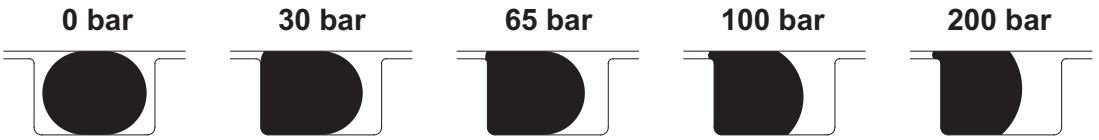


Figure 14.5
Principle of operation of O rings sealing a fluid against pressure.

Example 14.1

Specify suitable groove dimensions for a 0195-30 O ring to seal against a solid cylinder.

Solution

From Table 14.1 and with reference to Figure 14.6, $B = 3.7$ mm, $R = 1$ mm, and groove fillet radius $= 0.2$ mm.

A particular problem associated with O rings is the ability to cope with small movements of the housings and sealing faces. A wide range of solutions have been developed to produce seals that are resistant to for instance rotation within the seal groove. An X ring, also known as a quadring, and a rectangular seal are illustrated in Figure 14.8.

Aperture seals used for example for doors, windows, and cabriolet bodies are typically made from elastomeric extrusions, as production costs are low relative to fabricated mechanical seals and as their assembly can be automated. In the case of automobiles, the requirements are demanding with the need to seal against differential pressure and exclude dust, air, water,

Table 14.1: O ring seal dimensions for diametral sealing (mm) (limited range tabulated only for illustration).

Reference number	Internal diameter (mm)	Section diameter (mm)	$d_{nominal}$ (mm) (Figure 14.6a)	$D_{nominal}$ (mm) (Figure 14.6a)	$D_{nominal}$ (mm) (Figure 14.6b)	$d_{nominal}$ (mm) (Figure 14.6b)	*B* (mm)	*R* (mm)
0031-16	3.1	1.6	3.5	5.8	6	3.7	2.3	0.5
0041-16	4.1	1.6	4.5	6.8	7	4.7	2.3	0.5
0051-16	5.1	1.6	5.5	7.8	8	5.7	2.3	0.5
0061-16	6.1	1.6	6.5	8.8	9	6.7	2.3	0.5
0071-16	7.1	1.6	7.5	9.8	10	7.7	2.3	0.5
0081-16	8.1	1.6	8.5	10.8	11	8.7	2.3	0.5
0091-16	9.1	1.6	9.5	11.8	12	9.7	2.3	0.5
0101-16	10.1	1.6	10.5	12.8	13	10.7	2.3	0.5
0111-16	11.1	1.6	11.5	13.8	14	11.7	2.3	0.5
0036-24	3.6	2.4	4	7.7	8	4.3	3.1	0.5
0046-24	4.6	2.4	5	8.7	9	5.3	3.1	0.5
0195-30	19.5	3.0	20	24.8	25	20.2	3.7	1.0
0443-57	44.3	5.7	45	54.7	55	45.3	6.4	1.0
1441-84	144.1	8.4	145	160	160	145	9.0	1.0
2491-84	249.1	8.4	250	265	265	250	9.0	1.0

Source: British Standard BS 4518.

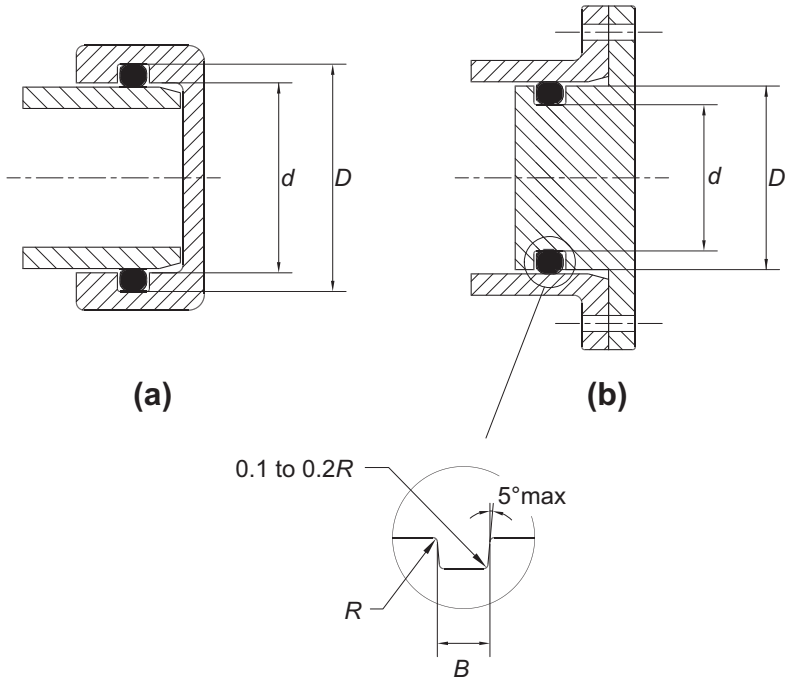


Figure 14.6
O ring groove dimensions for diametral sealing. (a) Groove on outer component; (b) groove on inner component.

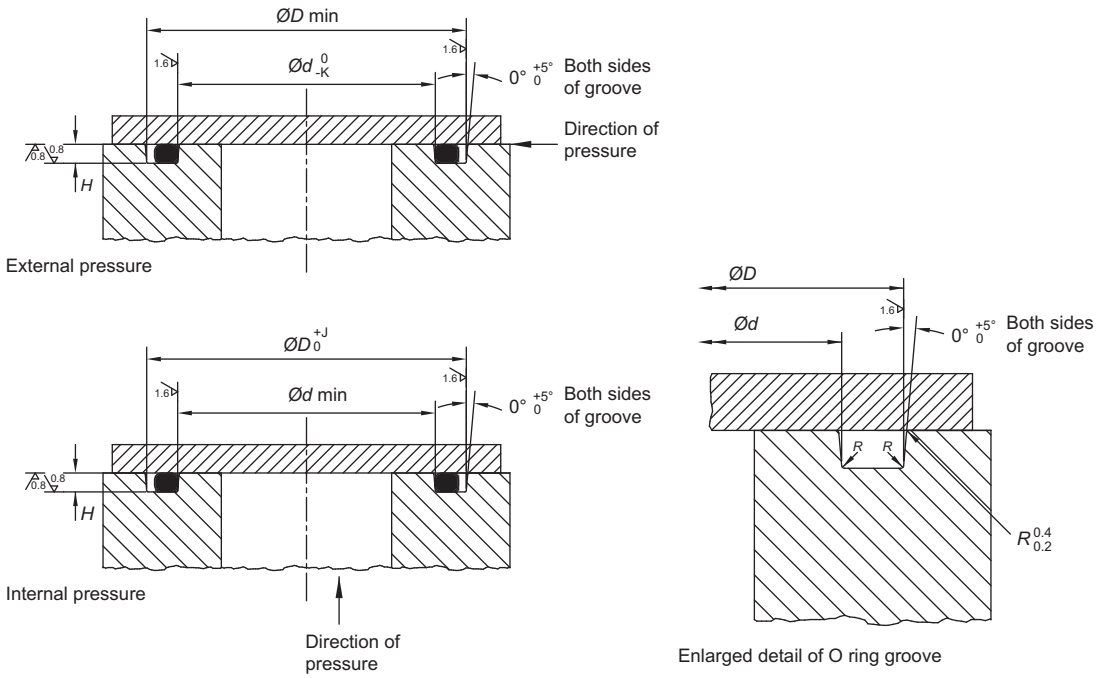


Figure 14.7
Groove for static face sealing. *BS 4518, 1982.*

Table 14.2: Groove dimensions for static face sealing. Selection only.

O ring ref. no.	*d* (max) (mm)	*D* (mm)	*J* (mm)	*D* (min)	*d* (mm)	*K* (mm)	*H* (mm)	*R* (mm)
0593-30	57	65	0.19	68	60	0.19	$2.2 \pm {}^{0.1}_{0}$	1.0
0625-30	60	68	0.19	71	63	0.19	$2.2 \pm {}^{0.1}_{0}$	1.0
0645-30	62	70	0.19	73	65	0.19	$2.2 \pm {}^{0.1}_{0}$	1.0
0695-30	67	75	0.19	78	70	0.19	$2.2 \pm {}^{0.1}_{0}$	1.0
0745-30	72	80	0.19	83	75	0.19	$2.2 \pm {}^{0.1}_{0}$	1.0
0795-30	77	85	0.22	88	80	0.19	$2.2 \pm {}^{0.1}_{0}$	1.0
0845-30	82	90	0.22	93	85	0.22	$2.2 \pm {}^{0.1}_{0}$	1.0
0895-30	87	95	0.22	98	90	0.22	$2.2 \pm {}^{0.1}_{0}$	1.0
0945-30	92	100	0.22	103	95	0.22	$2.2 \pm {}^{0.1}_{0}$	1.0
0995-30	97	105	0.22	108	100	0.22	$2.2 \pm {}^{0.1}_{0}$	1.0
1045-30	102	110	0.22	113	105	0.22	$2.2 \pm {}^{0.1}_{0}$	1.0
1095-30	107	115	0.22	118	110	0.22	$2.2 \pm {}^{0.1}_{0}$	1.0
1145-30	112	120	0.22	123	115	0.22	$2.2 \pm {}^{0.1}_{0}$	1.0
1195-30	117	125	0.25	128	120	0.22	$2.2 \pm {}^{0.1}_{0}$	1.0
1245-30	122	130	0.25	133	125	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1295-30	127	135	0.25	138	130	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1345-30	132	140	0.25	143	135	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1395-30	137	145	0.25	148	140	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1445-30	142	150	0.25	153	145	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1495-30	147	155	0.25	158	150	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1545-30	152	160	0.25	163	155	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1595-30	157	165	0.25	168	160	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1645-30	162	170	0.25	173	165	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1695-30	167	175	0.25	178	170	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1745-30	172	180	0.25	183	175	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1795-30	177	185	0.29	188	180	0.25	$2.2 \pm {}^{0.1}_{0}$	1.0
1845-30	182	190	0.29	193	185	0.29	$2.2 \pm {}^{0.1}_{0}$	1.0
1895-30	187	195	0.29	198	190	0.29	$2.2 \pm {}^{0.1}_{0}$	1.0
1945-30	192	200	0.29	203	195	0.29	$2.2 \pm {}^{0.1}_{0}$	1.0
1995-30	197	205	0.29	208	200	0.29	$2.2 \pm {}^{0.1}_{0}$	1.0
2095-30	207	215	0.29	218	210	0.29	$2.2 \pm {}^{0.1}_{0}$	1.0
2195-30	217	225	0.29	228	220	0.29	$2.2 \pm {}^{0.1}_{0}$	1.0
2295-30	227	235	0.29	238	230	0.29	$2.2 \pm {}^{0.1}_{0}$	1.0
2395-30	237	245	0.29	248	240	0.29	$2.2 \pm {}^{0.1}_{0}$	1.0
2445-30	242	250	0.29	253	245	0.29	$2.2 \pm {}^{0.1}_{0}$	1.0
2495-30	247	255	0.29	258	250	0.29	$2.2 \pm {}^{0.1}_{0}$	1.0

Source: BS 4518, 1982.



Figure 14.8
X ring and rectangular seal.

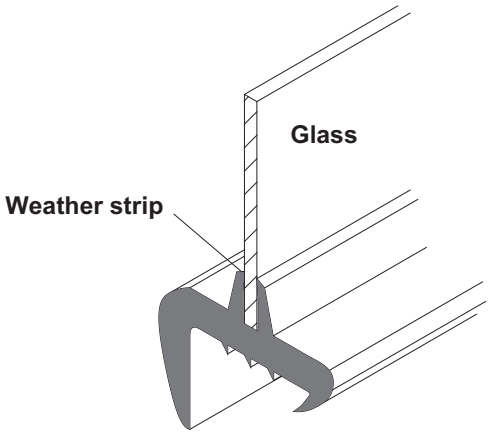


Figure 14.9
Aperture seal for automotive applications.

and noise. In addition, the components need to have a life compatible with that of the vehicle and to function over a wide range of operating temperatures, the cost of production and installation must be low, the function of the seal must not deteriorate significantly with time, and the seal must not significantly influence the drag of the vehicle. A typical seal for an automotive window application is illustrated in Figure 14.9.

14.2.2 Gaskets

A gasket is a material or composite of materials clamped between two components with the purpose of preventing fluid flow. Gaskets are typically made up of spacer rings, a sealing element, internal reinforcement, a compliant surface layer, and possibly some form of surface antistick treatment as shown in Figure 14.10.

Figure 14.11 shows a typical application for a gasket seal. When first closed, a gasket seal is subject to compressive stresses produced by the assembly. Under working conditions, however, the compressive load may be relieved by the pressures generated within the assembly or machine. This must be accounted for in the detailed design or by use of a factor to allow for the relaxation of gasket compression. Typical gasket designs are illustrated in

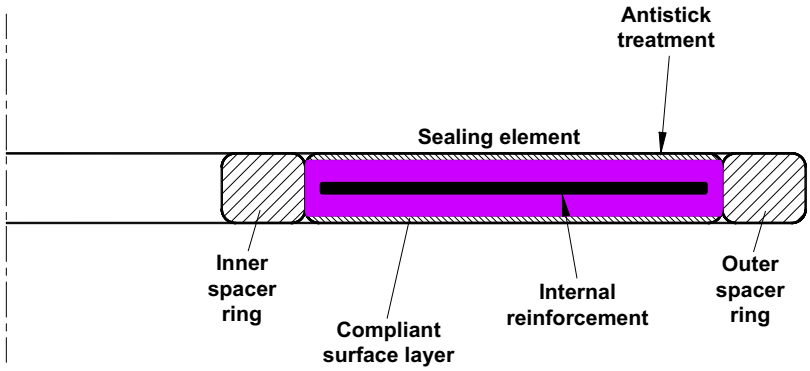


Figure 14.10
The constituent parts of a typical gasket.

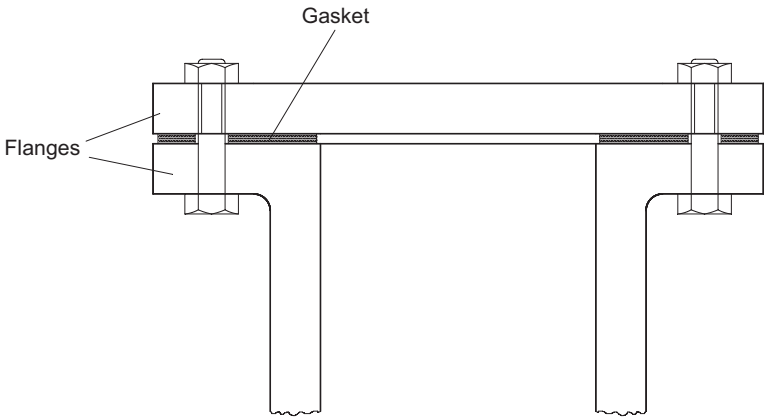


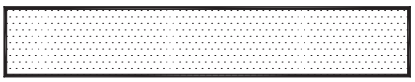
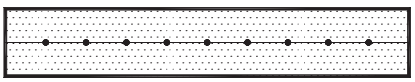
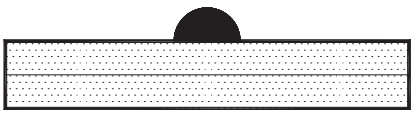
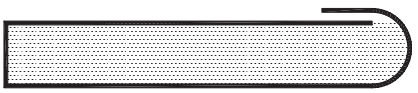
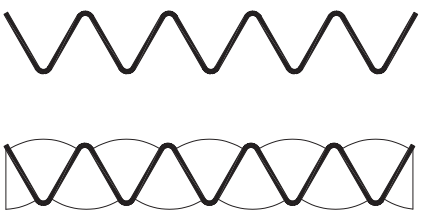
Figure 14.11
Typical gasket application.

Table 14.3. The choice of material depends on the temperature of operation, the type of fluid being contained and the leakage rate that can be tolerated. The order of magnitude of leakage for a range of materials is illustrated in Figure 14.12. Gasket selection is considered in further detail by Winter (1998).

14.2.3 Foodstuffs Containers

The sealing of foodstuffs amounts for a significant proportion of seal designs. Sealing of foodstuffs containers involves consideration of leakage of the contents, sealing against biological contamination by for instance bacteria and chemical odors. The typical diameter of bacteria is of the order of 1 μm; the challenge in designing foodstuffs containers is to exclude bacteria for the shelf life of the product, which may be a number of years. For

Table 14.3: Typical gasket designs.

Type	Cross section	Comment
Flat		Available in a wide variety of materials; easily formed into other shapes
Reinforced		Fabric or metal reinforced; improves torque retention and blowout resistance in comparison with flat types
Flat with rubber beads		Rubber beads located on flat or reinforced material; gives high unit sealing pressure
Flat with metal grommet		The metal grommet gives protection to the base material
Plain metal jacket		The metal jacket gives protection to the filler on one edge and across the surface
Corrugated or embossed		Corrugations provide increased sealing pressure capability
Profile		Multiple sealing surfaces
Spiral wound		Interleaving pattern of metal and filler

Source: Reproduced with alterations from Czernik (1996).

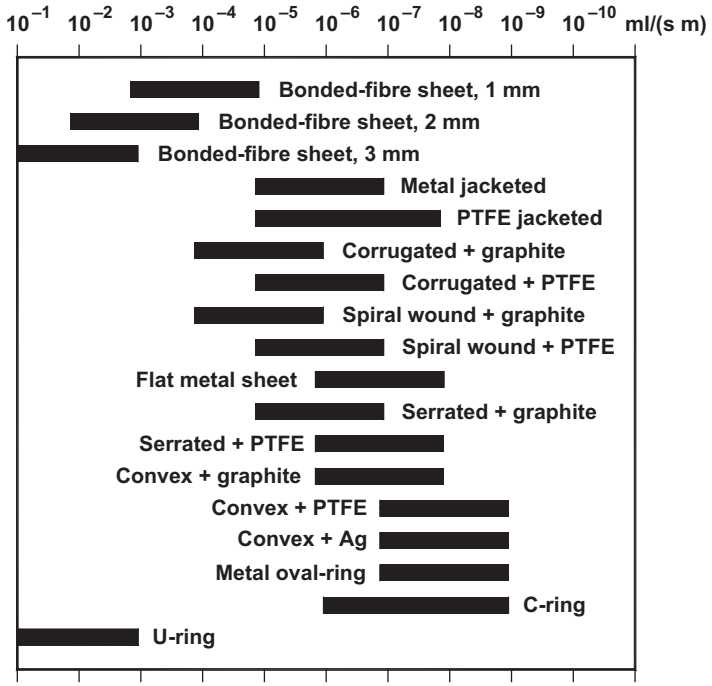


Figure 14.12
Typical order of magnitude of gasket leakage rate per unit diameter for a range of gasket materials. *Figure reproduced from Muller and Nau (1998).*

many designs, the joint must be reusable, as with carbonated drink and sauce bottles. A variety of typical joints for food containers are illustrated in Figure 14.13.

14.3 Dynamic Seals

The term "dynamic seal" is used to designate a device used to limit the flow of fluid between surfaces that move relative to each other. The range of dynamic seals is extensive, with devices for both rotary and reciprocating motion. The requirements of dynamic seals are often conflicting and require compromise. Effective sealing may require high contact pressure between a stationary component and a rotating component, but minimal wear is also desired for long seal life.

14.3.1 Seals for Rotating Machinery

The functions of seals on rotating shafts include retaining working fluids, retaining lubricants, and excluding contaminants such as dirt and dust. The selection of seal type depends on the shaft speed, working pressure, and desired sealing effectiveness. Seals for rotary

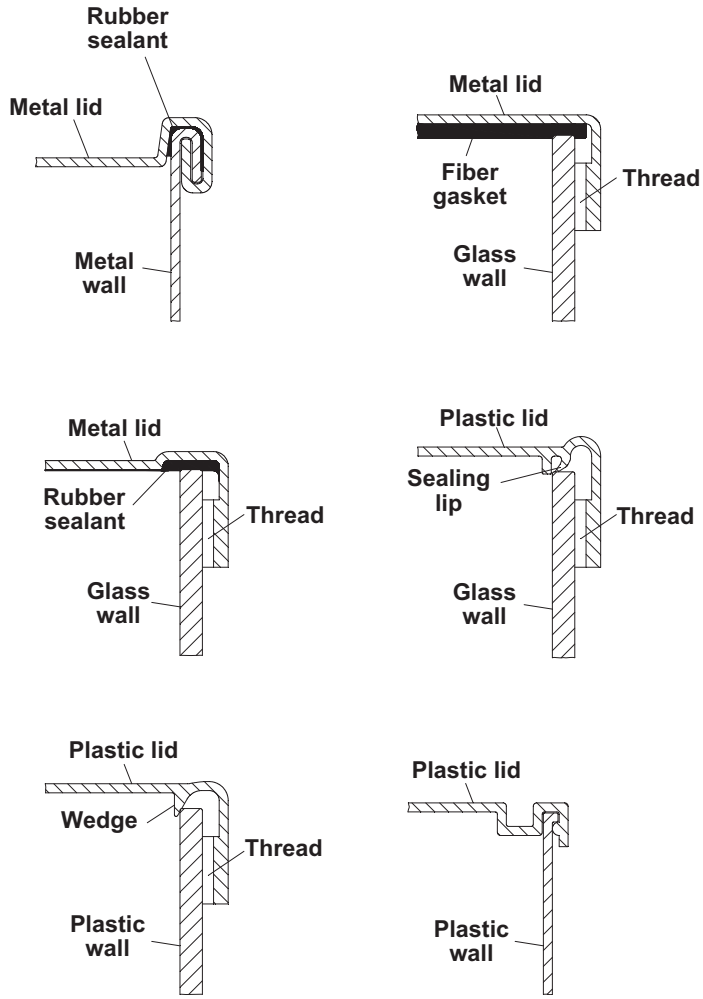


Figure 14.13
Seals for food containers. *Figure reproduced from Muller and Nau (1998).*

motion include O rings, lip seals, face seals, sealing rings, compression packings and noncontacting seals such as bush and labyrinth seals.

O rings were covered in Section 14.2.1. Their application to rotating shafts is generally limited to use when the shaft speed is below 3.8 m/s and seal pressures are below 14 bar.

The typical geometry for a radial lip seal, commonly known as an oil ring, is shown in Figure 14.14. These seals are used to retain lubricants and exclude dirt and are well suited to moderate speed and low pressure applications. The outer case should be retained in the housing by an interference fit. The purpose of the garter spring is to maintain a uniform radial force on the shaft, ensuring contact between the elastomeric sealing ring and the rotating

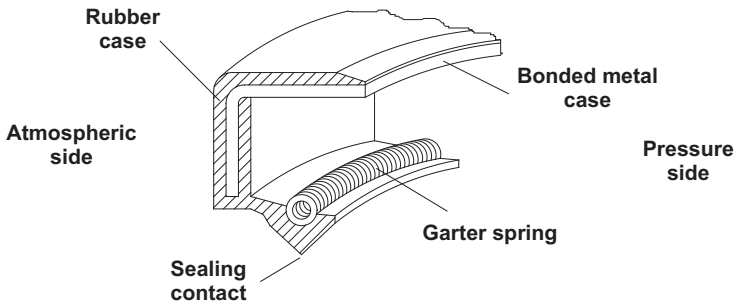


Figure 14.14
Radial lip seal.

shaft. The design of radial lip seals is detailed by Horve (1996) and in the SAE fluid sealing handbook.

For applications where low leakage, high reliability, and low wear are important, mechanical face seals may be appropriate. A mechanical face seal consists of two sealing rings, one attached to the rotating member and one attached to the stationary component to form a sealing surface, usually perpendicular to the shaft axis as illustrated in Figure 14.15. During

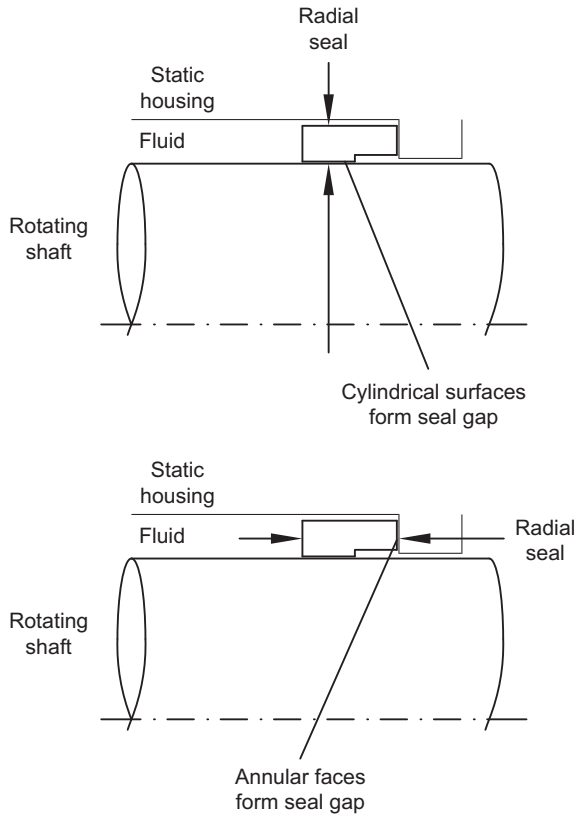


Figure 14.15
Operating principle of mechanical face seals.

rotation the primary sealing ring attached to the shaft rubs with its seal face against the counterseal face of the stationary ring. The two interface contact areas function like bearings and are subject to frictional wear. Any leakage flow must pass across this interface.

The overall assembly for a typical mechanical face seal is illustrated in Figure 14.16. Frictional rubbing contact between the sealing faces is maintained by forces acting axially caused by hydraulics or mechanically using, for example, a spring as shown in Figure 14.16. The rubbing action between the surfaces produces heat and wear. In order to minimize this, lubrication is used, which, as well as limiting wear and heat buildup, also serves to generate a fluid film, which assists in producing a tight seal. The range of face seal designs is extensive, and they are widely used in pumps, compressors, gearboxes, and domestic washing machines. The design of face seals is reviewed by Dolan (1992), Summers-Smith (1992), Muller and Nau (1998), and Lebeck (1991).

The term interstitial seal is used for seals that allow unrestricted relative motion between the stationary and moving components (i.e. no seal to shaft contact). Types include labyrinth, brush, and bush seals. Labyrinth seals and bush seals are described in Sections 14.4 and 14.5, respectively.

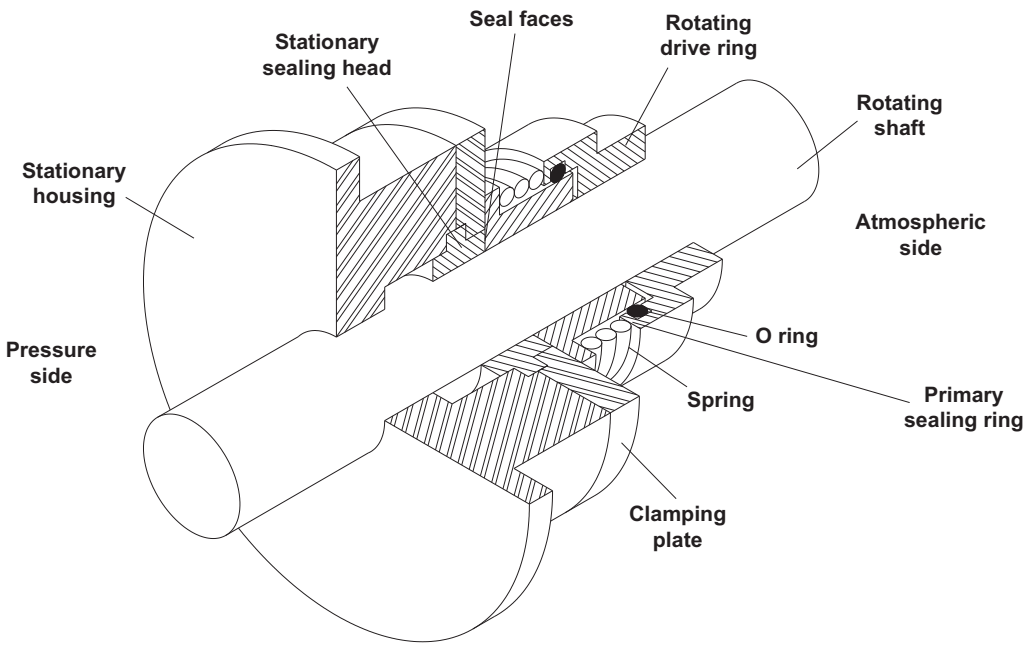


Figure 14.16
A typical mechanical face seal.

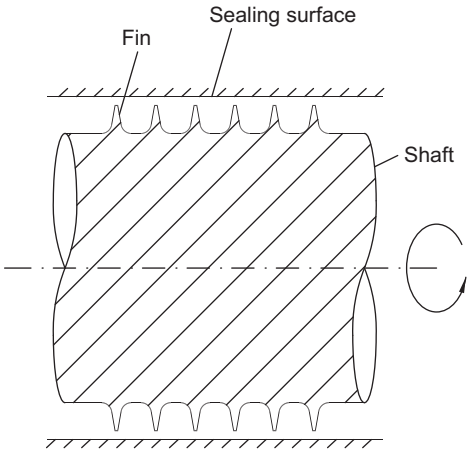


Figure 14.17
Typical straight labyrinth seal.

14.4 Labyrinth Seals

In its simplest form, a labyrinth seal consists of a series of fins and corresponding chambers, forming a restriction to the flow and a volume for expansion, respectively, as illustrated in Figure 14.17. Labyrinth seals work by throttling the flow through successive openings in series. In each throttle, static pressure difference accelerates the flow, and some of the kinetic energy associated with the flow is dissipated by turbulence induced by the intense shear stress and eddy motion in the next chamber, as illustrated in Figure 14.18. The use of a tortuous path between high and low static pressure regions, incorporating a series of

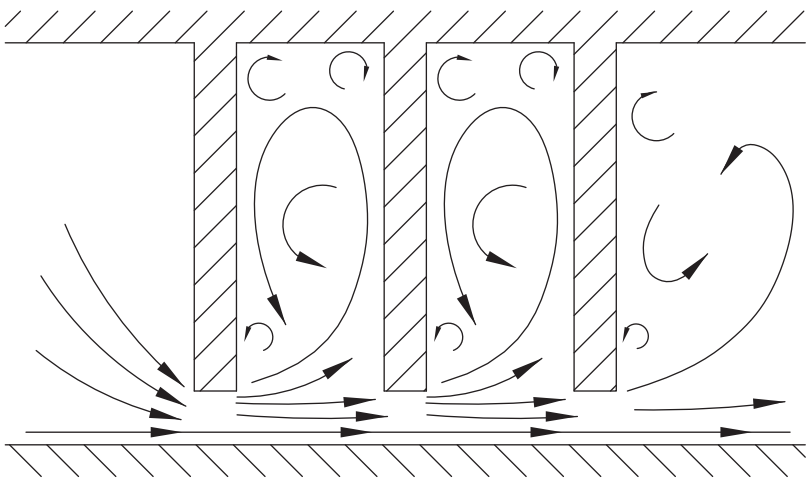


Figure 14.18
Characteristic flow through a labyrinth seal. *After Tipton et al. (1986).*

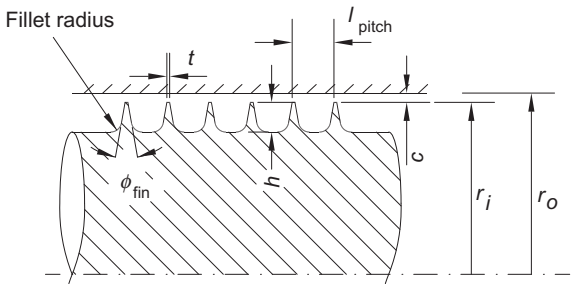


Figure 14.19
Principal labyrinth seal geometric parameters.

noncontacting restrictions and chambers in the form of a labyrinth seal, was implemented by Parsons for his steam turbine concept in 1892 (see Parsons, 1892; The Engineer, 1938). Labyrinth seals have a long history of proven reliability in a wide range of applications, including gas turbine engines (Chupp et al., 2006), with robust operation and developed technology. Labyrinth seals are considered in detail in ESDU 09008, upon which this section is based, Sneck (1974) and Trutnovsky and Komotori (1981).

The principal geometric parameters of a typical straight labyrinth seal are shown in Figures 14.19 and 14.20. The terms fin, knife, tooth, and blade are used interchangeably in industry and the technical literature. Here the term fin is principally used.

Common forms of labyrinth seal include the following:

- straight through;
- stepped; and
- staggered.

Each of these forms of labyrinth seal can be used in axial rotating flow applications, where the axis of the fins is coincident with the axis of the shaft, or in radial sealing applications, where the fins are perpendicular to the shaft axis. Schematic diagrams for each of these are illustrated in Figure 14.21. Fins can rotate, move in a reciprocating motion, or be stationary. In general, for the case of an axial application, it is desirable to have the fins on the inner member, as this improves the stability of the system and heat pickup.

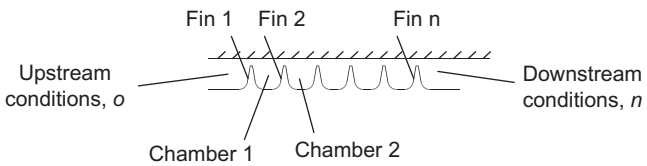


Figure 14.20
Labyrinth fin and chamber notation.

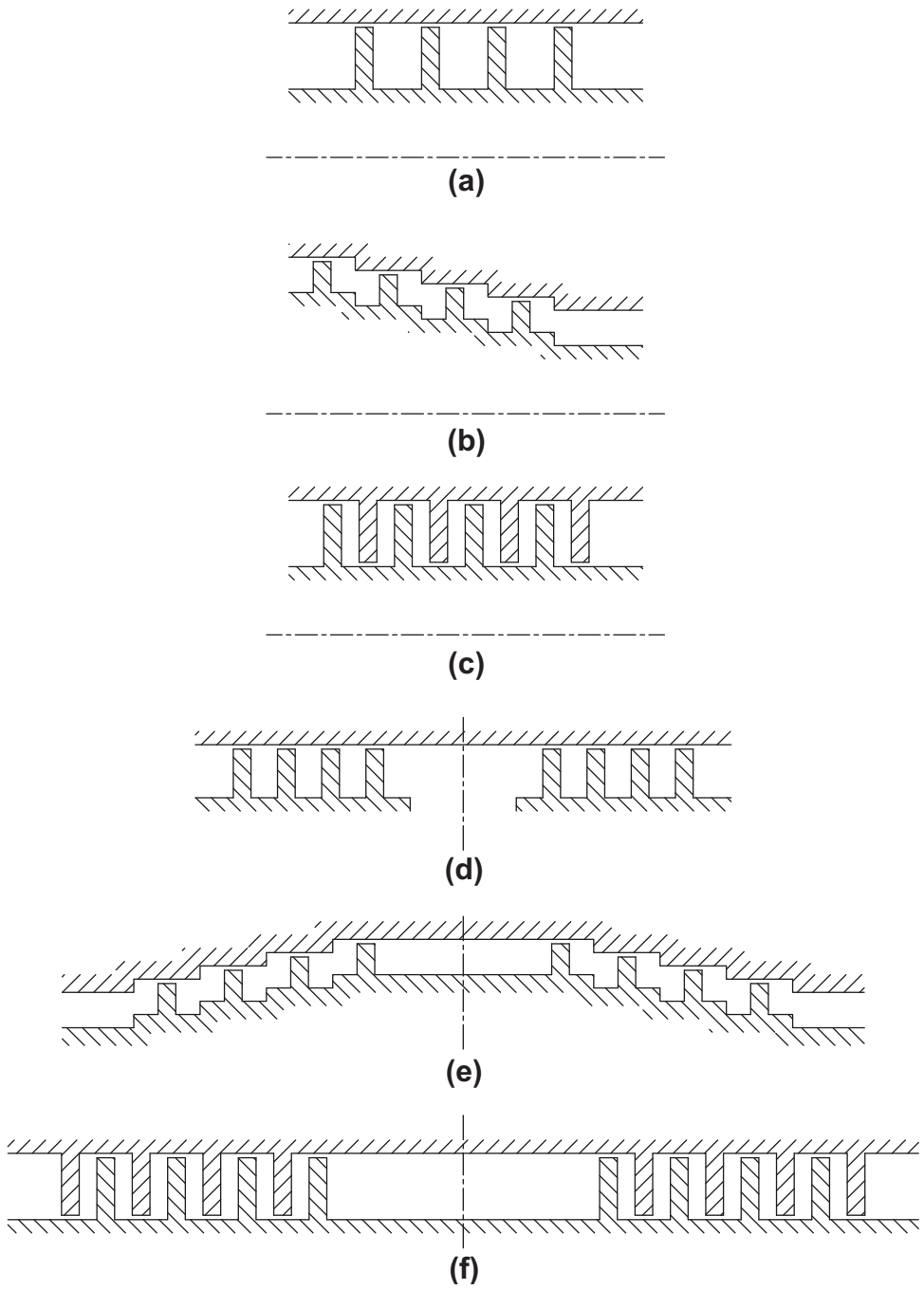


Figure 14.21
Schematic diagrams illustrating a selection of labyrinth seals. Axial applications: (a) straight through, (b) stepped, and (c) staggered. Radial applications: (d) straight through, (e) stepped, and (f) staggered.

Labyrinth seals do not provide a complete seal; if there is a static pressure difference across the seal, there will normally be a net flow of fluid across the seal. A typical application for a labyrinth seal for limiting the leakage of flow in a centrifugal pump is illustrated in Figure 14.22. In this application, a straight-through labyrinth seal is used for limiting the leakage flow on the inlet side of the impeller shaft, and a radial labyrinth is used on the rear face of the impeller.

The design of a straight-through labyrinth seal in most practical applications typically involves a compromise between the number of fins and a pitch that is large enough to ensure that the kinetic energy of the flow is dissipated in the cavity (chamber). There are a significant number of performance and design parameters to consider (see Figure 14.19 for the geometric parameters):

- clearance, c;
- pitch, l_{pitch};
- the number of fins, n;
- fin-tip land width, t;
- rotational speed, Ω;
- cavity volume;
- fin angle, ϕ_{fin};

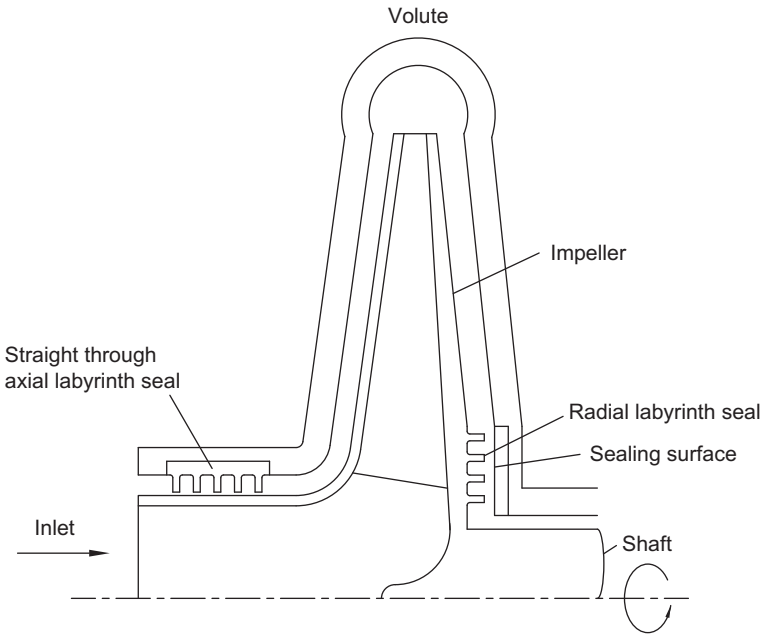


Figure 14.22
Application of labyrinth seals in a pump with a straight-through seal at the pump inlet and a radial labyrinth seal on the pump rear face.

- land surface roughness and porosity;
- fin height, h;
- eccentricity;
- pressure ratio; and
- axial Reynolds number, Re.

For a stepped seal additional parameters also include:

- step height, H;
- step configuration;
- distance from seal fin to step face;
- flow direction.

The principal fluid dynamic parameters that determine labyrinth seal performance and their effects are listed in Table 14.4. The corresponding geometric parameters are given in Table 14.5. In terms of modeling, the parameters identified do not operate independently; therefore, a significantly large matrix of parameter combinations affects labyrinth seal performance.

A particular challenge in determining labyrinth seal performance parameters arises with in-service and worn seals. In-service and worn seals can involve issues with the following:

- erosion of coatings on either the fin land or on the sealing surface;
- grooves in abradable coatings caused by wear;
- uneven fin wear;
- fin rounding;
- manufacturing errors;
- thermal distortions; and
- axial movements.

Table 14.4: Principal fluid dynamic parameters for labyrinth seals.

Parameter	Definition	Characteristic effect
Mass flow rate function	$Q = \dfrac{m\sqrt{T_{t0}}}{Ap_{t0}}$	Dependent variable
Pressure ratio	p_{t0}/p_n	Strong
Axial Reynolds number	$\mathrm{Re} = \dfrac{(m/A)c}{\mu}$	Moderate
Fin-tip speed	u_{ϕ,r_i}	Moderate
Rotational Reynolds number	$\mathrm{Re}_\phi = \dfrac{\rho\Omega rc}{\mu}$	Weak
Taylor number	$\mathrm{Ta} = \dfrac{\rho u_{\phi,r_i} c}{\mu}\sqrt{\dfrac{c}{r_i}}$	Weak

Source: After Tipton et al. (1986).

Table 14.5: Principal geometric parameters for labyrinth seals.

Parameter	Affected parameter	Characteristic influence on			
		Straight seals	Stepped seals	Staggered seals	Radial seals
Number of fins, n	Number of throttles	Strong	Strong	Strong	Strong
Fin angle	Orifice geometry	Moderate	Moderate	Moderate	Moderate
Fin-tip land width, t	Relative throat length	Moderate	Moderate	Moderate	Moderate
Clearance, c	Fin relative sharpness	Moderate	Moderate	Moderate	Moderate
Fin shape	Orifice geometry	Weak to moderate	Weak	Weak	Weak to moderate
Pitch, l_{pitch}	Relative throttle spacing	Moderate	Weak	Moderate	Moderate
Fin height, h	Relative chamber depth	Weak	Weak	Weak	Weak
Land surface roughness	Land relative roughness	Moderate	Weak	Moderate	Moderate
Land surface porosity	Land relative porosity	Moderate	Weak to moderate	Moderate	Moderate
Step height	Relative step height	—	Weak	—	—
Distance to contact	Rotor relative axial location	—	Weak to moderate	—	—
Flow direction	Flow-down stator step/flow-up stator step	—	Weak	—	—

Source: Adapted from Tipton et al. (1986).

Care must be taken in the design of labyrinth seals. If the cavity following a radial fin is too small, the flow will pass straight through without expanding and without the subsequent pressure drop. In any case despite careful labyrinth design and specification of labyrinth seal parameters, flow will inevitably be carried straight over from one fin to another. In order to reduce this effect, steps can be incorporated into a labyrinth seal design, and a wide range of labyrinth variants are possible.

Methods available for quantifying the leakage through a labyrinth seal include empirical, analytical, and numerical computational fluid dynamics (CFD) techniques. Quantification of fluid flow rates in various applications is fraught with difficulty, and labyrinth seals are no exception.

A straight labyrinth seal consists of a series of radial fins forming a restriction to an annular flow of fluid, as illustrated in Figures 14.17 and 14.19. In order for the fluid to pass through the annular restriction it must accelerate and, in this process, part of the energy associated with the static pressure of the fluid entering the restriction is converted into kinetic energy. Downstream of the restriction, the fluid jet decelerates and expands with the formation of separation eddies in the cavity downstream of the fin, as indicated in Figure 14.18. These eddies dissipate some of the energy of the flow by small scale turbulence viscosity action, reducing the total pressure. This process is repeated in subsequent cavities until the flow finally exhausts through the last restriction and the pressure reaches downstream conditions.

Under design operation, labyrinth seals are essentially a controlled-clearance seal without rubbing contact. As there is no surface-to-surface contact, very high relative speeds are possible, and the geometry can be arranged to limit leakage to tolerable levels. Leakage is, however, inevitable and can be quantified using the empirical methods, based on discharge coefficients and carryover correction factors or using CFD. Typically, the leakage through a labyrinth is modeled as an adiabatic throttling process. The neglect of real gas and heat transfer effects is generally of secondary importance in practical applications and an acceptable assumption. The ideal throttling assumption has led to two principal types of flow model:

1. Modeling the labyrinth as a series of discrete restrictions with associated local pressure losses;
2. Modeling the seal as a rough pipe with uniformly distributed wall friction with summation of the frictional losses of the fluid as it passes through the geometric configuration of the seal. The losses due to friction are typically described in terms of a friction factor that represents the seal's resistance to leakage.

In order to avoid damage to the components of the labyrinth, typically the fins and the adjacent seal surface, as a result of contact between the rotating and stationary components,

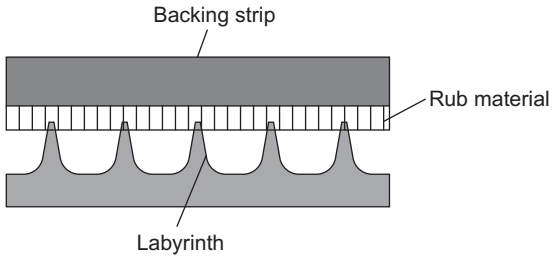


Figure 14.23
Labyrinth seal with abradable rub-in surface.

a minimum clearance needs to be maintained. Various configurations for the seal throttlings and cavities that attempt to reduce the kinetic-energy carryover effect have been developed, with similar benefits to reducing the clearance. These include step-up and step-down configurations, configurations that use inclined fins, staggered or interlocking configurations where successive labyrinth fins project from the rotating and stationary surfaces, respectively, and configurations that use abradable surfaces, see Figure 14.23.

An abradable surface enables tighter clearances to be maintained and is typically used on the stationary component rather than the fin tips. Any wear of the abradable surface can be mitigated against by replacement at service intervals in order to maintain a specific design clearance. One type of surface that is used regularly in labyrinth seals is the open-celled honeycomb, mounted with cell openings facing the labyrinth fins, Figure 14.24. The honeycomb surface has an enhanced frictional effect that reduces the kinetic-energy carryover effect of the seal. Some honeycomb seal surfaces are capable of running at high temperatures and have relatively stiff structural properties. Honeycombs, however, contribute to windage heating in labyrinth seals; this needs to be accounted for in a given application.

In order to minimize the leakage flow in a straight-through labyrinth seal, the radial clearance should be made as small as possible. However, in selecting and specifying the clearance,

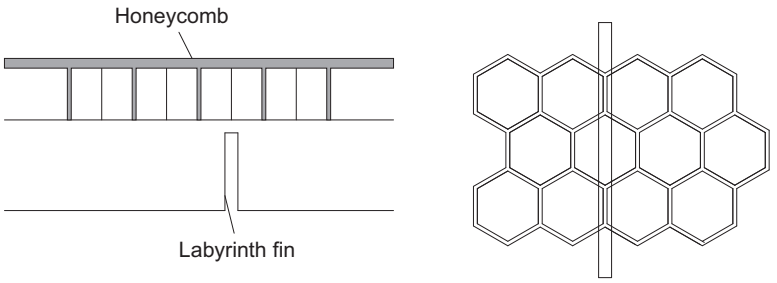


Figure 14.24
Open-celled honeycomb mounted with cell openings facing a labyrinth fin.

allowances must be made for thermal expansion, growth due to centripetal acceleration, and shaft misalignment. If allowance for these factors is not made, there is a risk of fin contact with the sealing surface with the potential for accompanying damage to the components. Abradable surfaces can be used in order to accommodate the possibility of contact between the seal fins and the seal surface. The clearance between the seal fin and abradable surface may be set such that contact between the two is deliberately caused, with the fins "cutting" a running clearance. Such an approach, provided any debris from this and the power required to overcome the frictional contact in the running-in period can be accommodated, can be used to provide a seal with the smallest practical running clearance for the conditions imposed.

The following general guidelines are noted by Morrison et al. (1983):

- Normally, a narrow fin width is preferable to wider fins;
- The fin width should always be less than one half of the value of the pitch, especially for applications where the angular velocity is less than circa 50 rad/s (about 500 rpm); and
- The fin height should generally be at least twice the value of the radial clearance. Measurements from Yamada (1962) showed that the flow coefficient is relatively constant for fin height to clearance ratios between 2 and 11.

Data from Stocker (1976) and Stocker et al. (1977) for stepped labyrinth seals indicated that the optimum fin angle for minimum leakage was between 50° and 70°, and that the optimum pitch for minimum leakage tended to decrease as the fin angle increased. Stocker's data also indicated that the leakage decreased as the surface roughness was increased up to a certain amount and then began to increase as the roughness was increased further.

Designing a seal to minimize the leakage of flow for a specific seal requires an iterative optimization technique. For a seal of fixed overall length, there is a tradeoff between selecting the pitch and the number of throttles. As the number of throttles increases, the flow coefficient decreases with accompanying reduction in the mass flow, but as the number of throttles increases, the pitch decreases, causing the carryover factor to increase. The increase in carryover factor causes an increase in the leakage, offsetting some or even all of the decrease in leakage achieved by decreasing the flow coefficient.

In addition to consideration of the geometric parameters, i.e. use of abradable materials and quantification of the seal leakage, the operating temperature of the seal needs to be assessed and the contributions to windage heating of the seal flow due to viscous dissipation of energy as a result of rotation of the seal's surfaces need to be quantified.

A further consideration to take into account in the design of straight-through labyrinth seals is rotordynamic instability. Fluid flow through the cavities of a seal can create net pressure and shear forces acting on the rotor. These forces may contribute to the destabilization of marginally stable rotors rotating at high speeds. A typical analysis for labyrinth seals involves

determining the rotordynamic force coefficients. These coefficients distinguish position- and speed-dependent force components in two directions, thereby describing the stiffness and damping effects of the fluid. In practical applications, the tendency of labyrinth seals to produce flow instabilities can be controlled by swirl brakes or intracavity slots or blocks and drum dampers. A review on the rotordynamic instability issues is given in Childs (1993); specific applications are considered in Childs and Rhode (1984) and Picardo and Childs (2005).

A guideline for the initial specification of geometric parameters for a labyrinth seal is given in Table 14.6, including typical values given in the open literature. The design principles are applicable to the majority of types of labyrinth seal.

Labyrinth seals have been the subject of extensive study, and a number of empirical and analytical models have been developed. A selection of these are summarized in Table 14.7.

Martin's equation, which has been widely used for determining the mass flow in a labyrinth seal, is discussed in Section 14.4.1. The use of compressible flow functions for modeling

Table 14.6: Guidelines for the initial specification of straight labyrinth seal geometric parameters.

Geometric parameters	Guidelines	Typical values
Clearance, c	Minimize subject to practical constraints of manufacture and machine movements	$0.2\ \text{mm} < c < 2\ \text{mm}$
Pitch, l_{pitch}	For a seal of fixed overall length, there is a tradeoff between selecting the pitch and the number of throttles	$2 < l_{\text{pitch}} < 10\ \text{mm}$
Number of fins, n	For a seal of fixed overall length, there is a tradeoff between pitch and number of throttles; as the number of throttles increases, the flow coefficient decreases with accompanying reduction in the mass flow but as the number of throttles increases, the pitch decreases, causing the carryover factor to increase and therefore raising the mass flow	Tradeoff with pitch, l_{pitch}
Fin width, t	Normally, a narrow fin width is preferable to wider fins The fin width should always be less than one half of the value of the pitch: $t < 0.5 l_{\text{pitch}}$	$0.1 < t < 0.4$
Fin height, h	The fin height should generally be at least twice the value of the radial clearance: $h > 2c$	$2 < h < 8$
Fin angle, ϕ_{fin}	—	$7 \leq \phi_{\text{fin}} \leq 40^\circ$
Fillet radius, r_{fillet}	—	$0.3 \leq r_{\text{fillet}} \leq 1.2\ \text{mm}$
Land roughness	—	$8.3\ \mu\text{m}$
Eccentricity	Minimize	

Table 14.7: Selected correlations for determining the mass flow for straight labyrinth seals.

Reference	Equation	Flow type	Application
Martin (1908, 1919)	$m = Ap_{t0}\sqrt{\dfrac{1-(p_n/p_{t0})^2}{RT_{t0}[n-\ln(p_n/p_{t0})]}}$	Compressible flow	Straight-through seals
Egli (1935)	$m = \alpha\varepsilon A\sqrt{\dfrac{p_{t0}\rho_0[1-(p_n/p_{t0})^2]}{n+\ln(p_{t0}/p_n)}}$ Where α is a flow coefficient and ε is a carryover correction factor.	Compressible flow	Straight-through and staggered seals
Hodkinson (1940)	$m = \alpha k_1 A\sqrt{\dfrac{p_{t0}\rho_0[1-(p_n/p_{t0})^2]}{n}}$ $k_1 = \left\{1-\dfrac{(n-1)(c/l_{\text{pitch}})}{n[(c/l_{\text{pitch}})+0.02]}\right\}^{-0.5}$ Where α is a flow coefficient.	Compressible flow	Straight-through seals
Vermes (1961)	$m = 5.76C_d kA\sqrt{\dfrac{p_{t0}\rho_0[1-(p_n/p_{t0})^2]}{n+\ln(p_{t0}/p_n)}}$ $k = \dfrac{8.52}{[(l_{\text{pitch}}-t)/c]+7.23}$	Compressible flow	Straight-through and stepped seals
Komotori and Mori (1971)	$m = \alpha\varepsilon_1 AF(n, p_{t0}/p_n, \gamma)\sqrt{\rho_0 p_0}$ Where α is a flow coefficient, ε_1 is a carryover factor and $F(n, p_{t0}/p_n, \gamma)$ is defined in the paper.	Compressible flow	Straight-through and radial seals

Continued

Table 14.7: Selected correlations for determining the mass flow for straight labyrinth seals.—cont'd

Reference	Equation	Flow type	Application
Rao and Sidheswar (1976)	$m = \alpha A\sqrt{\frac{p_{t0}\rho_0[1-(p_n/p_{t0})^2]}{n}}$ Where α is a flow coefficient.	Compressible flow	Straight-through seals
Han (1979)	$\Delta p = \frac{[(p_{t0}-p_n)-1.5(\rho u_{\text{mean}}^2/2)]}{n}$ $\text{Re} = \frac{\rho u c}{\mu};\ \text{Re}_1 = \frac{\rho(l_{\text{pitch}}-t)u_{\text{mean}}F(\text{geometry})}{\mu}$ Where Δp is the average driving pressure drop of each throttling and *F(*geometry*)* from paper.	Incompressible flow	Straight-through seals
Yamada (1962)	$f = f(\text{Re}, \text{Re}_\phi)$ from paper. $\text{Re} = \frac{\rho u c}{\mu};\ \text{Re}_\phi = \frac{\rho\Omega r c}{\mu}$	Incompressible flow	Straight-through seals
Dodge (1963)	$m = A\sqrt{\frac{2\rho(p_{t0}-p_n)}{fL/[2c+1.5+k(n-1)]}}$ $k = 1$ straight-through seals; $k = 2$ staggered seals. $f = 64/\text{Re}$ for $\text{Re} < 2320$ $f = 0.316/\text{Re}^{0.25}$ for $\text{Re} > 2320$ L is the total length of the seal.	Incompressible flow	Straight-through and staggered seals
Zimmermann and Wolff (1998)	$m = k_2 C_d A p_{t0}\sqrt{\frac{1-(p_n/p_{t0})^2}{RT_{t0}[n+\ln(p_{t0}/p_n)]}}$	Compressible flow	Straight-through and stepped seals

a labyrinth seal by a series of orifices is introduced in Section 14.4.2. The model recommended, in this item, due to its simplicity, accuracy, and versatility in accommodating practical parameters for straight labyrinth seals, the Zimmermann–Wolff model, is described in Section 14.4.3.

Numerical solution of the Navier–Stokes equations using CFD can provide valuable insight into detailed labyrinth seal flow characteristics and quantification of global quantities such as mass flow rate. The value of a CFD solution is that it can analyze the effects of most geometric and fluid dynamic parameters. In addition to the correlation-based approaches indicated in Table 14.7 and CFD models, various application specific software codes have been produced for modeling labyrinth seals. These include the knife-to-knife (KTK) and Labflow codes produced by the former Allison Gas Turbine Division (now Rolls-Royce) and the Rotating Machinery and Controls group, respectively. The KTK model is based on Tipton et al. (1986), Chupp et al. (2006) and Shapiro and Chupp (2004). The KTK provides modeling capability for both straight and stepped labyrinth seals.

14.4.1 Martin's Equation

An analytical approach based on a simplified gas dynamics approach can be applied to a labyrinth seal that models the leakage in the seal as flow of an ideal gas through a series of orifices and was originally developed by Martin (1908, 1919). Martin assumed that all of the kinetic energy in each cavity or throttling chamber was completely dissipated before the flow passed into the next cavity and that the flow was isothermal. The resulting equation following this analysis is presented in Eqn (14.1) and is generally known as Martin's equation:

$$m = Ap_{t0}\sqrt{\frac{1-(p_n/p_{t0})^2}{RT_{t0}[n-\ln(p_n/p_{t0})]}} \tag{14.1}$$

where m is the mass flow rate, A is the annular clearance area of the seal, p_{t0} is the upstream total pressure, p_n is the downstream static pressure, R is the characteristic gas constant, T_{t0} is the upstream total temperature, and n is the number of labyrinth fins.

The mass flow function, Q, provides a convenient means for comparing the relative performance of different seals. This is defined by

$$Q = \frac{m\sqrt{T_{t0}}}{Ap_{t0}} \tag{14.2}$$

Substitution for the mass flow function in Eqn (14.1) gives

$$Q = \sqrt{\frac{1 - (p_n/p_{t0})^2}{R[n - \ln(p_n/p_{t0})]}} \tag{14.3}$$

The flow characteristics for a straight labyrinth for compressible flow, using Eqn (14.3), are illustrated in Figure 14.25 for a range of numbers of fins, where the mass flow function, given in Eqn (14.2), is plotted against the pressure ratio, p_{t0}/p_n.

In real practical conditions, labyrinth seal flow deviates from the assumptions made in the development of Eqn (14.1). For example, typically, only a fraction of a dynamic head is lost across an individual fin. The effects of contractions and coatings also need to be taken into account. These factors can be modeled by use of a carryover factor and a discharge coefficient. As a result of such factors the Martin equation tends to overestimate the mass flow in a labyrinth seal. Zimmermann and Wolff (1998) demonstrated that this can be by approximately 35% for a typical gas turbine engine labyrinth.

In modeling the flow through a labyrinth seal it is often assumed that all of the dynamic pressure is dissipated as the flow expands into a cavity downstream of a fin. A carryover factor is used to account for the practical condition where not all of the dynamic pressure is dissipated and some kinetic energy is transferred to the following constriction.

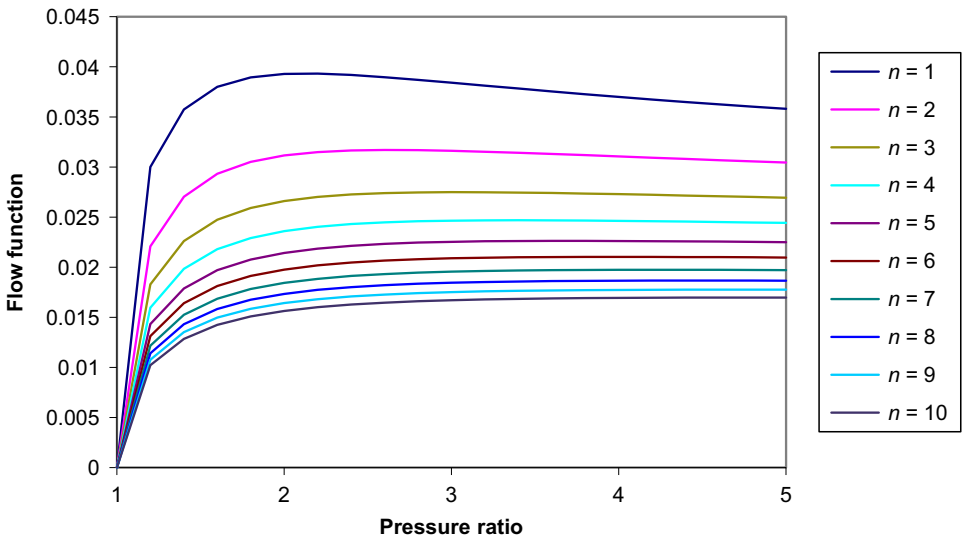


Figure 14.25
Labyrinth mass flow function modeled using Eqn (14.3) with $R = 287$ J/kg K.

The discharge coefficient, C_d, represents the ratio of the actual and ideal mass flow rate through the seal clearance:

$$C_d = \frac{m_{\text{actual}}}{m_{\text{ideal}}} \tag{14.4}$$

Taking practical factors into account using a carryover factor and a discharge coefficient, the mass flow in a labyrinth seal can be estimated by

$$m = kC_d A p_{t0} \sqrt{\frac{1 - (p_n/p_{t0})^2}{RT_{t0}[n - \ln(p_n/p_{t0})]}} \tag{14.5}$$

where m is the mass flow rate through the seal clearance, k is the carryover factor, and C_d is the discharge coefficient.

14.4.2 Orifice Flow Approach

A real labyrinth seal can be modeled as a series of successive nozzles and chambers. If the nozzles are small and the chambers large, it can be assumed that there is little or no carryover of kinetic energy from one nozzle to the next. The flow through each nozzle can be modeled by an adiabatic polytropic expansion, and a discharge coefficient can be applied. In each cavity the flow is modeled as isentropic and the kinetic energy of the flow is recovered. Following these assumptions, use can be made of the standard compressible flow relationships for each nozzle.

The flow function in terms of the static pressure either side of a single orifice is given by

$$Q = C_{d,i} \sqrt{\frac{2\gamma_{i-1}}{R(\gamma_{i-1} - 1)} \left[\left(\frac{p_i}{p_{i-1}} \right)^{2/\gamma_{i-1}} - \left(\frac{p_i}{p_{i-1}} \right)^{(\gamma_{i-1}+1)/\gamma_{i-1}} \right]} \tag{14.6}$$

where $C_{d,i}$ is the discharge coefficient for the ith chamber.

When the pressure ratio of the last stage, p_n/p_{n-1}, is less than the critical pressure ratio, the flow will be choked and the Mach number, Ma, through the last restriction will be unity. The critical pressure ratio, $\lambda_c = \left(\frac{p_n}{p_{n-1}}\right)_{\text{critical}}$, is given by

$$\lambda_c = \left(\frac{p_n}{p_{n-1}} \right)_{\text{critical}} = \left(\frac{2}{\gamma_{n-1} + 1} \right)^{\gamma_{n-1}/(\gamma_{n-1}-1)} \tag{14.7}$$

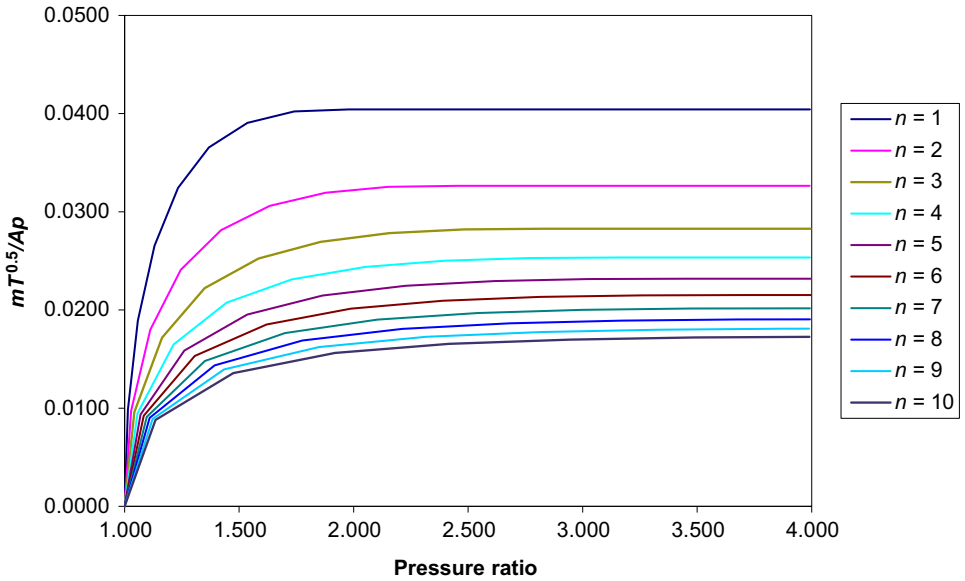


Figure 14.26

Flow function versus pressure ratio for an isentropic throttle-based model based on Eqns (14.6)–(14.8), with $C_d = 1$.

and the flow function becomes

$$Q = C_{d,n}\sqrt{\frac{2\gamma_{n-1}}{R(\gamma_{n-1}-1)}\left[(\lambda_c)^{2/\gamma_{n-1}} - (\lambda_c)^{(\gamma_{n-1}+1)/\gamma_{n-1}}\right]} \tag{14.8}$$

Such a scheme has been applied to a labyrinth seal; the results are illustrated in Figure 14.26, where $C_{d,1} = C_{d,2}$ $C_{d,i} = C_{d,n} = 1$ is assumed.

14.4.3 Zimmermann and Wolff's Model

As indicated by the preceding discussion and the methods presented in Table 14.7, there have been a large number of modeling approaches proposed for labyrinth seals. Certain of these models have been validated for a specific application, such as gas turbine engines or steam turbines. For practical applications where an estimate for flow leakage in a labyrinth is required and accurate experimental data are not available, the model proposed by Zimmermann and Wolff (1998) for straight seals, Eqn (14.9), is suggested, as the model has been validated for a wide range of turbomachinery applications and is comparatively simple to use:

$$m = k_2 C_d A p_{t0}\sqrt{\frac{1-(p_n/p_{t0})^2}{RT_{t0}[n-\ln(p_n/p_{t0})]}} \tag{14.9}$$

In this equation k_2 is the carryover factor and is defined in Eqn 14.11.

The model accounts for a wide range of practical situations encountered in labyrinth seal applications, including variation of the following:

- carryover factor with pitch and clearance;
- discharge coefficient with tip width and clearance;
- grooves;
- honeycombs;
- corner radius; and
- chamber depth.

In an ideal straight labyrinth, one unit of dynamic pressure is dissipated in each of the fin seal cavities. This does not occur in practice, and only a fraction of dynamic pressure is lost downstream of each fin. The carryover factor is used to account for some of the dynamic pressure not being dissipated as the flow expands in the seal cavity. Hence the leakage flow is greater than expected in the ideal model, and the carryover factor is greater than unity.

Measurements and modeling results have revealed that the proportion of dynamic pressure carried over is a function of the number of fins. The carryover factor can be modeled by Hodkinson (1940):

$$k_1 = \sqrt{\frac{1}{1 - \left(\frac{n-1}{n}\right)\left[\frac{c/l_{\text{pitch}}}{\left(c/l_{\text{pitch}}\right)+0.02}\right]}} \tag{14.10}$$

where k_1 is Hodkinson's carryover factor, c is the seal clearance, and l_{pitch} is the pitch of the fins.

A revised carryover factor, k_2, recommended by Zimmermann and Wolff (1998), is given in Eqn (14.11):

$$k_2 = \sqrt{\frac{n/(n-1)}{\left(\frac{n}{n-1}\right) - \left[\frac{c/l_{\text{pitch}}}{\left(c/l_{\text{pitch}}\right)+0.02}\right]}} \tag{14.11}$$

where k_2 is Zimmerman's carryover factor.

It is recommended that Zimmerman's carryover factor, k_2, in Eqn (14.11) is used to determine the labyrinth seal flow rates in Eqns 14.4 and 14.9. Values for the carryover factor, given by Eqn (14.11), are plotted in Figure 14.27.

For the case of modeling a labyrinth seal by means of a series of successive expansion chambers, Yucel and Kazakia (2001) recommended taking the carryover factor as unity for the first fin, i.e. $i = 1$, and as given in Eqn (14.12) for subsequent fins, i.e. for $i = 2$ to n.

$$k_i = 1 + 0.0179(i - 1) \tag{14.12}$$

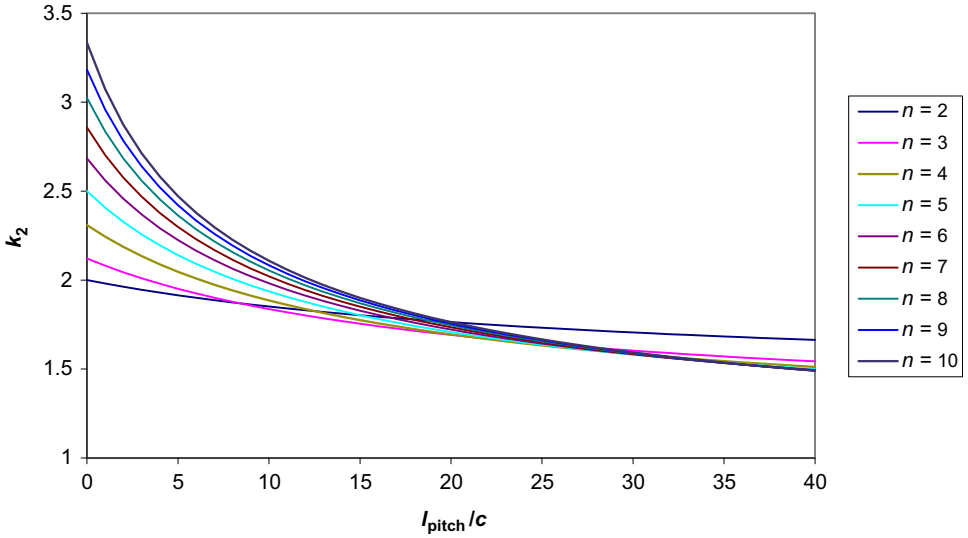


Figure 14.27
Zimmerman's carryover factor as a function of the ratio l_{pitch}/c for different numbers of fins.

The discharge coefficient is principally used to model the effects of flow contraction in a passage. Except at very low Reynolds numbers (Re less than about 10) and at conditions for which compressibility is significant, flow through a sharp-edged orifice will separate from the sharp entrance edge. This separation restricts the available flow area and the effective reduction in flow area can be modeled by means of a discharge coefficient (see ESDU 82009; ESDU TN 07007).

The flow contraction of a jet from a high pressure chamber through a planar orifice was studied by Chaplygin (see Gurevich, 1966) who derived the formula for the discharge coefficient in Eqn (14.13). This equation has been used by several investigators (e.g. see Eser and Kazakia, 1995; Yucel and Kazakia, 2001) to model the flow coefficient for a particular fin constriction. The discharge coefficient C_d depends on the pressures in the two cavities adjacent to a particular fin i and is given by

$$C_d = \frac{\pi}{\pi + 2 - 5S_i + 2S_i^2} \tag{14.13}$$

where

$$S_i = \left(\frac{p_{i-1}}{p_i}\right)^{(\gamma-1)/\gamma} - 1 \tag{14.14}$$

In order to use Eqn (14.13) in conjunction with Eqn (14.4) or (14.9) an iterative procedure must be employed to predict the leakage flow through the seals.

Plots for the discharge coefficient for a two-fin labyrinth and a labyrinth seal with more than two fins are presented in Figures 14.28 and 14.29. These are based on the data based on measurements presented by Zimmermann and Wolff (1998) attained from a proprietary database. As indicated in these figures, typical values for the discharge coefficient for multiple fin labyrinths lie in the range $0.59 < C_d < 0.63$ at high Reynolds numbers, where the Reynolds number, Re_{D_h}, is based on the seal leakage based and hydraulic diameter, D_h:

$$\mathrm{Re}_{D_h} = \frac{\rho u_{\mathrm{mean}} D_h}{\mu} \tag{14.15}$$

u_{mean} is the mean velocity across the sealing clearance and D_h is defined as

$$D_h = 2(r_o - r_i) \tag{14.16}$$

The discharge coefficient reduces with decreasing Reynolds number. For low values of Reynolds number, where the flow is dominated by friction, a friction factor would actually provide a more suitable match to the flow physics.

During transient operation of for example gas turbine engines, the rotor and stator assemblies expand at different rates, altering the relative radial clearance and axial position of the fins in any labyrinth seal used. A change in radial clearance and axial position, in the

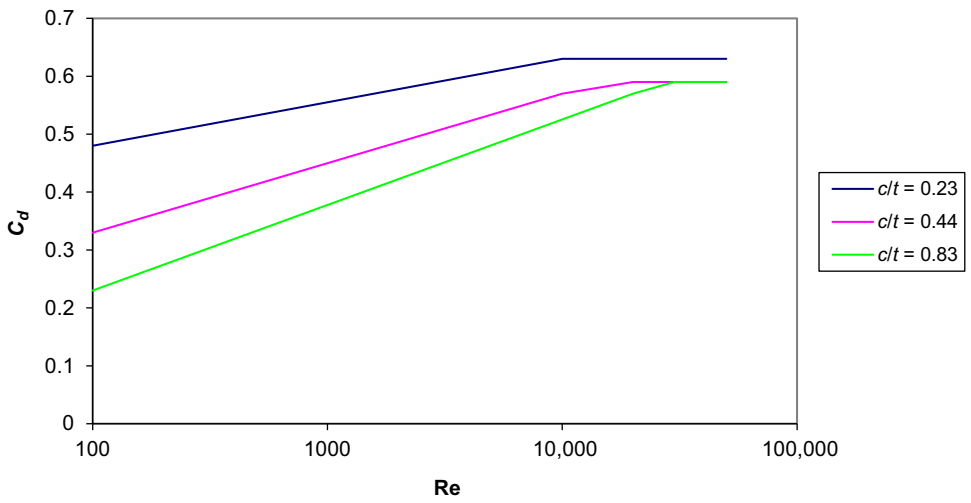


Figure 14.28
Discharge coefficient for a straight labyrinth seal with two fins ($n = 2$).

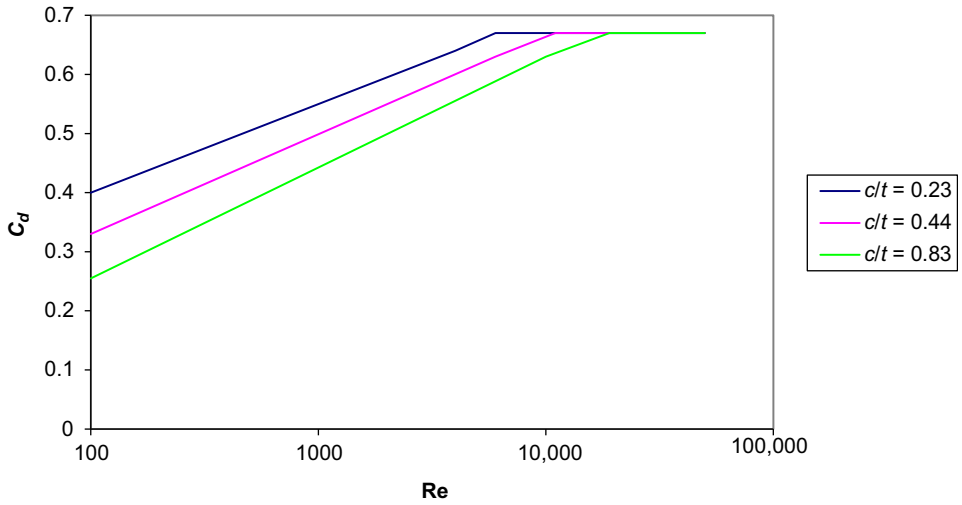


Figure 14.29
Discharge coefficient for a straight labyrinth seal with more than two fins.

case of for example a stepped seal, can significantly alter the leakage flow rate through a labyrinth seal. Rubbing of the fins on the stator can potentially occur in many rotating flow applications.

The inner surface of a labyrinth can be coated with an abradable coating (see Figure 14.23), in order to accommodate potential rubs between the labyrinth fins and stator surface or in order to provide tight running clearances by allowing the fins to "machine" a running clearance in service or at a machine commissioning phase. Contact between the fin tips and stator surface results in a series of grooves along the length of the seal, as illustrated in Figure 14.30, and a correction to the discharge coefficient needs to be made.

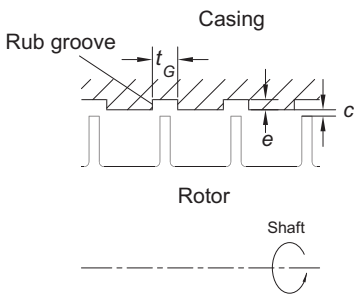


Figure 14.30
Straight labyrinth seal with rub grooves.

Rhode and Adams (2004) investigated the effects of varying the prerub radial clearance, rub-groove width, and rub-groove depth in order to determine the effect of rub-groove geometry on compressible in straight labyrinth seals. In the cases considered with each fin tip outside of its rub groove, the presence of a rub-groove increases the effective fin clearance. It was found that both the size of the rub-groove and the prerub radial clearance are important in determining changes in the local and global flow pattern as well as leakage. The presence of the rub-groove generally increases the leakage, primarily due to an increased effective fin radial clearance. For cases with a large prerub radial clearance and narrow rub-groove width, however, the increase was negligible. The negligible increase, for the case of a narrow rub-groove width, was attributed to low through-flow deflection into the rub-groove as a result of a minimized effective fin clearance. Rhode and Adams surmised that for operating conditions for which the fin tip is located inside of its rub groove that the presence of the groove may act to decrease the leakage. A large width rub-groove generally gives slightly more leakage than an intermediate or narrow width, with intermediate and large width giving an almost identical leakage. The presence of the large or intermediate width rub grooves allows the rub-groove depth to exert a fairly large effect on the leakage, especially for the smallest prerub clearance considered. The presence of a narrow rub-groove with the smallest prerub clearance considered allows the rub-groove depth to exert a moderate effect on the leakage, but a dramatic effect on the overall flow pattern.

The ratio between the discharge coefficient with and without stator grooves as a function of fin tip-to-groove width ratio and groove depth and clearance is plotted in Figure 14.31 following the data of Zimmermann and Wolff (1998).

In order to improve the leakage characteristics, the inner surface of the labyrinth seal, i.e. the stator surface, can be treated with a honeycomb layer. The leakage is a function of the clearance and the honeycomb cell dimension, L_{cell}, see Figure 14.32. A correction factor, Δm, see Eqn (14.17), for the change in the leakage flow in a labyrinth seal with a honeycomb on the stator, relative to that for a smooth land, is illustrated in Figure 14.33 using experimentally derived data from Stocker (1978).

$$\Delta m = \frac{Q_{\text{with honeycomb}} - Q_{\text{smooth}}}{Q_{\text{smooth}}} \times 100\% \tag{14.17}$$

where $Q_{\text{with honeycomb}}$ is the mass flow function with a honeycomb surface and Q_{smooth} is the mass flow function without the honeycomb.

Chougule et al. (2008) report the use of CFD to optimize the configuration of a labyrinth seal with a honeycomb land, and a staggered honeycomb land and straight teeth with an inclined notch gave approximately a 17% reduction in seal leakage in comparison to their baseline conditions. The performance of honeycombs to reduce labyrinth seal leakage and on wear is reported in the theses by Allcock (1999) and Collins (2006), respectively.

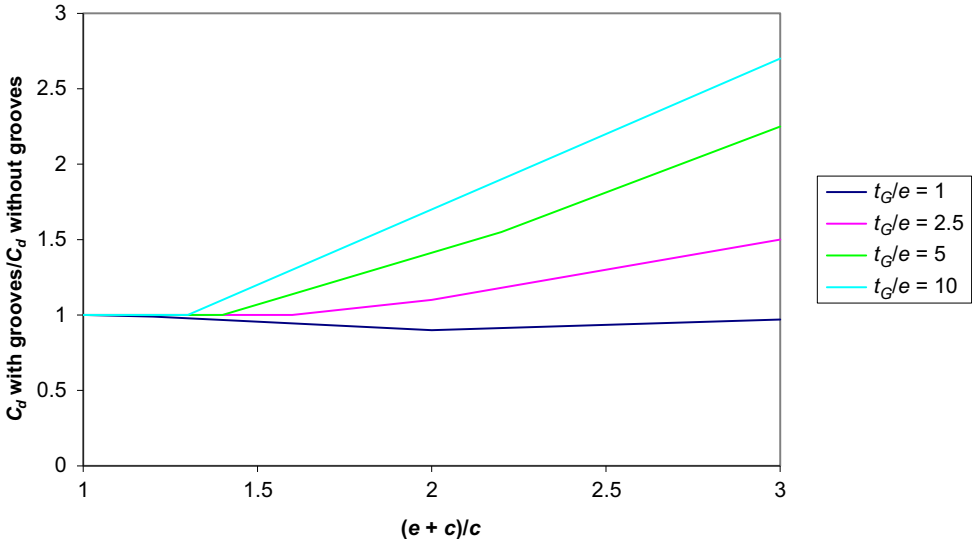


Figure 14.31
Ratio of the discharge coefficient with and without stator grooves as a function of fin tip-to-groove width ratio and groove depth and clearance, $(e + c)/c$.

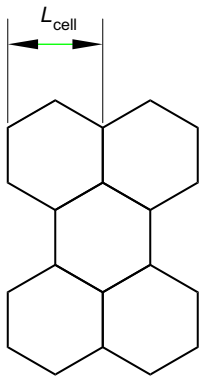


Figure 14.32
Honeycomb cell definition.

Corner rounding, see Figure 14.34, also known as fin-tip rounding, or rounded knife tips due to wear and abrasion, has a relatively minor impact (up to 12%) on the discharge coefficient for flow through a straight labyrinth seal. Corner rounding has been studied by Zimmermann et al. (1994) and Rhode and Allen (1999). Data, derived from experimental studies and proprietary information, from Zimmermann et al. (1994) for the ratio of the discharge coefficient with rounding to that with a sharp edge are plotted in Figure 14.35 for

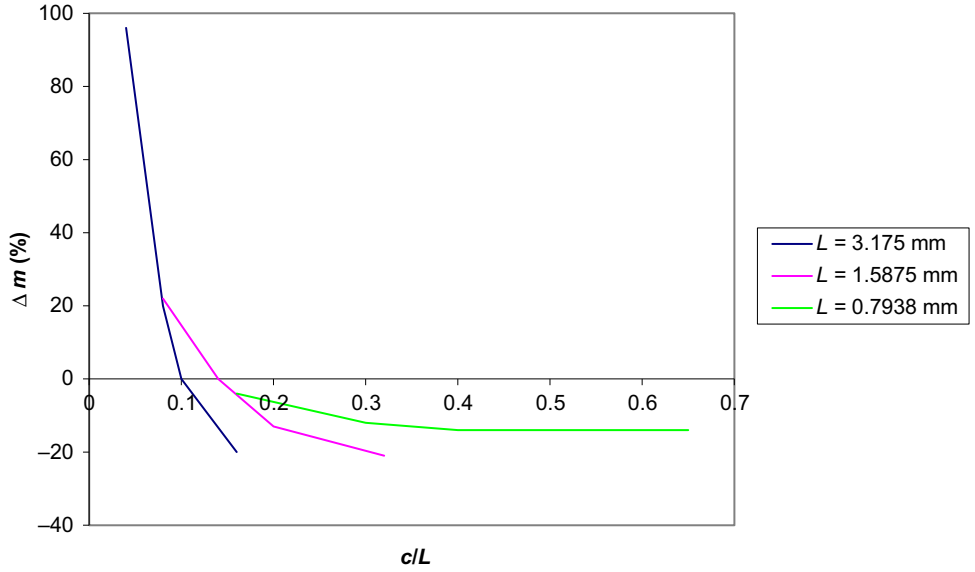


Figure 14.33
Variation of leakage with cell size induced by a honeycomb layer on a labyrinth stator.

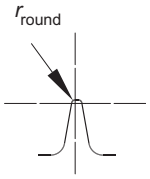


Figure 14.34
Corner rounding definition.

both straight and stepped labyrinth seals. Because there is more carryover in a straight labyrinth in comparison to a stepped labyrinth, the effect of corner radius is much smaller for a straight labyrinth.

The discharge coefficient is a function of the labyrinth chamber depth, which is defined as the ratio of the fin height, h, and pitch l_{pitch}. This is plotted in Figure 14.36 based on the experimental data from Trutnovsky and Komotori (1981). A minimum in the value of the discharge coefficient occurs approximately for $h/l_{\text{pitch}} = 0.25$; this can therefore be used as a guideline for the specification of the optimal fin height in preliminary labyrinth design:

$$h = 0.25 l_{\text{pitch}} \tag{14.18}$$

In general, a narrow fin-tip land width (or fin width), t, is preferable. Yamada (1962) found that the flow coefficient generally decreased slightly with either the fin width-to-clearance

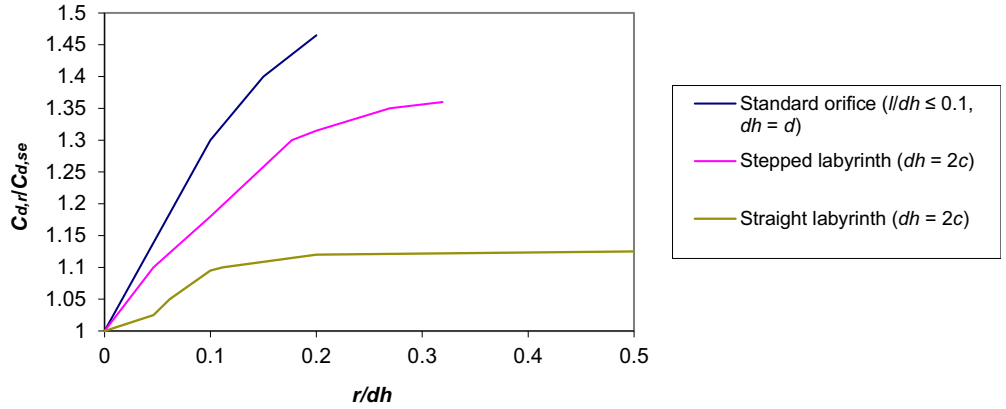


Figure 14.35
Effect of rounding radius.

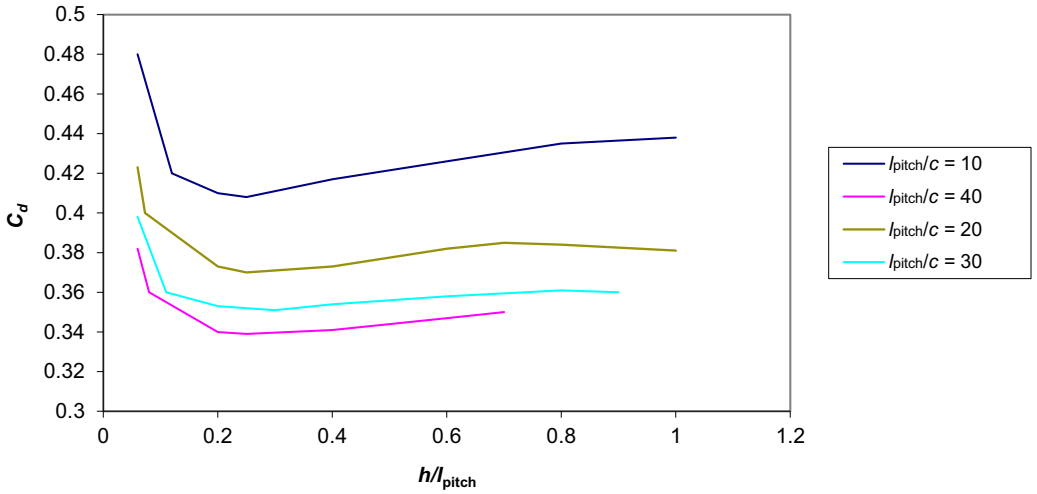


Figure 14.36
Effect of labyrinth chamber depth on discharge coefficient for different pitch-to-clearance ratios.

ratio or the fin width-to-pitch ratio. At very low rotational speeds, about 300 rpm and less in the tests concerned, a significant dependence of the flow coefficient on the fin width-to-clearance ratio was identified. As a maximum, it is recommended that the fin width should be less than half of the pitch ($t < 0.5l_{\text{pitch}}$), especially for low rotation rates (Morrison et al., 1983).

Stocker et al. (1977) performed tests for labyrinth seals with smooth, medium (8.3 μm), and rough (22.9 μm) sealing surfaces. It was identified that as the roughness increases up to 8.3 μm, leakage decreased. If the roughness is increased further, to 22.9 μm in the tests

concerned, the leakage increased. Specifically, a medium surface roughness of 8.3 μm reduced the leakage in a straight labyrinth seal by approximately 23% in comparison to a seal with a smooth land for a 0.13-mm clearance and by 5.0% for a clearance of 0.51 mm.

The effect of cylindrical eccentricity of the seal shaft has been investigated by Yeh and Cochran (1970), Bell and Bergelin (1957), Nikitin and Ipatov (1973), and Gamal and Vance (2007). In general, it was found that eccentricity did not have a significant impact on the flow coefficient, with less than 5% change in the flow coefficient for the cases considered. Hodkinson (1940), however, found that full eccentricity of the seal increased the flow rate by a factor of 2.5 for some of the tests. The available data are therefore inconsistent. Gamal and Vance (2007) found that leakage through the seals they tested increased with increasing eccentricity and that this effect was more pronounced at lower supply pressures.

The influence of rotation on leakage flow in a labyrinth seal is negligible at axial Reynolds numbers $\mathrm{Re} > 10^4$ (e.g. see Waschka et al., 1992). For laminar flow ($\mathrm{Re} < 200$), however, the effect is significant. Waschka et al. (1992) reported results for the variation of the discharge coefficient with Reynolds number for a range of rotational speeds and clearances. The discharge coefficient increased from about 0.34 to 0.45. At $\mathrm{Re} = 2000$ for a clearance of 1 mm the rotational speed is increased to 10,000 rpm for the seal tested. Pressure losses for laminar flow result from increasing friction with decreasing Reynolds number and depend on surface area and form.

Miyake and Duh (1990) investigated the relationship between mass flow rate and rotation. They found that the mass flow rate decreased in straight-through labyrinth seals with increasing peripheral velocity. This decrease was found to be larger when the restriction tip thickness was large and the chamber depth was shallow. The effects of rotational speed were found to be largest for straight as opposed to stepped seals, with the step-down seal performing better than the step-up seal.

Rotation of a labyrinth seal also results in frictional heating of the fluid through viscous dissipation of energy known as windage heating. This raises the temperature of the flow through the labyrinth and needs to be accounted for in any labyrinth design procedure in order to ensure that operating temperature limits are acceptable. The windage in labyrinth seals is primarily dependent upon rotational velocity and surface roughness; methods for its quantification have been studied by McGreehan and Ko (1989) and Millward and Edwards (1996).

Example 14.2

Determine the mass flow rate through a straight labyrinth seal with the following geometric characteristics:

- four fins, $n = 4$;
- outer diameter $D = 2 \times r_i = 215.9$ mm;

- fin height $h = 2.286$ mm;
- pitch $l_{\text{pitch}} = 3.175$ mm;
- fin taper angle $\phi_{\text{fin}} = 7.5^{\circ}$;
- radial clearance $c = 0.127$ mm; and
- fin-tip land width $t = 0.305$ mm.

The total pressure drop across the seal is 200 kPa, the downstream pressure is 1.01 bar, and the average seal inlet temperature is $T_{t0} = 300$ K. Take the gas constant R as $R = 287$ J/(kg K) and the dynamic viscosity $\mu = 1.8 \times 10^{-5}$ Pa s. Example after ESDU 09004.

Solution

From Eqn (14.11), the carryover factor, k_2, is given by

$$k_2 = \sqrt{\frac{n/(n-1)}{1 - \left(\frac{n-1}{n}\right)\left[\frac{c/l_{\text{pitch}}}{(c/l_{\text{pitch}})+0.02}\right]}} = \sqrt{\frac{4/(4-1)}{1 - \left(\frac{4-1}{4}\right)\left[\frac{0.127/3.175}{(0.127/3.175)+0.02}\right]}} = 1.633$$

The discharge coefficient, C_d, depends on the Reynolds number, Re, and thus on the leakage mass flow rate, which is not known at this stage of the calculation.

From Figure 14.29, the value of the discharge coefficient, C_d, can be taken as $C_d = 0.67$, using the value for which the discharge coefficient becomes invariant with Reynolds number; this value can be revised once an estimate of the Reynolds number in the seal has been obtained.

$$r_i = D/2 = 215.9/2 = 107.95 \text{ mm}$$

$$r_o = r_i + c = 107.95 + 0.127 = 108.077 \text{ mm}$$

The annular seal cross-sectional flow area is given by

$$A = \pi\left(r_o^2 - r_i^2\right) = \pi\left(0.108077^2 - 0.10795^2\right) = 8.619 \times 10^{-5} \text{ m}^2$$

Substituting into Eqn (14.9), the mass flow rate through the clearance area can then be calculated as

$$m = k_2 C_d A p_{t0} \sqrt{\frac{1 - (p_n/p_{t0})^2}{RT0_{t0}[n - \ln(p_n/p_{t0})]}}$$

$$= 1.633 \times 0.67 \times 8.619 \times 10^{-5} \times 3.01 \times 10^5 \sqrt{\frac{1 - \left(1.01 \times 10^5/3.01 \times 10^5\right)^2}{287 \times 300[4 - \ln(1.01 \times 10^5/3.01 \times 10^5)]}}$$

$$= 0.04038 \text{ kg/s}$$

The Reynolds number now needs to be determined.

The hydraulic diameter for an annulus is given by

$$D_h = 2(r_o - r_i) = 2.54 \times 10^{-4}\ \text{m}$$

The Reynolds number based on the hydraulic diameter, D_h, is given by

$$\text{Re}_{D_h} = \frac{mD_h}{A\mu} = \frac{0.04038 \times 2.54 \times 10^{-4}}{8.619 \times 10^{-5} \times 1.8 \times 10^{-5}} = 6611$$

The clearance to fin-tip land width ratio is

$$c/t = 0.127/0.305 = 0.4164.$$

Calculating C_d, from Figure 14.29, the revised value of the discharge coefficient, C_d, is approximately 0.64.

The calculations for the mass flow now need to be repeated with this updated value for the discharge coefficient and the results examined to see whether they have converged.

Recalculating the mass flow, m, and Re_{D_h}

Substituting these values into Eqn (14.9), the revised mass flow rate is calculated to be

$$\begin{aligned} m =& 1.633 \times 0.64 \times 8.619 \times 10^{-5} \\ & \times 3.01 \times 10^5 \sqrt{\frac{1 - (1.01 \times 10^5/3.01 \times 10^5)^2}{287 \times 300[4 - \ln(1.01 \times 10^5/3.01 \times 10^5)]}} \\ =& 0.03857\ \text{kg/s} \end{aligned}$$

Hence, the Reynolds number is given by

$$\text{Re}_{D_h} = \frac{mD_h}{A\mu} = \frac{0.03857 \times 2.54 \times 10^{-4}}{8.619 \times 10^{-5} \times 1.8 \times 10^{-5}} = 6315$$

These revised values for the mass flow rate and Reynolds numbers are significantly different from those obtained previously. Therefore, further calculations with the updated estimate for the Reynolds number are necessary.

From Figure 14.29, with $\text{Re}_{D_h} = 6315$, the value of the discharge coefficient, C_d, is approximately 0.63. The calculations for the mass flow, velocity, and Reynolds number give

$$m = 0.03797\ \text{kg/s}$$

$$\text{Re}_{D_h} = 6216$$

Examination of Figure 14.29 with $\mathrm{Re}_{D_h} = 6216$ gives a value of the discharge coefficient, C_d, of approximately 0.63. Since this value is close to that found previously, the solution, therefore, can be assumed to have converged. The estimate for the mass flow through the seal for these conditions is approximately 0.038 kg/s.

14.4.4 Stepped Labyrinth Seals

A step can be incorporated into a labyrinth seal design to prevent carryover of kinetic energy from occurring, as illustrated in Figure 14.37. This provides improved sealing characteristics in comparison to straight labyrinth seals. However, radial space must be available in machines where sealing is required to accommodate a stepped seal. In addition, axial pressure gradients acting on step faces can cause undesirable axial loads. The geometric form of the seal can be defined to enhance the dissipation of kinetic energy; examples of this are illustrated in Figure 14.38 (see Stocker, 1978). Such stepped seals have been applied to gas turbine engine applications.

Studies on stepped labyrinth seals have been reported by a number of researchers, including Stocker (1976), Stocker et al. (1977), Vermes (1961), Benckert and Wachter (1979), Stepanoff (1931), Isaacson (1957) and Yamada (1962); a variety of flow models have been developed for stepped labyrinth seals, as indicated in Table 14.8. The detailed flow structure of flow in stepped labyrinth configurations has been studied by Rhode et al. (1997a,b).

In the absence of a validated database or code, the model proposed by Zimmermann and Wolff (1998) is suggested for stepped labyrinths, based on the following equation:

$$m = k_s C_d A p_{t0} \sqrt{\frac{1 - (p_n/p_{t0})^2}{RT_{t0}[n + \ln(p_{t0}/p_n)]}} \tag{14.19}$$

where k_s is a correction factor for C_d (see Figure 14.39).

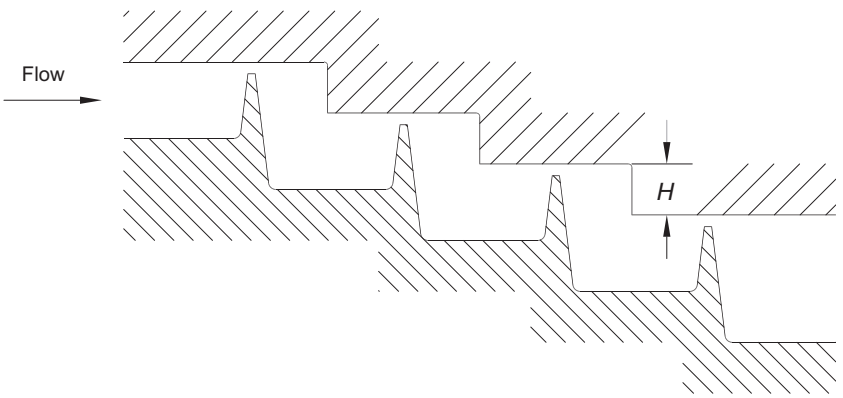


Figure 14.37
Typical stepped labyrinth seal.

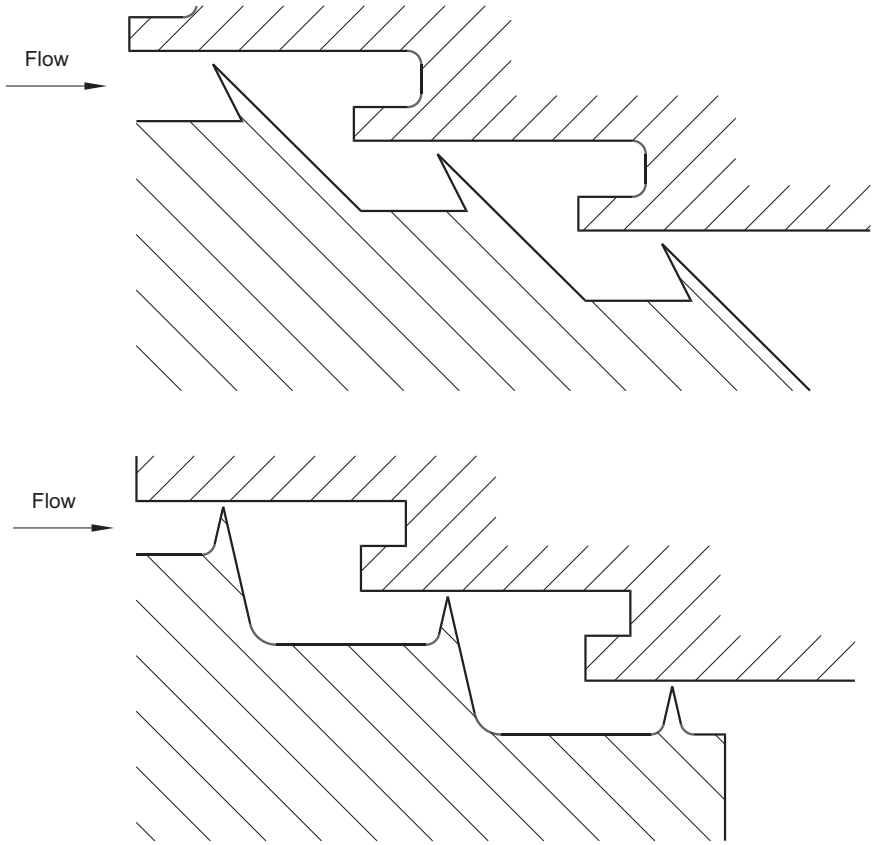


Figure 14.38
Alternative configurations for stepped labyrinth seals.

Table 14.8: Selected correlations for determining the mass flow for stepped labyrinth seals.

Reference	Equation	Flow type	Application
Stocker (1976), Stocker et al. (1977)	$m = \frac{Ap_{t0}}{\sqrt{T}} \times F(\text{geometry, flow})$ F(geometry, flow) from paper.	Compressible flow	Stepped seals
Vermes (1961)	$m = 5.76 C_d k A \sqrt{\frac{p_{t0}\rho_0[1-(p_n/p_{t0})^2]}{n+\ln(p_{t0}/p_n)}}$ $k = \frac{8.52}{[(l_{\text{pitch}}-t)/c]+7.23}$	Compressible flow	Straight-through and stepped seals
Zimmermann and Wolff (1998)	$m = kC_d A p_{t0} \sqrt{\frac{1-(p_n/p_{t0})^2}{RT_{t0}[n+\ln(p_{t0}/p_n)]}}$	Compressible flow	Straight-through and stepped seals

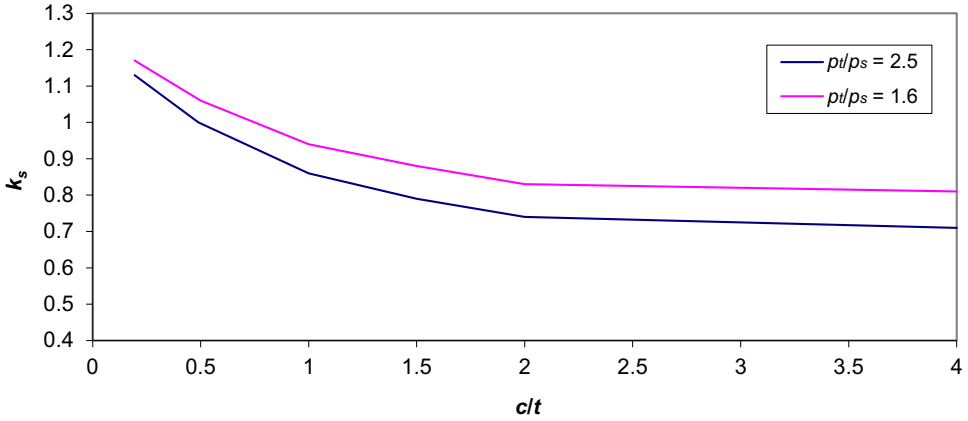


Figure 14.39
C_d correction factor.

For a stepped labyrinth seal Yucel (2004) reported using a discharge coefficient of unity for the first fin and 0.7 for subsequent fins.

Willenborg et al. (2001) provided experimental data, from a test rig representative of a labyrinth seal, for the discharge coefficient as a function of c/t, pressure ratio, and Reynolds number, where the Reynolds number was defined by $\mathrm{Re} = 2mc/A\mu$. Due to the combined influence of contraction and friction effects, a general dependence of the discharge coefficient on Reynolds number was identified. For low Reynolds numbers, the discharge coefficient increased by up to 9% for the 1:1 scale model tested as the Reynolds number increased from 2500 to 8050. For the 1:1 model tested, the following ranges of values of the discharge coefficient were found:

$$0.53 \leq C_d \leq 0.64 \quad \text{for } c/t = 0.909;$$

$$0.46 \leq C_d \leq 0.57 \quad \text{for } c/t = 1.515;$$

$$0.46 \leq C_d \leq 0.53 \quad \text{for } c/t = 2.424.$$

At sufficiently high Reynolds numbers, between about 30,000 and 40,000, the discharge coefficients for a particular seal configuration were found to depend only on the pressure ratio.

Zimmermann and Wolff (1998) proposed the use of discharge coefficient data from Snow (1952) for a single fin (Figure 14.40), in combination with a correction factor, k_s. Their data based on measurements and proprietary information for the discharge coefficient correction factor, k_s, are in Figure 14.39. Zimmermann and Wolff reported that these data represent test results within $\pm 5\%$.

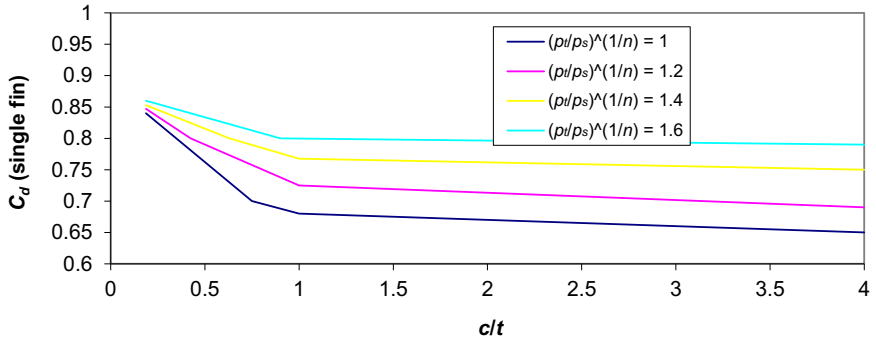


Figure 14.40
Discharge coefficients for a stepped labyrinth seal (single fin).

Corner rounding (see Figure 14.34) can have a significant impact (up to 36%) on the discharge coefficient for flow through a stepped labyrinth seal. The data based on measurements and proprietary information from Zimmermann and Wolff (1998) for the ratio of the discharge coefficient with rounding to that with a sharp edge, plotted in Figure 14.35 for both straight and stepped labyrinth seals, can be used to model the effect of corner rounding.

Zimmermann and Wolff (1998) presented proprietary data for the influence of step height (Figure 14.37). A minimum at $H/l_{\text{pitch}} = 0.075$ is recommended if practicable.

14.4.5 Staggered Labyrinths

In a staggered labyrinth seal, the fins are alternatively attached to the shaft and the casing. Staggered labyrinths give lower leakage than corresponding straight labyrinth seals with the same clearance, as illustrated in Figure 14.41. The improved sealing characteristics must, however, be assessed against the increased complexity of their assembly and limitations on axial movement. In order to assemble a staggered labyrinth seal, the case must normally

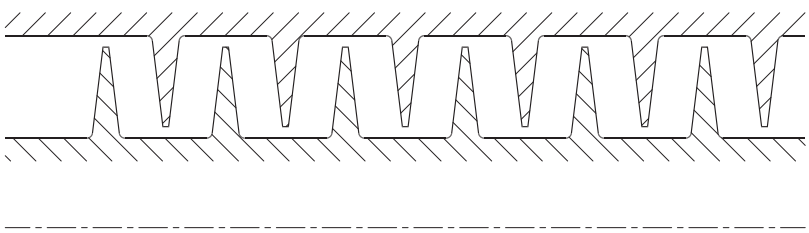


Figure 14.41
Staggered labyrinth seal.

be split, typically into two parts. Axial growth of the shaft or casing must be accommodated in the design to ensure no contact between alternate fins.

Studies on staggered labyrinth seals have been reported by a number of researchers, including Martin (1908), Stodola (1927), Dollin and Brown (1937), Egli (1935), Kearton and Keh (1952), Komotori and Mori (1971), Deych et al. (1980a,b), Samsonov et al. (1978) and Dodge (1963). A variety of flow models have been developed for staggered labyrinth seals, as indicated in Table 14.9, enabling an appropriate model for a given type of flow and application to be identified.

14.4.6 Radial Labyrinths

In a radial labyrinth seal, the flow direction is orthogonal to the machine axis or shaft as illustrated in Figure 14.21(d)–(f). The flow can be radially inward, toward the shaft axis, or radially outward, away from the shaft axis. A radial labyrinth seal may be suitable in applications where the axial space available in a machine is limited but use of radial space can be accommodated. Radial labyrinth seals can have a straight, staggered, or stepped configuration.

Studies on radial labyrinth seals have been reported by a number of researchers, including Martin (1908), Stodola (1927), Kearton (1955), Solomko et al. (1979) and Ueda and Kubo (1967). A variety of flow models have been developed for radial labyrinth seals as indicated in Table 14.10, enabling an appropriate model for a given type of flow and application to be identified.

For the case of radially outward flow, the data of Kearton (1955) showed that the flow coefficient decreased slightly with pressure ratio for decreasing clearances except at small clearances where the flow coefficient maintained a fairly constant value. For the case of radial inflow, the flow coefficient decreased only slightly as the clearance was decreased across a range of pressure ratios.

14.5 Axial and Bush Seals

Simple axial and radial bush seals are illustrated in Figures 14.42 and 14.43 and can be used for sealing both liquids and gases. Leakage through a concentric axial bush seal can be estimated using Eqn (14.20) for incompressible flow (flow where the density can be considered constant) and Eqn (14.21) for compressible flow. The leakage flow through a radial bush seal can be estimated by Eqn (14.22) for incompressible flow and Eqn (14.23) for compressible flow.

$$Q = \frac{\pi \phi c^3 (p_o - p_a)}{12 \mu L} \tag{14.20}$$

Table 14.9: Selected correlations for determining the mass flow for staggered labyrinth seals.

Reference	Equation	Flow type	Application
Martin (1908, 1919)	$m = Ap_{t0}\sqrt{\dfrac{1-(p_n/p_{t0})^2}{RT_{t0}[n+\ln(p_{t0}/p_n)]}}$	Compressible flow	Staggered seals
Stodola (1927)	$m = Ap_{t0}\sqrt{\dfrac{1-(p_n/p_{t0})^2}{RT_{t0}n}}$	Compressible flow	Staggered seals
Dollin and Brown (1937)	$m = A\sqrt{\dfrac{2\gamma\rho_0 p_{t0}\left[1-(p_n/p_{t0})^{(1+\gamma)/\gamma}\right]}{(\gamma+1)[n+(1/\gamma)\ln(p_n/p_{t0})]}}$	Polytropic process; incompressible flow	Staggered seals
	$m = A\sqrt{\dfrac{2\rho(p_{t0}-p_n)}{n}}$		
Egli (1935)	$m = \alpha\varepsilon A\sqrt{\dfrac{p_{t0}\rho_0[1-(p_n/p_{t0})^2]}{n+\ln(p_{t0}/p_n)}}$	Compressible flow	Straight-through and staggered seals
Kearton and Keh (1952)	$m = C_d A\sqrt{\dfrac{F(p_0^2-p_n^2)}{nRT_{t0}}}$ for $p_n/p_{t0} >$ critical	Compressible flow	Staggered seals
	F is a function of the pressure ratio and is defined in [10]		
	$m = \dfrac{3.873C_d A p_{n-1}}{\sqrt{RT_{t0}}}$ for $p_n/p_{t0} >$ critical (air)		

Continued

Table 14.9: Selected correlations for determining the mass flow for staggered labyrinth seals.—cont'd

Reference	Equation	Flow type	Application
Komotori and Mori (1971)	$m = \alpha\varepsilon_1 A_0 F(n,\beta,\gamma)\sqrt{\rho_0 p_{t0}}$ $F(n, \beta, \gamma)$ from paper	Compressible flow	Straight-through and radial seals
Deych et al. (1980a,b)	$m = \left[\frac{\sigma}{x_0 + x_0^2}\right]^{-0.5} A\sqrt{\frac{\rho_o p_{t0}[1-(p_n/p_{t0})^2]}{n}}$ Where x_o is the steam quality and σ can be found from a graph in the original paper	Superheated and wet (two-phase) steam flow	Staggered seals
Samsonov et al. (1978)	$m = \alpha A\sqrt{\frac{\rho_0 p_{t0}[1-(p_n/p_{t0})^2]}{n}}$	Superheated and wet (two-phase) steam flow	Staggered seals
Dodge (1963)	$m = A\sqrt{\frac{2\rho(p_{t0}-p_n)}{fL/[2c+1.5+k(n-1)]}}$ $k = 1$ straight-through seals $k = 2$ staggered seals $f = 64/\mathrm{Re}$ for $\mathrm{Re} < 2320$ $f = 0.316/\mathrm{Re}^{0.25}$ for $\mathrm{Re} > 2320$	Incompressible flow	Straight-through and staggered seals

Table 14.10: Selected correlations for determining the mass flow for radial labyrinth seals.

Reference	Equation	Flow type	Application
Martin (1908, 1919)	$m = p_{t0}\sqrt{\dfrac{A_0 A_n[1-(p_n/p_{t0})^2]}{RT_{t0}[n+\ln(A_0 p_{t0}/A_n p_{tn})]}}$	Compressible flow	Radial seals
Stodola (1927)	$m = p_{t0}\sqrt{\dfrac{A_0 A_n[1-(p_n/p_{t0})^2]}{RT_{t0}n}}$	Compressible flow	Radial seals
Kearton (1955)	$m = \alpha A\sqrt{\dfrac{\rho_0 p_{t0} F(\beta)[1-(p_n/p_{t0})^2]}{n^{\#}}}$ $F(\beta)$ from paper $n^{\#} = \dfrac{n+2l_{\text{pitch}}n(n-1)}{2r_i - l_{\text{pitch}}(2n-1)}$ inward flow $n^{\#} = \dfrac{n-2l_{\text{pitch}}n(n-1)}{2r_i + l_{\text{pitch}}(2n-1)}$ outward flow	Compressible flow	Radial seals
Ueda and Kubo (1967)	$m = \alpha\sqrt{\dfrac{A_0 A_n \rho_0 p_{t0}^2 g\zeta_m[1-(p_n/p_{t0})^2]}{(1-\nu^2)p_n\left[\left(n-1\right)+\frac{2\ln(p_{t0}/p_n)}{\gamma\zeta_m(1-\nu^2)}\right]}}$ Where ζ_m represents the mean number of fins	Compressible flow	Radial seals

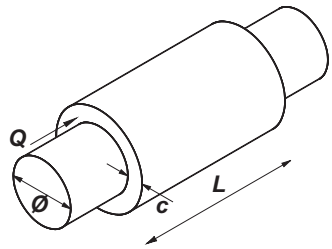


Figure 14.42
Axial bush seal.

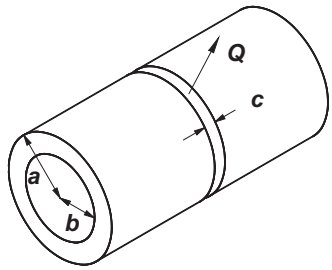


Figure 14.43
Radial bush seal.

$$Q = \frac{\pi \phi c^3 \left(p_o^2 - p_a^2\right)}{24 \mu L p_a} \tag{14.21}$$

where

Q = volumetric flow rate (m^3/s),
ϕ = diameter of the shaft (m),
c = radial clearance (m),
p_o = upstream pressure (Pa),
p_a = downstream pressure (Pa),
μ = absolute viscosity (Pa s),
L = length of the axial bush seal (m).

$$Q = \frac{\pi c^3 (p_o - p_a)}{6 \mu \ln(a/b)} \tag{14.22}$$

$$Q = \frac{\pi c^3 \left(p_o^2 - p_a^2\right)}{12 \mu p_a} \tag{14.23}$$

where

c = axial clearance (m),
a = outer radius (m),
b = inner radius (m).

Example 14.3

An axial bush seal consists of an annular gap with inner and outer radii of 50 and 50.5 mm, respectively. The length of the seal is 40 mm. Determine the flow rate of oil through the seal if the pressures upstream and downstream of the seal are 7 and 5.5 bar, respectively. The viscosity of the oil can be taken as 0.025 Pa s.

Solution

The radial clearance is $0.0505 - 0.05 = 0.0005$ m, $\phi = 0.1$ m, $L = 0.04$ m.

The volumetric flow rate is given by

$$Q = \frac{\pi \times 0.1 \times \left(0.5 \times 10^{-3}\right)^3 \left(7 \times 10^5 - 5.5 \times 10^5\right)}{12 \times 0.025 \times 0.04} = 4.909 \times 10^{-4} \text{ m}^3/\text{s}.$$

14.6 Seals for Reciprocating Components

The seals principally used for reciprocating motion are packings and piston rings.

Packing seals are illustrated in Figure 14.44. The seal essentially consists of a cup, V, U, or X section of leather, solid rubber, or fabric reinforced rubber. The sealing principle is by direct

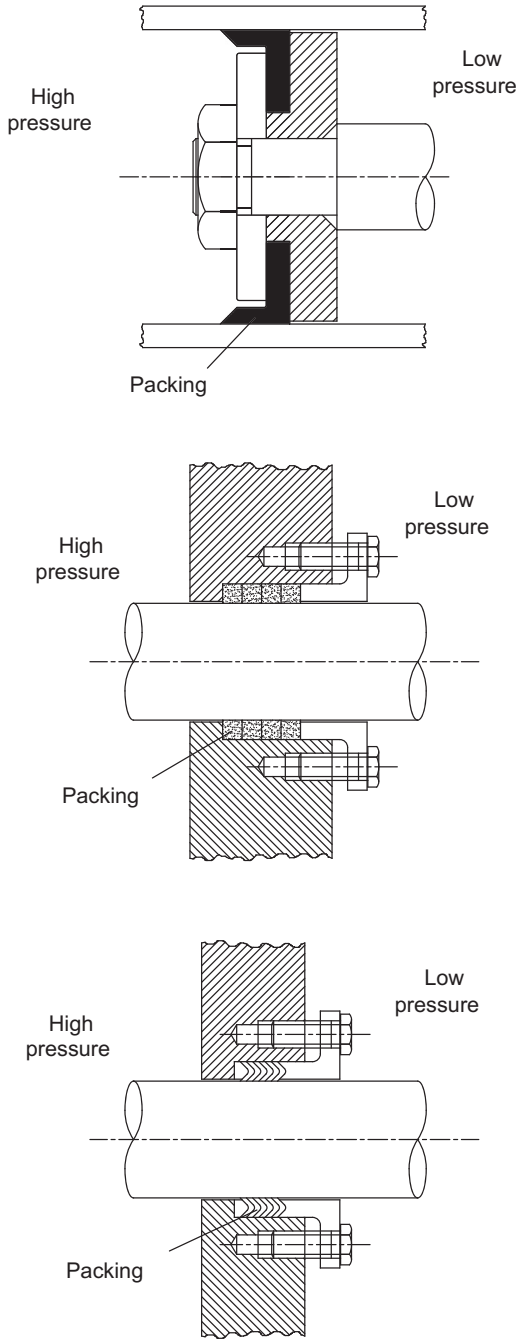


Figure 14.44
Packing seals.

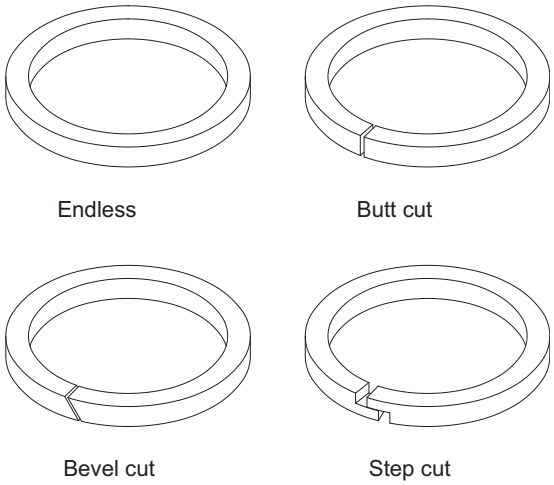


Figure 14.45
Piston rings.

contact with the reciprocating component. The contact pressure can be increased in the case of V packings by axial compression of the seals, although this obviously increases the friction on the shaft and wear rate. The principal uses of cup packings are as piston seals in hydraulic and pneumatic applications, U packings for piston rods, and V packings for sealing piston rods or reciprocating shafts.

Piston rings are used to seal cylinders where the operating temperature is above the limit of elastomeric, fabric, or polymeric materials. Piston rings are used in automotive cylinders for three purposes:

1. to seal the combustion chamber/cylinder head;
2. to transfer heat from the piston to the cylinder walls; and
3. to control the flow of oil.

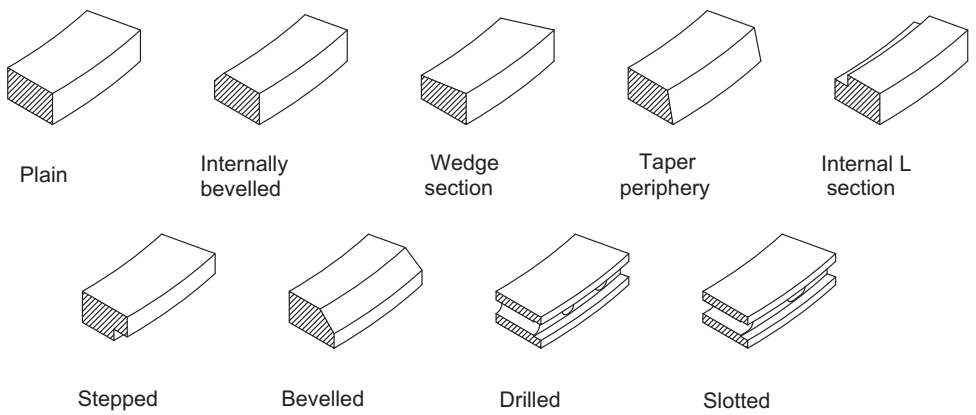


Figure 14.46
Piston ring sections.

Table 14.11: Number of piston rings required to seal a given pressure.

P_o (bar)	Number of rings
<20	2
$20 < p_o < 60$	3
$60 < p_o < 100$	4
$100 < p_o < 200$	5
>200	6+

Piston rings are usually machined from a fine grain alloy cast iron and must be split to allow for assembly over the piston. Conventional practice is to use three piston rings, with two compression rings sealing the high pressure and one controlling the flow of oil. The range of piston ring available is extensive, as illustrated in Figures 14.45 and 14.46. For other applications, the number of piston rings required can be determined using Table 14.11 assuming normal piston temperatures and running speeds.

14.7 Conclusions

It is frequently necessary in machine design to provide some means of containing or limiting the flow of fluid from one region to another. Because of the very small nature of fluid molecules, this is a challenging task. This chapter has reviewed a range of static and dynamic seals. Because of the wide range of applications, seals tend not to be available as stock items and instead must be designed and fit for purpose.

References

Books and Papers

Allcock, D., 1999. Abradable Stator Gas Turbine Labyrinth Seals (Ph.D. thesis). Cranfield University.

Bell, K.J., Bergelin, O.P., 1957. Flow through annular orifices. Trans. ASME 79, 593–601.

Benckert, H., Wachter, J., 1979. Investigations on the mass flow and flow induced forces in contactless seals of turbomachines. In: Proceedings of the Sixth Conference on Fluid Machinery. Scientific Society of Mechanical Engineers, Budapest, pp. 57–66.

Childs, D., 1993. Turbomachinery Rotordynamics: Phenomena, Modeling and Analysis. Wiley.

Childs, D., Rhode, D., 1984. Rotordynamic Forces Developed by Labyrinth Seals. Turbomachinery Laboratories Mechanical Engineering Department. AFOSR-TR-85-1070.

Chougule, H.H., Ramerth, D., Ramachandrandran, D., 2008. Low Leakage Designs for Labyrinth Seals. ASME. Paper GT2008 51024.

Chupp, R.E., Hendricks, R.C., Lattime, S.B., Steinetz, B.M., 2006. Sealing in Turbomachinery. NASA/TM—2006-214341.

Collins, D., 2006. The Effects of Wear on Abradable Honeycomb Labyrinth Seals (EngD. thesis). Department of Power, Propulsion & Aerospace Engineering, School of Engineering Cranfield University.

Czernik, D.E., 1996. Gaskets. In: Shigley, J.E., Mischke, C.R. (Eds.), Standard Handbook of Machine Design. McGraw Hill.

Deych, M.Y., Supunov, O.G., Shanin, V.K., Sabri, T.I., 1980a. Effect of geometric dimensions of labyrinth seal annuli on the rate of flow of superheated and wet steam through them. Fluid Mech. Sov. Res. 9 (4), 72–77.

Deych, M.Y., Solomko, V.I., Zezyulinskiy, G.S., 1980b. Rate of leakage of wet steam through a labyrinth seal. Fluid Mech. Sov. Res. 9 (4), 64–71.
Dodge, L., 1963. Labyrinth shaft seals. Prod. Eng. 34, 75–79.
Dolan, P.J., 1992. Mechanical seal review. In: Nau, B.S. (Ed.), Fluid Sealing. Kluwer, pp. 413–427.
Dollin, F., Brown, W., 1937. Flow of fluids through openings in series. The Engineer 164 (4259), 223–224.
Egli, A., 1935. The leakage of steam through labyrinth seals. Trans. ASME 57, 115–122, 445–446.
ESDU, 1982. Compressible Flow of Gases. Pressure Losses and Discharge Coefficients of Orifice Plates, Perforated Plates and Thick Orifice Plates in Ducts. ESDU 82009. ESDU International Plc, London.
ESDU, 2007. Incompressible Flow through Orifice Plates — a Review of the Data in the Literature. ESDU TN 07007. ESDU International Plc.
ESDU, 2009. Labyrinth seal flow. ESDU 09004. ISBN 978 1 86246 639 5.
Eser, D., Kazakia, J.Y., 1995. Air flow in cavities of labyrinth seals. Int. J. Eng. Sci. 33, 2309–2326.
Flitney, R., 2007. Seals and Sealing Handbook. Elsevier.
Gamal, A.M., Vance, J.M., 2007. Labyrinth Seal Leakage Tests: Tooth Profile, Tooth Thickness, and Eccentricity Effects. ASME. Paper GT2007-27223.
Gurevich, M.I., 1966. The Theory of Jets in an Ideal Fluid. Pergamon.
Hamilton, P., 1994. Seals. In: Hurst, K. (Ed.), Rotary Power Transmission. McGraw Hill.
Han, J.T., 1979. A Fluid Mechanics Model to Estimate the Leakage of Incompressible Fluids through Labyrinth Seals. ASME. Paper 79-FE-4.
Hodkinson, B., 1940. Estimation of the leakage through a labyrinth gland. Proc. Inst. Mech. Eng. 141, 283–288.
Horve, L., 1996. Shaft Seals for Dynamic Applications. Marcel Dekker.
Isaacson, J., 1957. Pump Impeller Labyrinth Seal Study. DDR-712-3012.
Kearton, W.J., Keh, T.H., 1952. The flow of air through labyrinth glands of staggered type. Proc. Inst. Mech. Eng. 166, 180–195.
Kearton, W.J., 1955. The flow of air through radial labyrinth glands. Proc. Inst. Mech. Eng. 169, 539–550.
Komotori, K., Mori, H., 1971. Leakage characteristics of labyrinth seals. Fifth International Conference on Fluid Sealing. Paper E4, pp. 45–63.
Lebeck, A.O., 1991. Principles and Design of Mechanical Face Seals. Wiley Interscience.
Martin, H., 1908. Labyrinth packings. Engineering 85, 35–38.
Martin, H., 1919. Steam leakage in dummies of the Ljungstrom type. Engineering 107, 1–3.
McGreehan, W., Ko, S., 1989. Power Dissipation in Smooth and Honeycomb Labyrinth Seals. ASME. Paper 89-GT-220.
Millward, J., Edwards, M., 1996. Windage heating of air passing through labyrinth seals. Trans. ASME J. Turbomach. 118, 414–419.
Miyake, K., Duh, W.C., 1990. Leakage characteristics of labyrinth seal. J. Chin. Soc. Mech. Eng. 11, 334–348.
Morrison, G.L., Rhode, D.L., Cogan, K.C., Chi, D., Demko, J., 1983. Labyrinth seals for incompressible flow. Final Report. NASA CR 170938.
Muller, H.K., Nau, B.S., 1998. Fluid Sealing Technology. Marcel Dekker.
Neale, M.J., 1994. Drives and Seals. Butterworth Heinemann.
Nikitin, G.A., Ipatov, A.M., 1973. Design of labyrinth seals in hydraulic equipment (In Russian). Russ. Eng. J. 53, 26–30.
Parsons, C.A., 4 Nov, 1892. Electrical lighting at Cambridge. The Engineer.
Picardo, A., Childs, D.W., 2005. Rotordynamic coefficients for a tooth-on-stator labyrinth seal at 70 bar supply pressures: measurements versus theory and comparisons to a hole-pattern stator seal. Trans. ASME J. Eng. Gas Turbines Power 127, 843–856.
Rao, N.B., Sidheswar, N., 1976. Influence of straight through type of labyrinth gland parameters on the amount of air leakage. India Eng. J. ME 56, 176–181.
Rhode, D.L., Adams, R.G., 2004. Rub-groove width and depth effects on flow predictions for straight-through labyrinth seals. Trans. ASME J. Tribol. 126, 781–787.

Rhode, D.L., Allen, B.F., 1999. Measurement and Visualization of Leakage Effects of Rounded Teeth Tips and Rub-grooves on Stepped Labyrinths. ASME. Paper 99-GT-377.
Rhode, D.L., Johnson, J.W., Broussard, D.H., 1997a. Flow visualization and leakage measurements of stepped labyrinth seals: Part 1 — annular groove. Trans. ASME J. Turbomach. 119, 839–843.
Rhode, D.L., Younger, J.S., Wernig, M.D., 1997b. Flow visualization and leakage measurements of stepped labyrinth seals: Part 2 — sloping surfaces. Trans. ASME J. Turbomach. 119, 844–848.
Samsonov, Y.F., Solomko, V.I., Zezyulinskiy, G.S., 1978. Investigation of leakage of superheated and wet steam in labyrinth seals. Heat Transfer Sov. Res. 10, 80–87.
Shapiro, W., Chupp, R., 2004. Numerical, Analytical, Experimental Study of Fluid Dynamic Forces in Seals. In: Description of Seal Dynamics Code Dyseal and Labyrinth Seals Code KTK, vol. 5. NASA/CR-2004-213199/VOL5.
Sneck, H.J., 1974. Labyrinth seal literature survey. J. Lubr. Technol. Trans. ASME 96, 579–582.
Snow, E.W., 1952. Discussion. Proc. Inst. Mech. Eng. 166.
Society of Automotive Engineers, 1998. Sealing for Automotive Applications. SAE.
Solomko, V.I., Sivakov, V.I., Samsonov, V.F., 1979. Investigation of the leakage of superheated steam from radial labyrinth seals. Fluid Mechanics Soviet Research 8 (2), 18–25.
Steinetz, B.M., Hendricks, R.C., 1997. Engine seals. In: Zaretsky, E.V. (Ed.), Tribology for Aerospace Applications, Society of Tribologists and Lubrication Engineers.
Stepanoff, A.J., 1931. Leakage Loss and Axial Thrust in Centrifugal Pumps. ASME. Paper HYD-54-5.
Stocker, H.L., 1978. Determining and Improving Labyrinth Seal Performance in Current and Advanced High Performance Gas Turbines. AGARD CP-237.
Stocker, H.L., 1976. Advanced labyrinth seal design performance for high-pressure ratio gas-turbines. Mech. Eng. 98 (2), 83–83.
Stocker, H.L., Cox, D.M., and Holle, G.F., 1977. Aerodynamic performance of conventional and advanced design labyrinth seals with solid-smooth, abradable and honeycomb lands. NASA Report, CR-135307.
Stodola, A., 1927. Steam and Gas Turbines, sixth ed. McGraw Hill.
Summers-Smith, J.D. (Ed.), 1992. Mechanical Seal Practice for Improved Performance. IMechE, MEP.
The Engineer, Jan 1938. The labyrinth packing. The Engineer 165 (4280).
Tipton, D.L., Scott, T.E., Vogel, R.E., 1986. Labyrinth Seal Analysis. AFWAL-TR-85-2103. In: Volume III — Analytical and Experimental Development of a Design Model for Labyrinth Seals. AF Wright Aeronautical Laboratories.
Trutnovsky, K., Komotori, K., 1981. Berührungsfreie Dichtungen. VDI Verlag.
Ueda, T., Kubo, T., 1967. The leakage of air through radial labyrinth glands. Bull. JSME 1 (38), 298–307.
Vermes, G., 1961. A fluid mechanics approach to labyrinth seal leakage problem. J. Basic Eng. 83, 161–169.
Waschka, W., Wittig, S., Kim, S., 1992. Influence of high rotational speeds on the heat transfer and discharge coefficients in labyrinth seals. Trans. ASME J. Turbomach. 114, 462–468.
Willenborg, K., Kim, S., Wittig, S., 2001. Effects of Reynolds number and pressure ratio on leakage loss and heat transfer in a stepped labyrinth seal. Trans. ASME J. Turbomach. 123, 815–822.
Winter, J.R., 1998. Gasket selection — a flowchart approach. In: Bickford, J.H. (Ed.), Gaskets and Gasketed Joints. Marcel Dekker.
Yamada, Y., 1962. On the pressure loss of flow between rotating co-axial cylinders with rectangular grooves. Bull. JSME 5, 642–651.
Yeh, F.C., Cochran, R.P., 1970. Comparison of Experimental and Ideal Leakage Flows through Labyrinth Seals for Very Small Pressure Differences. NASA TMX-1958.
Yucel, U., 2004. Calculation of leakage and dynamic coefficients of stepped labyrinth gas seals. J. Appl. Math. Comput. 152, 521–533.
Yucel, U., Kazakia, J.Y., 2001. Analytical prediction techniques for axisymmetric flow in gas labyrinth seals. Trans. ASME J. Eng. Gas Turbines Power 123, 255–257.
Zimmermann, H., Wolff, K.H., 1998. Air System Correlations Part 1: Labyrinth Seals. Paper 98-GT-206. ASME.
Zimmermann, H., Kammerer, A., Wolff, K.H., 1994. Performance of Worn Labyrinth Seals. Paper 94-GT-131. ASME.
Zuk, J., 1976. Fundamentals of Fluid Sealing. NASA TN D-8151.

Further Reading

Buchter, H.H., 1979. Industrial Sealing Technology. Wiley.
Engineering Sciences Data Unit, 1980. Dynamic sealing of fluids 1: guide to selection of rotary seals. ESDU 80012.
Engineering Sciences Data Unit, 1983. Dynamic sealing of fluids 2: guide to selection of reciprocating seals. ESDU 83031.
Hopkins, R.B., 1996. Seals. In: Shigley, J.E., Mischke, C.R. (Eds.), Standard Handbook of Machine Design. McGraw Hill.
Morse, W., 1969. Seals. Morgan-Grampian.
Society of Automotive Engineers, 1996. SAE Fluid Sealing Handbook Radial Lip Seals. SAE.
Stone, R., 1992. Introduction to Internal Combustion Engines. Macmillan.
Warring, R.H., 1981. Seals and Sealing Handbook. Latty International.

Standards

British Standards Institution. BS 5341: Part 5 1976. Piston rings up to 200 mm diameter for reciprocating internal combustion engines. Ring grooves.
British Standards Institution. BS 4518: 1982. Specification for metric dimensions of toroidal sealing rings ("O" rings) and their housings.
British Standards Institution. BS 2492: 1990. Specification for elastomeric seals for joints in pipework and pipelines.
British Standards Institution. BS 5341: Part 7: Section 7.4: 1992. Piston rings up to 200 mm diameter for reciprocating internal combustion engines. Designs, dimensions and designations for single piece rings. Specifications for oil control rings.
British Standards Institution. BS 7780: Part 1: 1994. Specification for rotary shaft lip type seals. Nominal dimensions and tolerances.
British Standards Institution. BS 7780: Part 2: 1994. Specification for rotary shaft lip type seals. Vocabulary.
British Standards Institution. 98/706003 DC. Fluid systems. Sealing devices. O-rings. Part 2. Housing dimensions for general applications. (ISO/CD 3601-2:1997).
British Standards Institution. 99/714538 DC. ISO/DIS 3601-1. Fluid power systems. O-rings. Part 1. Inside diameters, cross sections, tolerances and size identification codes. 1999.
British Standards Institution. BS ISO 6194-4:1999. Rotary shaft lip type seals. Performance and test procedures.
British Standards Institution. BS EN 12756:2001. Mechanical seals: Principal dimensions, designations and material codes.
British Standards Institution. BS EN 3748:2001. Aerospace series. O-ring grooves. Dimensions.
British Standards Institution. BS ISO 16589-1:2001. Rotary shaft lip-type seals incorporating thermoplastic sealing elements. Nominal dimensions and tolerances.
British Standards Institution. BS ISO 16589-2:2001. Rotary shaft lip-type seals incorporating thermoplastic sealing elements. Vocabulary.

Web Sites

At the time of going to press the world-wide-web contained useful information relating to this chapter at the following sites.

fluidsciences.perkinelmer.com
oringswest.com/
www.allorings.com/
www.chesterton.com
www.china-dongya.com
www.claron.co.uk
www.coni-seal.com
www.federal-mogul.com
www.flexibox.com/

www.garlock-inc.com/
www.goodway-rubber.com
www.jenp.com.tw
www.mamat.com
www.nicholsons.co.uk
www.orings.com
www.o-rings-ez.com
www.pspglobal.com
www.samsunsegman.com
www.sealsales.com
www.ses-seal.com.tw
www.superseal.com
www.totalseal.com
www.trisunltd.com
www.uniquerubber.com
www.usseal.com
www.ussealmfg.com
www.vercoseals.com

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

a	outer radius (m)
A	annular cross-sectional flow area of the seal (m^2)
b	inner radius (m)
c	seal clearance (m)
C_d	discharge coefficient (−)
$C_{d,r}$	discharge coefficient for a labyrinth fin with rounded tip edge (−)
$C_{d,se}$	discharge coefficient for a labyrinth fin with a sharp tip edge (−)
D	diameter of outer fin (m)
D_h	hydraulic diameter, defined as $D_h = 2(r_o - r_i)$ (m)
e	groove depth (m)
f	friction factor, defined as $f = \Delta p/[(L/D)0.5]\rho u_{\text{mean}}^2$ (−)
g	acceleration due to gravity (m/s^2)
h	fin height (m)
H	step height (m)
k	carryover factor (−)
k_1	Hodkinson's carryover factor (−)
k_2	Zimmerman's carryover factor (−)
k_i	carryover factor (−)
k_s	correction factor for C_d (−)
l_{pitch}	pitch (m)
L	total length of seal (m)
L_{cell}	honeycomb cell dimension (m)
m	mass flow rate across the seal clearance area A, defined as $m = \rho u_{\text{mean}} A$ (kg/s)

$\boldsymbol{m}_{\mathbf{actual}}$ actual mass flow rate (kg/s)
$\boldsymbol{m}_{\mathbf{ideal}}$ ideal mass flow rate (kg/s)
Ma Mach number (−)
$\boldsymbol{n}$ number of labyrinth fins (−)
$\boldsymbol{p}$ static pressure (Pa)
$\boldsymbol{p_n}$ downstream static pressure (Pa)
$\boldsymbol{p_t}$ total pressure (Pa)
$\boldsymbol{p_{t0}}$ upstream total pressure (Pa)
$\boldsymbol{Q}$ mass flow function (kg $K^{0.5}$/(N s)), volumetric flow rate (m^3/s)
$\boldsymbol{r}$ radial coordinate (m)
$\boldsymbol{r}_{\mathbf{fillet}}$ fillet radius (m)
$\boldsymbol{r_i}$ radius of labyrinth fin tip (m)
$\boldsymbol{r_o}$ inner radius of outer cylinder (m)
$\boldsymbol{r}_{\mathbf{round}}$ rounding radius (m)
$\boldsymbol{R}$ characteristic gas constant (J/(kg K))
Re axial Reynolds number (−)
$\mathbf{Re}_{D_h}$ Reynolds number based on hydraulic diameter (−)
$\mathbf{Re}_{\phi}$ rotational Reynolds number (−)
$\boldsymbol{S_i}$ pressure factor (−)
$\boldsymbol{t}$ fin-tip land width (m)
$\boldsymbol{t_G}$ groove width (m)
$\boldsymbol{T}$ fluid temperature (K)
$\boldsymbol{T_t}$ fluid total temperature (K)
$\boldsymbol{T_{t0}}$ upstream fluid total temperature (K)
Ta Taylor number (−)
$\boldsymbol{u}_{\mathbf{mean}}$ mean velocity across the seal clearance (m/s)
$\boldsymbol{u_{\phi,ri}}$ fin-tip angular velocity component or fin-tip speed (m/s)
$\boldsymbol{x}$ axial coordinate (m)
$\boldsymbol{\varphi}$ diameter of the shaft (m)
$\boldsymbol{\phi}_{\mathbf{fin}}$ fin angle (degree)
$\boldsymbol{\gamma}$ isentropic index (−)
$\boldsymbol{\mu}$ fluid dynamic viscosity (Pa s)
$\boldsymbol{\rho}$ fluid density (kg/m^3)
$\boldsymbol{\rho_0}$ upstream fluid density (kg/m^3)
$\boldsymbol{\Delta m}$ mass flow change induced by honeycomb (kg/s)
$\boldsymbol{\Delta p}$ pressure difference (Pa)
$\boldsymbol{\Omega}$ shaft angular velocity (rad/s).

Subscripts

0 refers to flow conditions upstream of a labyrinth seal
i refers to flow conditions in the *i*th chamber or *i*th fin
n refers to flow conditions in the *n*th chamber or *n*th fin or downstream of a labyrinth seal.

CHAPTER 15

Springs

Chapter Outline

15.1 Introduction 625
15.2 Helical Compression Springs 629
15.3 Helical Extension Springs 648
15.4 Helical Torsion Springs 659
15.5 Leaf Springs 661
15.6 Belleville Spring Washers 664
15.7 Conclusions 671
References 672
Further Reading 672
Nomenclature 674

Abstract

Springs are used to store elastic energy and to release it when required. There is a wide range of types of spring that are readily available from specialist suppliers or that can be designed and manufactured fit-for-purpose. The aim of this chapter is to introduce spring technology and to outline the principal steps in the design of helical compression, extension, and torsion springs, leaf springs, and Belleville washer springs.

15.1 Introduction

Springs are flexible devices used to exert force or torque and store energy. The force produced by a spring can be compressive or tensile and linear or radial, as in the case of a helical torsion spring clip used to hold a tube on the end of a pipe. Alternatively, springs can be configured to produce a torque with applications including door closers. The process of deflecting a spring involves the transfer of energy into stored spring energy. When the force causing the spring deflection is removed, the stored spring energy will be returned.

Springs can be classified by the direction and nature of the force exerted when they are deflected. Several types of spring are listed in Table 15.1 according to the nature of the force or torque exerted. The principal characteristics of the various classes of springs are summarized in Table 15.2.

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00015-0
Copyright © 2014 Elsevier Ltd. All rights reserved.

Table 15.1: Classification of springs according to the nature of force or torque exerted.

Actuation	Type of spring
Compressive	Helical compression springs Belleville springs Flat springs; e.g. cantilever or leaf springs
Tensile	Helical extension springs Flat springs; e.g. cantilever or leaf springs Drawbar springs Constant force springs
Radial	Garter springs Elastomeric bands Spring clamps
Torque	Torsion springs Power springs

Table 15.2: Principal characteristics of a variety of types of spring.

Type of spring	Principal characteristics
Helical compression springs	These are usually made from round wire wrapped into a straight cylindrical form with a constant pitch between adjacent coils.
Helical extension springs	These are usually made from round wire wrapped into a straight cylindrical form but with the coils closely spaced in the no-load condition. As an axial load is applied, the spring will extend but resisting the motion.
Drawbar springs	A helical spring is incorporated into an assembly with two loops of wire. As a load is applied, the spring is compressed in the assembly resisting the motion.
Torsion springs	These exert a torque as the spring is deflected by rotation about their axes. A common example of the application of a torsion spring is a clothes peg.
Leaf springs	Leaf springs are made from flat strips of material and loaded as cantilever beams. They can produce a tensile or compressive force depending on the mode of loading applied.
Belleville springs	These comprise shallow conical discs with a central hole.
Garter springs	These consist of coiled wire formed into a continuous ring so that they can exert a radial inward force when stretched.
Volute springs	These consist of a spiral wound strip that functions during compression. They are subject to significant friction and hysteresis.

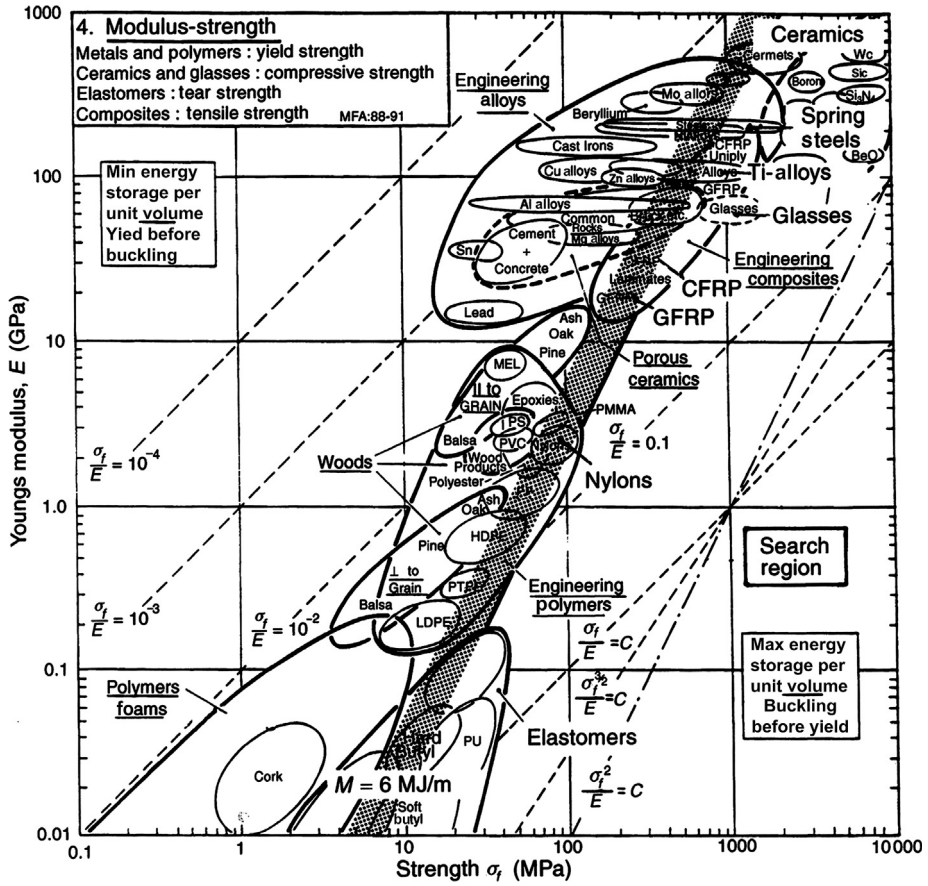


Figure 15.1
Materials for springs of minimum volume. *Reproduced from Ashby (1999).*

Virtually any material can be used to make springs. However, the ideal material would have a high ultimate strength and yield point and a low modulus of elasticity in order to provide maximum energy storage. Ashby (1999) developed two selection charts for springs, one based on physical size and the other on weight. Both of these charts are reproduced here as Figures 15.1 and 15.2. The highlighted line in Figure 15.1 illustrates the best choices for springs of minimum volume. These include high strength steels; glass-fiber reinforced plastic (GFRP), which is used for certain truck springs; titanium alloys; glass, which is used in galvanometers; and nylon, which is used in some toys. The choice for most applications tends to be limited to plain carbon steels, alloy steels, stainless steels, high-nickel steels, and copper-based alloys. Most spring materials are manufactured to American Society for Testing and Materials (ASTM), British Standards (BS), or Deutsche Industrie Normen (DIN) specifications. Typical properties of common spring materials are listed in Table 15.3. Round

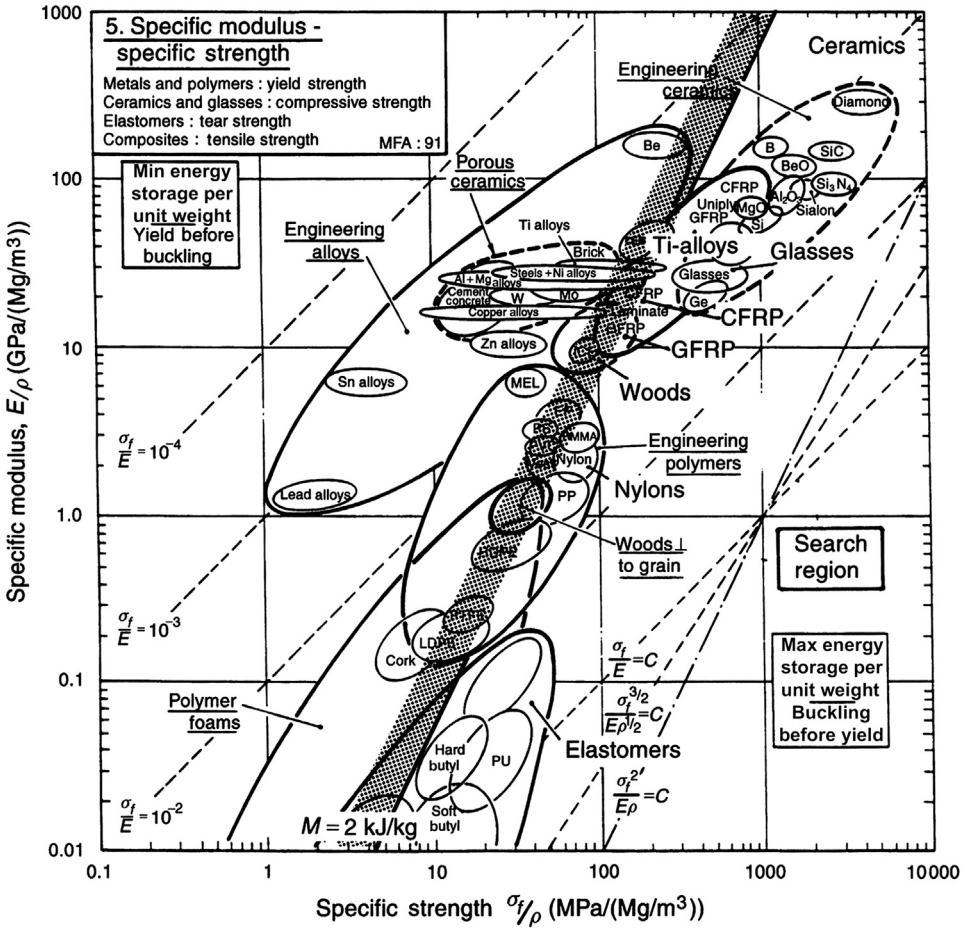


Figure 15.2
Materials for springs of minimum weight. *Reproduced from Ashby (1999).*

wire is the most common choice for spring material, and preferred diameters are listed in Table 15.4, although any diameter can be produced if necessary.

Important aspects in spring design include the determination of the spring material and dimensions in order to ensure that it will not fail due to either static or fluctuating loads for the lifetime required for the application, that it will not buckle or deform beyond allowable limits, that the natural frequencies of vibration are sufficiently in excess of the frequency of motion that they control, and that cost and aesthetic aspirations are met.

All materials deform with load. For a spring, the parameter modeling this is called the spring rate and is defined as the slope of the force-deflection curve of a spring. If the slope is constant, then

$$k = \frac{F}{\delta} \tag{15.1}$$

where k is the spring rate (N/m), F is the applied load (N), and δ is the deflection (m).

Table 15.3: Typical properties of common spring materials.

Material	Young's modulus (GPa)	Modulus of rigidity (GPa)	Density (kg/m^3)	Maximum service temperature (°C)
Music wire	207	79.3	7860	120
Hard drawn wire	207	79.3	7860	150
Oil tempered	207	79.3	7860	150
Valve spring	207	79.3	7860	150
Chrome vanadium alloy steel wire	207	79.3	7860	220
Chrome silicon alloy steel wire	207	79.3	7860	245
302 stainless steel	193	69	7920	260
17-7 PH stainless steel	203	75.8	7810	315
Phosphor bronze (A)	103	43.4	8860	95
Silicon bronze (A)	103	38.6	8530	95
Silicon bronze (B)	117	44.1	8750	95
Beryllium copper	128	48.3	8260	205
Inconel 600	214	75.8	8430	320
Inconel X750	214	79.3	8250	595
AISI 1050	207	79.3	7860	95
AISI 1065	207	79.3	7860	95
AISI 1074	207	79.3	7860	120
AISI 1095	207	79.3	7860	120

Source: Selected data reproduced from Joerres (1996).

The spring rate is also known as the spring constant or spring scale. Spring rates can be linear or nonlinear.

Springs can be combined in parallel, Figure 15.3, series, Figure 15.4, or some combination of these. For springs in parallel, the spring rates add directly,

$$k_{\text{total}} = \sum_{i=1}^{n} k_i = k_1 + k_2 + k_3 + \ldots + k_n. \tag{15.2}$$

For springs in series, the spring rates combine reciprocally:

$$\frac{1}{k_{\text{total}}} = \sum_{i=1}^{n} \frac{1}{k_i} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_n}. \tag{15.3}$$

15.2 Helical Compression Springs

The most familiar type of spring is the helical compression spring. In its most common form, it is made from constant diameter round wire with a constant pitch, as shown in Figure 15.5. Other forms are possible, such as the variable pitch, barrel, hourglass, and conical helical compression springs shown in Figure 15.6. In addition to variations on the

Table 15.4: Preferred wire diameters.

Diameter (in)	Diameter (mm)
0.004	0.10
0.005	0.12
0.006	0.16
0.008	0.20
0.010	0.25
0.012	0.30
0.014	0.35
0.016	0.40
0.018	0.45
0.020	0.50
0.022	0.55
0.024	0.60
0.026	0.65
0.028	0.70
0.030	0.80
0.035	0.90
0.038	1.00
0.042	1.10
0.045	
0.048	1.20
0.051	
0.055	1.40
0.059	
0.063	1.60
0.067	
0.072	1.80
0.076	
0.081	2.00
0.085	2.20
0.092	
0.098	2.50
0.105	
0.112	2.80
0.125	3.00
0.135	3.50
0.148	
0.162	4.00
0.177	4.50
0.192	5.00
0.207	5.50
0.225	6.00
0.250	6.50
0.281	7.00
0.312	8.00
0.343	9.00
0.362	
0.375	

Table 15.4: Preferred wire diameters.—cont'd

Diameter (in)	Diameter (mm)
0.406	10.0
0.437	11.0
0.469	12.0
0.500	13.0
0.531	14.0
0.562	15.0
0.625	16.0

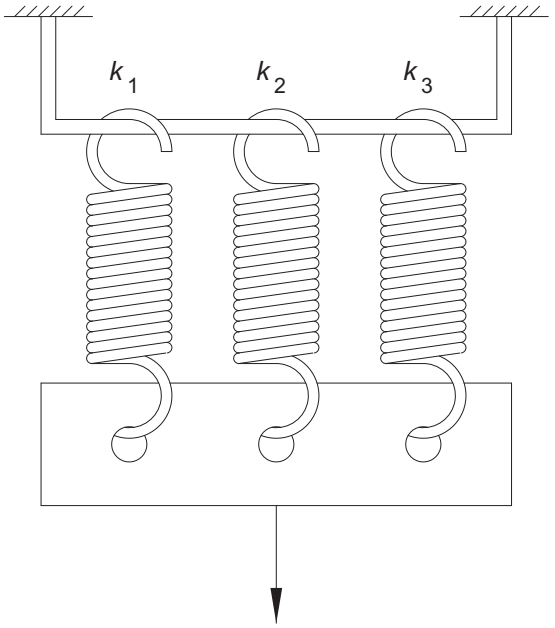


Figure 15.3
Springs in parallel.

pitch of the coil and diameter, the formation of the end is important. A variety of common end treatments is illustrated in Figure 15.7. Plain ends result from cutting the spring stock and leaving the spring with a constant pitch. Treatment of the end by some form of machining or pressing can facilitate alignment, and this is the purpose of options (b)—(d) illustrated in Figure 15.7, each of which adds to the cost of production of the spring and influences the performance. The end of a spring can also be formed to improve the connection to mating components by the incorporation of, for example, hooks and rings.

The principal dimensions for a constant pitch helical compression spring are illustrated in Figure 15.8. The wire diameter d, mean diameter D, free length L_f, and either the number of

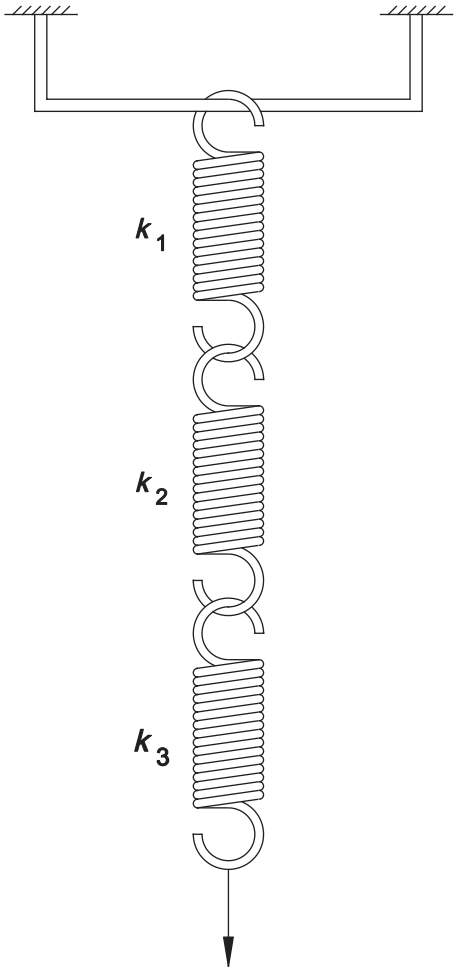


Figure 15.4
Springs in series.

coils N or the pitch p are used to define a helical spring's geometry and in associated analysis. The inner and outer diameter are useful in designing the mating and locating components. The minimum recommended diametral clearance between the outer diameter and a hole or between the inner diameter and a pin, according to Associated Spring (1987), is given by 0.10 D for $D < 13$ mm or 0.05 D for $D > 13$ mm.

In addition to the geometrical parameters identified in Figure 15.8 for an unloaded spring, there are a number of useful lengths defined for a spring in use, as illustrated in Figure 15.9. The installed length is the length after installation with initial deflection δ_{initial}. The operating length is the shortest dimension to which the spring is compressed in use. The shut height or solid length is the length of the spring when the spring is loaded such that the coils

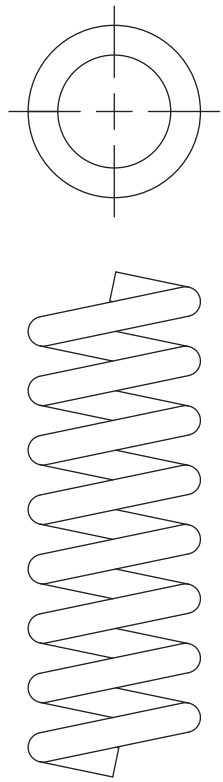


Figure 15.5
A constant pitch helical compression spring.

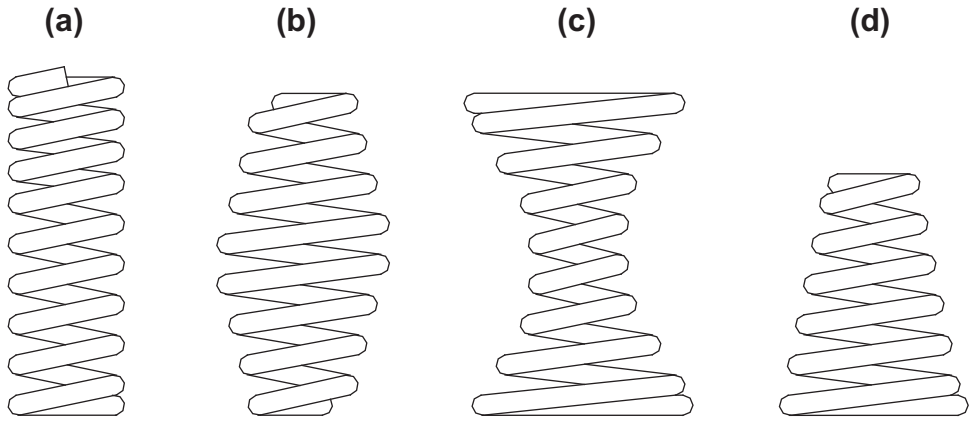


Figure 15.6
Some additional helical spring configurations. (a) Variable pitch. (b) Barrel. (c) Hourglass. (d) Conical.

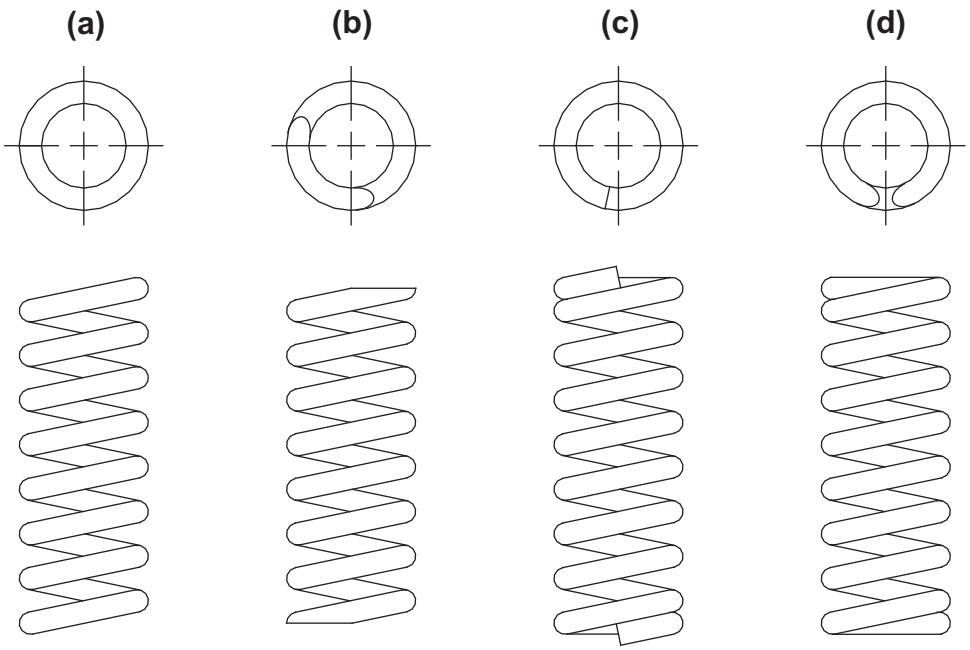


Figure 15.7
Common styles of end treatments for helical compression springs. (a) Plain ends. (b) Plain ground ends. (c) Squared ends. (d) Squared ground ends.

are actually touching. This is the shortest possible length for the spring without crushing it beyond all recognition.

Springs are subject to failure by yielding due to too high a stress in the case of static loading or by fatigue in the case of dynamic loading. In order to determine the geometry of a spring to avoid such failure, or to determine when failure will occur, it is necessary to consider the stresses experienced by a spring under loading.

The free-body diagram for a helical spring loaded with force F is illustrated in Figure 15.10. There are two components of stress on any cross section of coil: a torsional shear stress due to the torque and a direct shear stress due to the force. The stresses accumulate and the maximum shear stress will occur at the inner fiber of the wire's cross section.

$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A} = \frac{F(D/2)(d/2)}{\pi d^4/32} + \frac{F}{\pi d^2/4} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2} \tag{15.4}$$

where $\tau_{\max}$ is the maximum shear stress (N/m^2), T is the torque (N m), r is the radius (m), J is the polar second moment of area (m^4) $= \pi d^4/32$ for a solid circular cylinder, F is

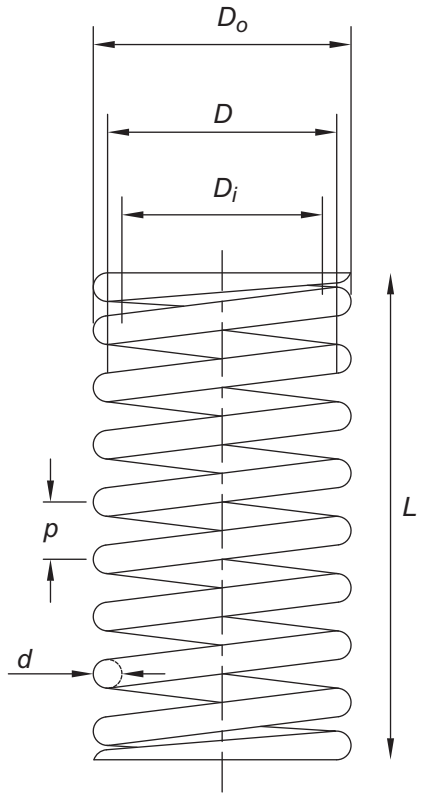


Figure 15.8
Dimensional parameters for helical compression springs.

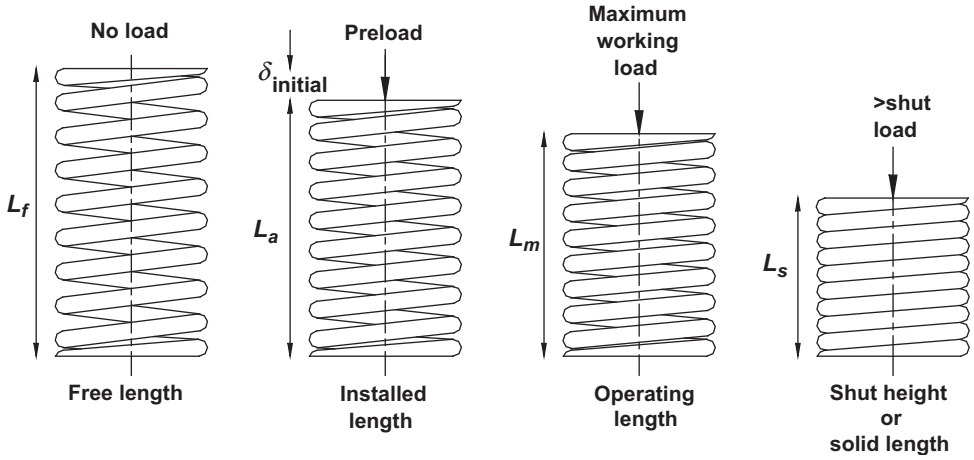


Figure 15.9
Various lengths associated with a spring in use.

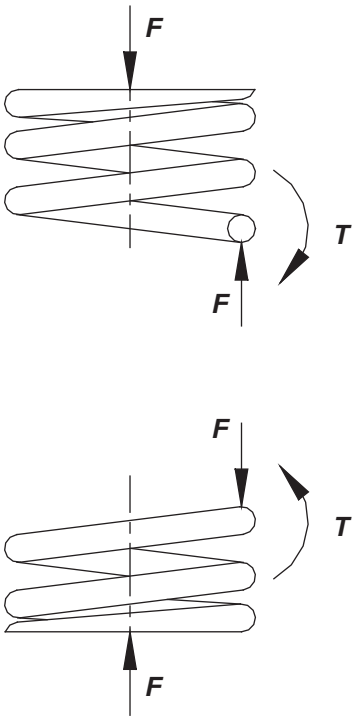


Figure 15.10
Free-body diagram for a helical compression spring loaded with force *F*.

the load (N), A is the area (m^2), d is the wire diameter (m), and D is the mean coil diameter (m).

The spring index is defined by

$$C = \frac{D}{d}. \tag{15.5}$$

Preferred values for the spring index are between 4 and 12. For values of the spring index below 4, springs are difficult to manufacture, and for values greater than 12, they are prone to buckling. Buckling can be prevented by providing either internal or external support by, for instance, placing the spring in a hole or on a rod. However, frictional contact between the spring and the support may reduce the force that can be delivered at the spring end. The tendency of a helical spring to buckle is proportional to the slenderness factor, which is the ratio of the free length to the mean coil diameter, L_f/D, the ratio of the spring's deflection to its free length, δ/L_f, and the type of end support used. Figure 15.11 provides guidance for determining whether a particular combination of these factors results in a stable spring or a spring prone to buckling.

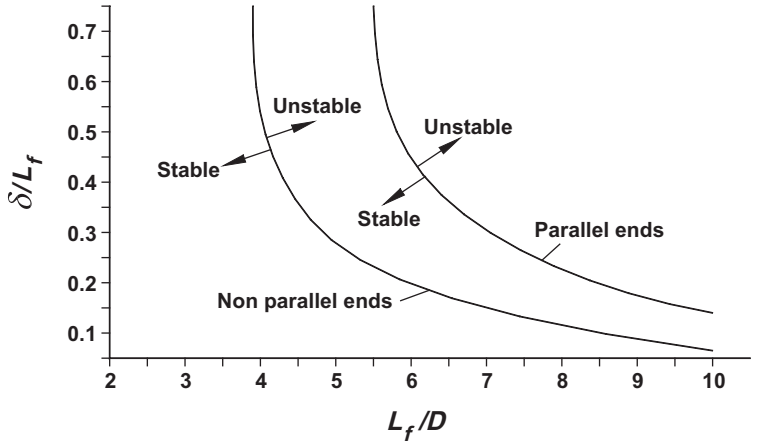


Figure 15.11
Critical buckling curves for two types of end condition. *Adapted from Associated Spring, Barnes Group (1987).*

Substituting for the spring index in Eqn (15.4) gives

$$\tau_{\max} = \frac{8FD}{\pi d^3}\left(1 + \frac{1}{2C}\right) = K_s \frac{8FD}{\pi d^3} \tag{15.6}$$

where K_s is called the direct shear factor and is given by

$$K_s = 1 + \frac{1}{2C}. \tag{15.7}$$

Curved beams have a stress concentration on the inner surface of curvature. The Wahl factor, K_w, includes both direct shear stress effects and the stress concentration factor due to curvature and is given by

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \tag{15.8}$$

$$\tau_{\max} = K_w \frac{8FD}{\pi d^3} = K_w \frac{8FC}{\pi d^2}. \tag{15.9}$$

It is commonly recommended that Eqn (15.9) be used for modeling fatigue and Eqn (15.6) for modeling a spring under static loading only.

For springs with end treatments, it is necessary to account for whether some of the coils are inactive and can be neglected in calculations for stress and deflection. For a spring with squared and ground ends or simply squared ends, each end coil is inactive and the number of active coils is given by

$$N_a = N - 2 \tag{15.10}$$

where N is the total number of coils in the spring.

For plain coils with ground ends,

$$N_a = N - 1. \tag{15.11}$$

The deflection of a helical spring under load F is given by

$$\delta = \frac{8FD^3N_a}{d^4G} = \frac{8FC^3N_a}{dG} \tag{15.12}$$

where N_a is the number of active coils in the spring.

Thus the spring rate for a helical spring is given by

$$k = \frac{F}{\delta} = \frac{d^4G}{8D^3N_a} = \frac{dG}{8C^3N_a}. \tag{15.13}$$

It should be noted that it is unlikely that a load on a spring will be directly on its geometric axis. Eccentric loading of a spring causes the stresses on one side of the spring to be higher than indicated in Eqns (15.6) and (15.9).

Springs can vibrate both laterally and longitudinally when excited near their natural frequencies. If a helical spring, fixed at one end, is given a sufficiently rapid compression at the other, the end coil will be pushed against its neighbor before the remaining coils have time to respond to the displacement. This compression then propagates down the spring with first coils one and two in contact, then coils two and three in contact, and so on until the compression wave reaches the other end, where the disturbance will be reflected back. This process repeats itself until the motion is damped out. This phenomenon is known as spring surge and causes very high stresses in the spring, which are approximately equal to those when the spring is compressed to its solid length. The natural frequency, f_n, of the spring surge depends on the boundary conditions. For the fixed-fixed case,

$$f_n = \frac{1}{2}\sqrt{\frac{k}{m}} \tag{15.14}$$

where f_n is the natural frequency (Hz), k is the spring rate (N/m), and m is the mass (kg).

The mass of a helical spring is equal to the product of density and volume, so, for N coils in a spring, this is given by

$$m = \rho V = \rho N(\pi D)\left(\pi d^2/4\right) = \rho N\pi^2 d^2 D/4. \tag{15.15}$$

Substitution for the mass and spring rate in Eqn (15.14) gives

$$f_n = \frac{d}{2\pi D^2 N}\sqrt{\frac{G}{2\rho}}. \tag{15.16}$$

For steel springs with a modulus of rigidity of 79.3 GPa and density 7860 kg/m^3,

$$f_n = \frac{357d}{D^2 N} \tag{15.17}$$

where d is the wire diameter (m), D is the coil diameter (m), and N is the number of coils.

In order to avoid surge, the spring should not be cycled at a frequency close to its natural frequency. The natural frequency of spring surge should usually be made higher than the highest significant harmonic of the motion involved, which is typically about the 13th harmonic. So the natural frequency should be at least 13 times the forcing frequency of the load on the spring in order to avoid resonance. Designing springs with high natural frequencies typically involves operating at the highest possible stress levels as this minimizes the mass of the spring and thereby maximizes f_n, which is proportional to $m^{-0.5}$.

Springs are subject to creep under load. This is sometimes evident in old cars where the sustained weight on the suspension springs over the years has caused a permanent shortening of the spring's overall length and the car body's ground clearance is reduced. This shortening by creep is known as set. Set is directly related to yield stress. Unfortunately data for yield stress are not as readily available as those for the ultimate tensile strength. Values for the ultimate tensile strength of wire are found to vary with diameter and are illustrated in Figure 15.12 for a range of materials. Juvinall and Marshek (1991) recommend the approximations given in Table 15.5 for shear stress in order to limit set in compression coil springs to 2%.

Prestressing, also known as presetting, of a spring can be used to improve a spring's ability to withstand stress, increasing its load-carrying capability and fatigue resistance. For example, compression springs manufactured from BS EN10270-1 cold drawn carbon steel without prestressing can to be loaded to 49% of the material's ultimate tensile strength. With prestressing it can be loaded to 70% of its ultimate tensile strength. Prestressing takes place after the spring has been coiled, stress-relieved, and ground. It involves compressing the spring to its solid length or a fixed position that is greater than its maximum working length. This process is repeated a number of times, typically no less than three. During prestressing, the spring's dimensions will alter. For a particular desired final length, the spring's dimensional changes must be accounted for by the manufacturer.

There are a number of strategies that can be followed in designing helical compression springs (see Mott, 1999). One approach, knowing the force and length of the spring, is to

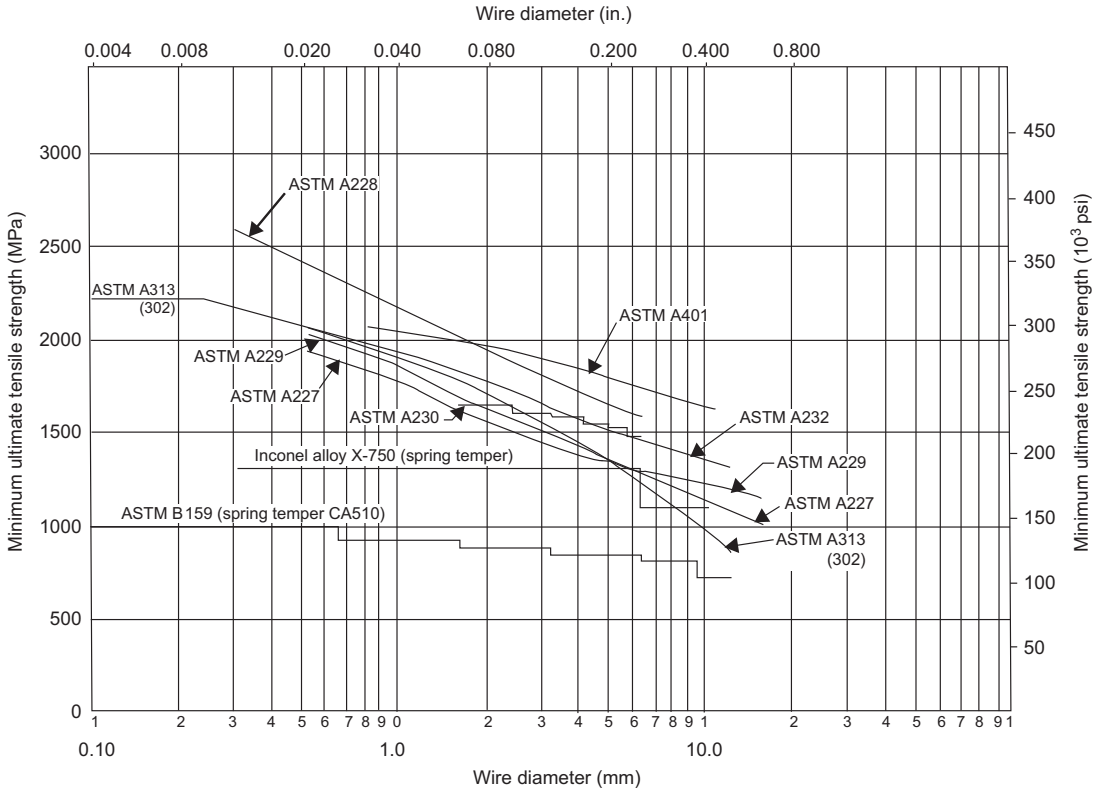


Figure 15.12
Minimum tensile strengths of spring wire. *Associated Spring, Barnes Group Inc.*

Table 15.5: Solid shear stress to limit long-term set in compression coil springs to less than 2%.

τ_{solid}	Material condition
$0.45\sigma_{uts}$	Ferrous without presetting.
$0.35\sigma_{uts}$	Nonferrous and austenitic stainless without presetting.
$0.65\sigma_{uts}$	Ferrous with presetting.
$0.55\sigma_{uts}$	Nonferrous and austenitic stainless with presetting.

Source: Juvinall and Marshek, 1991.

specify a material, guess a trial diameter for the spring considering the space available, check the values calculated for spring rate and free length, and if necessary try a new wire diameter. This approach is outlined below and in the following example. The design procedure requires access to tables of data for material properties and wire diameters.

1. Select a material and identify its shear modulus of elasticity G.
2. Identify the operating force, F_o, operating length, L_o, the installed force, F_i, and the installed length, L_i.

3. Determine the spring rate, $k = (F_o - F_i)/(L_i - L_o)$.
4. Calculate the free length, $L_f = L_i + (F_i/k)$.
5. Specify an initial estimate for the mean diameter.
6. Specify an initial design stress. An estimate for the initial design stress can be made using Table 15.5 and Figure 15.12.
7. Calculate a trial wire diameter, d, by rearranging Eqn (15.6) or Eqn (15.9) and assuming a value for K_s or K_w, which is unknown at this stage. $K_w = 1.2$ is generally a suitable estimate at this stage.
8. Based on the value determined in (7), select a standard wire diameter from a wire manufacturer's catalog or using Table 15.4 as a guide.
9. Calculate the spring index, C, and the Wahl factor, K_w.
10. Determine the expected stress due to the operating force and compare with the design stress using Eqn (15.6) or Eqn (15.9).
11. Determine the number of active coils required to give the desired deflection characteristics for the spring, $N_a = Gd/(8kC^3)$.
12. Calculate the solid length, the force on the spring at the solid length, and the shear stress in the spring at the solid length. Compare this value with the allowable shear stress and see whether it is safe. If the value for the shear stress is too high, alter the design parameters set above, such as the wire diameter or material, and reanalyze the results.
13. Check whether the spring is likely to buckle.
14. Specify the spring dimensions.

Example 15.1

A helical compression spring is required to exert a force of 35 N when compressed to a length of 60 mm. At a length of 48 mm, the force must be 50 N. The spring is to be installed in a hole with a diameter of 24 mm. The application involves slow cycling, and a total life of 250,000 cycles is required. The maximum temperature of operation is 80 °C.

Solution

A standard material such as ASTM A232 chromium vanadium steel wire is suggested as an initial starting point for this application. The maximum service temperature for this material is approximately 120 °C, which is greater than the operating temperature for the application. The application involves slow cyclic variation of the load so Eqn (15.9) can be used for determining the spring's stress level.

From Table 15.3 for chrome vanadium steel, $G = 79.3$ GPa.

The maximum operating force, F_o, is 50 N with an operating length, L_o, of 48 mm. The installed force, F_i, is 35 N at an installed length, L_i, of 60 mm.

The spring rate k is given by

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{50 - 35}{0.06 - 0.048} = 1250 \text{ N/m}.$$

The free length L_f is given by

$$L_f = L_i + \frac{F_i}{k} = 0.06 + \frac{35}{1250} = 0.088 \text{ m}.$$

A mean diameter of 18 mm is suggested, as this should locate in the 24 mm diameter hole available, allowing room for the diameter of the wire itself, which is currently undetermined.

A design shear stress of $0.45\sigma_{\text{uts}}$ is suggested. σ_{uts} for ASTM A232 chrome vanadium steel with an assumed wire diameter of 2 mm is approximately 1700 MPa (see Figure 15.12). The design shear stress will therefore be $0.45 \times 1700 = 765$ MPa. If the wire diameter calculated in the design procedure is significantly different, then the estimate for the design shear stress will need to be modified.

The equation for shear stress requires a value for K_w. K_w is a function of $C = d/D$, but the wire diameter d is unknown at this stage. A value of $K_w = 1.2$ can be used as an initial estimate here, although this will be checked and modified later in the procedure.

$$d = \left(\frac{8FDK_w}{\pi\tau_{\max}}\right)^{1/3} = \left(\frac{8 \times 50 \times 0.018 \times 1.2}{\pi 765 \times 10^6}\right)^{1/3} = 1.532 \times 10^{-3} \text{ m}.$$

The estimated wire diameter is therefore 1.53 mm. Examination of Table 15.3 or a stock spring supplier's catalog will identify a suitably close wire diameter to select. In this case from Table 15.3, the nearest larger wire diameter is 1.6 mm.

The spring index can now be calculated,

$$C = \frac{D}{d} = \frac{0.018}{0.0016} = 11.25.$$

The Wahl factor is

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.25 - 1}{4 \times 11.25 - 4} + \frac{0.615}{11.25} = 1.128.$$

The maximum shear stress is given by

$$\tau_{\max} = K_w \frac{8FD}{\pi d^3} = 1.128 \frac{8 \times 50 \times 0.018}{\pi (0.0016)^3} = 631 \times 10^6 \text{ Pa}.$$

This is significantly lower than the allowable maximum shear stress, so the design seems acceptable at this stage.

The number of active coils can be determined from

$$N_a = \frac{Gd}{8kC^3} = \frac{79.3 \times 10^9 \times 0.0016}{8 \times 1250 \times (11.25)^3} = 8.91 \text{ coils.}$$

The solid length of a spring occurs when all the coils are touching. If the spring is squared and ground, then there will be two inactive coils.

$$L_s = d(N + 2) = 0.0016(8.91 + 2) = 0.0175 \text{ m.}$$

The force at the solid length would be

$$F_s = k\left(L_f - L_s\right) = 1250(0.088 - 0.0175) = 88.2 \text{ N.}$$

As stress in a spring is directly proportional to force, the solid length shear stress can be determined by

$$\tau_{\text{solid}} = \tau_{\max}\frac{F_s}{F_o} = 631 \times 10^6 \frac{88.2}{50} = 1113 \times 10^6 \text{ Pa.}$$

This value is greater than the maximum allowable stress of 765 MPa indicating that failure is likely. It is necessary to alter one or more of the design parameters in order to develop a more suitable design. One approach is to try a different wire diameter.

If $d = 0.0018$ m is tried, then

$$C = \frac{D}{d} = \frac{0.018}{0.0018} = 10$$

$$K_w = \frac{4 \times 10 - 1}{4 \times 10 - 4} + \frac{0.615}{10} = 1.145$$

$$\tau_{\max} = 1.145\frac{8 \times 50 \times 0.018}{\pi(0.0018)^3} = 450 \times 10^6 \text{ Pa.}$$

This is significantly lower than the allowable maximum shear stress, so the design seems acceptable at this stage.

$$N_a = \frac{79.3 \times 10^9 \times 0.0018}{8 \times 1250 \times (10)^3} = 14.27 \text{ coils}$$

$$L_s = 0.0018(14.27 + 2) = 0.0293 \text{ m}$$

$$F_s = 1250(0.088 - 0.0293) = 73.4 \text{ N}$$

$$\tau_{\text{solid}} = 450 \times 10^6 \frac{73.4}{50} = 660 \times 10^6 \text{ Pa.}$$

This is below the maximum allowable stress, indicating an acceptable design.

The ratio of the free length to the mean coil diameter is $L_f/D = 0.088/0.018 = 4.89$. The deflection of the spring under the operating force is 40 mm. Thus the ratio of deflection to free length is $\delta/L_f = 0.04/0.088 = 0.455$. Examination of Figure 15.11 for squared and ground ends shows that this design is stable and unlikely to buckle.

The outer diameter of the spring can be determined from

$$D_o = d + D = 0.0018 + 0.018 = 0.0198 \text{ m.}$$

The inner diameter of the spring will be

$$D_i = D - d = 0.018 - 0.0018 = 0.0162 \text{ m.}$$

Nominal specification:

ASTM A232 chromium vanadium steel wire helical compression spring,
$k = 1250$ N/m,
$L_f = 0.088$ m,
$L_s = 0.0293$ m.
Squared and ground ends, 16.27 coils,
$d = 1.8$ mm,
$D = 18$ mm, $D_i = 16.2$ mm, $D_o = 19.8$ mm.

If there are no constraints on the spring diameter, then the procedure can be modified as outlined below.

1. Select a material and identify its shear modulus of elasticity G and an estimate of the design stress.
2. Calculate a value for the wire diameter based on the spring material properties and assuming approximate values for C and K_w. Typical estimates for C and K_w of 7 and 1.2, respectively, are generally suitable.

$$d = \sqrt{\frac{8K_w F_o C}{\pi \tau_{\max}}}. \tag{15.18}$$

3. Select a standard wire diameter.

4. Determine the maximum number of active coils possible for the spring. The rationale here is that the solid length must be less than the operating length. The relationship between the solid length and the number of coils is a function of the end treatment. For squared and ground ends,

$$L_s = d(N_a + 2). \tag{15.19}$$

Substituting $L_s = L_o$

$$N_{\max} = \frac{L_o}{d} - 2. \tag{15.20}$$

5. The designer can now select any number of active coils less than the maximum calculated value. Choosing a small value will result in larger clearances between adjacent coils and will use less wire per spring but will entail higher stresses for a given load. One approach is to try progressively fewer coils until the maximum permissible design stress is approached.
6. Calculate the spring index using

$$C = \left(\frac{Gd}{8kN_a}\right)^{1/3}. \tag{15.21}$$

7. The mean diameter can now be calculated from the definition of the spring index, $D = Cd$.
8. Calculate the Wahl factor, K_w.
9. Determine the expected stress due to the operating force and compare with the design stress.
10. Calculate the solid length, the force on the spring at the solid length, and the shear stress in the spring at the solid length. Compare this value with the allowable shear stress and see whether it is safe.
11. Check whether the spring is likely to buckle.
12. Specify the spring dimensions.

Example 15.2

A helical spring is required to exert a force of 160 N at a length of 170 mm and 200 N at a length of 150 mm. Specify a suitable spring for this application. The maximum operating temperature is 50 °C, the load varies slowly with time, and a total life of 200,000 cycles is required.

Solution

A standard material such as ASTM A232 chromium vanadium steel wire is suggested as an initial starting point for this application. The maximum service temperature for this material is approximately 120 °C, which is greater than the operating temperature for the application. The application involves slow cyclic variation of the load, so Eqn (15.9) should be used for determining the spring's stress level.

From Table 15.3 for chromium vanadium steel, $G = 79.3$ GPa.

The maximum operating force, F_o, is 200 N with an operating length, L_o, of 150 mm. The installed force, F_i, is 160 N at an installed length, L_i, of 170 mm.

The spring rate k is given by

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{200 - 160}{0.17 - 0.15} = 2000 \text{ N/m}.$$

The free length L_f is given by

$$L_f = L_i + \frac{F_i}{k} = 0.17 + \frac{160}{2000} = 0.25 \text{ m}.$$

A design shear stress of $0.45\sigma_{\text{uts}}$ is suggested. σ_{uts} for ASTM A232 chrome vanadium steel with an assumed wire diameter of 3 mm is approximately 1600 MPa (see Figure 15.12). The design shear stress will therefore be $0.45 \times 1700 = 720$ MPa

$$d = \sqrt{\frac{8K_wF_oC}{\pi\tau_{\max}}} = \sqrt{\frac{8 \times 1.2 \times 200 \times 7}{\pi 7.2 \times 10^8}} = 2.438 \times 10^{-3} \text{ m}.$$

Choosing $d = 3$ mm,

$$N_{\max} = \frac{L_o}{d} - 2 = \frac{0.15}{0.003} - 2 = 48.$$

Trying $N = 35$ gives

$$C = \left(\frac{Gd}{8kN_a}\right)^{1/3} = \left(\frac{79.3 \times 10^9 \times 0.003}{8 \times 2000 \times 35}\right)^{1/3} = 7.517.$$

$$D = Cd = 0.02255 \text{ m},$$
$$K_w = 1.197,$$

$$\tau_{\max} = K_w\frac{8FD}{\pi d^3} = 1.197\frac{8 \times 200 \times 0.02255}{\pi(0.003)^3} = 509 \times 10^6 \text{ Pa}.$$

This is significantly lower than the allowable maximum shear stress, so the design seems acceptable at this stage.

$$L_s = d(N_a + 2) = 0.003(35 + 2) = 0.111 \text{ m}.$$

The force at the solid length would be

$$F_s = k\left(L_f - L_s\right) = 2000(0.25 - 0.111) = 278 \text{ N}.$$

The solid length shear stress can be determined by

$$\tau_{\text{solid}} = \tau_{\max}\frac{F_s}{F_o} = 509 \times 10^6 \frac{278}{200} = 708 \times 10^6 \text{ Pa}.$$

This value is just below the maximum allowable stress of 720 MPa, indicating that failure is unlikely.

The ratio of the free length to the mean coil diameter is $L_f/D = 0.25/0.02255 = 11.09$. The deflection of the spring under the operating force is 100 mm. Thus the ratio of deflection to free length is $\delta/L_f = 0.1/0.25 = 0.4$. Examination of Figure 15.11 for squared and ground ends shows that this design is unstable and likely to buckle.

Therefore, the design parameters need to be changed in order to ensure the spring does not buckle. Decreasing the number of coils will increase the stress, which is already approaching the limit at the solid length. A different wire diameter is proposed.

If $d = 0.004$ m,

$$N_{\max} = \frac{L_o}{d} - 2 = \frac{0.15}{0.004} - 2 = 35.5.$$

Trying a low value for the number of active coils, say $N_a = 16$, gives

$$C = \left(\frac{Gd}{8kN_a}\right)^{1/3} = \left(\frac{79.3 \times 10^9 \times 0.004}{8 \times 2000 \times 16}\right)^{1/3} = 10.74.$$

$$D = Cd = 0.04296 \text{ m},$$
$$K_w = 1.134.$$

σ_{uts} for ASTM A232 chrome vanadium steel with an assumed wire diameter of 4 mm is approximately 1550 MPa (see Figure 15.12). The design shear stress will therefore be $0.45 \times 1550 = 698$ MPa

$$\tau_{\max} = K_w\frac{8FD}{\pi d^3} = 1.134\frac{8 \times 200 \times 0.04296}{\pi(0.004)^3} = 388 \times 10^6 \text{ Pa}.$$

This is significantly lower than the allowable maximum shear stress, so the design seems acceptable at this stage.

$$L_s = d(N_a + 2) = 0.004(16 + 2) = 0.072 \text{ m}.$$

The force at the solid length would be

$$F_s = k\left(L_f - L_s\right) = 2000(0.25 - 0.072) = 356 \text{ N}.$$

The solid length shear stress can be determined by

$$\tau_{\text{solid}} = \tau_{\max}\frac{F_s}{F_o} = 388 \times 10^6 \frac{356}{200} = 691 \times 10^6 \text{ Pa}.$$

This value is below the maximum allowable stress of 720 MPa, indicating that failure is unlikely.

The ratio of free length to the mean coil diameter, $L_f/D = 0.25/0.04296 = 5.819$. The deflection of the spring under the operating force is 100 mm. Thus the ratio of deflection to free length is $\delta/L_f = 0.1/0.25 = 0.4$. Examination of Figure 15.11 for squared and ground ends shows that this design is stable and unlikely to buckle.

The outer diameter of the spring can be determined from

$$D_o = d + D = 0.004 + 0.04296 = 0.04696 \text{ m}.$$

The inner diameter of the spring will be

$$D_i = D - d = 0.04296 - 0.004 = 0.03896 \text{ m}.$$

Nominal specification:

ASTM A232 chromium vanadium steel wire helical compression spring,
$k = 2000$ N/m,
$L_f = 0.25$ m,
$L_s = 0.072$ m.
Squared and ground ends, 18 coils,
$d = 4$ mm,
$D = 42.96$ mm, $D_i = 38.96$ mm, $D_o = 46.96$ mm.

15.3 Helical Extension Springs

Helical extension springs are loaded in tension and exert a pulling force as well as storing energy. They are similar in appearance to helical compression springs but tend to be manufactured so that adjacent coils touch when at their free length. Hooks or loops are incorporated as part of the spring to facilitate attachment. The principal dimensions of an extension spring are illustrated in Figure 15.13.

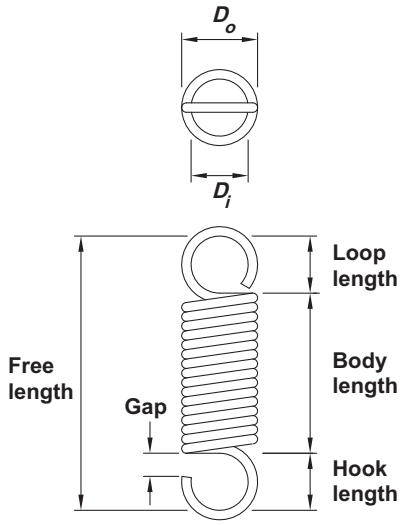


Figure 15.13
Principal dimensions for a helical extension spring with hook and loop end configurations.

The coils of extension springs are tightly wound together and the wire is twisted as it is wound, creating a preload in the coils that must be overcome in order to separate them. The spring rate for an extension coil is linear after the initial coil separation section and can be modeled by

$$k = \frac{F_o - F_i}{\delta} = \frac{Gd^4}{8N_aD^3}. \tag{15.22}$$

The spring index is given by $C = D/d$ and should be kept in the range of 4−12.

The deflection of an extension spring can be determined by modifying Eqn (15.10) to account for the preload

$$\delta = \frac{8(F_o - F_i)D^3N_a}{d^4G}. \tag{15.23}$$

In addition to accounting for stresses in the spring coils, it is also necessary to consider the stresses in the hooks and loops used in an extension spring. The standard loop end illustrated in Figure 15.14 has a high bending stress at point A and a high torsional stress at point B.

The bending stress at point A can be determined using

$$\sigma_A = K_b\frac{16DF}{\pi d^3} + \frac{4F}{\pi d^2} \tag{15.24}$$

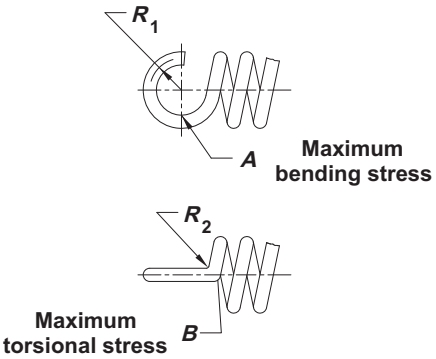


Figure 15.14
Locations of maximum stress in the loop of an extension spring.

where

$$K_b = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} \tag{15.25}$$

$$C_1 = \frac{2R_1}{d}. \tag{15.26}$$

The torsional stress at B can be found from Wahl (1963)

$$\tau_B = K_{w2}\frac{8DF}{\pi d^3} \tag{15.27}$$

where

$$K_{w2} = \frac{4C_2 - 1}{4C_2 - 4} \tag{15.28}$$

$$C_2 = \frac{2R_2}{d}. \tag{15.29}$$

C_2 should be greater than 4 to avoid high stresses.

In a helical extension spring, all the coils are considered active and in addition a fictitious coil is added to the number of coils in order to calculate the required body length.

Design procedures for extension springs can be developed in a similar fashion to the ones outlined for compression springs. The procedure given here relies upon the designer having access to data for the maximum allowable torsional shear stress. If at any step the values arrived at are deemed unacceptable, then parameters previously determined need to be reconsidered.

1. Select a suitable material for the spring and determine the relevant material parameters, G, τ.
2. Assume a trial mean diameter for the spring.
3. Calculate a diameter for the spring wire, using Eqn (15.6) or Eqn (15.9), assuming a value for K of 1.2.
4. Select the nearest larger diameter wire from Table 15.4 or a spring supplier.
5. Determine the design stress for the wire diameter selected.
6. Determine the outer, mean, and inner diameters for the spring.
7. Determine the spring index and the Wahl factor.
8. Calculate the maximum shear stress in the spring under the operating load.
9. Determine the number of coils necessary to produce the desired deflection characteristics.
10. Calculate the body length of the spring, propose the end configurations, and determine the free length.
11. Calculate the deflection from the free length.
12. Calculate the initial force at which the spring coils just begin to separate. The initial tension in an extension spring is typically 10%−25% of the maximum design force.
13. Determine the stress in the spring under the initial tension and compare it with the recommended values. Recommended values for the initial shear stress are in the range $0.4\sigma_{uts}/C \leq \tau_{initial} \leq 0.8\sigma_{uts}/C$.

Example 15.3

A helical extension spring is required to exert a force of 30 N when the length between attachment locations is 70 mm and a force of 20 N at a length of 60 mm. The spring will be cycled through its load 100 times a day. ASTM A228 music wire steel has been proposed for the spring. The diameter of the spring should not exceed 20 mm. Determine the suitable dimensions for the spring wire diameter, the number of coils, and the mean diameter.

Solution

$$k = \frac{30 - 20}{70 - 60} = 1000 \text{ N/m}.$$

Assuming a nominal wire diameter of 1.8 mm, the approximate ultimate tensile strength for A228 music wire is $\sigma_{uts} = 1900$ MPa.

$$\tau_{max} = 0.45 \times 1900 = 855 \text{ MPa}.$$

A mean coil diameter of 15 mm is proposed.

$$d = \left(\frac{8 \times 30 \times 0.015 \times 1.2}{\pi 855 \times 10^6}\right)^{0.33} = 1.172 \times 10^{-3} \text{ m}.$$

Try $d = 1.4$ mm.
Design stress is $\tau_{max} = 0.45 \times 2100 = 945$ MPa. This looks acceptable.

$$D_i = 0.015 - 0.0014 = 0.0136 \text{ m},$$
$$D_o = 0.015 + 0.0014 = 0.0164 \text{ m},$$
$$C = D/d = 0.015/0.0014 = 10.71,$$
$$K_w = 1.135,$$

$$\tau = \frac{8K_wF_oD}{\pi d^3} = \frac{8 \times 1.135 \times 30 \times 0.015}{\pi 0.0014^3} = 474 \times 10^6 \text{ Pa},$$

$$N_a = \frac{Gd}{8C^3k} = \frac{79.3 \times 10^9 \times 0.0014}{8 \times 10.71^3 \times 1000} = 11.3.$$

Body length $= d(N_a + 1) = 0.0172$ m. An additional fictitious coil has been added to obtain the body length.

If we use a full loop of inside diameter 0.0136 m at each end,

$$L_f = 0.0172 + 2 \times 0.0136 = 0.04442 \text{ m},$$
$$\delta = L_o - L_f = 0.07 - 0.04442 = 0.02558 \text{ m},$$
$$F_{\text{initial}} = F_o - k\delta = 30 - 1000 \times 0.02558 = 4.42 \text{ N}.$$

This value is between 10% and 25% of F_o, so it is acceptable.

$$\tau_{\text{initial}} = \tau \frac{F_{\text{initial}}}{F_o} = 474 \frac{14.42}{30} = 69.8 \text{ MPa}.$$

This is less than $0.4\sigma_{\text{uts}}/C$, so it is acceptable.

Example 15.4

A helical compression spring is required to exert a force of 400 N when compressed to a length of 140 mm. At a length of 170 mm, the force must be 160 N. A nominal spring mean diameter of 35 mm has been proposed. The application involves slow cycling, and a total life of 150,000 cycles is required. The maximum temperature of operation is 60 °C. Determine a suitable wire diameter for this application if ASTM A232 is to be used.

Solution

A standard material such as ASTM A232 chromium vanadium steel wire is suggested as an initial starting point for this application.

The maximum service temperature for this material is approximately 120 °C, which is greater than the operating temperature for the application.

The application involves slow cyclic variation of the load, so Eqn (15.9) can be used for determining the spring's stress level.

From Table 15.3 for chrome vanadium steel, $G = 79.3$ GPa.

The maximum operating force, F_o, is 400 N with an operating length, L_o, of 140 mm.

The installed force, F_i, is 160 N at an installed length, L_i, of 170 mm.

The spring rate k is given by

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{400 - 160}{0.17 - 0.16} = 8000 \text{ N/m}.$$

The free length L_f is given by

$$L_f = L_i + \frac{F_i}{k} = 0.17 + \frac{160}{8000} = 0.19 \text{ m}.$$

A nominal mean diameter of 35 has been proposed.

A design shear stress of $0.45\sigma_{\text{uts}}$ is suggested. σ_{uts} for ASTM A232 chrome vanadium steel with an assumed wire diameter of 5 mm is approximately 1500 MPa (see Figure 15.12).

The design shear stress will therefore be $0.45 \times 1500 = 675$ MPa.

If the wire diameter calculated in the design procedure is significantly different, then the estimate for the design shear stress will need to be modified.

The equation for shear stress requires a value for K_w. K_w is a function of $C = d/D$, but the wire diameter d is unknown at this stage.

A value of $K_w = 1.2$ can be used as an initial estimate here, although this will be checked and modified later in the procedure.

$$d = \left(\frac{8FDK_w}{\pi\tau_{\max}}\right)^{1/3} = \left(\frac{8 \times 400 \times 0.035 \times 1.2}{\pi 675 \times 10^6}\right)^{1/3} = 3.987 \times 10^{-3} \text{ m}.$$

The estimated wire diameter is therefore 3.987 mm.

Examination of Table 15.3 or a stock spring supplier's catalog will identify a suitably close wire diameter to select.

In this case from Table 15.3, the nearest larger wire diameter is 5 mm.

The spring index can now be calculated.

$$C = \frac{D}{d} = \frac{0.035}{0.005} = 7.$$

The Wahl factor is

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 7 - 1}{4 \times 7 - 4} + \frac{0.615}{7} = 1.213.$$

The maximum shear stress is given by

$$\tau_{\max} = K_w \frac{8FD}{\pi d^3} = 1.213 \frac{8 \times 400 \times 0.035}{\pi (0.005)^3} = 346 \times 10^6 \text{ Pa.}$$

This is significantly lower than the allowable maximum shear stress, so the design seems acceptable at this stage.

The number of active coils can be determined from

$$N_a = \frac{Gd}{8kC^3} = \frac{79.3 \times 10^9 \times 0.005}{8 \times 8000 \times (7)^3} = 18.06 \text{ coils.}$$

The solid length of a spring occurs when all the coils are touching.

If the spring is squared and ground, then there will be two inactive coils.

$$L_s = d(N + 2) = 0.005(18.06 + 2) = 0.1003 \text{ m.}$$

The force at the solid length would be

$$F_s = k\left(L_f - L_s\right) = 8000(0.19 - 0.1003) = 717.5 \text{ N.}$$

As stress in a spring is directly proportional to force, the solid length shear stress can be determined by

$$\tau_{\text{solid}} = \tau_{\max} \frac{F_s}{F_o} = 346 \times 10^6 \frac{717.5}{400} = 621 \times 10^6 \text{ Pa.}$$

This is below the maximum allowable stress, indicating an acceptable design.

The ratio of free length to the mean coil diameter is $L_f/D = 0.19/0.035 = 5.43$.

The deflection of the spring under the operating force is 50 mm.

Thus the ratio of deflection to the free length is $\delta/L_f = 0.05/0.19 = 0.263$.

Examination of Figure 15.11 for squared and ground ends shows that this design is stable and unlikely to buckle.

The outer diameter of the spring can be determined from

$$D_o = d + D = 0.005 + 0.035 = 0.04 \text{ m.}$$

The inner diameter of the spring will be

$$D_i = D - d = 0.035 - 0.005 = 0.03 \text{ m.}$$

Nominal specification:

ASTM A232 chromium vanadium steel wire helical compression spring,
$k = 8000$ N/m,
$L_f = 0.19$ m,
$L_s = 0.1003$ m.
Squared and ground ends, 20.06 coils,
$d = 5$ mm,
$D = 35$ mm, $D_i = 30$ mm, $D_o = 40$ mm.

Example 15.5

A helical spring is required to exert a force of 2000 N at a length of 200 mm and 1500 N at a length of 250 mm. Specify a suitable spring for this application.

Solution

As there are no constraints on the spring diameter, method 2 can be used.

A standard material such as ASTM A232 chromium vanadium steel wire is suggested as an initial starting point for this application.

The maximum service temperature for this material is approximately 120 °C, which is greater than the assumed operating temperature for the application.

The application involves slow cyclic variation of the load, so Eqn (15.9) should be used for determining the spring's stress level.

From Table 15.3 for chromium vanadium steel, $G = 79.3$ GPa.

The maximum operating force, F_o, is 2000 N with an operating length, L_o, of 200 mm.

The installed force, F_i, is 1500 N at an installed length, L_i, of 250 mm.

The spring rate k is given by

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{2000 - 1500}{0.25 - 0.2} = 10{,}000 \text{ N/m}.$$

The free length L_f is given by

$$L_f = L_i + \frac{F_i}{k} = 0.25 + \frac{1500}{10{,}000} = 0.4 \text{ m}.$$

A design shear stress of $0.45\sigma_{\text{uts}}$ is suggested.

σ_{uts} for ASTM A232 chrome vanadium steel with an assumed wire diameter of 4 mm is approximately 1350 MPa (see Figure 15.12).

The design shear stress will therefore be $0.45 \times 1350 = 608$ MPa

$$d = \sqrt{\frac{8K_w F_o C}{\pi \tau_{\max}}} = \sqrt{\frac{8 \times 1.2 \times 2000 \times 7}{\pi 6.08 \times 10^8}} = 8.392 \times 10^{-3} \text{ m}.$$

The next standard diameter available is $d = 9$ mm.

However, selecting such a small diameter for the spring wire could result in very high stresses at solid height.

A larger diameter is suggested for consideration initially.

Choosing $d = 11$ mm,

$$N_{\max} = \frac{L_o}{d} - 2 = \frac{0.2}{0.011} - 2 = 16.18.$$

Trying $N = 12$ gives

$$C = \left(\frac{Gd}{8kN_a}\right)^{1/3} = \left(\frac{79.3 \times 10^9 \times 0.011}{8 \times 10{,}000 \times 12}\right)^{1/3} = 9.685.$$

$$D = Cd = 0.1065 \text{ m},$$
$$K_w = 1.15,$$

$$\tau_{\max} = K_w \frac{8FD}{\pi d^3} = 1.15 \frac{8 \times 2000 \times 0.1065}{\pi (0.011)^3} = 469 \times 10^6 \text{ Pa}.$$

This is significantly lower than the allowable maximum shear stress, so the design seems acceptable at this stage.

$$L_s = d(N_a + 2) = 0.011(12 + 2) = 0.154 \text{ m}.$$

The force at the solid length would be

$$F_s = k\left(L_f - L_s\right) = 10{,}000(0.4 - 0.154) = 2460 \text{ N}.$$

The solid length shear stress can be determined by

$$\tau_{\text{solid}} = \tau_{\max} \frac{F_s}{F_o} = 469 \times 10^6 \frac{2460}{2000} = 577 \times 10^6 \text{ Pa}.$$

This value is just below the maximum allowable stress of 608 MPa, indicating that failure is unlikely.

The ratio of the free length to the mean coil diameter is $L_f/D = 0.4/0.1065 = 3.75$.

The deflection of the spring under the operating force is 200 mm.

Thus the ratio of deflection to free length is $\delta/L_f = 0.2/0.4 = 0.5$.

Examination of Figure 15.11 for squared and ground ends shows that this design is stable and unlikely to buckle.

The outer diameter of the spring can be determined from

$$D_o = d + D = 0.011 + 0.1065 = 0.1175 \text{ m}.$$

The inner diameter of the spring will be

$$D_i = D - d = 0.1065 - 0.011 = 0.09554 \text{ m}.$$

Nominal specification:

ASTM A232 chromium vanadium steel wire helical compression spring,
$k = 10{,}000$ N/m,
$L_f = 0.4$ m,
$L_s = 0.154$ m.
Squared and ground ends, 14 coils,
$d = 11$ mm,
$D = 106.5$ mm, $D_i = 95.54$ mm, $D_o = 117.5$ mm.

Example 15.6

A helical spring is required for a pogo stick to exert a force of 975 N at a length of 140 mm and 300 N at a length of 220 mm. Specify a suitable spring for this application if the spring material is ASTM A232.

Solution

As there are no constraints on the spring diameter, method 2 can be used.

A standard material such as ASTM A232 chromium vanadium steel wire is suggested as an initial starting point for this application.

The maximum service temperature for this material is approximately 120 °C, which is greater than the operating temperature for the application.

The application involves slow cyclic variation of the load, so Eqn (15.9) should be used for determining the spring's stress level.

From Table 15.3 for chromium vanadium steel, $G = 79.3$ GPa.

The maximum operating force, F_o, is 975 N with an operating length, L_o, of 140 mm.

The installed force, F_i, is 300 N at an installed length, L_i, of 220 mm.

The spring rate k is given by

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{975 - 300}{0.22 - 0.14} = 8438 \text{ N/m}.$$

The free length L_f is given by

$$L_f = L_i + \frac{F_i}{k} = 0.22 + \frac{300}{8438} = 0.2556 \text{ m}.$$

A design shear stress of $0.45\sigma_{\text{uts}}$ is suggested. σ_{uts} for ASTM A232 chrome vanadium steel with an assumed wire diameter of 7 mm is approximately 1450 MPa (see Figure 15.12).

The design shear stress will therefore be $0.45 \times 1450 = 653$ MPa

$$d = \sqrt{\frac{8K_wF_oC}{\pi\tau_{\max}}} = \sqrt{\frac{8 \times 1.2 \times 975 \times 7}{\pi 6.53 \times 10^8}} = 5.654 \times 10^{-3} \text{ m}.$$

Choosing $d = 7$ mm,

$$N_{\max} = \frac{L_o}{d} - 2 = \frac{0.14}{0.007} - 2 = 18.$$

Trying $N = 14$ gives

$$C = \left(\frac{Gd}{8kN_a}\right)^{1/3} = \left(\frac{79.3 \times 10^9 \times 0.007}{8 \times 8438 \times 14}\right)^{1/3} = 8.375.$$

$$D = Cd = 0.05862 \text{ m},$$
$$K_w = 1.175,$$

$$\tau_{\max} = K_w\frac{8FD}{\pi d^3} = 1.175\frac{8 \times 975 \times 0.05862}{\pi(0.007)^3} = 499 \times 10^6 \text{ Pa}.$$

This is significantly lower than the allowable maximum shear stress, so the design seems acceptable at this stage.

$$L_s = d(N_a + 2) = 0.007(14 + 2) = 0.112 \text{ m}.$$

The force at the solid length would be

$$F_s = k\left(L_f - L_s\right) = 8438(0.2556 - 0.112) = 1211 \text{ N}.$$

The solid length shear stress can be determined by

$$\tau_{\text{solid}} = \tau_{\max}\frac{F_s}{F_o} = 499 \times 10^6 \frac{1211}{975} = 630 \times 10^6 \text{ Pa}.$$

This value is just below the maximum allowable stress of 653 MPa, indicating that failure is unlikely.

The ratio of the free length to the mean coil diameter is $L_f/D = 0.2556/0.05862 = 4.359$.

The deflection of the spring under the operating force is 116 mm.

Thus the ratio of deflection to free length is $\delta/L_f = 0.116/0.2556 = 0.452$.

Examination of Figure 15.11 for squared and ground ends shows that this design is stable and unlikely to buckle.

The outer diameter of the spring can be determined from

$$D_o = d + D = 0.007 + 0.05862 = 0.06562 \text{ m}.$$

The inner diameter of the spring will be

$$D_i = D - d = 0.05862 - 0.007 = 0.05162 \text{ m}.$$

Nominal specification:

ASTM A232 chromium vanadium steel wire helical compression spring,
$k = 8438$ N/m,
$L_f = 0.2556$ m,
$L_s = 0.112$ m.
Squared and ground ends, 16 coils,
$d = 7$ mm,
$D = 58.62$ mm, $D_i = 51.62$ mm, $D_o = 65.62$ mm.

15.4 Helical Torsion Springs

Many applications require a spring that exerts a torsional moment. A helical coil spring can be loaded in torsion to fulfill this requirement. Various configurations of torsional springs are shown in Figure 15.15. The ends of the coil tend to be extended to provide levers to apply the torque.

The basic equation governing the angular deflection of a torsional spring is

$$\theta = \frac{ML}{EI} \tag{15.30}$$

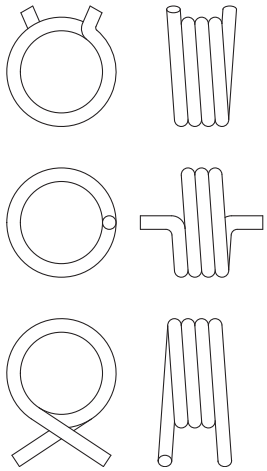


Figure 15.15
Various configurations for torsional springs.

where M is the applied moment (N m), L is the length of wire in the spring (m), E is Young's modulus (N/m^2), I is the second moment of area (m^4), and θ is the angular deflection (rad).

Substituting for the second moment of the area and the length and converting the angular deflection to revolutions gives a more convenient relationship:

$$\theta' = \frac{10.2MDN_a}{d^4E} \tag{15.31}$$

where θ' is the angular deflection (revolutions).

The spring rate for a torsional spring is defined as the moment per unit of angular deflection and is given by

$$k = \frac{M}{\theta'} \cong \frac{d^4E}{10.2DN}. \tag{15.32}$$

When a coil is loaded, the coil diameter decreases and its length increases as the coil is wound up. The minimum inner diameter of the coil at full deflection is given by

$$D_{i\ \min} = \frac{DN_b}{N_b + \theta'} - d \tag{15.33}$$

where $D_{i\ \min}$ is the minimum inner diameter (m) and N_b is the number of coils in the body.

The maximum coil body length when the spring is fully wound up is given by

$$L_{\max} = d(N_b + 1 + \theta) \tag{15.34}$$

where $L_{\max}$ is the maximum coil body length (m).

The number of active coils in a torsion spring is equal to the number of body turns, N_b, plus some contribution from the ends, which also bend. For straight ends, this contribution can be expressed by the number of equivalent coils, N_e.

$$N_e = \frac{L_1 + L_2}{3\pi D} \tag{15.35}$$

where L_1 and L_2 are the lengths of the ends. The number of active coils is given by

$$N_a = N_b + N_e. \tag{15.36}$$

15.5 Leaf Springs

Leaf springs consist of one or more flat strips of material loaded as cantilevers or simple beams, as illustrated in Figure 15.16. They can be designed to provide a compressive or tensile force as they are deflected from their free condition. Leaf springs are capable of exerting large forces within comparably small spaces.

Particular force-deflection characteristics can be achieved for a spring by careful dimensioning of the strips and nesting of a number of components. Various forms for cantilever beams are illustrated in Figure 15.17. Each type shown has special characteristics. Figure 15.17a illustrates a generalized beam. If it is desired to maintain uniform bending stresses over the length of the beam, then the width of the cantilever needs to vary linearly

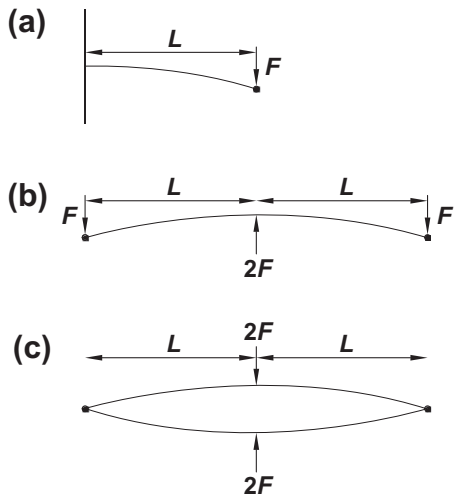


Figure 15.16
Basic types of leaf spring. (a) Simple cantilever. (b) Semielliptic. (c) Full elliptic.

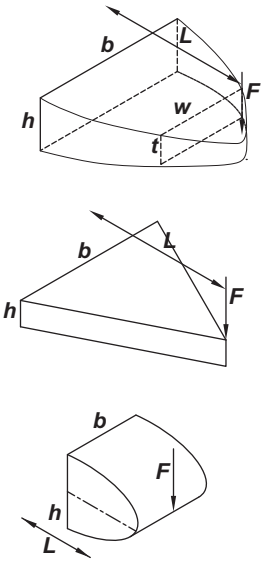


Figure 15.17
Cantilever beams. (a) General form. (b) Uniform bending stress. (c) Uniform stress cantilever of constant width.

with location, as illustrated in Figure 15.17b. This triangular form for the cantilever is the basic building block for leaf springs in common use. An alternative requirement might be to maintain uniform stress in a cantilever of constant width. In this case, the profile of the beam would need to be parabolic, as illustrated in Figure 15.17c. This is the basis used for analyzing the bending strength of teeth in spur gears.

The concept used in producing compact cantilever springs of uniform bending stress is to chop the triangular form illustrated in Figure 15.17b into a number of strips and recombine them as illustrated in Figure 15.18. The multileaf spring shown and the single triangular section beam both have the same stress and deflection characteristics, with the exceptions being that the multileaf spring is subject to additional damping due to friction between the leaves and that it can carry a full load in only one direction due to the tendency for the leaves to separate. Leaf separation can be partially overcome by the provision of clips around the leaves.

The deflection of a triangular leaf spring is given by

$$\delta = \frac{FL^3}{2EI} \tag{15.37}$$

where F is the force (N), L is the length (m), E is Young's modulus (N/m^2), and I is the second moment of area (m^4).

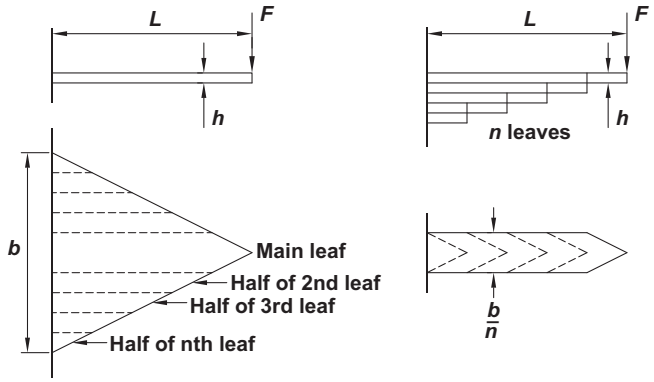


Figure 15.18
Triangular plan view cantilever and equivalent multileaf spring.

For a rectangular cross section,

$$I = \frac{bh^3}{12} \tag{15.38}$$

where b is the width (m) and h is the thickness (m).

The spring rate is given by

$$k = \frac{F}{\delta} = \frac{Ebh^3}{6L^3}. \tag{15.39}$$

The corresponding bending stress is given by

$$\sigma = \frac{6FL}{bh^2}. \tag{15.40}$$

For a semielliptic beam, the maximum deflection at the center is given by

$$\delta = \frac{6FL^3}{Ebh^3}. \tag{15.41}$$

The corresponding bending stress for a semielliptic beam is given by Eqn (15.40).

For a full-elliptic beam, the maximum deflection at the center is given by

$$\delta = \frac{12FL^3}{Ebh^3}. \tag{15.42}$$

The corresponding bending stress for a full-elliptic beam is given by Eqn (15.40).

15.6 Belleville Spring Washers

Belleville spring washers comprise a conical-shaped disc with a hole through the center, as illustrated in Figure 15.19. They have nonlinear force-deflection characteristic, which makes them useful in certain applications. Belleville springs are compact and are capable of large compressive forces, but their deflections are limited. Examples of their use include gun recoil mechanisms, pipe flanges, small machine tools, and applications where differential expansion could cause relaxation of a bolt.

The force-deflection characteristics for a Belleville washer depend on material properties and the dimensions illustrated in Figure 15.20. The force-deflection relationship is nonlinear, so it is not stated in the form of a spring rate and is given (Norton, 1996) by

$$F = \frac{4E\delta}{K_1 D_o^2 (1-\mu^2)} \left[(h-\delta)\left(h - \frac{\delta}{2}\right) t + t^3 \right] \tag{15.43}$$

where

$$K_1 = \frac{6}{\pi \ln R_d} \left[\frac{(R_d - 1)^2}{R_d^2} \right] \tag{15.44}$$

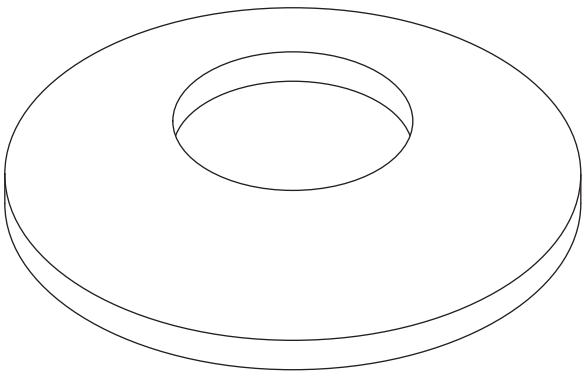


Figure 15.19
A Belleville spring washer.

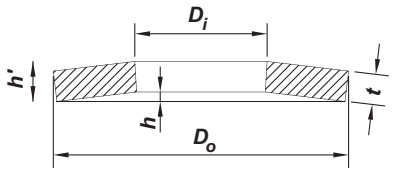


Figure 15.20
Principal Belleville washer dimensions.

$$R_d = \frac{D_o}{D_i}. \tag{15.45}$$

The load at the flat position is given by

$$F_{\text{flat}} = \frac{4Eht^3}{K_1 D_o^2 (1 - \mu^2)}. \tag{15.46}$$

Equation (15.43) has been used to illustrate the variation of force with deflection in Figure 15.21 for a variety of h/t ratios. The curves have been normalized to the spring's condition when flat for a steel Belleville spring. For a ratio of h/t of 0.4, the ratio of force to deflection is close to linear. As the h/t ratio is increased, it becomes increasingly nonlinear. For an h/t ratio of 1.414, there is a nearly constant region for the force. For values of h/t greater than 1.414, the curves become bimodal where a given force could cause more than one possible deflection. Once installed and in the flat position, a trivial force would be required to trip it in one direction or the other. The deviation of force from the force to flat value with deflection is shown in Figure 15.22 for a spring with $R_d = 2$ and $h/t = 1.414$. The force produced by an $h/t = 1.414$ ratio Belleville spring is within $\pm 10\%$ of the force to flat value for deflections of 55%−145% of the deflection to flat value. This is a useful characteristic if a nearly constant force is required over this range of deflection.

The stresses in a Belleville spring are concentrated at the edges of the inside and outside diameters. The largest stress is compressive and occurs at the inside radius on the convex side. The edges on the concave side have tensile stresses given by Eqns (15.48) and (15.49). These equations are quoted from Norton (1996).

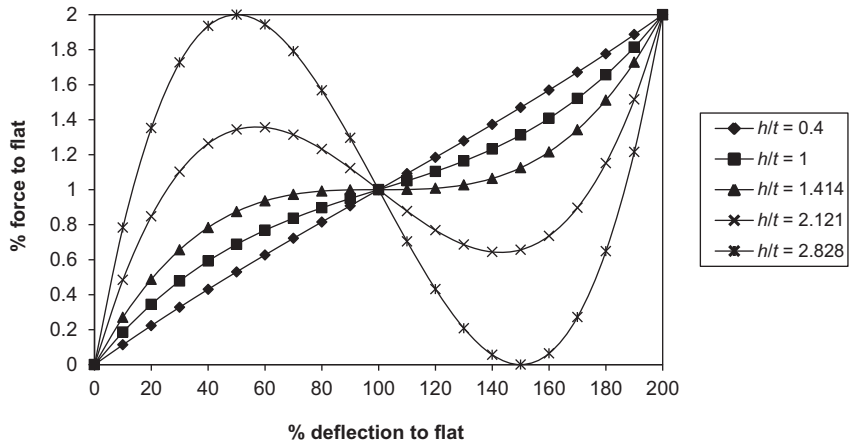


Figure 15.21
Normalized force-deflection characteristics for Belleville washer springs for various h/t ratios. Calculations performed with $E = 207$ GPa and $\mu = 0.3$.

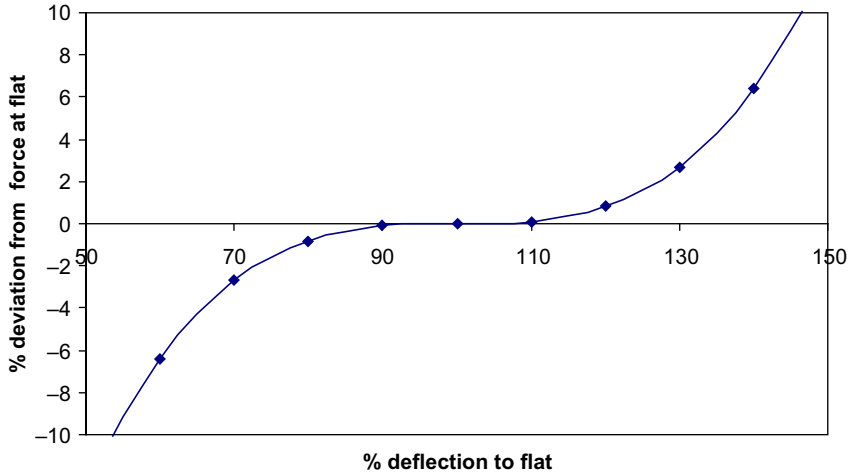


Figure 15.22
Percentage error in a constant force Belleville spring around its flat position. $R_d = 2$, $h/t = 1.414$.

$$\sigma_c = -\frac{4E\delta}{K_1 D_o^2(1-\mu^2)}\left[K_2\left(h-\frac{\delta}{2}\right)+K_3 t\right] \tag{15.47}$$

$$\sigma_{t\,i} = \frac{4E\delta}{K_1 D_o^2(1-\mu^2)}\left[-K_2\left(h-\frac{\delta}{2}\right)+K_3 t\right] \tag{15.48}$$

$$\sigma_{t\,o} = \frac{4E\delta}{K_1 D_o^2(1-\mu^2)}\left[K_4\left(h-\frac{\delta}{2}\right)+K_5 t\right] \tag{15.49}$$

where

$$K_2 = \frac{6}{\pi \ln R_d}\left[\frac{R_d-1}{\ln R_d}-1\right] \tag{15.50}$$

$$K_3 = \frac{6}{\pi \ln R_d}\left[\frac{R_d-1}{2}\right] \tag{15.51}$$

$$K_4 = \left[\frac{R_d \ln R_d-(R_d-1)}{\ln R_d}\right]\left[\frac{R_d}{(R_d-1)^2}\right] \tag{15.52}$$

$$K_5 = \frac{R_d}{2(R_d-1)}. \tag{15.53}$$

The maximum deflection of a single Belleville spring tends to be small. They can, however, be stacked in combinations, as illustrated in Figure 15.23. If stacked in series, the total

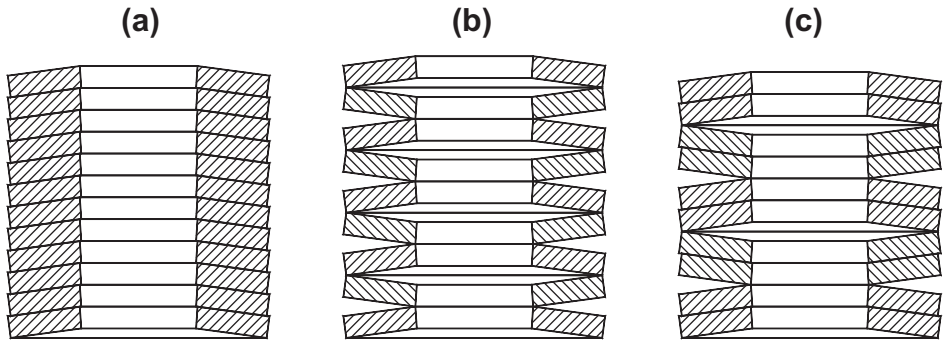


Figure 15.23
Stacking combinations for Belleville springs. (a) Parallel. (b) Series. (c) Series parallel.

force will be the same as for a single Belleville spring but the deflections will increase. If they are stacked in parallel, the total deflection will be the same as for a single spring but the forces will increase. Series and parallel combinations are also possible. It should be noted that Belleville spring towers need some form of support, either by inserting them into a hole or over a rod. This, however, will reduce the available load due to friction.

The design or selection of a Belleville spring for a given application requires consideration of the diameter and h/t ratio as well as the type of material to be used in order to give the desired force-deflection characteristics. This process invariably involves some iteration. A material is selected, values for the diameter ratio are proposed, often based on given constraints, and values of the h/t ratio proposed either for a single spring or some combination.

In designing Belleville springs, it can be useful to estimate the thickness required to give a particular force in the flat position which can be found, substituting $E = 207$ GPa, $\mu = 0.3$, and $K_1 = 0.69$ in Eqn (15.46), from

$$t = \frac{1}{1072}\left(\frac{D_o^2 F_{\text{flat}}}{h/t}\right)^{0.25} \tag{15.54}$$

where t is the thickness (m), h is the deflection to flat (m), D_o is the outer diameter (m), and F_{flat} is the load at flat position (N).

Dimensions for a selection of Belleville springs manufactured to DIN 6796 are given in Table 15.6.

Example 15.7

A Belleville spring is required to give a constant force of 200 N $\pm$ 10 N over a deflection of $\pm$0.3 mm. The spring must fit within a 62-mm-diameter hole. A carbon spring steel with $\sigma_{\text{uts}} = 1700$ MPa has been proposed.

Table 15.6: Dimensions for a selection of Belleville washer springs manufactured to DIN 6796 from DIN 17222 spring steel.

Notation	D_i (mm)	D_o (mm)	h' max (mm)	h' min (mm)	t (mm)	Force (N)*	Test force (N)**	Mass kg/1000	Core diameter (mm)
2	2.2	5	0.6	0.5	0.4	628	700	0.05	2
2.5	2.7	6	0.72	0.61	0.5	946	1100	0.09	2.5
3	3.2	7	0.85	0.72	0.6	1320	1500	0.14	3
3.5	3.7	8	1.06	0.92	0.8	2410	2700	0.25	3.5
4	4.3	9	1.3	1.12	1	3770	4000	0.38	4
5	5.3	11	1.55	1.35	1.2	5480	6550	0.69	5
6	6.4	14	2	1.7	1.5	8590	9250	1.43	6
7	7.4	17	2.3	2	1.75	11,300	13,600	2.53	7
8	8.4	18	2.6	2.24	2	14,900	17,000	3.13	8
10	10.5	23	3.2	2.8	2.5	22,100	27,100	6.45	10
12	13	29	3.95	3.43	3	34,100	39,500	12.4	12
14	15	35	4.65	4.04	3.5	46,000	54,000	21.6	14
16	17	39	5.25	4.58	4	59,700	75,000	30.4	16
18	19	42	5.8	5.08	4.5	74,400	90,500	38.9	18
20	21	45	6.4	5.6	5	93,200	117,000	48.8	20
22	23	49	7.05	6.15	5.5	113,700	145,000	63.5	22
24	25	56	7.75	6.77	6	131,000	169,000	92.9	24
27	28	60	8.35	7.3	6.5	154,000	221,000	113	27
30	31	70	9.2	8	7	172,000	269,000	170	30

*Force applies to the pressed flat condition and corresponds to twice the calculated value at a deflection h'-t.
**Test force applies for loading tests to DIN 6796.

Solution

Assume a 60 mm outer diameter to allow some clearance in the hole.

In order to provide a constant force, an h/t ratio of 1.414 is selected.

The variation of force of $\pm 5\%$ can be met by choosing an appropriate deflection range to operate in from Figure 15.22. If the deflection is limited to between 65% and 135% of the flat deflection, then the tolerance on force can be achieved. The nominal force of 200 N will occur in the flat position and the spring will provide a similar force, between 210 and 190 N, operating on both sides of its center.

From Eqn (15.54),

$$t = \frac{1}{1072}\left(\frac{D_o^2 F_{\text{flat}}}{h/t}\right)^{0.25} = \frac{1}{1072}\left(\frac{0.06^2 \times 200}{1.414}\right)^{0.25} = 7.88 \times 10^{-4}\ \text{m}$$

$$h = 1.414t = 1.414 \times 0.788 = 1.114\ \text{mm}.$$

The minimum and maximum deflections are

$\delta_{min} = 0.65\ h = 0.65 \times 1.114 = 0.724$ mm,
$\delta_{max} = 1.35\ h = 1.35 \times 1.114 = 1.504$ mm.
$\delta_{max} - \delta_{min}$ is greater than the required deflection range of 0.6 mm, so the force tolerance can be met.

From Eqs (15.44) and (15.50)−(15.53),

$K_1 = 0.689$,
$K_2 = 1.220$,
$K_3 = 1.378$,
$K_4 = 1.115$,
$K_5 = 1$.

From Eqn (15.47), $\sigma_c = -840$ MPa.

From Eqn (15.48), $\sigma_{t\ i} = 355$ MPa.

From Eqn (15.49), $\sigma_{t\ o} = 658$ MPa.

These stresses are well within the capability of a 1700 MPa uts material.

Example 15.8

A Belleville spring is required to give a constant force of 50 N ± 5 N over a deflection of ±0.2 mm. The spring must fit within a 40-mm-diameter hole. A carbon spring steel with $\sigma_{uts} = 1700$ MPa has been proposed.

Solution

Assume a 35 mm outer diameter to allow some clearance in the hole.

In order to provide a constant force, an *h*/*t* ratio of 1.414 is selected.

The variation of force of ±10% can be readily met by choosing an appropriate deflection range to operate in from Figure 15.22.

If the deflection is limited to between 65% and 135% of the flat deflection, then the tolerance on force can be achieved.

The nominal force of 50 N will occur in the flat position and the spring will provide a similar force, between 55 and 45 N, operating on both sides of its center.

From Eqn (15.52),

$$t = \frac{1}{1072}\left(\frac{D_o^2 F_{\text{flat}}}{h/t}\right)^{0.25} = \frac{1}{1072}\left(\frac{0.035^2 \times 50}{1.414}\right)^{0.25} = 4.26 \times 10^{-4}\ \text{m}.$$

$$h = 1.414t = 1.414 \times 0.426 = 0.602\ \text{mm}.$$

The minimum and maximum deflections are

$\delta_{\min} = 0.65\ \text{h} = 0.65 \times 0.602 = 0.391$ mm,
$\delta_{\max} = 1.35\ \text{h} = 1.35 \times 0.602 = 0.812$ mm.
$\delta_{\max} - \delta_{\min}$ is greater than the required deflection range of 0.4 mm, so the force tolerance can be met.

From Eqns (15.44) and (15.50)–(15.53),

$K_1 = 0.689$,
$K_2 = 1.220$,
$K_3 = 1.378$,
$K_4 = 1.115$,
$K_5 = 1$.

From Eqn (15.47), $\sigma_c = -720$ MPa.

From Eqn (15.48), $\sigma_{t\,i} = 305$ MPa.

From Eqn (15.49), $\sigma_{t\,o} = 564$ MPa.

These stresses are well within the capability of a 1700 MPa uts material.

Example 15.9

A Belleville spring is required to give a constant force of 10 N ± 1 N over a deflection of ±0.15 mm. The spring must fit within a 16-mm-diameter hole. A carbon spring steel with $\sigma_{\text{uts}} = 1700$ MPa has been proposed.

Solution

Assume a 14 mm outer diameter to allow some clearance in the hole.

In order to provide a constant force, an *h*/*t* ratio of 1.414 is selected.

The variation of force of ±10% can be met by choosing an appropriate deflection range to operate in from Figure 15.22.

If the deflection is limited to between 65% and 135% of the flat deflection, then the tolerance on force can be achieved.

The nominal force of 10 N will occur in the flat position and the spring will provide a similar force, between 11 and 9 N, operating on both sides of its center.

From Eqn (15.52),

$$t = \frac{1}{1072}\left(\frac{D_o^2 F_{\text{flat}}}{h/t}\right)^{0.25} = \frac{1}{1072}\left(\frac{0.014^2 \times 10}{1.414}\right)^{0.25} = 1.8 \times 10^{-4} \text{ m}.$$

$$h = 1.414t = 1.414 \times 0.18 = 0.255 \text{ mm}.$$

The minimum and maximum deflections are

$$\delta_{\min} = 0.65 \text{ h} = 0.65 \times 0.255 = 0.165 \text{ mm},$$
$$\delta_{\max} = 1.35 \text{ h} = 1.35 \times 0.255 = 0.344 \text{ mm}.$$

From Eqns (15.44) and (15.50)–(15.53),

$$K_1 = 0.689,$$
$$K_2 = 1.220,$$
$$K_3 = 1.378,$$
$$K_4 = 1.115,$$
$$K_5 = 1.$$

From Eqn (15.47), $\sigma_c = -810$ MPa.

From Eqn (15.48), $\sigma_{t\,i} = 341$ MPa.

From Eqn (15.49), $\sigma_{t\,o} = 630$ MPa.

These stresses are well within the capability of a 1700 MPa uts material.

$\delta_{\max} - \delta_{\min}$ is not, however, greater than the required deflection range of 0.3 mm, so the force tolerance is not met.

Altering the diameter of the spring, however, to, say, 15 or 13 mm does not give $\delta_{\max} - \delta_{\min} > 0.3$ mm. So a different h/t ratio would need to be considered.

15.7 Conclusions

This chapter has introduced the function and characteristics of a range of spring technologies. Design methods have been outlined for helical compression, extension, and torsion springs, leaf springs, and Belleville spring washers. There is no single design procedure that is suitable for all types of spring, and the procedures outlined here have not considered all of the important design parameters, which also include cost, appearance, and environmental

considerations. Spring design invariably becomes a process of optimization in which various tradeoffs need to be balanced in order to provide the best possible total solution.

References

Books and Papers

Ashby, M.F., 1999. Materials Selection for Mechanical Design, second ed. Butterworth Heinemann.
Associated Spring, Barnes Group, 1987. Engineering Guide to Spring Design. Associated Spring.
Joerres, R.G., 1996. Springs (Chapter 24). In: Shigley, J.E., Mischke, C.R. (Eds.), Standard Handbook of Machine Design. McGraw Hill.
Juvinall, R.C., Marshek, K.M., 1991. Fundamentals of Machine Component Design. Wiley.
Mott, R.L., 1999. Machine Elements in Mechanical Design, third ed. Prentice Hall.
Norton, R.L., 1996. Machine Design. Prentice Hall.
Wahl, A.M., 1963. Mechanical Springs, second ed. McGraw Hill.

Further Reading

Almen, J.O., Laszlo, A., 1936. The uniform section disk spring. Trans. ASME 58, 305–314.
Carlson, H., 1978. Spring Designers Handbook. Marcel Dekker.
Carlson, H., 1982. Spring Manufacturing Handbook. Marcel Dekker.
Society of Automotive Engineers, 1988. Manual on Design and Manufacture of Coned Disk Springs (Belleville Springs) and Spring Washers. SAE.
Spring Committee Society of Automotive Engineers, 1996. Spring Design Manual, second ed. SAE.

Standards

ASTM International. ASTM A125–96(2001). Standard Specification for Steel Springs, Helical Heat Treated.
ASTM International. ASTM A227/A227M-99. Standard Specification for Steel Wire, Cold-Drawn for Mechanical Springs.
ASTM International. ASTM A228/A228M-02. Standard Specification for Steel Wire, Music Spring Quality.
ASTM International. ASTM A229/A229M-99. Standard Specification for Steel Wire, Oil-Tempered for Mechanical Springs.
ASTM International. ASTM A230/A230M-99. Standard Specification for Steel Wire, Oil Tempered Carbon Valve Spring Quality.
ASTM International. ASTM A231/A231M-96(2002). Standard Specification for Chromium Vanadium Alloy Steel Spring Wire.
ASTM International. ASTM A232/A232M-99. Standard Specification for Chromium Vanadium Alloy Steel Valve Spring Quality Wire.
ASTM International. ASTM A313/A313M-98. Standard Specification for Stainless Spring Wire.
ASTM International. ASTM A401/A401M-98. Standard Specification for Steel Wire, Chromium Silicon Alloy.
ASTM International. ASTM B159M-01. Standard Specification for Phosphor Bronze Wire.
British Standards Institution. BS EN 10270–1:2001. Steel wire for mechanical springs. Patented cold-drawn unalloyed steel spring wire.
British Standards Institution. BS EN 10270–2:2001. Steel Wire for Mechanical Springs. Oil Hardened and Tempered Spring Steel Wire.
British Standards Institution. BS EN 10270–3:2001. Steel Wire for Mechanical Springs. Stainless Spring Steel Wire.
British Standards Institution. BS EN 12166:1988. Copper and Copper Alloys. Wire for General Purposes.

British Standards Institution. BS 1429: 1980. Specification for Annealed Round Steel Wire for General Engineering Springs.
British Standards Institution. BS 1726−1:2002. Cylindrical Helical Springs Made from Round Wire and Bar. Guide to Methods of Specifying, Tolerances and Testing. Compression Springs.
British Standards Institution. BS 1726−2:2002. Cylindrical Helical Springs Made from Round Wire and Bar. Guide to Methods of Specifying, Tolerances and Testing. Extension Springs.
British Standards Institution. BS 1726−3:2002. Cylindrical Helical Springs Made from Round Wire and Bar. Guide to Methods of Specifying, Tolerances and Testing. Torsion Springs.
British Standards Institution. BS 8726−1:2002. Cylindrical Helical Springs Made from Rectangular and Square Section Wire and Bar. Guide to Calculation and Design. Compression Springs.
British Standards Institution. BS 8726−2:2002. Cylindrical Helical Springs Made from Rectangular and Square Section Wire and Bar. Guide to Calculation and Design. Torsion Springs.
British Standards Institution. BS EN ISO 2162−1:1996. Technical product documentation. Springs. Simplified representation.
British Standards Institution. BS EN 13906−1:2002. Cylindrical Helical Springs Made from Round Wire and Bar. Calculation and Design. Compression Springs.
British Standards Institution. BS EN 13906−2:2002. Cylindrical Helical Springs Made from Round Wire and Bar. Calculation and Design. Extension Springs.
British Standards Institution. BS EN 13906−3:2002. Cylindrical Helical Springs Made from Round Wire and Bar. Calculation and Design. Torsion Springs.
DIN. DIN Handbook 140 Fasteners 4. Dimensional Standards for Nuts and Accessories for Bolt/Nut Assemblies. 2001.
DIN. DIN 6796. Conical Spring Washers for Bolt/Nut Assemblies. 1987.

Web Sites

At the time of going to press, the world-wide-web contained useful information relating to this chapter at the following links.

www.acewirespring.com
www.asbg.com
www.ashfield-springs.com
www.assocspring.co.uk
www.astm.org
www.automaticsprings.com.au
www.bestspring.com
www.bsi.org.uk
www.harris-springs.com
www.hzap.cz
www.indiamart.com/paragonindustries/
www.leespring.co.uk
www.sae.org
www.smihq.org
www.springmasters.com
www.uzelotomotiv.com
www.wmhughes.co.uk
www.wolverinecoilspring.com
www.din.de

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

A	area (m^2)
b	width (m)
C	spring index
d	wire diameter (m)
D	mean coil diameter (m)
D_i	inner diameter (m)
$D_{i\ \min}$	minimum inner diameter (m)
D_o	outer diameter (m)
E	Young's modulus (N/m^2)
f_n	natural frequency (Hz)
F	load (N)
F_{flat}	load at flat position (m)
F_i	installed force (N)
F_o	operating force (N)
G	modulus of rigidity (N/m^2)
h	thickness (m), deflection to flat (m)
h'	height (m)
I	second moment of area (m^4)
J	polar second moment of area (m^4)
k	spring rate (N/m)
k_{total}	total spring rate (N/m)
K_b	stress concentration factor
K_s	direct shear factor
K_w	Wahl factor
L	length (m)
L_a	assembled length (m)
L_i	installed length (m)
L_f	free length (m)
L_m	minimum working length (m)
$L_{\max}$	maximum coil body length (m)
L_o	operating length (m)
L_s	shut length (m)
m	mass (kg)
M	applied moment (N m)
N	number of coils

N_a	number of active coils
N_b	number of coils in the body
N_e	number of equivalent coils
N_{max}	maximum number of coils
p	pitch (m)
PH	precipitated hardened
r	radius (m)
R_d	diameter ratio
R_1	mean loop radius (m)
R_2	side bend radius (m)
t	thickness (m)
T	torque (N m)
δ	deflection (m)
$\delta_{initial}$	initial deflection (m)
μ	Poisson's ratio
θ	angular deflection (rad)
θ'	angular deflection (revolutions)
ρ	density (kg/m^3)
τ	shear stress (N/m^2)
$\tau_{initial}$	shear stress under initial tension (N/m^2)
τ_{max}	maximum shear stress (N/m^2)
τ_{solid}	shear stress at solid (N/m^2)
σ	bending stress (N/m^2)
σ_{uts}	ultimate tensile strength (N/m^2)

CHAPTER 16

Fastening and Power Screws

Chapter Outline

16.1 Introduction to Permanent and Nonpermanent Fastening 677
16.2 Threaded Fasteners 679
16.3 Power Screws 693
16.4 Rivets 699
16.5 Adhesives 708
16.6 Welding 714
16.7 Snap Fasteners 714
16.8 Conclusions 717
References 717
Further Reading 717
Nomenclature 718

Abstract

A fastener is a device used to connect or join two or more components. Traditional forms of fastening include nuts, bolts, screws, and rivets. In addition, welding and adhesives can be used to form permanent joins between components. The aim of this chapter is to introduce a wide selection of fastening techniques. While considering threaded fasteners, the subject of power screws, which are used for converting rotary motion to linear motion, is also presented.

16.1 Introduction to Permanent and Nonpermanent Fastening

The joining of components is a frequent necessity in the design of products. For example, the Boeing 747 has over 2.5 million fasteners. Fastening techniques can also be a major feature in design. Current styling for automotive vehicles dictates an absence of the means to fasten components together under a cursory inspection. The range of fastening techniques is extensive. It includes adhesives, welding, brazing, soldering, threaded and unthreaded fasteners, special purpose fasteners, and friction joints. Some of these techniques are permanent in nature and some allow the joint to be dismantled. A variety of basic types of joint is illustrated in Figure 16.1.

Design considerations include the following:

- whether the joint should be permanent or nonpermanent
- cost

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00016-2
Copyright © 2014 Elsevier Ltd. All rights reserved.

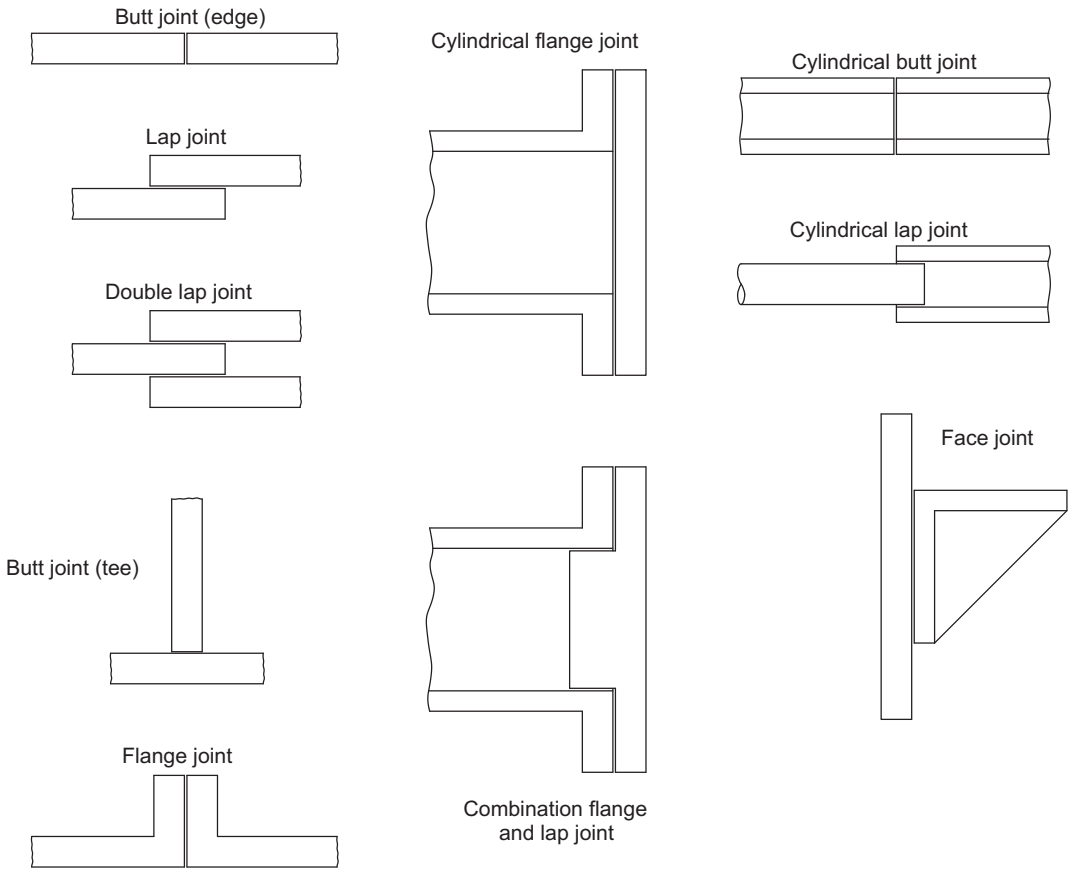


Figure 16.1
A variety of types of joint.

- loads in the fastener or power screw and the associated components
- life
- tooling
- assembly
- tolerances
- aesthetics
- size
- corrosion

Threaded fasteners are introduced in Section 16.2, power screws in Section 16.3, rivets in Section 16.4, adhesives in Section 16.5, and welding and the related subjects of soldering and brazing in Section 16.6.

16.2 Threaded Fasteners

There is a large variety of fasteners available that use a threaded form to connect components. The common element of screw fasteners is the helical thread that causes a screw to advance into a component or nut when rotated.

In the case of a two-part threaded fastener, common notation used for the two components is as follows:

- A male component, the screw, with a helical thread or groove formed on its outer diameter
- A female component, the nut, with a helical thread or groove formed on its inner diameter

For a two-part threaded fastener, the two helical grooves are defined with the same pitch, hand, and nominal diameter. A slight clearance is required to allow assembly between the diameter for the screw and nut. Rotation of, for example, the screw component causes it to advance into the nut component forming a threaded assembly. Screw threads can be either left-handed or right-handed depending on the direction of rotation desired for advancing the thread, as illustrated in Figure 16.2. Generally, right-hand threads are normally used, with left-handed threads being reserved for specialist applications. The shape of the groove forming the helix is called the thread form or profile. A large number of thread forms exist and are available, but the vast majority of threads and threaded fasteners use the so-called "vee" thread. The detailed aspects of a thread and the specialist terminology used are illustrated in Figure 16.3 and defined in Table 16.1.

Specific thread forms, angle of helix, etc., vary according to specific standards. Common standards developed include Unified National Standard (UNS) series threads and International Standards Organization (ISO) threads. Both of these use a 60° included angle but are not interchangeable. The form for an ISO metric thread for a nut is illustrated in

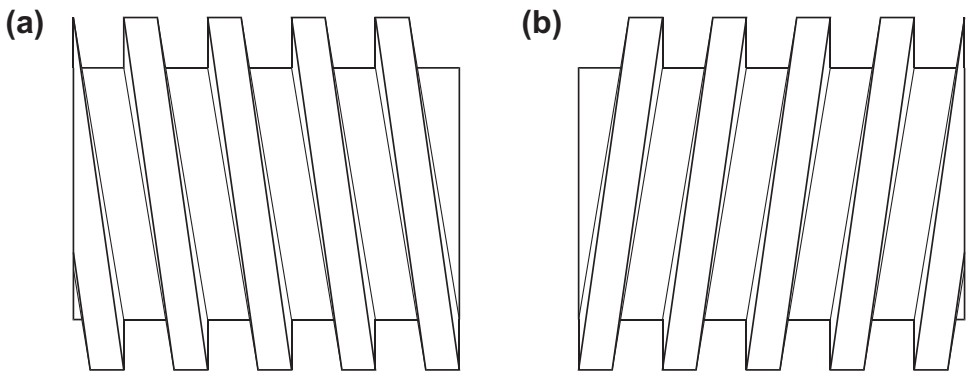


Figure 16.2
(a) Right-hand thread; (b) left-hand thread.

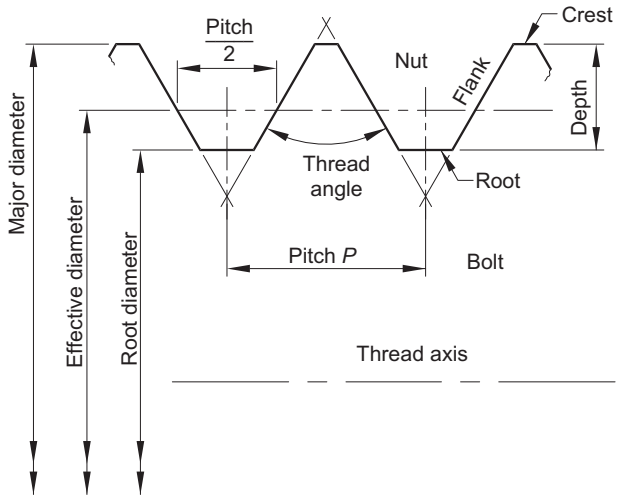


Figure 16.3
Specialist terminology used for describing threads.

Table 16.1: Thread terminology.

Term	Description
Pitch	The thread pitch is the distance between corresponding points on adjacent threads. Measurements must be made parallel to the thread axis.
Outside diameter	The outside or major diameter is the diameter over the crests of the thread measured at right angles to the thread axis.
Crest	The crest is the most prominent part of the thread, either external or internal.
Root	The root lies at the bottom of the groove between two adjacent threads.
Flank	The flank of a thread is the straight side of the thread between the root and the crest.
Root diameter	The root, minor, or core diameter is the smallest diameter of the thread measured at right angles to the thread axis.
Effective diameter	The effective diameter is the diameter in which the width of the spaces is equal to the width of the threads. It is measured at right angles to the thread axis.
Lead	The lead of a thread is the axial movement of the screw in one revolution.

Figure 16.4. In practice, the root of the nut and the crest of the mating bolt are rounded. Both male and female ISO threads are subject to manufacturing tolerances, which are detailed in BS 3643. A coarse series thread and fine series threads are defined in the ISO standard, but fine series threads tend to be more expensive and may not be readily available from all stockists. Table 16.2 gives the standard sizes for a selection of ISO coarse series hexagon bolts, screws, and nuts. ISO metric threads are designated by the letter M, followed by the nominal diameter and the pitch required, for example, M6 $\times$ 1.5.

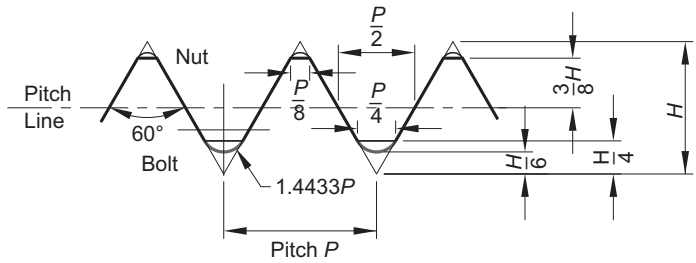


Figure 16.4
ISO metric thread.

Table 16.2: Selected dimensions for a selection of British Standard ISO Metric Precision Hexagon Bolts.

Nominal size and thread diameter	Pitch of thread (coarse pitch series)	Width across flats		Height of head		Tapping drill	Clearance drill
		(max)	(min)	(max)	(min)		
M1.6	0.35	3.2	3.08	1.225	0.975	1.25	1.65
M2	0.4	4.0	3.88	1.525	1.275	1.60	2.05
M2.5	0.45	5.0	4.88	1.825	1.575	2.05	2.60
M3	0.5	5.5	5.38	2.125	1.875	2.50	3.10
M4	0.7	7.0	6.85	2.925	2.675	3.30	4.10
M5	0.8	8.0	7.85	3.650	3.35	4.20	5.10
M6	1	10.0	9.78	4.15	3.85	5.00	6.10
M8	1.25	13.0	12.73	5.65	5.35	6.80	8.20
M10	1.5	17.0	16.73	7.18	6.82	8.50	10.20
M12	1.75	19.0	18.67	8.18	7.82	10.20	12.20
M14	2	22.0	21.67	9.18	8.82	12.00	14.25
M16	2	24.0	23.67	10.18	9.82	14.00	16.25
M18	2.5	27.0	26.67	12.215	11.785	15.50	18.25
M20	2.5	30.0	29.67	13.215	12.785	17.50	20.25
M22	2.5	32.0	31.61	14.215	13.785	19.50	22.25
M24	3	36.0	35.38	15.215	14.785	21.00	24.25
M27	3	41.0	40.38	17.215	16.785	24.00	27.25
M30	3.5	46.0	45.38	19.26	18.74	26.50	30.50
M33	3.5	50.0	49.38	21.26	20.74	29.50	33.50
M36	4	55.0	54.26	23.26	22.74	32.00	36.50
M39	4	60.0	59.26	25.26	24.74	35.00	39.50
M42	4.5	65.0	64.26	26.26	25.74	37.50	42.50
M45	4.5	70.0	69.26	28.26	27.74	40.50	45.50
M48	5	75.0	74.26	30.26	29.74	43.00	48.75
M52	5	80.0	79.26	33.31	32.69	47.00	52.75
M56	5.5	85.0	84.13	35.31	34.69	50.50	56.75
M60	5.5	90.0	89.13	38.31	37.69	54.50	60.75
M64	6	95.0	94.13	40.31	39.69	58.00	64.75
M68	6	100.0	99.13	43.31	42.96	62.00	68.75

All dimensions in millimeters.
Source: BS 3692:1967.

The Unified system of screw threads was originally introduced in the United Kingdom, Canada, and the United States in order to provide a common standard for use by the three countries. Types of unified threads in common use include the Unified coarse pitch thread series (UNC) and the Unified fine pitch thread series (UNF). Pertinent dimensions for selected UNC and UNF threads are given in Tables 16.3 and 16.4. Unified threads are specified by notation, in the case of a ½ bolt, in the form "½ in.-13UNC" or "½ in.-20UNF" depending on whether a coarse or a fine thread is being used.

The range of threaded fasteners available is extensive. It includes nuts and bolts, machine screws, set screws, and sheet metal screws. A variety of machine screws is illustrated in Figure 16.5.

Table 16.3: American Standard thread dimensions for UNC screw threads.

Size designation	Nominal major diameter (in.)	Threads per inch	Tensile stress area ($in.^2$)
0	0.0600		
1	0.0730	64	0.00263
2	0.0860	56	0.00370
3	0.0990	48	0.0487
4	0.1120	40	0.00604
5	0.1250	40	0.00796
6	0.1380	32	0.00909
8	0.1640	32	0.0140
10	0.1900	24	0.0175
12	0.2160	24	0.0242
Fractional sizes			
¼	0.2500	20	0.0318
$^5/_{16}$	0.3125	18	0.0524
$^3/_8$	0.3750	16	0.0775
$^7/_{16}$	0.4375	14	0.1063
½	0.5000	13	0.1419
$^9/_{16}$	0.5625	12	0.182
$^5/_8$	0.6250	11	0.226
$^3/_4$	0.7500	10	0.334
$^7/_8$	0.8750	9	0.462
1	1.000	8	0.606
$1^1/_8$	1.125	7	0.763
1¼	1.250	7	0.969
$1^3/_8$	1.375	6	1.155
1½	1.500	6	1.405
$1^3/_4$	1.750	5	1.90
2	2.000	4.5	2.50

Table 16.4: American Standard thread dimensions for UNF screw threads.

Size designation	Nominal major diameter (in.)	Threads per inch	Tensile stress area (in.^{2})
0	0.0600	80	0.00180
1	0.0730	72	0.00278
2	0.0860	64	0.00394
3	0.0990	56	0.00523
4	0.1120	48	0.00661
5	0.1250	44	0.00830
6	0.1380	40	0.01015
8	0.1640	36	0.01474
10	0.1900	32	0.0200
12	0.2160	28	0.0258
	Fractional sizes		
¼	0.2500	28	0.0364
$^5/_{16}$	0.3125	24	0.0580
$^3/_8$	0.3750	24	0.0878
$^7/_{16}$	0.4375	20	0.1187
½	0.5000	20	0.1599
$^9/_{16}$	0.5625	18	0.203
$^5/_8$	0.6250	18	0.256
$^3/_4$	0.7500	16	0.373
$^7/_8$	0.8750	14	0.509
1	1.000	12	0.663
$1^1/_8$	1.125	12	0.856
1¼	1.250	12	1.073
$1^3/_8$	1.375	12	1.315
1½	1.500	12	1.581

With such an array of types of fastener, the task of selecting the appropriate type for a given application can be time-consuming.

Washers can be used either under the bolt head or the nut or both in order to distribute the clamping load over a wide area and to provide a bearing surface for rotation of the nut. The most basic form of a washer is a simple disc with a hole through which the bolt or screw passes. There are, however, many additional types with particular attributes such as lock washers, which have projections that deform when compressed. This produces additional forces on the assembly, decreasing the possibility that the fastener assembly will loosen during use. Various forms of washer are illustrated in Figure 16.6.

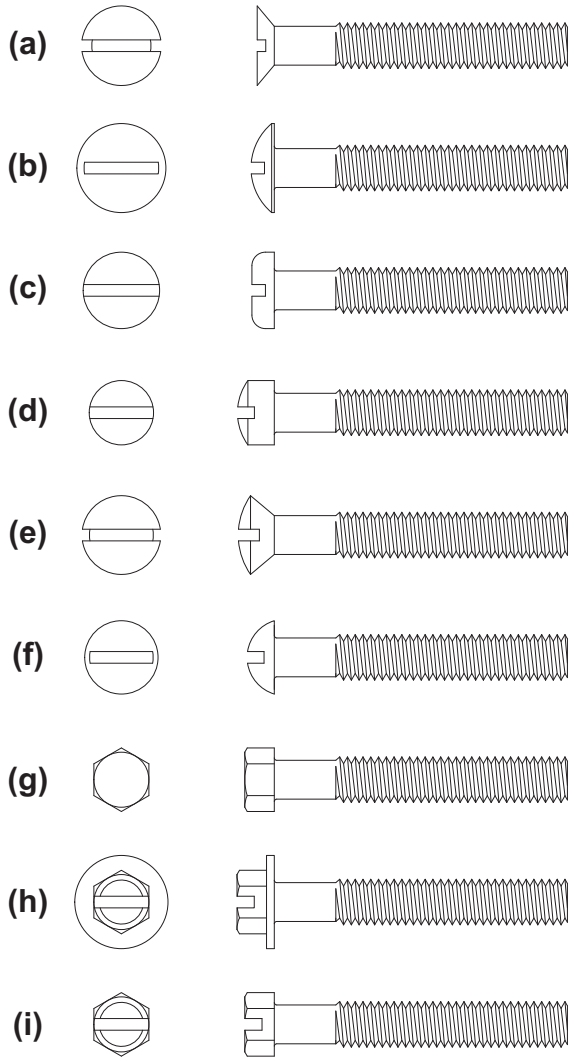


Figure 16.5
Various machine screw styles. (a) Flat countersunk head. (b) Slotted truss head. (c) Slotted pan head. (d) Slotted fillister head. (e) Slotted oval countersunk. (f) Round head. (g) Hex. (h) Hex washer. (i) Slotted hexagon head.

Selection of a particular fastener will depend upon many different criteria, including:

- strength at the operating temperatures concerned
- weight
- cost
- corrosion resistance
- magnetic properties

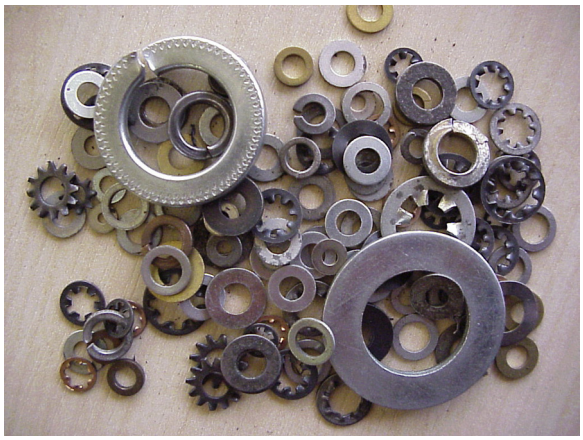


Figure 16.6
Washers.

- life expectancy
- assembly considerations

Bolted joints tend to consist of a series of components, such as flanges or plates, which need to be restrained from moving relative to one another. To prevent or limit movement, a tension member (the bolt or screw) is tightened, generating a tensile preload in the bolt. The preload causes an interface pressure between the components. Friction between the components as a result of the pressure then prevents, or limits, relative movement of the components. Although several or many components can be clamped in this way, generally the number is just two, as shown in Figure 16.7.

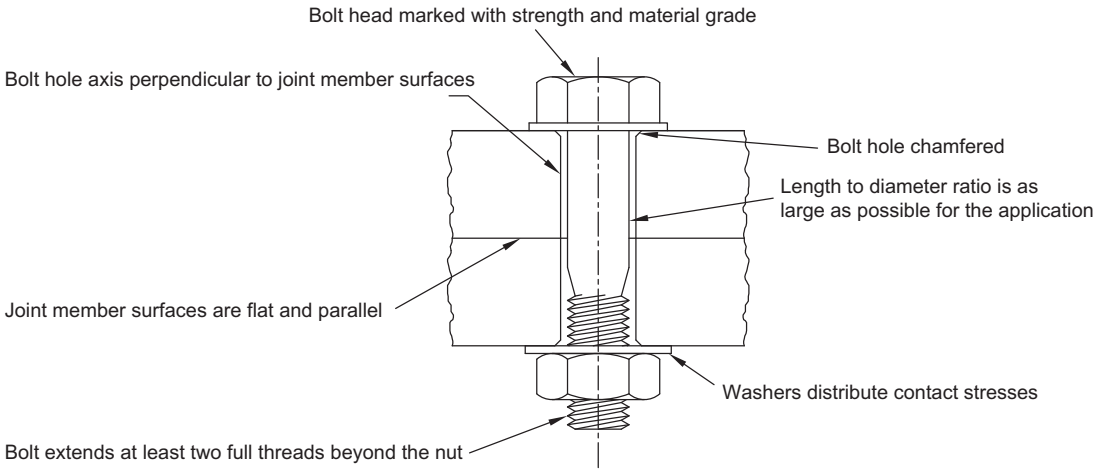


Figure 16.7
Features of a typical bolted joint.

For a bolted joint, the tensile stresses formed in the minimum section of the bolt's shank need to be considered. The minimum section is normally taken as the core area of the thread, although the necking down area of the shank can be used in high-stress or safety-critical applications. A further issue is the inherent stress concentration caused by the thread itself. In most applications, the nut is in compression while the bolt is in tension. This has the effect of slightly shortening the nut and hence the pitch of its thread. Conversely, the bolt stretches slightly and there is a consequential increase in its thread pitch. The combined effect of these small changes in pitch is that most of the bolt load is taken on the first few threads of the bolt, or perhaps just one thread, even if the nut is many bolt diameters long. This effect greatly increases the stress intensity. The maximum tensile stress in fasteners is therefore often high, and in cyclically loaded machines methods of avoiding fatigue of male threaded components (and occasionally of the female threaded component as well) are often required. For any structure, once the stress concentration effect is established, the likelihood of fatigue occurring can be determined by both the stress amplitude and the average stress. As a general rule, it is desirable to minimize the stress amplitude. In bolted joints, this can be achieved using a number of techniques, e.g. by maximizing the flexibility of the bolt compared to the joint. In practice, this means using many thin, long bolts rather than a few short, fat bolts of equivalent strength.

The compressive stresses in a bolted joint caused by the preload of the bolt can be assumed to act over a restricted volume of material. This is usually taken as a conical form, as shown in Figure 16.8, and the surrounding material is ignored in the analysis. Assuming the bolt and conical slugs of the material are of a similar Young's modulus, the bolt will be more flexible than the joint due to the bolt's smaller cross-sectional area.

Once tightened, any strain of the joint, due to cyclic loading, will be equal to the strain of the bolt; and due to the lower stiffness of the bolt, the stress amplitude in the bolt is

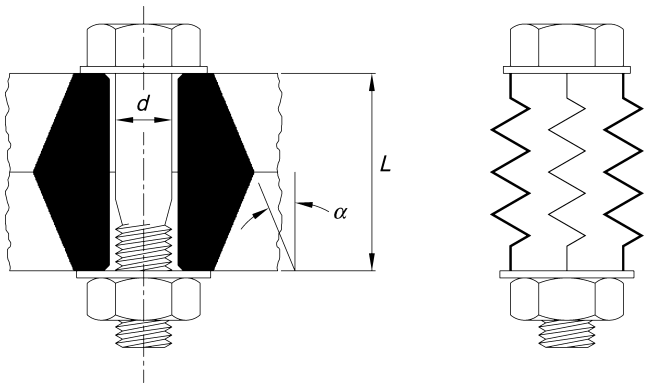


Figure 16.8
Bolted joint modeling.

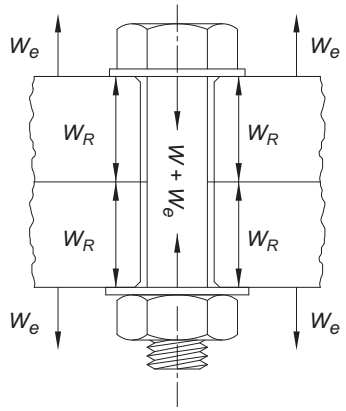


Figure 16.9
Typical bolted joint forces.

lower than the applied load variation. To analyze this, the forces acting on the bolt and joint, Figure 16.9, can be equated.

$$W_e = W_b + W_R \tag{16.1}$$

where

W_e is the applied external load (N),
W_b is the force on the bolt (N),
W_R is the force on the clamped members (N).

The load and deflection for the bolted joint shown in Figure 16.9, when there is no external load, $W_e = 0$, is illustrated in Figure 16.10. The loads are equal in magnitude and

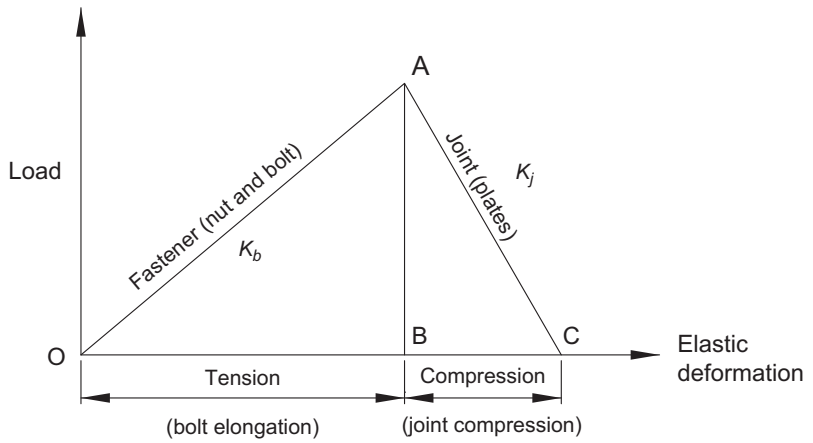


Figure 16.10
Load and deflection in a bolt and joint, respectively.

opposite in sign. The initial load is called the preload, and this is an important quantity specified by the engineer or the designer. The deflection of the bolt tends to be much greater than that of the joint. This is because the bolt is normally less stiff than the joint. The stiffness of the bolt and joint, given by the slope of the load/deflection lines, is stated in Eqns (16.2) and (16.3).

For a bolt, the stiffness is given by

$$k_b = \frac{\pi d^2 E}{4L} \tag{16.2}$$

where

k_b is the axial spring rate for the bolt (N/m),
d is the nominal bolt diameter (m),
E is Young's modulus for the bolt material (N/m^2),
L is the effective load-carrying length (m).

For flanges of equal thickness, using $\alpha = 30°$, the stiffness of the members is given by

$$k_j = \frac{0.5774\pi E d}{2\ln\left(5 \times \frac{0.5774L + 0.5d}{0.5774L + 2d}\right)} \tag{16.3}$$

where k_j is the spring rate for the members (N/m).

For the case of an external load, $W_e > 0$, applied to the joint, additional deflection will occur, altering the stresses and strains of both the bolt and the joint. If the applied load, W_e, is sufficiently high, separation of the two flanges will occur, potentially leading to failure of the machine or degradation of machine function by, for example, leakage (as in the case of a flanged pipe containing fluid). The resulting change in bolt tension for a given change in W_e is also of interest as this will enable calculation of the stress amplitude and hence characterization of the fatigue of the bolt, if the external load varies cyclically.

The changes in loads and strains are illustrated in Figure 16.11. The change in bolt load, W_b, is much smaller than the applied load, W_e. It is also worth noting that to cause separation of the joint, a load higher than the preload must be applied. The so-called separation force, W_{sep}, is given by Eqn (16.4). Once the threshold value W_{sep} is reached, the bolt will see all of the applied load and the joint will see none of it. W_{sep} represents the highest load the bolt should be allowed to experience, assuming that separation is not allowable.

$$W_{\text{sep}} = \left(1 + \frac{k_b}{k_j}\right) W_{\text{preload}} \tag{16.4}$$

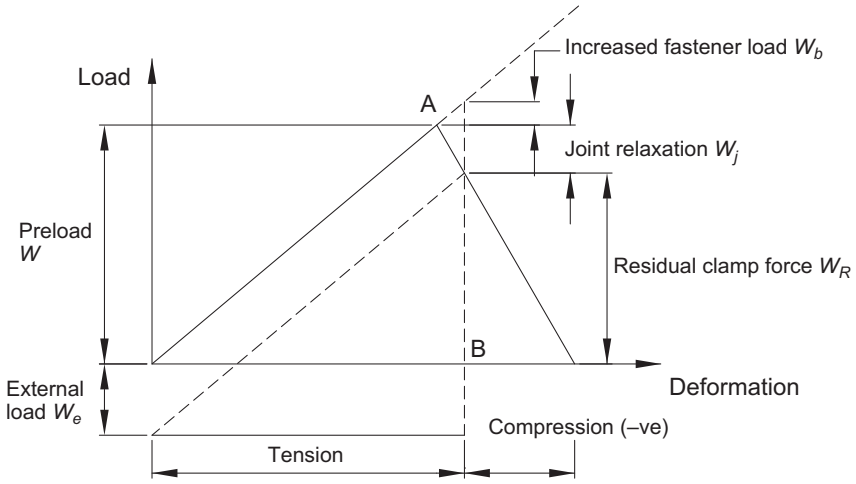


Figure 16.11
Load and deflection in a bolt and joint, respectively.

where

W_{sep} is the separation force (N),
$W_{preload}$ is the preload force (N).

As previously indicated, threaded fasteners tend to be used such that they are predominantly loaded in tension. An example is the bolts shown in Figure 16.12 that are used to fasten a flanged joint. As the fastener is tightened, the tension on the bolt will increase. It might be envisaged that the strength of a threaded fastener would be limited by the area of its

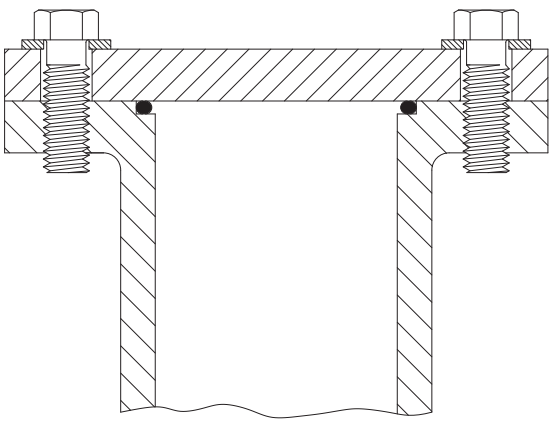


Figure 16.12
Flanged joint.

minor diameter. Testing, however, shows that the tensile strength is better defined using an area based on an average of the minor and pitch diameters.

$$A_t = \frac{\pi}{16}\left(d_p + d_r\right)^2 \tag{16.5}$$

For UNS threads,

$$d_p = d - \frac{0.649519}{N} \text{ and } d_r = d - \frac{1.299038}{N} \tag{16.6}$$

For ISO threads,

$$d_p = d - 0.649519p \tag{16.7}$$

and

$$d_r = d - 1.226869p \tag{16.8}$$

The stress in a threaded rod due to a tensile load is

$$\sigma_t = \frac{F}{A_t} \tag{16.9}$$

Theoretically one might think that when a nut engages a thread, all the threads in the engagement would share the load. However, inaccuracies in thread spacing cause virtually all the load to be taken by the first pair of threads.

Bolts are normally tightened by applying torque to the head or nut, which causes the bolt to stretch. This stretching results in bolt tension, known as preload, which is the force that holds a joint together. Torque is relatively easy to measure using a torque meter during assembly, so this is the most frequently used indicator of bolt tension. High preload tension helps to keep bolts tight, increases the strength of a joint, generates friction between parts to resist shear, and improves the fatigue resistance of bolted connections. The recommended preload for reusable connections can be determined by

$$F_i = 0.75A_t\sigma_p \tag{16.10}$$

and for permanent joints by

$$F_i = 0.9A_t\sigma_p \tag{16.11}$$

where

A_t is the tensile stress area of the bolt (m^2),
σ_p is the proof strength of the bolt (N/m^2).

Material properties for steel bolts are given in SAE standard J1199 and by bolt manufacturers.

If detailed information concerning the proof strength is unavailable, then it can be approximated by

$$\sigma_p = 0.85\sigma_y \tag{16.12}$$

Once the preload has been determined, the torque required to tighten the bolt can be estimated from

$$T = KF_i d \tag{16.13}$$

where

T is the wrench torque (N m),
K is a constant,
F_i is the preload (N),
d is the nominal bolt diameter (m).

The value of K depends on the bolt material and size. In the absence of data from manufacturers or detailed analysis, values for K are given in Table 16.5 for a variety of materials and bolt sizes.

Example 16.1

An M10 bolt has been selected for a reusable application. The proof stress of the low carbon steel bolt material is 310 MPa. Determine the recommended preload on the bolt and the torque setting.

Solution

From Table 16.2, the pitch for a coarse series M10 bolt is 1.5 mm.

$d_p = 10 - 0.649519 \times 1.5 = 9.026$ mm,
$d_r = 10 - 1.226869 \times 1.5 = 8.160$ mm,

Table 16.5: Values for the constant *K* for determining the torque required to tighten a bolt.

Conditions	*K*
¼ in. to 1 in. mild steel bolts	0.2
Nonplated black finish steel bolts	0.3
Zinc-plated steel bolts	0.2
Lubricated steel bolts	0.18
Cadmium-plated steel bolts	0.16

Source: Machinery's Handbook 25.

$$A_t = \frac{\pi}{16}(9.026 + 8.16)^2 = 57.99 \text{ mm}^2$$

For a reusable connection, the recommended preload is

$$F_i = 0.75 A_t \sigma_p = 13.48 \text{ kN}.$$

From Table 16.5, $K = 0.2$.

The torque required to tighten the bolt is given by

$$T = KF_i d = 0.2 \times 13.48 \times 10^3 \times 0.01 = 26.96 \text{ N m}$$

The principal application of threaded fasteners such as bolts and nuts is clamping components together. In such situations, the bolt is predominantly in tension. Both the bolt and the clamped components will behave as elastic members, provided material limits are not exceeded. If a load is applied to a bolted joint that is above the clamping load, then the behavior of the joint itself needs to be considered. As the bolt stretches, the compressive load on the joint will decrease, alleviating some of the load on the bolt. If a very stiff bolt is used to clamp a flexible member, such as a soft gasket, most of the additional force, above the clamping load, is taken by the bolt and the bolt should be designed to take the clamping force and any additional force. Such a joint can be classified as a soft joint. If, however, the bolt is relatively flexible compared to the joint, then nearly all the externally applied load will initially go toward decreasing the clamping force until the components separate. The bolt will then carry all of the external load. This kind of joint is classified as a hard joint.

Practical joints normally fall between the two extremes of hard and soft joints. The clamped components of a typical hard joint have a stiffness of approximately three times that of the bolt. An externally applied load will be shared by the bolt and the clamped components according to the relative stiffnesses, which can be modeled by

$$F_b = F_i + \frac{k_b}{k_b + k_c} F_e \tag{16.14}$$

$$F_c = F_i - \frac{k_c}{k_b + k_c} F_e \tag{16.15}$$

where

F_b is the final force in the bolt (N),
F_i is the initial clamping load (N),
k_b is the stiffness of the bolt (N/m),
k_c is the stiffness of the clamped components (N/m),
F_e is the externally applied load (N),
F_c is the final force on the clamped components (N).

Example 16.2

A set of six M8 bolts are used to provide a clamping force of 20 kN between two components in a machine. If the joint is subjected to an additional load of 18 kN after the initial preload of 8.5 kN per bolt has been applied, determine the stress in the bolts. The stiffness of the clamped components can be assumed to be three times that of the bolt material. The proof stress of the low carbon steel bolt material is 310 MPa.

Solution

Taking $k_c = 3k_b$,

$$F_b = F_i + \frac{k_b}{k_b + k_c}F_e = F_i + \frac{k_b}{k_b + 3k_b}F_e = F_i + \frac{1}{4}F_e = 8500 + \frac{18{,}000/6}{4} = 9250 \text{ N}$$

$$F_c = F_i - \frac{k_c}{k_b + k_c}F_e = F_i - \frac{3k_b}{k_b + 3k_b}F_e = F_i - \frac{3}{4}F_e = 8500 - \frac{3(18{,}000/6)}{4} = 6250 \text{ N}$$

As F_c is greater than zero, the joint remains tight. The tensile stress area for the M8 bolt can be determined from

$d_p = 8 - 0.649519 \times 1.25 = 7.188$ mm,
$d_r = 8 - 1.226869 \times 1.25 = 6.466$ mm,

$$A_t = \frac{\pi}{16}(7.188 + 6.466)^2 = 36.61 \text{ mm}^2$$

The stress in each bolt is given by

$$\sigma = \frac{F_b}{A_t} = \frac{9250}{36.61 \times 10^{-6}} = 252.7 \text{ MPa}$$

This is 82% of the proof stress. The bolts are therefore safe.

16.3 Power Screws

Power screws, which are also known as lead screws, are used to convert rotary motion into linear movement. With suitably sized threads, they are capable of a large mechanical advantage and can lift or move large loads. Applications include screw jacks and traverses in production machines. Although suited to fasteners, thread forms such as the ISO metric standard screw threads, UNC, and UNF series described in Section 16.2 may not be strong enough for power screw applications. Instead square, Acme, and buttress thread forms have been developed and standardized; see Figures 16.13–16.15, respectively. Some of the principal dimensions for standard Acme threads are given in Table 16.6.

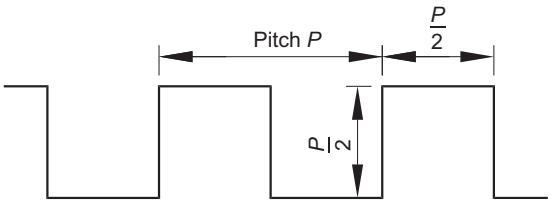


Figure 16.13
Square thread.

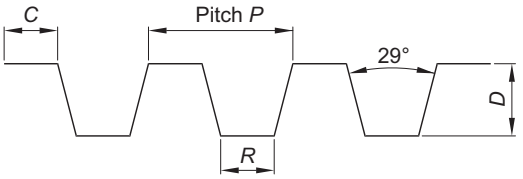


Figure 16.14
Acme thread.

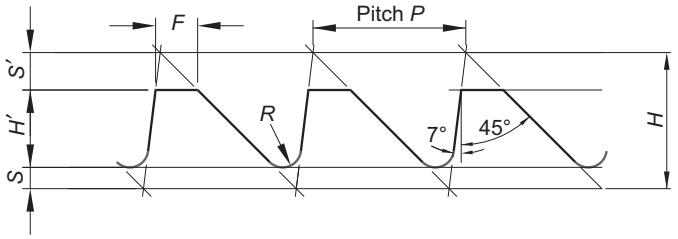


Figure 16.15
Buttress thread.

Self-locking in power screws refers to the condition in which a screw cannot be turned by the application of axial force on the nut. This is very useful in that a power screw that is self-locking will hold its position and load unless a torque is applied. As an example, most screw jacks for cars are self-locking and do not run down when the handle is released. The opposite condition to self-locking is a screw that can be back-driven. This means that an axial force applied to the nut will cause the screw to turn. A product application of this is the Yankee screwdriver, which has a high-lead thread on its barrel that is attached to the screwdriver bit. The handle acts as the nut and when pushed the barrel will turn, driving the bit around. Back-drivable power screws are a useful form of turning linear motion into rotary motion.

A screw thread can be considered to be an inclined plane wrapped around a cylinder to form a helix. The forces on a single thread of a power screw are illustrated for the case of lifting a load and lowering a load in Figure 16.16.

Table 16.6: Principal dimensions for ACME threads.

Major diameter (in.)	Threads per inch	Thread pitch (in.)	Pitch diameter (in.)	Minor diameter (in.)	Tensile stress area (in.^{2})
0.25	16	0.063	0.219	0.188	0.032
0.313	14	0.071	0.277	0.241	0.053
0.375	12	0.083	0.333	0.292	0.077
0.438	12	0.083	0.396	0.354	0.110
0.500	10	0.100	0.450	0.400	0.142
0.625	8	0.125	0.563	0.500	0.222
0.750	6	0.167	0.667	0.583	0.307
0.875	6	0.167	0.792	0.708	0.442
1.000	5	0.200	0.900	0.800	0.568
1.125	5	0.200	1.025	0.925	0.747
1.250	5	0.200	1.150	1.050	0.950
1.375	4	0.250	1.250	1.125	1.108
1.500	4	0.250	1.375	1.250	1.353
1.750	4	0.250	1.625	1.500	1.918
2.000	4	0.250	1.875	1.750	2.580
2.250	3	0.333	2.083	1.917	3.142
2.500	3	0.333	2.333	2.167	3.976
2.750	3	0.333	2.583	2.417	4.909
3.000	2	0.500	2.750	2.500	5.412
3.500	2	0.500	3.250	3.000	7.670
4.000	2	0.500	3.750	3.500	10.321
4.500	2	0.500	4.250	4.000	13.364
5.000	2	0.500	4.750	4.500	16.800

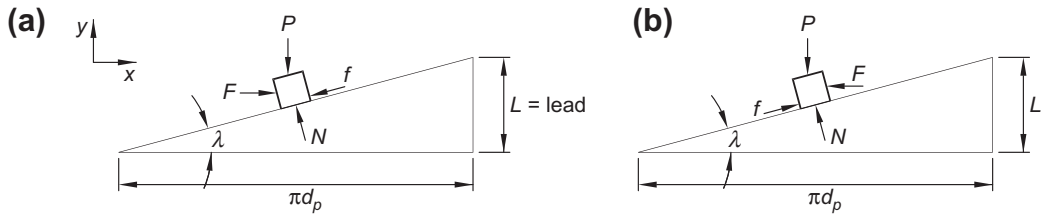


Figure 16.16
Force analysis at the interface of a lead screw and nut (a) lifting a load and (b) lowering a load.

The inclination of the plane is called the lead angle λ.

$$\tan \lambda = \frac{L}{\pi d_p} \tag{16.16}$$

In the case of lifting a load, summing the forces gives

$$\sum F_x = 0 = F - f \cos \lambda - N \sin \lambda = F - \mu N \cos \lambda - N \sin \lambda \tag{16.17}$$

Thus

$$F = N(\mu \cos \lambda + \sin \lambda) \tag{16.18}$$

$$\sum F_y = 0 = N \cos \lambda - f \sin \lambda - P = N \cos \lambda - \mu N \sin \lambda - P \tag{16.19}$$

and

$$N = \frac{P}{\cos \lambda - \mu \sin \lambda} \tag{16.20}$$

where μ is the coefficient of friction between the screw and the nut. The coefficient of friction can typically be taken as 0.15.

Solving Eqns (16.18) and (16.20) to give an expression for F gives

$$F = \frac{P(\mu \cos \lambda + \sin \lambda)}{\cos \lambda - \mu \sin \lambda} \tag{16.21}$$

The screw torque required to lift the load is given by

$$T_s = F\frac{d_p}{2} = \frac{Pd_p(\mu \cos \lambda + \sin \lambda)}{2(\cos \lambda - \mu \sin \lambda)} \tag{16.22}$$

It is convenient to rewrite this equation in terms of the lead L, so substituting with Eqn (16.12) and rearranging gives

$$T_s = \frac{Pd_p(\mu \pi d_p + L)}{2(\pi d_p - \mu L)} \tag{16.23}$$

The thrust collar also contributes a friction torque that must be accounted for. The torque required to turn the thrust collar is given by

$$T_c = \mu_c P\frac{d_c}{2} \tag{16.24}$$

where

d_c is the diameter of the thrust collar (m),
μ_c is the coefficient of friction in the thrust bearing.

The total torque to lift the load for a square thread is

$$T_u = \frac{Pd_p(\mu \pi d_p + L)}{2(\pi d_p - \mu L)} + \mu_c P\frac{d_c}{2} \tag{16.25}$$

Similar analysis can be performed for lowering a load, in which case

$$T_d = \frac{Pd_p(\mu\pi d_p - L)}{2(\pi d_p + \mu L)} + \mu_c P\frac{d_c}{2} \tag{16.26}$$

For Acme threads, the equivalent torque relationship for lifting a load is

$$T_u = \frac{Pd_p(\mu\pi d_p + L\cos\alpha)}{2(\pi d_p\cos\alpha - \mu L)} + \mu_c P\frac{d_c}{2} \tag{16.27}$$

and for lowering

$$T_d = \frac{Pd_p(\mu\pi d_p - L\cos\alpha)}{2(\pi d_p\cos\alpha + \mu L)} + \mu_c P\frac{d_c}{2} \tag{16.28}$$

The work done on a power screw is the product of the torque and angular displacement. For one revolution of a screw,

$$W_{\text{in}} = 2\pi T \tag{16.29}$$

The work delivered for one revolution is

$$W_{\text{out}} = PL \tag{16.30}$$

The efficiency of the system defined as work out/work in is

$$\eta = \frac{PL}{2\pi T} \tag{16.31}$$

Substituting for the torque for an Acme thread using Eqn (16.23), neglecting collar friction, gives

$$\eta = \frac{L(\pi d_p\cos\alpha - \mu L)}{\pi d_p(\pi\mu d_p + L\cos\alpha)} \tag{16.32}$$

Simplifying using Eqn (16.16) gives

$$\eta = \frac{\cos\alpha - \mu\tan\lambda}{\cos\alpha + \mu\cot\lambda} \tag{16.33}$$

For a square thread, $\alpha = 0$, so $\cos \alpha = 1$, giving

$$\eta = \frac{1 - \mu \tan \lambda}{1 + \mu \cot \lambda} \tag{16.34}$$

Standard Acme screws have lead angles between about 2° and 5°. Assuming a coefficient of friction of 0.15 gives the efficiency between 18% and 36%. This shows the disadvantage of power screws. Higher efficiencies can, however, be attained by reducing the friction, and one of the ways of doing this has been found to be the use of ball screws, although these potentially add to the cost of production.

A screw will self-lock if

$$\mu \geq \frac{L}{\pi d_p} \cos \alpha \tag{16.35}$$

or

$$\mu \geq \tan \lambda \cos \alpha \tag{16.36}$$

For a square thread, where $\cos \alpha = 1$,

$$\mu \geq \frac{L}{\pi d_p} \tag{16.37}$$

or

$$\mu \geq \tan \lambda \tag{16.38}$$

It should be noted that Eqns (16.35)−(16.38) are for a statically loaded screw. Dynamic loading such as vibration can reduce the effective friction and cause a screw to back-drive.

In the design of a power screw, consideration should also be given to buckling of the screw and choice of material for the screw and nut.

Example 16.3

A self-locking power screw is required for a screw jack. An initial proposal is to use a single start 1.25−5 Acme power screw. The axial load is 4000 N and collar mean diameter is 1.75 in. Determine the lifting and lowering torques, the efficiency of the power screw, and whether the design proposal is self-locking.

Solution

Single start thread, so the lead, L, is equal to the pitch, P.

$N = 5$ teeth per inch,

$$P = \frac{1}{N} = \frac{1}{5} = 0.2 \text{ in.}$$

$L = 0.2$ in. $= 5.08$ mm,
$d_p = 1.15$ in. $= 29.21$ mm,
$d_c = 1.75$ in. $= 44.45$ mm.

Assume sliding friction, $\mu = 0.15$.

The torque to lift the load is given by Eqn (16.27).

$$T = \frac{4000 \times 0.02921}{2}\left(\frac{0.15\pi 0.02921 + 0.00508\cos 14.5}{\pi 0.02921\cos 14.5 - 0.15 \times 0.00508}\right) + 0.15 \times 4000\frac{0.04445}{2}$$

$$= 58.42\,\frac{0.01376 + 4.9182 \times 10^{-3}}{0.08884 - 7.62 \times 10^{-4}} + 13.34 = 12.39 + 13.34 = 25.73 \text{ N m}$$

The torque to lower the load is given by Eqn (16.28),

$$T = 58.42\,\frac{0.01376 - 4.9182 \times 10^{-3}}{0.08884 + 7.62 \times 10^{-4}} + 13.34 = 5.765 + 13.34 = 19.11 \text{ N m}$$

$$\eta = \frac{PL}{2\pi T}$$

$$\eta_{\text{screw}} = \frac{4000 \times 0.00508}{2\pi 12.39} = 0.261$$

$$\eta_{\text{both}} = \frac{4000 \times 0.00508}{2\pi 25.73} = 0.126$$

The design will be self-locking if

$$\mu \geq \frac{L}{\pi d_p}\cos\alpha = \frac{0.00508}{\pi 0.02921}\cos 14.5 = 0.05359$$

$\mu = 0.15$ so the design is self-locking.

16.4 Rivets

Rivets are nonthreaded fasteners that are usually manufactured from steel or aluminum. They consist of a preformed head and shank, which are inserted into the material to be joined, and a second head that enables the rivet to function as a fastener is formed on the free end by a

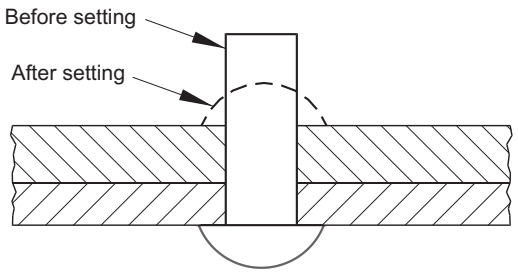


Figure 16.17
Conventional rivet before and after setting.

variety of means known as setting. A conventional rivet before and after setting is illustrated in Figure 16.17.

Rivets are widely used to join components in aircraft, boilers, ships, boxes, and other enclosures. Rivets tend to be much cheaper to install than bolts, and the process can be readily automated with single riveting machines capable of installing thousands of rivets per hour.

Rivets can be made from any ductile material, such as carbon steel, aluminum, and brass. A variety of coatings are available to improve corrosion resistance. Care needs to be taken in the selection of material and coating in order to avoid the possibility of corrosion by galvanic action. In general, a given size rivet will be not as strong as the equivalent threaded fastener.

The two main types of rivet are tubular and blind, and each type is available in a multitude of varieties. The advantage of blind rivets is that they require access to only one side of the joint. Another type of rivet with potentially many overall advantages, from a production perspective, is self-piercing rivets that do not require predrilled holes. The rivet is driven into the target materials with high force, piercing the top sheets and spreading outward into the bottom sheet of the material under the influence of an upsetting die to form the joint.

Factors in the design and specification of rivets include the size, type, and material for the rivet, the type of joint, and the spacing between rivets. There are two main types of riveted joint: lap joints and butt joints; see Figure 16.18. In lap joints, the components to be joined overlap each other, while for butt joints an additional piece of material is used to bridge the two components to be joined, which are butted up against each other. Rivets can fail by shearing through one cross section known as single shear, shearing through two cross sections known as double shear, and crushing. Riveted plates can fail by shearing, tearing, and crushing.

For many applications, the correct use of rivets is safety-critical and their use is governed by construction codes. For information and data concerning joints for pressure vessels, reference to the appropriate standards should be made, such as the ASME Boiler Code.

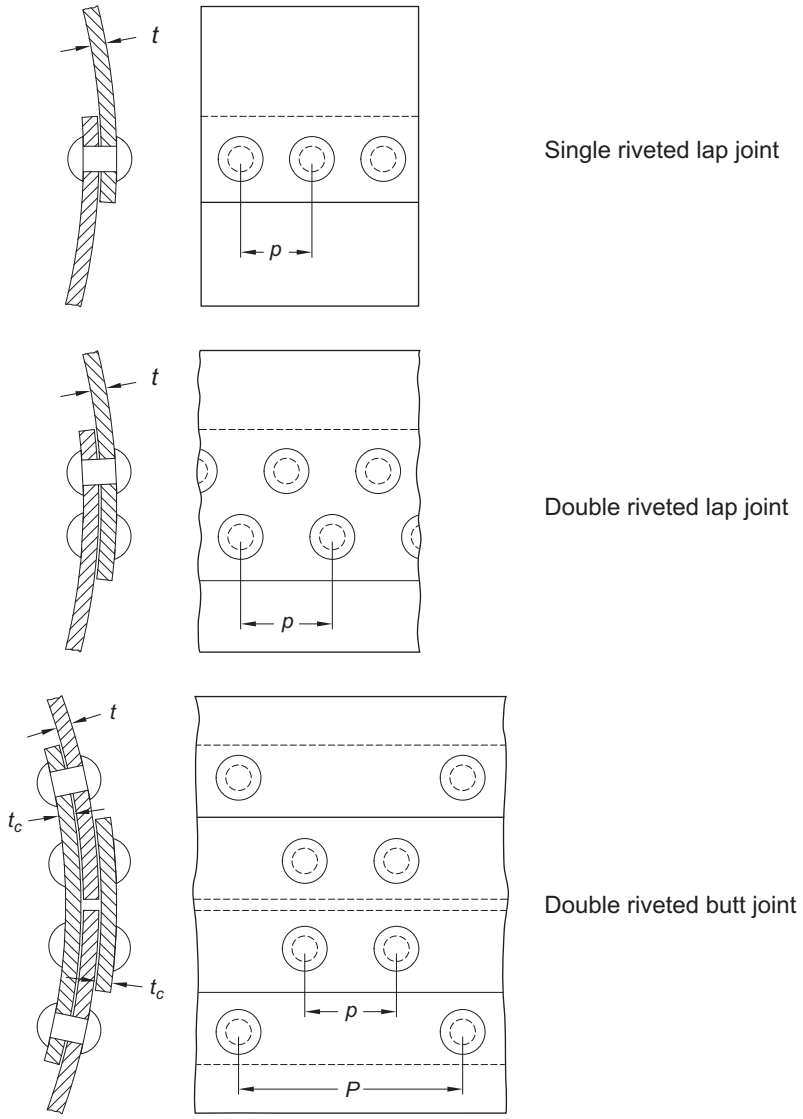


Figure 16.18
Some types of riveted joints.

Riveted joints can be designed using a simple procedure (*Machinery's Handbook*) assuming that:

- the load is carried equally by the rivets
- no combined stresses act on a rivet to cause failure
- the shearing stress in a rivet is uniform across the cross section
- the load that would cause failure in single shear would have to be double to cause failure in double shear

- the bearing stress of the rivet and plate is distributed equally over the projected area of the rivet
- the tensile stress is uniform in the section of metal between the rivets.

The allowable stress for a rivet is generally defined in the relevant standard. For example, the ASME Boiler Code lists an ultimate tensile stress for rivets of 379 MPa, an ultimate shearing stress of 303 MPa, and an ultimate compressive or bearing stress of 655 MPa. Design stresses are usually 20% of these values, i.e., for tensile, shear, and bearing stresses, the design limits are 75, 60, and 131 MPa, respectively.

For a single lap joint, the safe tensile load based on shear is given by

$$L = nA_r\tau_d \tag{16.39}$$

For a single lap joint, the safe tensile load based on compressive or bearing stress is given by

$$L = nA_b\sigma_c \tag{16.40}$$

$$A_b = td \tag{16.41}$$

For a single lap joint, the safe tensile load based on tensile stress is given by

$$L = A_p\sigma_t \tag{16.42}$$

where

L is the load (N),
A_r is the cross-sectional area of the rivet (m^2),
A_b is the projected bearing area of the rivet (m^2),
A_p is the cross-sectional area of the plate between rivet holes (m^2),
t is the thickness of the plate (m),
d is the diameter of the rivet (m),
τ_d is the allowable shear stress (N/m^2),
σ_c is the allowable bearing or compressive stress (N/m^2),
σ_t is the allowable tensile stress (N/m^2).

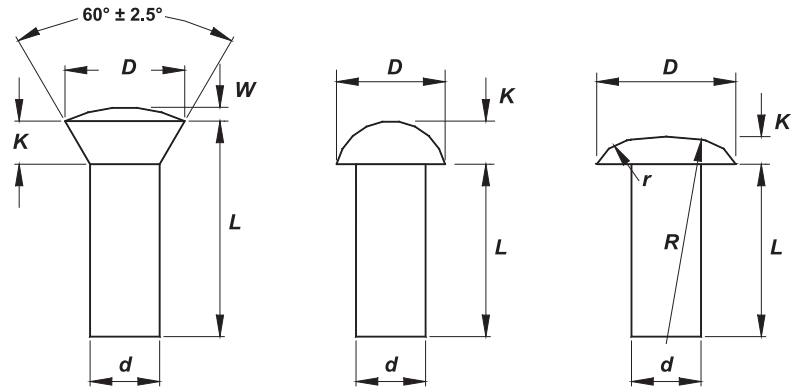
The efficiency of a riveted joint is given by

$$\eta = \frac{\text{least safe load}}{\text{ultimate tensile strength of unperforated section}} \tag{16.43}$$

A selection of rivets specified in BS 4620:1970 and ANSI B18.1.2-1972 is included in Tables 16.7−16.10 as examples. Rivets, however, are available as stock items from specialist manufacturers and suppliers in a much wider variety than the small selection presented in Tables 16.7−16.10. For any given application, the relevant standard should be referenced and a range of manufacturer's products considered.

Table 16.7: British Standard hot forged rivets for general engineering purposes.

Hot forged rivets



60° Csk. and raised Csk. head | Snap head | Universal head

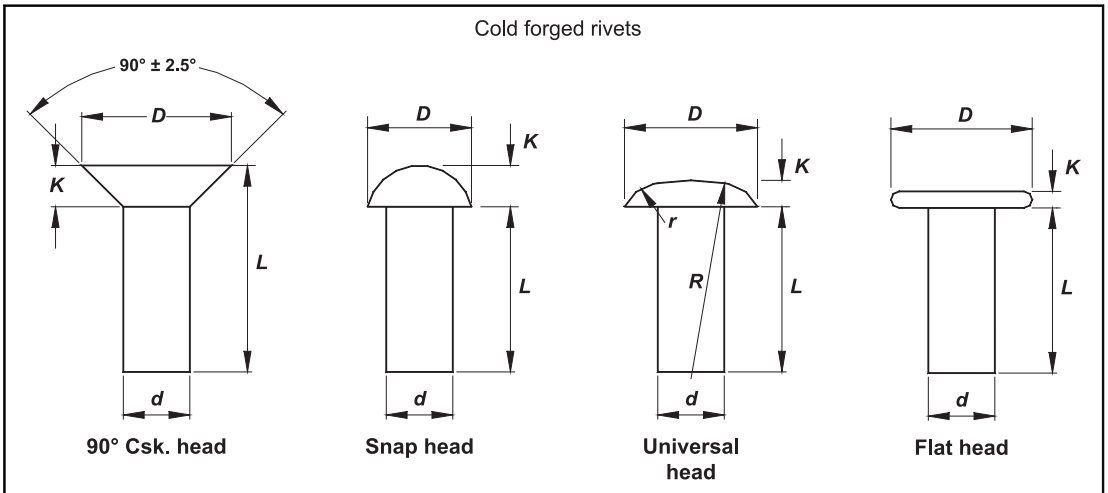
Nom. shank diam. *d* (mm)	Tol. on diam. *d* (mm)	60° Csk. and raised Csk. head		Snap head		Universal head			
		Head dimensions							
		Nom. diam. *D* (mm)	Height of raise *W* (mm)	Nom. diam. *D* (mm)	Nom. depth *K* (mm)	Nom. diam. *D* (mm)	Nom. depth *K* (mm)	Rad. *R* (mm)	Rad. *r* (mm)
(14)	±0.43	21	2.8	22	9	28	5.6	42	8.4
16		24	3.2	25	10	32	6.4	48	9.6
(18)		27	3.6	28	11.5	36	7.2	54	11
20	±0.52	30	4.0	32	13	40	8.0	60	12
(22)		33	4.4	36	14	44	8.8	66	13
24		36	4.8	40	16	48	9.6	72	14
(27)	±0.62	40	5.4	43	17	54	10.8	81	16
30		45	6.0	48	19	60	12.0	90	18
(33)		50	6.6	53	21	66	13.2	99	20
36		55	7.2	58	23	72	14.4	108	22
(39)		59	7.8	62	25	78	15.6	117	23

All dimensions are in millimeters. Sizes shown in parentheses are nonpreferred.
Source: Extracted from BS 4620:1970. Note: see the standard for full ranges.

Example 16.4

Determine the safe tensile, shear, and bearing loads and the efficiency for a 300 mm section of single-riveted lap joint made from ¼ in. plates using six 16-mm-diameter rivets. Assume that the drilled holes are 1.5 mm larger in diameter than the rivets. The values for the design limits for tensile, shear, and bearing stress can be taken as 75, 60, and 131 MPa, respectively.

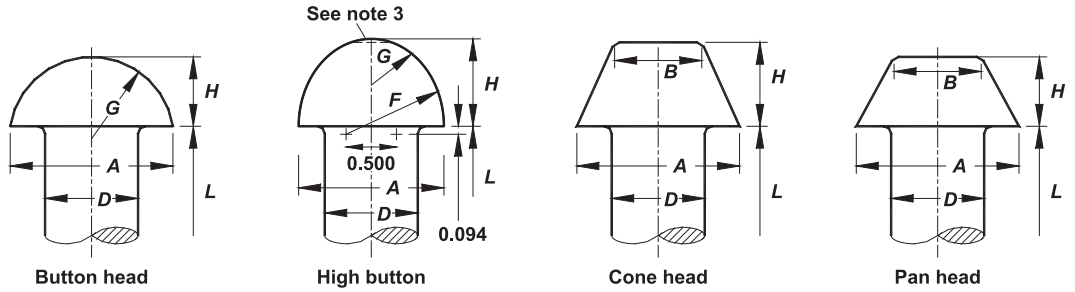
Table 16.8: British Standard cold forged rivets for general engineering purposes.



Nom. shank diam. *d* (mm)	Tol. on diam. *d* (mm)	90° Csk. head	Snap head		Universal head				Flat head	
		Head dimensions								
		Nom. diam. *D* (mm)	Nom. diam. *D* (mm)	Nom. depth *K* (mm)	Nom. diam. *D* (mm)	Nom. depth *K* (mm)	Rad. *R* (mm)	Rad. *r* (mm)	Nom. diam. *D* (mm)	Nom. depth *K* (mm)
1	±0.07	2	1.8	0.6	2	0.4	3.0	0.6	2	0.25
1.2		2.4	2.1	0.7	2.4	0.5	3.6	0.7	2.4	0.3
1.6		3.2	2.8	1.0	3.2	0.6	4.8	1.0	3.2	0.4
2		4	3.5	1.2	4	0.8	6.0	1.2	4	0.5
2.5		5	4.4	1.5	5	1.0	7.5	1.5	5	0.6
3		6	5.3	1.8	6	1.2	9.0	1.8	6	0.8
(3.5)	±0.09	7	6.1	2.1	7	1.4	10.5	2.1	7	0.9
4		8	7	2.4	8	1.6	12	2.4	8	1.0
5		10	8.8	3.0	10	2.0	15	3.0	10	1.3
6		12	10.5	3.6	12	2.4	18	3.6	12	1.5
(7)	±0.11	14	12.3	4.2	14	2.8	21	4.2	14	1.8
8		16	14	4.8	16	3.2	24	4.8	16	2
10		20	18	6.0	20	4.0	30	6	20	2.5
12	±0.14	24	21	7.2	24	4.8	36	7.2	—	—
(14)		—	25	8.4	28	5.6	42	8.4	—	—
16		—	28	9.6	32	6.4	48	9.6	—	—

All dimensions are in millimeters. Sizes shown in parentheses are nonpreferred.
Source: Extracted from BS 4620:1970. Note: see the standard for full ranges.

Table 16.9: Selected American National Standard large button, high button, cone, and pan head rivets.



Nom. body diam. *D**	Head diam. *A* (in.)		Height *H* (in.)		Head diam. *A* (in.)		Height *H* (in.)	
	Mfd. Note 1	Driven Note 2	Mfd. Note 1	Driven Note 2	Mfd. Note 1	Driven Note 2	Mfd. Note 1	Driven Note 2
	Button head				**High button head (Acorn)**			
½	0.875	0.922	0.375	0.344	0.781	0.875	0.500	0.375
5/8	1.094	1.141	0.469	0.438	0.969	1.062	0.594	0.453
3/4	1.312	1.375	0.562	0.516	1.156	1.250	0.688	0.531
7/8	1.531	1.594	0.656	0.609	1.344	1.438	0.781	0.609
1	1.750	1.828	0.750	0.688	1.531	1.625	0.875	0.688
1 1/8	1.969	2.062	0.844	0.781	1.719	1.812	0.969	0.766
1¼	2.188	2.281	0.938	0.859	1.906	2.000	1.062	0.844
1 3/8	2.406	2.516	1.031	0.953	2.094	2.188	1.156	0.938
1½	2.625	2.734	1.125	1.031	2.281	2.375	1.250	1.000
1 5/8	2.844	2.969	1.219	1.125	2.469	2.562	1.344	1.094
1 3/4	3.062	3.203	1.312	1.203	2.656	2.750	1.438	1.172
	Cone head				**Pan head**			
½	0.875	0.922	0.438	0.406	0.800	0.844	0.350	0.328
5/8	1.094	1.141	0.547	0.516	1.000	1.047	0.438	0.406
3/4	1.312	1.375	0.656	0.625	1.200	1.266	0.525	0.484
7/8	1.531	1.594	0.766	0.719	1.400	1.469	0.612	0.578
1	1.750	1.828	0.875	0.828	1.600	1.687	0.700	0.656
1 1/8	1.969	2.063	0.984	0.938	1.800	1.891	0.788	0.734
1¼	2.188	2.281	1.094	1.031	2.000	2.094	0.875	0.812
1 3/8	2.406	2.516	1.203	1.141	2.200	2.312	0.962	0.906
1½	2.625	2.734	1.312	1.250	2.400	2.516	1.050	0.984
1 5/8	2.844	2.969	1.422	1.344	2.600	2.734	1.138	1.062
1 3/4	3.062	3.203	1.531	1.453	2.800	2.938	1.225	1.141

All dimensions are given in inches.

Note 1: Basic dimensions of head as manufactured.

Note 2: Dimensions of manufactured head after driving and also of driven head.

Note 3: Slight flat permissible within the specified head-height tolerance.

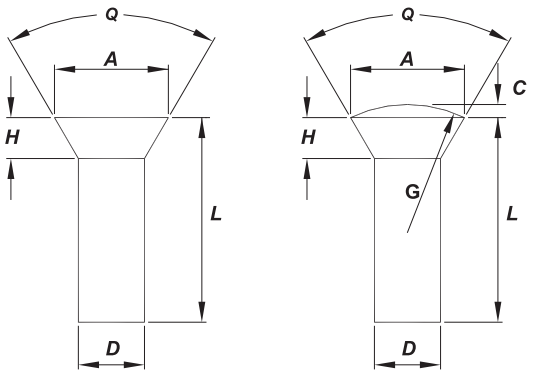
The following formulas give the basic dimensions for manufactured shapes: *Button head*, $A = 1.750D$; $H = 0.750D$; $G = 0.885D$. *High button head*, $A = 1.500D + 0.031$; $H = 0.750D + 0.125$; $F = 0.750D + 0.281$; $G = 0.750D - 0.281$. *Cone head*, $A = 1.750D$; $B = 0.938D$; $H = 0.875D$. *Pan head*, $A = 1.600D$; $B = 1.000D$; $H = 0.700D$. Length L is measured parallel to the rivet axis, from the extreme end to the bearing surface plane for flat bearing surface head type rivets, or to the intersection of the head top surface with the head diameter for countersunk head type rivets.

*The tolerance for the diameter of the body is plus and minus from nominal and for ½-in. size equals +0.020, −0.022; for sizes 5/8 to 1 in. it equals +0.030, −0.025; for sizes 1 1/8 and 1¼ in. it equals +0.035, −0.027; for sizes 1 3/8 and 1½ in. it equals +0.040, −0.030; for sizes 1 5/8 and 1 3/4 in. it equals +0.040, −0.037.

Source: Extracted from ANSI B18.1.2-1972, R1989. Note: see the standard for full ranges.

Table 16.10: Selected American National Standard large flat and oval countersunk rivets.

Flat and oval countersunk head



Flat Csk. head Oval Csk. head

Body diameter *D* (in.)			Head diam. *A* (in.)		Head depth *H* (in.)	Oval crown	Oval crown
Nominal*	Max.	Min.	Max.**	Min.§	Ref.	height* *C* (in.)	radius* *G* (in.)
½ 0.500	0.520	0.478	0.936	0.872	0.260	0.095	1.125
$^5/_8$ 0.625	0.655	0.600	1.194	1.112	0.339	0.119	1.406
$^3/_4$ 0.750	0.780	0.725	1.421	1.322	0.400	0.142	1.688
$^7/_8$ 0.875	0.905	0.850	1.647	1.532	0.460	0.166	1.969
1 1. 000	1.030	0.975	1.873	1.745	0.520	0.190	2.250
$1^1/_8$ 1.125	1.160	1.098	2.114	1.973	0.589	0.214	2.531
1¼ 1.250	1.285	1.223	2.340	2.199	0.650	0.238	2.812
$1^3/_8$ 1.375	1.415	1.345	2.567	2.426	0.710	0.261	3.094
1½ 1.500	1.540	1.470	2.793	2.652	0.771	0.285	3.375
$1^5/_8$ 1.625	1.665	1.588	3.019	2.878	0.831	0.309	3.656
$1^3/_4$ 1.750	1.790	1.713	3.262	3.121	0.901	0.332	3.938

All dimensions are given in inches.
*Basic dimension as manufactured. For tolerances, see table footnote in Table 16.9.
**Sharp-edged head.
§Rounded or flat-edged irregularly shaped head (heads are not machined or trimmed).
Source: Extracted from ANSI B18.1.2-1972, R1989. Note: see the standard for full ranges.

Solution

The safe tensile load, L, based of shear of the rivets is given by

$$L = nA_r\tau_d = 6\pi\frac{0.016^2}{4} \times 75 \times 10^6 = 90.48\,\text{kN}$$

The safe tensile load based on bearing or compressive stress is given by

$$L = 6 \times 0.016 \times 6.35 \times 10^{-3} \times 131 \times 10^6 = 79.86\,\text{kN}$$

The safe load based on tensile load is given by $L = A_p\sigma_t$.

The area of the plate between the rivet holes, A_p, is given by

$$A_p = 0.00635\left(0.3 - 6\left(16 \times 10^{-3} + 1.5 \times 10^{-3}\right)\right) = 1.238 \times 10^{-3}\ \text{m}^2$$

$$L = A_p\sigma_t = 1.238 \times 10^{-3} \times 131 \times 10^6 = 162.2\ \text{kN}$$

The safe tensile load would be the least of the three values determined, i.e. $L = 79.86$ kN.

$$\eta = \frac{79.86 \times 10^3}{0.00635 \times 0.3 \times 75 \times 10^6} = 0.56$$

The efficiency is 56%.

Example 16.5

Determine the maximum safe tensile load that can be supported by a 1 m section of a double-riveted butt joint with 15-mm-thick main plates and two 8-mm-thick cover plates. There are six rivets in each of the outer rows and seven rivets in each of the inner rows. The rivets are all 20 mm in diameter. Assume that the drilled holes are 1.5 mm larger in diameter than the rivets. The values for the design limits for tensile, shear, and bearing stress can be taken as 75, 60, and 131 MPa, respectively.

Solution

In analyzing a double-riveted joint, it is only necessary to analyze one side due to symmetry.

The safe tensile load based on double shearing of the rivets is equal to the number of rivets times the number of shearing planes per rivet times the cross-sectional area of the rivet times the allowable shearing stress.

$$L = n \times 2 \times A_r\tau_d$$

$$n = 6 + 7 = 13$$

$$L = 13 \times 2 \times \pi\frac{0.02^2}{4} \times 60 \times 10^6 = 490.1\ \text{kN}$$

The safe tensile load based on bearing stress is given by $L = nA_b\sigma_c$

$$L = 13 \times 0.02 \times 0.015 \times 131 \times 10^6 = 510.9\ \text{kN}$$

The cover plates have a combined thickness greater than that of the main plates, so detailed analysis need not be considered here.

The safe tensile load based on tensile stress, $L = A_p\sigma_t$, is

$$A_p = 0.015(1 - 6(0.02 + 0.0015)) = 0.01307 \text{ m}^2$$

$$L = 0.01307 \times 75 \times 10^6 = 980.3 \text{ kN}$$

To complete the analysis, it is necessary to consider the sum of the load that would cause tearing between rivets in the inner section plus the load carried by the rivets in the outer section. The sum of the loads is used because if the joint is to fail, it must fail at both sections simultaneously.

$$L = nA_b\sigma_c = 6 \times 0.02 \times 0.015 \times 131 \times 10^6 = 235.8 \text{ kN}$$

The safe tensile load based on the tensile strength of the main plate between the holes in the inner section is

$$L = A_p\sigma_t = 0.015(1 - 7(0.02 + 0.0015)) \times 75 \times 10^6 = 955.7 \text{ kN}$$

The total safe tensile load based on the sum of the above two values is 1.191 MN.

The safe tensile load for the riveted joints would be the least of the values calculated, i.e. 490.1 kN.

The efficiency of the riveted joint is given by

$$\eta = \frac{490.1 \times 10^3}{0.015 \times 1 \times 75 \times 10^6} = 0.436 = 43.6\%$$

16.5 Adhesives

Adhesive bonding involves the joining of similar or dissimilar materials using natural or synthetic substances, which form a rigid or semirigid interface without the use of mechanical fastening. Adhesives can be used to bond metals to nonmetals such as plastics, rubber, glass, and other ceramics and in applications in which welding would cause adverse effects on material properties. The advantages of adhesives in comparison to discrete fastening mechanisms include more uniform distribution of stresses over the bonded area, stiffer structures, smooth surface finishes, fatigue resistance, low-temperature fabrication, connection of dissimilar materials, and sealing. The limitations of adhesives include decreasing strength of the joint with increasing temperature, oxidation or other forms of chemical reaction reducing the strength of the adhesive joint over time, the necessity to cure and/or clamp components during assembly, and the fact that bonded structures are not easy to dismantle.

The range of adhesives is extensive, including acrylics, cyanoacrylates, epoxies, anaerobics, silicones, phenolics, plastisols, polyester hot melts, polyvinyl acetates, and polyurethane.

The merits of a variety of adhesives are listed in Table 16.11. Adhesives can be purchased from specialist manufacturers and suppliers; however, it should be noted that no one manufacturer is likely to produce all the different types available. The choice of which adhesive to use for a given application depends on the type of materials to be joined, the type of joint to be used, the environmental factors for the application (such as temperature and chemicals), the method of production, and the cost.

Bonded joints can fail by means of tensile, compressive, shear, or peel stresses or some combination of these; see Figure 16.19. Bonded joints tend to perform poorly under peel or

Table 16.11: Types of adhesives and principal characteristics.

Adhesive	Principal characteristics	Typical applications
Anaerobics	Cures in the absence of oxygen	Locking, and sealing of close-fitting components such as threaded fasteners
Cyanoacrylates	Cures through reaction with moisture held on surfaces to be bonded. Flows easily between close-fitting components. Usually solidify within seconds	Small plastic components, metals, ceramics, rubber, and wood
Acrylics	Fast curing. High strength. Tough. Supplied in two parts, resin and catalyst	Metals and plastics
Ceramics	High-temperature capability	Metals and ceramics
Epoxies	Consist of an epoxy resin and a hardener. Form strong and durable bonds	Metals, glass, plastics, and ceramics
Hot melts	Based on thermoplastics and epoxy and polyurethane thermosetting adhesives	Fast assembly of lightly loaded components
Phenolics	Require heat and pressure to cure bond	Metal to metal, metal to wood, and metal to brake-liners
Plastisols	Modified PVC dispersion that requires heat to harden. Resilient and tough	
Polyurethanes	Two part and fast curing. Form strong, resilient joints that are resistant to impact	Glass-fiber reinforced plastic. Certain thermoplastics
Polyvinyl acetates		Porous materials such as paper or wood and general packaging
Rubber adhesives	Based on solutions or latexes, they solidify through loss of the solvent or water medium	
UV-curable adhesives	Can be cured rapidly by exposure to ultraviolet radiation	
Silicones	Flexible adhesive with good high-temperature performance (up to approximately 200 °C)	Metals, glass, plastics, rubber, wood, and ceramics

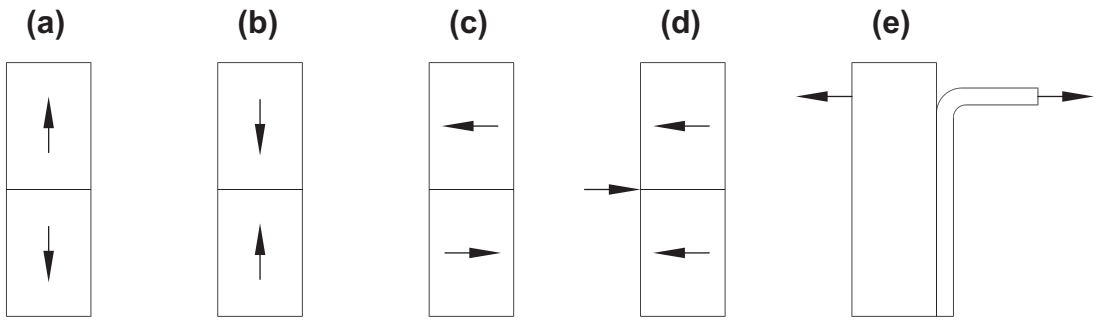


Figure 16.19
Loading of adhesive joints. (a) Tension. (b) Compression. (c) Shear. (d) Cleavage. (e) Peel.

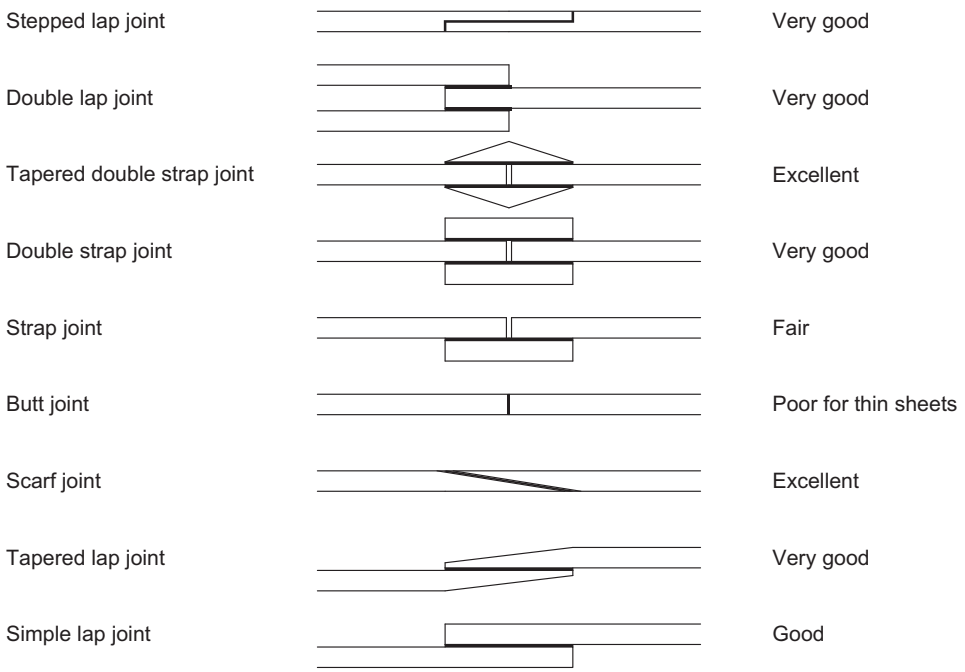


Figure 16.20
Relative performance of a variety of joint designs.

cleavage loading, and generally a bonded joint needs to be designed so that the loading is along the lines of the adhesive's strongest bond. Examples of joint designs and their relative performance are illustrated in Figure 16.20 and practical assemblies are shown in Figure 16.21. Adhesives can be useful for adding stiffeners to a sheet of material or adding strength to an aperture, as illustrated in Figures 16.22 and 16.23.

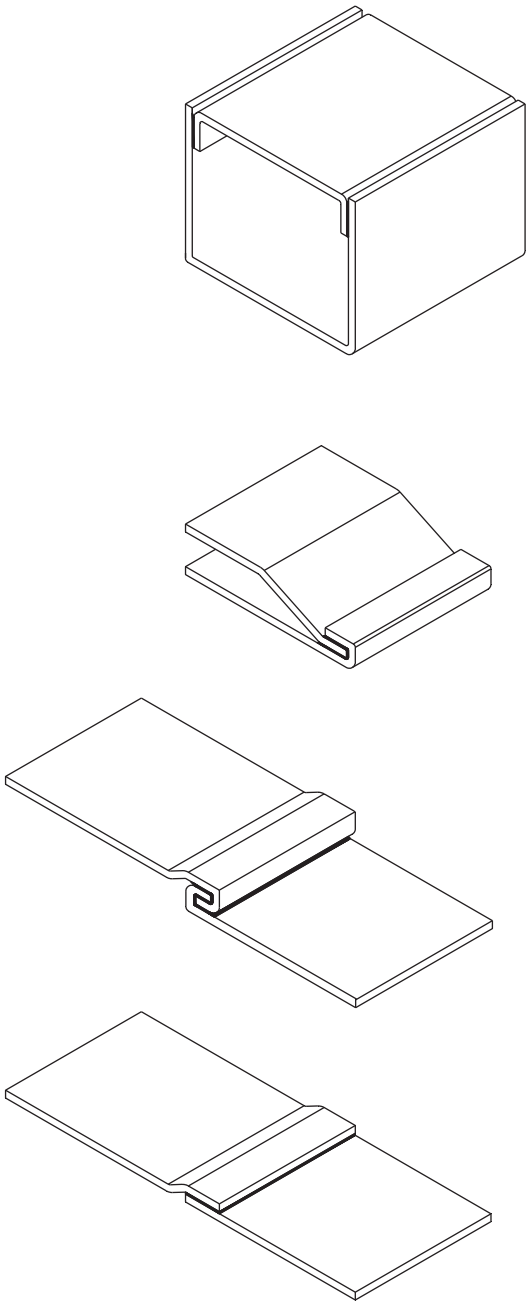


Figure 16.21
Practical joints for sheet materials.

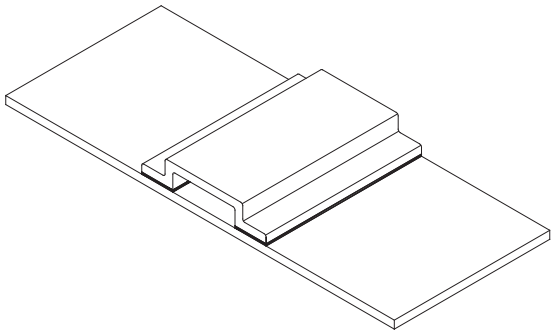


Figure 16.22
Stiffening of sheets.

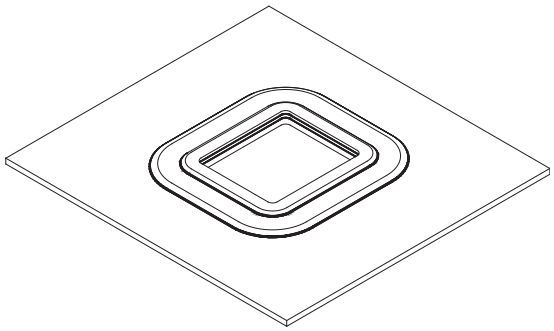


Figure 16.23
Strengthening an aperture.

The strength of a lap joint depends on the nature of the materials, the type of adhesive, the thickness of the sheets, and the area of the overlap. Test data are available for a variety of combinations of these variables, as illustrated in Figure 16.24. The tensile stress for the sheet material is given by

$$\sigma = \frac{P}{tw} \tag{16.44}$$

The shear stress in the joint is given by

$$\tau = \frac{P}{Lw} \tag{16.45}$$

Substituting for the load gives

$$\tau = \sigma \frac{t}{L} \tag{16.47}$$

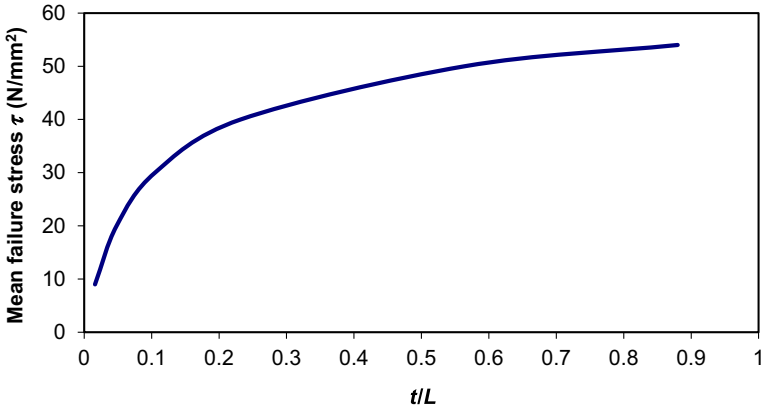


Figure 16.24

Shear strength versus t/L for simple lap joints. Data relate to Araldite bonded BS 1470-HS30 lap joints. *After Ciba-Geigy (1995).*

This equation can be used to determine the overlap required for a given sheet thickness, load, and joint strength.

A substantial source of data and design hints for the use of adhesives is given in the Loctite World Design Handbook, 1998.

Example 16.6

Determine the optimum overlap for a simple lap joint for 1/16 in. sheet metal if the maximum load is 250 N/mm wide. Use the Araldite for which data are given in Figure 16.24.

Solution

Selecting a safety factor of 2 to accommodate uncertainty in the loading conditions, the design load becomes 500 N/mm. The tensile stress in the metal is given by

$$\sigma = \frac{P}{t} = \frac{500 \times 10^3}{(1/16) \times 25.4 \times 10^{-3}} = 315.0 \text{ MPa}$$

$$\sigma = \frac{\tau}{t/L} = 315 \text{ MPa}$$

$$\tau = 315 t/L$$

Plotting this line in Figure 16.24 gives the intercept shear stress of approximately 28 MPa and $t/L = 0.09$.

With $t = 1.59$ mm, $L = 17.6$ mm.

The optimum overlap is therefore 17.6 mm.

Example 16.7

The Araldite for which data are given in Figure 16.24 is to be used for joining two sheets of metal. The joint should support loads of up to 180 N/mm width. If an overlap of 10 mm can be accommodated, determine the optimum sheet thickness.

Solution

Selecting a safety factor of 2 to accommodate uncertainty in the loading conditions, the design load becomes 360 N/mm.

The failure stress in the adhesive is given by

$$\tau = \frac{F/w}{L} = \frac{360 \times 10^3}{0.01} = 36 \text{ MPa}$$

Examination of Figure 16.24 gives $t/L = 0.16$ for $\tau = 36$ MPa.

If $L = 0.01$ m, then $t = 1.6 \times 10^{-3}$ m $= 1.6$ mm.

16.6 Welding

Welding can be described as the process of joining material together by raising the temperature of the surfaces to be joined so that they become plastic or molten.

For metals, welding involves the metallurgical bonding of components usually by application of heat by an electric arc, gas flame, or resistance heating under heavy pressure. There are numerous types of welding, including tungsten inert gas welding (TIG), metal inert gas welding (MIG), manual metal arc welding (MMA), submerged arc welding (SAW), resistance welding, and gas welding; see Swift and Booker (1997). A wide number of types of joint are possible with welding, a few of which are illustrated in Figure 16.25. The design of component joints is considered by Sampath (1997).

Thermoplastics can also be welded. Heat can be applied by means of a hot gas, which is usually inert. Other types of welding for plastics include inertia and ultrasonic and vibration welding; see Oberg et al. (1996).

Soldering and brazing involve the joining of heated metals while the components are in their solid state by means of molten filler metals (solders).

16.7 Snap Fasteners

Snap fasteners comprise a feature that deforms during assembly and then positively locates against a feature. Various examples are illustrated in Figure 16.26. Snap fasteners are

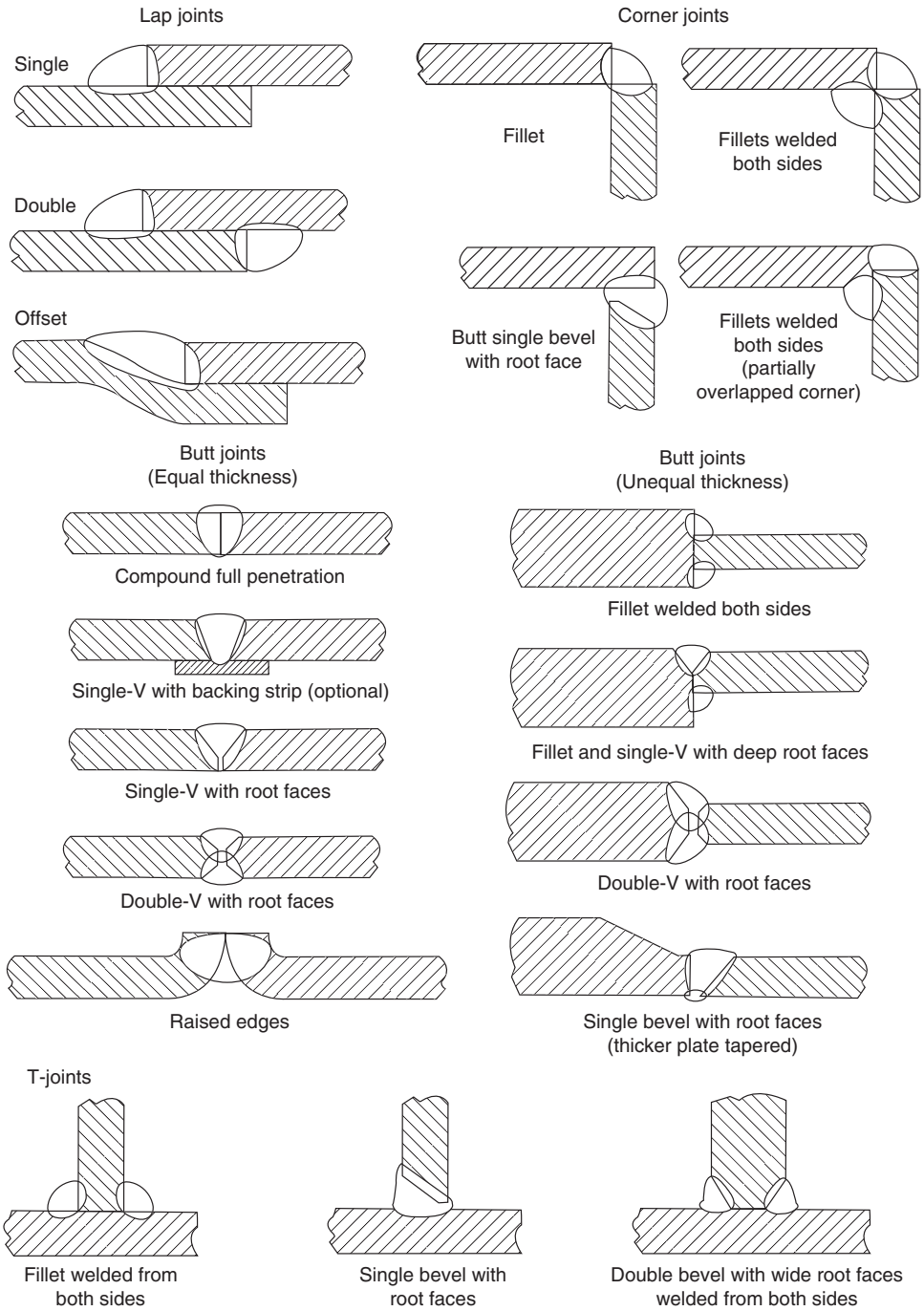


Figure 16.25
Weld joint design. *Reproduced from Swift and Booker (2013).*

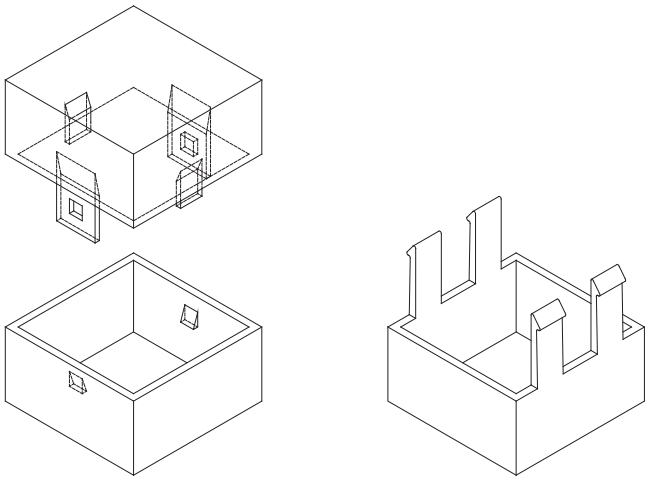


Figure 16.26
Examples of snap fasteners integrated onto moldings.

extremely popular because they can be molded directly into a component, which reduces the number of parts and the need to purchase separate fasteners, and due to their ease of assembly and associated savings in costs.

The analysis of snap fasteners is complex due to the nonlinear elastic behavior of the polymers typically used in the moldings concerned; see Hamrock et al. (1999) for further information. The permissible deflection for a rectangular-section cantilever snap joint is illustrated in Figure 16.27.

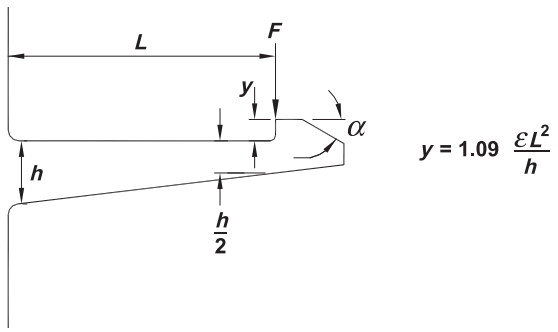


Figure 16.27
Permissible deflection for a rectangular-section cantilever snap joint.

16.8 Conclusions

The use of fasteners is critical to most machine designs. Their selection and design are frequently left too late in the design process. This can result in the use of too many fasteners or inconvenient locations with resultant component and machining costs. Instead, the means to fasten components should be considered within the total design activity, the interrelationships with other components should be identified, and appropriate fastening methods should be selected at an early stage in the design process. The range of fastening techniques available is extensive. Frequently, they can be obtained as a stock item with accompanying economies. This chapter has provided an overview of some of the fastening techniques available. It is, however, merely an introduction to an important subject.

References

Books and Papers

Ciba-Geigy, 1995. User's guide to adhesives.
Hamrock, B.J., Jacobson, B., Schmid, S.R., 1999. Fundamentals of Machine Elements. McGraw Hill.
Loctite, 1998. World Design Handbook, second ed. Loctite European Group.
Oberg, E., Jones, F., Horton, H.L., Ryffel, H.H., 1996. Machinery's Handbook, twenty fifth ed. Industrial Press Inc..
Sampath, K., 1997. Design for joining. In: ASM Handbook. Material Selection and Design, vol. 20. ASM International, pp. 762–773.
Swift, K.G., Booker, J.D., 2013. Manufacturing process selection handbook. Butterworth Heinemann.

Further Reading

Bayer, 1996. Bayer Snap-Fit Joints for Plastics. A Design Guide. Bayer Polymers Division.
Bickford, J.H., 1995. An Introduction to the Design and Behaviour of Bolted Joints, third ed. Marcel Dekker.
Industrial Fastener Institute, 1999. Metric Fastener Standards, third ed. Industrial Fastener Institute, Cleveland, Ohio, USA.
International Organization for Standardization, 2001. ISO Standards Handbook: Fasteners and Screw Threads. In: Terminology and Nomenclature, General Reference Standards, fifth ed. vol. 1.
Parmley, R.O., 1997. Standard Handbook of Fastening and Joining. McGraw Hill.
Renaud, J.E., Karr, P., 1996. Development of an integrated plastic snap fastener design advisor. Eng. Des. Autom. 2, 291–304.

Standards

British Standards Institution. BS 4620:1970. Specification for rivets for general engineering purposes. BSI, 1970.
British Standards Institution. BS 3643. ISO metric screw threads. BSI.
British Standards Institution. BS 3692. Metric precision hexagon bolts, screws and nuts. BSI.
British Standards Institution. BS 4168-3/4/5. Cap and socket headed setscrews. BSI.
British Standards Institution. BS 4183. ISO metric machine screws and machine screw nuts. BSI.
British Standards Institution. BS 4320. Metric washers. BSI.
British Standards Institution. BS 4464. Metric washers. BSI.
EN 24032/3/5. ISO metric hexagon bolts, screws and nuts.
EN ISO 2009. Slotted head bolts.
EN ISO 7046-1. Recessed head set screws.

Web Sites

At the time of going to press, the world-wide-web contained useful information relating to this chapter at the following sites:

fastenertech.com
www.apexfasteners.com
www.associatedfastening.com
www.atlanticfasteners.com
www.avdel.textron.com
www.boltscience.com
www.earnestmachine.com
www.fabory.com
www.fastenal.com
www.fastener-manufacturers.co.uk
www.fastenertechnology.com
www.libertyfastener.com
www.machinedesign.com/bde/fastening/fastening.html
www.nutsandbolts.net
www.plastic-fastener.com
www.rivet.com
www.rotaloc.com
www.seac.uk.com
www.sinofastener.com
www.skybolt.com
www.ssfast.co.uk
www.toleeto.com
www.vaughanjones.co.uk
www.watai.com
www.yerd.com.hk

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

A_r	cross-sectional area of the rivet (m^2)
A_b	projected bearing area of rivet (m^2)
A_p	cross-sectional area of the plate between rivet holes (m^2)
A_t	tensile stress area of the bolt (m^2)
Csk.	countersunk
d	diameter (m)
d_c	diameter of the thrust collar (m)
f	frictional force (N)
F	force (N)
F_b	final force in the bolt (N)
F_c	final force on the clamped components (N)
F_e	externally applied load (N)
F_i	preload (N)

h	maximum cantilever thickness (m)
K	constant
k_b	stiffness of the bolt (N/m)
k_c	stiffness of the clamped components (N/m)
L	length (m), safe load (N)
MIG	metal inert gas
MMA	manual metal arc
n	number
N	reaction (N)
P	pitch (m)
r, R	radius (m)
SAW	submerged arc welding
t	thickness (m)
T	torque (N m)
T_c	torque required to turn the thrust collar (N m)
T_d	torque for lowering a load (N m)
T_s	screw torque (N m)
T_u	torque for lifting a load (N m)
TIG	tungsten inert gas
UNC	Unified coarse pitch thread series
UNF	Unified fine pitch thread series
W	work done (N m)
y	deflection (m)
α	angle
ε	permissible strain
η	efficiency
λ	angle
μ	coefficient of friction
μ_c	coefficient of friction in the thrust bearing
σ	stress (N/m^2)
σ_p	proof strength of the bolt (N/m^2)
σ_t	tensile stress (N/m^2)
σ_y	yield stress (N/m^2)
τ	shear stress (N/m^2)
τ_d	allowable shear stress (N/m^2).

CHAPTER 17

Wire Rope

Chapter Outline

17.1 Introduction 721
17.2 Wire Rope Selection 725
17.3 Conclusions 732
References 732
Further Reading 732
Nomenclature 733

Abstract

Flexible wire rope is formed by using many small diameter wires twisted around a central core. Wire rope is widely used for hoists, haulage, and conveyor applications, where the wire rope supports tensile loading along its length. This chapter introduces the technology and outlines a selection procedure for typical applications.

17.1 Introduction

Wire rope is widely used in hoisting, haulage, and conveyer applications, with a NASA heavy lift winch example shown in Figure 17.1. By using many small diameter wires, twisted around a central core, some flexibility in the wire rope can be achieved, and it is possible for the wire rope to articulate drums and other radial segments.

Wire rope is manufactured by helically twisting many small wires together to form a strand, as illustrated in Figure 17.2. Typically 7, 19, or 37 small wires are twisted into a strand. Subsequently, a number, often six or eight, of multiwire strands are twisted about a core to form the flexible wire rope. Some common multiwire strand wire rope configurations are indicated in Table 17.1.

The central core is usually saturated with a lubricant in order to facilitate the sliding movement of the strands relative to each other.

The lay of a wire rope is designated by direction and type, with examples shown in Figure 17.3. The direction is right or left according to how the strands have been laid around the core. The type of lay is either regular or lang, depending on whether the wires in the strands are laid in the opposite direction of the strands or in the same direction as the strands.

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00017-4
Copyright © 2014 Elsevier Ltd. All rights reserved.



Figure 17.1
Use of wire rope in a winch application for the NASA 325 ton hoist.

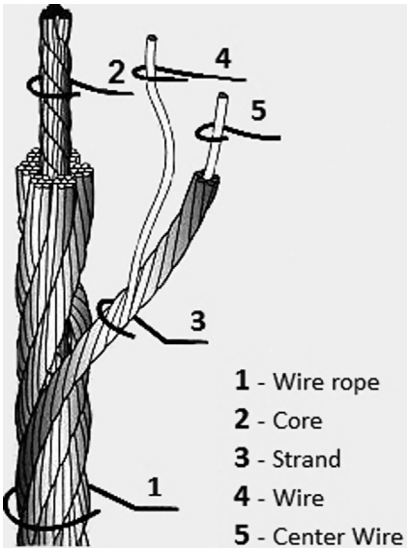


Figure 17.2
Wire rope construction. *Reproduced from AISI (1979).*

Table 17.1: Common multiwire strand wire rope configurations.

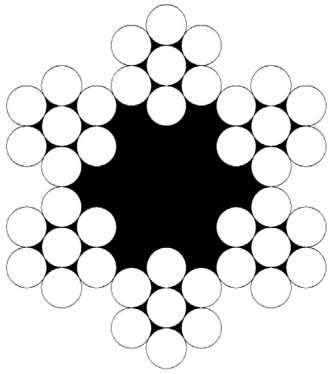
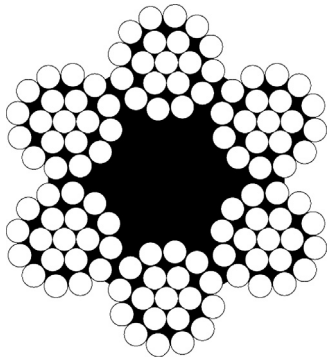
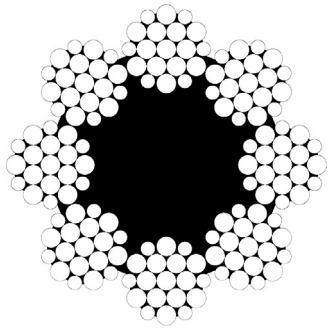
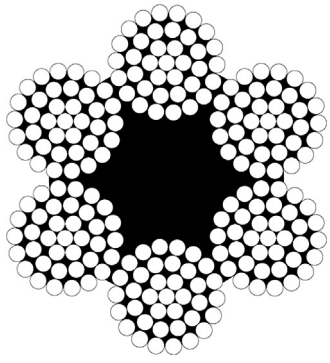
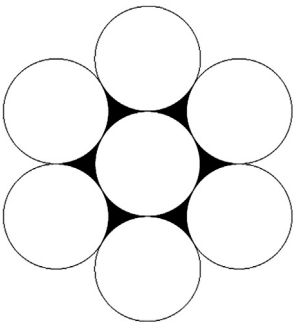
Wire	Typical application	
6 × 7	Haulage	
6 × 19	Standard hoisting	
8 × 19	Extra flexible	
6 × 37	Specially flexible	

Table 17.1: Common multiwire strand wire rope configurations.—cont'd

Wire	Typical application	
7-wire	Haulage	

Regular lay wire rope is used for the widest range of applications. It generally has better resistance to crushing than lang lay wire rope and does not rotate as severely under load when used in an application, where either end of the rope is not fixed. In regular lay, the wires appear to be nominally aligned with the axis of the rope.

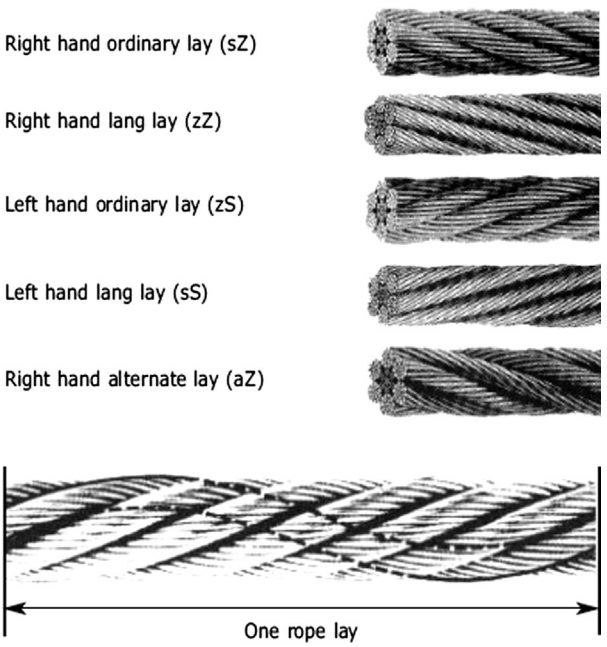


Figure 17.3
Wire rope lay. *Image courtesy of Noble & Son Ltd.*

In lang lay, the wires appear to make an angle with respect to the axis of the rope. Lang lay provides some advantages with possibly 15%−20% advantage for fatigue and wear resistance. This is because of the following two factors:

1. Smaller strains occur in the outer layers as the rope passes over a drum or radius; and
2. The contact areas between wires are larger and, therefore, the contact stresses are smaller.

However lang lay rope has some disadvantages. It tends to rotate when axial loads are applied, unless the rope is secured at both ends. Lang lay rope has less resistance against crushing against a drum or sheave.

When a tensile load is applied to helically twisted wire rope, the wires tend to stretch and the helixes "tighten." This causes Hertzian contact stresses and sliding motion between the wires. With load cycling and as wire rope is bent around formers and drums, the wire rope can experience failure due to:

- Tensile fatigue;
- Bending fatigue;
- Fretting fatigue;
- Surface fatigue wear;
- Abrasive wear;
- Yielding; or
- Rupture.

17.2 Wire Rope Selection

Aspects that require consideration in rope wire system design include the following:

- Materials;
- Wire rope dimensions;
- Construction type;
- Rope attachment;
- Drum or mandrel dimensions; and
- Inspection and maintenance.

Striking a balance between abrasion resistance and resistance to bending fatigue is an important aspect of wire rope selection. Typically, empirical guidance for this is available in the form of use of an industry selection chart, known as an X plot or an X chart, Figure 17.4.

Usually wire rope selection is an iterative process. It requires selection of an initial set of parameters, followed by an analysis of the proposed design, and then revision of the proposal and re-analysis, as necessary.

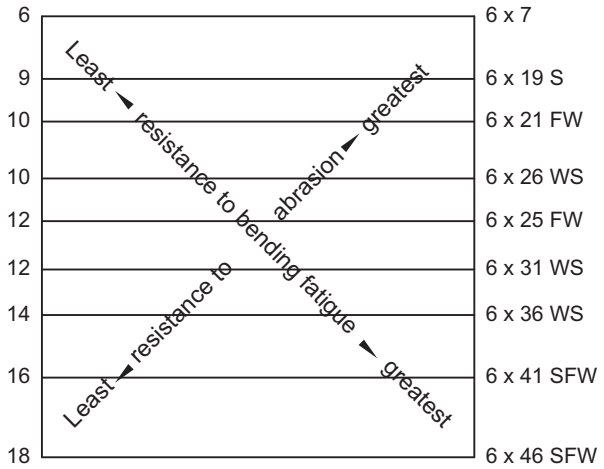


Figure 17.4
X plot comparing bending fatigue resistance versus abrasion resistance for selected wire rope constructions. *Adapted from Wire Rope Technical Board (2005).*

The following procedure provides a basis for wire rope selection.

1. Establish the principal design specifications for the system such as loads, failure modes, life, safety, cost, and maintenance.
2. Select an initial wire rope construction using the X plot (Figure 17.4).
3. Select a wire rope material.
4. Select a safety factor.
5. Determine the rope size using $\sigma_t = T/A$.
6. Using the rope diameter, determine the minimum recommended sheave diameter (see manufacturer data or Table 17.3 for indicative data).
7. Estimate the approximate bending stress in the outer wires (using Eqn (17.2)).
8. Determine the fatigue strength parameter, R_N (see Figure 17.5).
9. Determine the required rope diameter based on fatigue.
10. Determine the wear based limiting pressure for the rope class and sheave or drum material (see manufacturer data or Table 17.2 for indicative data).
11. Calculate the necessary rope diameter based on wear.
12. Identify the larger of the wire rope diameters determined from consideration of $d_{r\ \text{static}}$, $d_{r\ \text{fatigue}}$, and $d_{r\ \text{wear}}$.
13. Check all of the calculations with this new diameter and reconsider or modify the selections if necessary.

The direct tensile stress can be estimated using

$$\sigma_t = \frac{T}{A_{\text{rope}}} \tag{17.1}$$

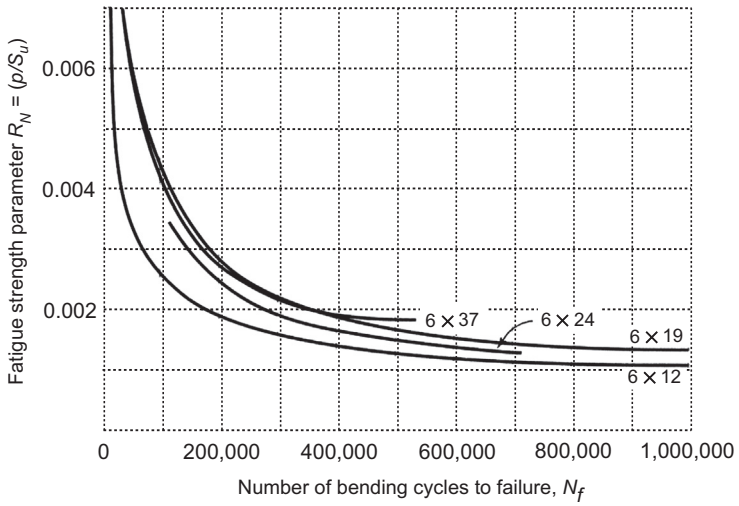


Figure 17.5
Fatigue lives for several wire rope constructions as a function of fatigue strength parameter. *Collins et al. (2010)*.

where T is the resultant tensile force (N) and A_{rope} is the approximate metallic cross-sectional area of the rope (m^2) as a function of the rope diameter, d_r.

Force components that may need to be considered include

- Load to be lifted,
- Weight of the wire rope,
- Inertial effects from accelerating the load to the operational velocity (these forces can be a few times the static load),
- Impact loading, and
- Frictional resistance.

Table 17.2: Maximum permissible pressures (Bridon).

Number of outer wires in strand	Cast iron (kgf/cm^2)	Low carbon steel (kgf/cm^2)	11%–13% Mn steel or equivalent (kgf/cm^2)
5–8 ordinary lay	20	40	105
5–8 lang lay	25	45	120
9–13 ordinary lay	35	60	175
9–13 lang lay	40	70	200
14–18 ordinary lay	42	75	210
14–18 lang lay	47	85	240
Triangular strand	55	100	280

Table 17.3: Material and construction data for selected wire rope classes.

Classification	6 × 7	6 × 19	6 × 37	8 × 19
Number of outer strands	6	6	6	8
Number of wires per strand	3—14	15—26	27—49	15—26
Maximum number of outer wires	9	12	18	12
Approx diameter of outer wires	$d_r/9$	$d_r/13$ to $d_r/16$	$d_r/22$	$d_r/15$ to $d_r/19$
Material available (typically). Core: FC	IPS (200)	*I* (80) *T* (130) IPS (200)	IPS (200)	*I* (80) *T* (130) IPS (200)
Material available (typically). Core: IWRC	IPS (190)	*I* (190) *T* (220) IPS (255)	EIPS (220) EIPS (255)	IPS (190) EIPS (200)
Approx metallic cross-section of rope. Core: FC	$0.384d_r^2$	$0.404d_r^2S^4$	$0.427d_r^2(\mathrm{FW})^4$	$0.366d_r^2(\mathrm{W})^4$
Approx metallic cross-section of rope. Core: IWRC	$0.451d_r^2$	$0.470d_r^2S^4$	$0.493d_r^2(\mathrm{FW})^4$	$0.497d_r^2(\mathrm{W})^4$
Standard nominal rope diameters	¼ to 5/8 × 1/16$^{\text{th}}$s $^3/_4$ to 1½ × 1/8$^{\text{th}}$s	¼ to 5/8 × 1/16$^{\text{th}}$s $^3/_4$ to $2^3/_4$ × 1/8$^{\text{th}}$s	¼ to 5/8 × 1/16$^{\text{th}}$s $^3/_4$ to 3¼ × 1/8$^{\text{th}}$s	¼ to 5/8 × 1/16$^{\text{th}}$s $^3/_4$ to 1½ × 1/8$^{\text{th}}$s
Unit weight of rope (lb/ft)	$1.5d_r^2$	$1.6d_r^2$	$1.55d_r^2$	$1.45d_r^2$
Approx modulus of elasticity of the rope (psi) 0% to 20% of S_u	11.7×10^6 (FC)	10.8×10^6 (FC) 13.5×10^6 (IWRC)	9.9×10^6 (FC) 12.6×10^6 (IWRC)	8.1×10^6
Approx modulus of elasticity of the rope (psi) 21% to 65% of S_u	13×10^6 (FC)	12×10^6 (FC) 15×10^6 (IWRC)	11.6×10^6 (FC) 14×10^6 (IWRC)	9×10^6
Recommended minimum sheave or drum diameter	$42d_r$	$34d_r$	$18d_r$	$26d_r$

Note: FC = fiber core; IPS = improved plow steel.
Source: Collins et al. (2010).

The bending stress (N/m^2) can be determined from

$$\sigma_b = \frac{M\left(\frac{d_w}{2}\right)}{\left(\frac{Md_s}{2E_r}\right)} = \frac{d_w}{d_s}E_r \tag{17.2}$$

Table 17.4: Maximum allowable bearing pressure between rope and sheave.

Rope	Sheave material			
	Wood	Cast iron	Cast steel	Manganese steel
		Regular lay		
6 × 7	1.0	2.1	3.8	10.1
6 × 19	1.7	3.3	6.2	16.6
6 × 37	2.1	4.0	7.4	20.7
8 × 19	2.4	4.7	8.7	24.1
		Lang lay		
6 × 7	1.1	2.4	4.1	11.4
6 × 19	1.9	3.8	6.9	19.0
6 × 37	2.3	4.6	8.1	22.8
8 × 19	2.7	5.5	10.0	27.6

Adapted from Wire Rope Users Manual.

where d_w is the wire diameter (m); d_s is the sheave or drum diameter (m); E_r is the modulus of elasticity of the wire rope (N/m^2); M is the applied moment (N m).

The compressive stress between the wire rope and the sheave can be estimated using

$$p = \frac{2T}{d_r d_s} \tag{17.3}$$

where p is the unit radial pressure (N/m^2).

The fatigue strength parameter, R_N, can be calculated from

$$R_N = \frac{p}{\sigma_{\text{uts}}} \tag{17.4}$$

Selected data for the maximum allowable bearing pressure between the rope and sheave is given in Table 17.4.

Example 17.1

Select a wire rope for a hoist required to lift 800 kg. The hoist design uses two lines to support the load attached to a vertically moving sheave, on which the swivel hook is mounted. The desired design life is 2 years, and the anticipated maximum use is 20 lifts per hour, 7.5 h a day for 220 days per year. A safety factor of 5 is required based on static ultimate strength and 1.5 based on fatigue.

Solution

Load = 400 × 9.81 = 3924 N.

From Figure 17.4, going for a balance of resistance to abrasion and fatigue, a 6 × 31 WS or a 6 × 25 FW wire rope construction might be appropriate.

From Table 17.3, the nearest classifications compatible with 6 × 31 WS and 6 × 25 FW are 6 × 37 and 6 × 19, respectively. As a first iteration for the design, a 6 × 37 WS wire rope construction will be explored.

A safety factor of 5 based on ultimate tensile strength has been defined.

For a suddenly applied load,

$$\sigma_{\text{max suddenly applied}} = 2\sigma_{\text{max static}}$$

From Eqn (17.1),

$$\sigma_{\text{max suddenly applied}} = 2\sigma_{\text{max static}} = 2\frac{T}{A_{\text{rope}}}$$

Each wire rope carries 3924/2 = 1962 N. From Table 17.3, $A_{\text{rope}} = 0.427d_r^2$

$$\sigma_{\text{max suddenly applied}} = 2\frac{T}{A_{\text{rope}}} = 2\frac{1962}{0.427d_{\text{rope}}^2}$$

From Table 17.3, the static ultimate tensile strength of improved plow steel (IPS) is 200,000 psi ≈ 1379 MPa.

With a static safety factor of 5, the design stress is given by

$$\sigma_d = 1379/5 = 275.8 \text{ MPa}$$

Equating the suddenly applied stress and the design stress, $\sigma_{\text{max suddenly applied}} = \sigma_{\text{design}}$, and solving for the rope diameter gives

$$d_{\text{rope}} = \sqrt{\frac{3924}{275.8 \times 10^6 \times 0.427}} = 8.164 \text{ mm}$$

From Table 17.3, the nearest larger standard rope diameter is 11/32 in or 12/32 in. Going for the larger of these, 12/32 in, $d_{r\ \text{static}} = 9.525$ mm.

From Table 17.3 the minimum recommended sheave diameter is given by

$$d_{\text{sheave}} = 18d_{\text{rope}} = 18 \times 9.525 = 171.5 \text{ mm}$$

The bending stress in the outer wires can be estimated from

$$\sigma_b = \frac{d_{\text{wire}}}{d_{\text{sheave}}} E_{\text{rope}}$$

The approximate diameter of the outer wires, in this case, from Table 17.3 is $d_{\text{wire}} = d_{\text{rope}}/22$. So,

$$\sigma_b = \frac{d_{\text{wire}}}{d_{\text{sheave}}} E_{\text{rope}} = \frac{8.164/22}{171.5} 75.8 \times 10^9 = 191.5 \text{ MPa}$$

The number of cycles is

$$N_d = 20 \times 7.5 \times 220 \times 2 = 66{,}000$$

From Figure 17.5, the fatigue strength parameter, $R_N = 0.0055$.

The pressure corresponding to failure in 66,000 cycles is

$$p_{N_f} = R_N \sigma_{\text{uts}} = 0.0055 \times 1379 = 7.584 \text{ MPa}$$

The fatigue safety facture is 1.5, so

$$p_{d \text{ fatigue}} = \frac{p_{N_f}}{1.5} = \frac{7.584}{1.5} = 5.056 \text{ MPa}$$

The wire rope diameter based on fatigue is given by

$$d_{\text{rope fatigue}} = \frac{2T}{p_{d \text{ fatigue}} d_{\text{sheave}}} = \frac{2 \times 1962}{5.056 \times 10^6 \times 0.1715} = 9.054 \text{ mm}$$

From Table 17.4, for a 6×37 wire rope on a cast-carbon steel sheave (BHN 160), the allowable bearing pressure load based on wear is 8.1 MPa.

The wire rope diameter based on wear is given by

$$d_{\text{rope wear}} = \frac{2T}{p_{d \text{ wear}} d_{\text{sheave}}} = \frac{2 \times 1962}{8.1 \times 10^6 \times 0.1715} = 5.627 \text{ mm}$$

In this case, examining the wire diameters based on static loading, fatigue and wear, the largest diameter is that for static loading.

In summary, the wire rope required is 3/8 in 6×37 WS IPS fiber core. The sheave is carbon steel (BHN-160) with a diameter of 171.45 mm.

17.3 Conclusions

Wire rope can be useful for hoist, haulage, and conveyer applications, where a force needs to be applied over a large distance and some flexibility is an advantage. The flexibility of wire rope enables it to be wound on a drum or sheave. Wire rope is available in a wide range of classes as a standard stock item. When a tensile load is applied to helically twisted wire rope, Hertzian contact stresses arise which may induce failure by fatigue, bending fatigue, fretting, surface wear, abrasion, yielding, or rupture. Failure due to corrosion may also be an issue. Step-by-step procedures for the selection of wire rope are available.

References

Collins, J.A., Busby, H., Staab, G., 2010. Mechanical Design of Machine Elements and Machines, second ed. Wiley.
Wire Rope Users Manual. fourth ed., 2005 Wire Rope Technical Board.
Wire Rope Users Manual, 1979. AISI.

Further Reading

Costello, G.A., 1997. Theory of Wire Rope, second ed. Springer.
Drucker, D.L., Tachau, H., 1945. A new design criterion for wire rope. Trans. ASME 67, A33–A38.
Starkey, W.L., Cress, H.A., 1959. An analysis of critical stresses and mode of failure of a wire rope. Trans. ASME, Eng. Ind. 81, 307–316.

Standards

ASTM A 1023/A 1023M. Standard Specification for Stranded Carbon Steel Wire Ropes for General Purposes.
BS 302–1: 1987, 1987. Stranded Steel Wire Ropes—Part 1: Specification for General Requirements. British Standards Institute.
BS 302–2:1987, 1987. Stranded Steel Wire Ropes—Part 2: Specification for Ropes for General Purposes. British Standards Institute.
BS 302–5:1987, 1987. Stranded Steel Wire Ropes—Part 5: Specification for Ropes for Hauling Purposes. British Standards Institute.
BS EN12385-4:2002 +A1:2008, 2008. Steel Wire Ropes—Safety—Part 4: Stranded Ropes for General Lifting Applications. British Standards Institute.
BS EN12385–2:2002 +A1:2008, 2008. Steel Wire Ropes—Safety—Part 2: Definitions, Designation and Classification. British Standards Institute.
BS EN12385–3:2004 +A1:2008, 2008. Steel Wire Ropes—Safety—Part 3: Information for Use and Maintenance. British Standards Institute.

Web Sites

At the time of press, the world-wide-web contained useful information relating to this chapter at the following sites.

www.bridon.com/
www.certex.co.uk/
www.fuchslubricants.com/
www.jakob.co.uk/

www.juststainless.co.uk/
www.midlandwirerope.com/
www.ormiston-wire.co.uk/
www.s3i.co.uk/
www.southwestwirerope.com/
www.stahlcranes.co.uk/
www.steelwirerope.com/
www.tecni-cable.co.uk/
www.wire-rope.co.uk/
www.wirerope.com/
www.wire-rope-direct.com/
www.wwirerope.com/
www.wwwrope.com/

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

A_{rope}	approximate metallic cross-sectional area of the rope (m^2)
d_{rope}	rope diameter (m)
d_{sheave}	sheave or drum diameter (m)
d_{wire}	wire diameter (m)
E_{rope}	modulus of elasticity of the wire rope (N/m^2)
M	applied moment (N m)
N_d	number of cycles
p	unit radial pressure (N/m^2)
R_N	fatigue strength parameter
T	resultant tensile force (N)
σ_b	bending stress (N/m^2)
σ_d	design stress (N/m^2)
σ_{uts}	ultimate tensile strength (N/m^2)

CHAPTER 18

Pneumatics and Hydraulics

Chapter Outline

18.1 Introduction 735
18.2 Pressure 737
18.2.1 Pressure Measurement 738
18.2.2 Fluid Flow Measurement 739
18.2.3 Temperature Measurement 742
18.3 Hydraulic Pumps 745
18.4 Air Compressors and Receivers 747
18.5 Filters 750
18.6 Control Valves 750
18.7 Pneumatic and Hydraulic Actuators 754
18.8 Conclusions 757
References 757
Further Reading 757
Nomenclature 758

Abstract

Pneumatics and hydraulics represent well-developed technologies where components are available from a wide range of suppliers in modular form, enabling the engineering design to specify a bespoke system for a particular application. Pneumatics and hydraulics are useful for the actuation of diverse elements from brakes and valves to robotics and machine tools. This chapter introduces both technologies and the associated principal components.

18.1 Introduction

A wide range of industrial applications require substances, objects, or components to be moved from one location to another. A further typical requirement is the application of a force to locate, hold, shape, or compress a component or material. These tasks can be achieved using a prime mover, with rotary motion being provided, for example by an electric motor and linear motion by screw jacks, rack and pinions, and solenoids. Liquids and gases can also be used to convey energy from one location to another and as a result produce rotary and linear motions and apply forces. Fluid based systems using a liquid as the transmission media are known as hydraulic, and those using a gas are known as pneumatic.

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00018-6
Copyright © 2014 Elsevier Ltd. All rights reserved.

The gases used in pneumatic systems tend to be low density and compressible, in comparison with the relatively high density and incompressible liquids used in hydraulic systems. As a result, pneumatic systems can be characterized as having a "softer" action in comparison with hydraulic systems, while hydraulic systems can generally be operated at much higher pressures, producing higher forces. Equivalent pneumatic and hydraulic systems for lifting a load are illustrated in Figure 18.1.

This chapter provides a brief overview of pressure in Section 18.2, along with the relevant subjects for pneumatic and hydraulic systems of pressure, flow, and temperature

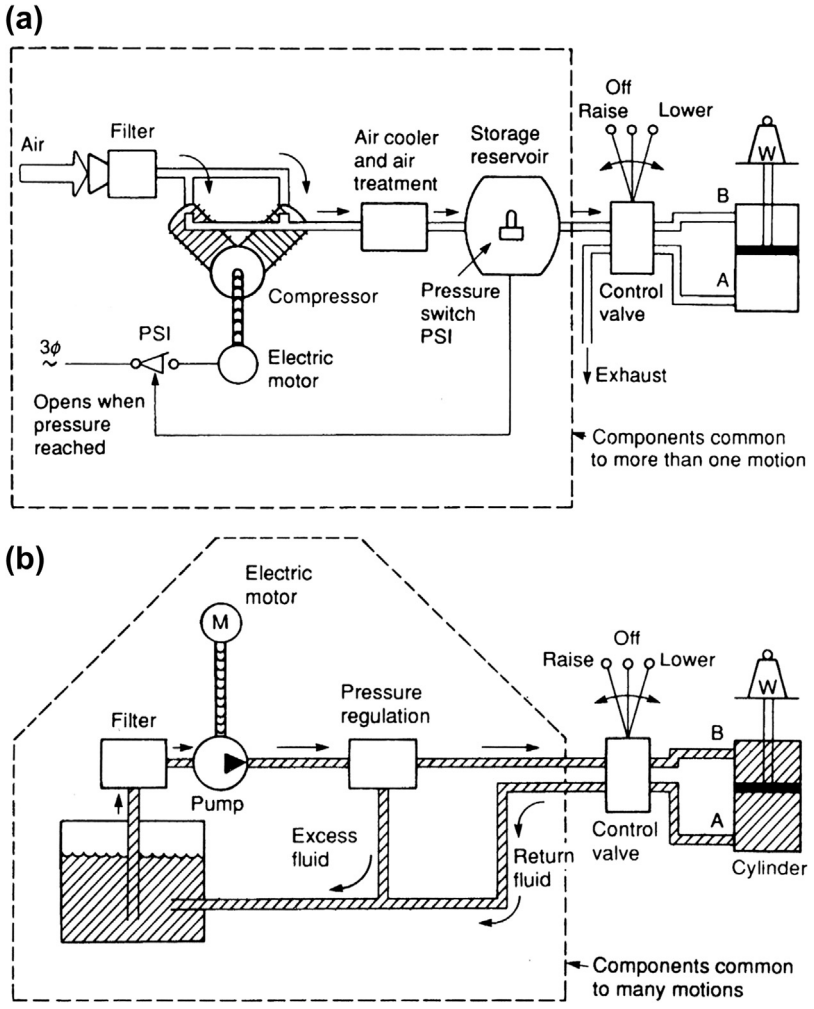


Figure 18.1

Comparison of the physical components for equivalent pneumatic and hydraulic systems for lifting a load using a ram: (a) pneumatic, (b) hydraulic. *Figures courtesy of Parr (2011).*

measurement. Hydraulic pumps and air compressors are introduced in Sections 18.3 and 18.4, respectively. Filters and control valves that represent fundamental building blocks for hydraulic and pneumatic systems are considered in Sections 18.5 and 18.6. Pneumatic and hydraulic actuators are introduced in Section 18.7.

18.2 Pressure

A fluid particle will respond to a force in the same way that a solid particle will. If a force is applied to a particle an acceleration will result, as governed by Newton's second law of motion, which states that the rate of change of momentum of a body is proportional to the unbalanced force acting on it and takes place in the direction of the force. It is useful to consider the forces that a fluid particle can experience:

- body forces such as gravity and electromagnetism,
- forces due to pressure,
- forces due to viscous action, and
- forces due to rotation.

The usual laws of mechanics, including conservation of mass and Newton's laws of motion, can be applied to a fluid. The resulting formulations provide the mathematical basis for modeling fluid flow and developing understanding and insight into fluid flow phenomena.

In the case of a practical fluid, the number of particles concerned is significantly high. For example, in each cubic meter of air at room temperature and pressure, there are approximately 2.549×10^{25} molecules. The term "pressure" is used to describe the force acting per unit area, Eqn (18.1). Pressure will occur in a fluid when it is subjected to a force, whether this is as a result of gravity, compression, viscous action, rotation, or a combination of these.

$$p = \frac{F}{A} \tag{18.1}$$

For a vessel containing a fluid with its surface open to the atmosphere, pressure arises in the fluid as a result of the weight of the fluid and depends on the height of the fluid. This pressure is referred to as the hydrostatic pressure and can be stated as absolute or relative to atmospheric pressure, in which case it is referred to as gauge pressure. Hydrostatic pressure is given by Eqn (18.2).

$$p = \rho g h \tag{18.2}$$

Pascal's law states that pressure at a point in a fluid is uniform. There may be slight variations in the hydrostatic pressure in a practical hydraulic or pneumatic system, but these are likely to be negligible in comparison to the system operating pressure.

18.2.1 Pressure Measurement

Pressure can be measured by a wide range of devices, such as McLeod gauges, barometers, and manometers, which are suitable for laboratory use; Bourdon gauges and bellows, which are suitable for some industrial applications; and piezoelectric transducers, which are useful for both scientific measurements and industrial control and monitoring.

The static pressure of a fluid flowing parallel to a surface can be measured by forming a small hole perpendicular to a solid wall, Figure 18.2. The hole must be perpendicular to the flow and free of burrs, and it must not be chamfered or have a fillet radius.

Piezoelectric pressure transducers convert an applied pressure into an electrical signal. This signal is proportional to the applied pressure and in many cases nearly linear. The output electrical signal can be used in control, displayed on a panel meter, or read by a data logger or voltmeter. Standard output signals available from transducer suppliers are a 4−20 mA two-wire current loop or a voltage signal such as 0−10 V direct current. All transducers require an excitation or supply voltage in order to power the internal circuitry, typically in the range of ±30 V DC, although any voltage within the published range for a particular transducer is acceptable.

Most pressure transducers are designed for a fixed range of pressure. The range should be equal to or greater than the highest working pressure expected for that particular application. There are four basic pressure references:

- Vacuum pressure transducers are vented to the atmosphere and typically produce a 20 mA output when no vacuum is applied. The output decreases as the vacuum increases.

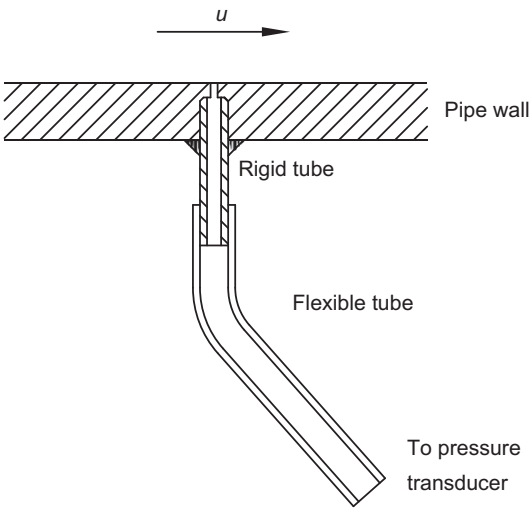


Figure 18.2
Measuring static pressure.

- Gauge pressure transducers are referenced to atmospheric pressure. With no pressure applied, the output is typically 4 mA or 0 V DC.
- Compound ranges are vented to the atmosphere and measure vacuum or pressure relative to atmospheric pressure.
- Absolute pressure transducers are referenced to a sealed vacuum chamber located within the sensor. These transducers sense changes in barometric pressure as the applied pressure is compared to a known vacuum. Absolute units are used where barometric changes affect the operation of the system being measured.

The maximum pressure defines the safety margin of pressure the transducer can tolerate for short periods of time without significant damage. Burst pressure defines the maximum pressure causing permanent damage or destruction of the transducer.

General purpose and industrial grade transducers are available as packaged units comprising amplification and conditioning circuitry and a plug connector, and are usually fitted with threads, depending on the country and application concerned, such as metric threads, British standard pipe fittings, 316 stainless-steel national pipe thread (NPT) process connections. These transducers are for use with nonclogging media only. When measuring the pressure of a medium that is highly viscous, crystallizing, or contains particulates, thread connection is not suitable because the connecting orifices may clog. In such applications a flush mounted transducer should be used. Alternatively, a chemical diaphragm seal can be used to protect the transducer.

18.2.2 Fluid Flow Measurement

Fluid flow measurements are necessary in a very wide range of applications, from the control of fuel flow in engine management systems to the regulation of drug delivery in ventilators. Fluid flow measurements involve determination of the flow velocity, the mass flow rate, or the volumetric flow rate.

The one-dimensional continuity equation for mass flow of a fluid is given by

$$\dot{m} = \rho \overline{u} A \tag{18.3}$$

where

$\dot{m}$ = mass flow rate (kg/s),
ρ = density (kg/m^3),
$\overline{u}$ = average flow speed (m/s), and
A = cross sectional area of flow channel (m^2).

Dividing through by density gives the volumetric flow rate

$$\dot{Q} = \overline{u} A \tag{18.4}$$

where

$\dot{Q}$ = volumetric flow rate (m^3/s).

From Eqns 18.3 and 18.4, measurements of the average flow speed can be used to determine the mass flow, if the density and cross-sectional area are known. Alternatively, if the mass flow is known, the average flow speed can be determined.

Fluid velocity can be measured by a variety of techniques, including Pitot tubes, thin film gauges, and laser Doppler anemometry. Pitot tubes consist of an L-shaped tube inserted into the flow so that the end stagnates the flow, Figure 18.3. The flow speed is given by

$$u = \sqrt{\frac{2(p_o - p)}{\rho}} \tag{18.5}$$

where

u = speed (m/s),
p_o = stagnation pressure (N/m^2),
p = static pressure (N/m^2), and
ρ = density (kg/m^3).

A number of devices measure mass flow based on the pressure loss across a constriction or flow feature. These include orifice plates and Venturi meters. The installation and use of orifice plates is documented in ISO 5167-1:1991(E) (also BS1042: Section 1.1:1992). The information given here has been sourced from this document and refers to devices with static

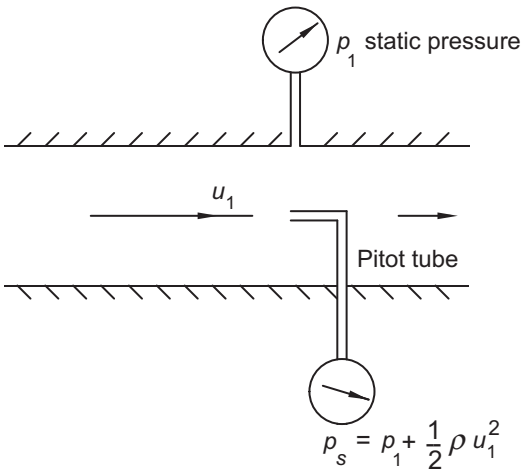


Figure 18.3
Velocity measurement with a Pitot tube.

pressure tappings at a distance of one diameter upstream of the plate and half a diameter downstream only (known as D and $D/2$ tappings). The mass flow rate through an orifice plate installation can be determined from

$$q_m = \frac{C}{\sqrt{1-\beta^4}} \varepsilon_1 \frac{\pi}{4} d^2 \sqrt{2\Delta p \rho_1} \tag{18.6}$$

where

q_m = mass flow rate (kg/s),
C = coefficient of discharge,
β = diameter ratio = d/D,
d = diameter of orifice (d),
D = upstream internal pipe diameter (d),
ε_1 = expansibility factor,
Δp = differential pressure (Pa),
ρ_1 = density of the fluid (kg/m^3),
$l_1 = D \pm 0.1D$,
$l_2 = 0.5D \pm 0.02D$ for $d/D \leq 0.6$, and
$l_2 = 0.5D \pm 0.01D$ for $d/D > 0.6$.

The density can be evaluated from conditions at the upstream pressure tapping ($\rho_1 = p_1/RT$). The temperature should be measured downstream of the orifice plate at a distance of between 5D and 15D. The temperature of fluid upstream and downstream of the orifice plate is assumed constant. The minimum straight lengths of pipe upstream and downstream of the orifice plate are listed in ISO 5167-1:1991(E). For a wide range of conditions for air, ε_1 and C can be approximated by $\varepsilon_1 = 0.99$, $C = 0.61$.

A common device for flow measurement is the convergent-divergent nozzle or Venturi meter, see Figure 18.4. This device involves the contraction of fluid and subsequent expansion. In comparison to other flow measurement devices such as an orifice plate, the smooth

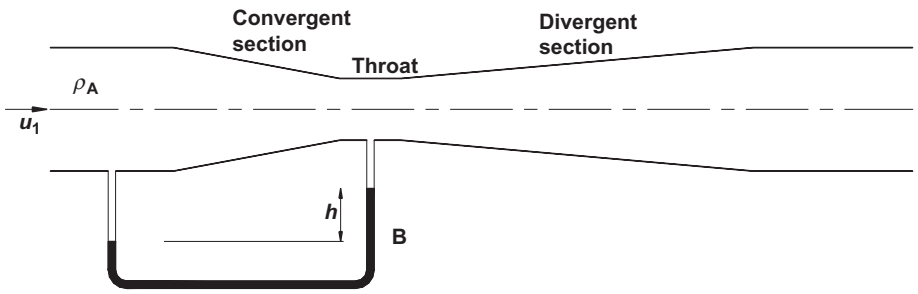


Figure 18.4
Convergent-divergent nozzle (Venturi meter).

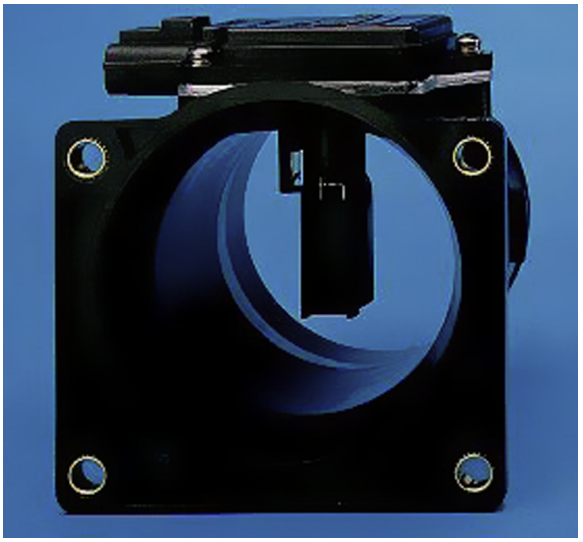


Figure 18.5
Hot film air mass flow sensor. Image courtesy of Visteon (Forster, 2002).

contraction and expansion does not cause a significant pressure loss as fluid streamlines are able to conform to the geometry.

Assuming the flow is incompressible flow (ρ is constant) and that pressure variations due to area change dominate the effects of friction,

$$Q = u_1 A_1 = A_1 \sqrt{\frac{2(p_1 - p_t)}{\rho\left[(A_1/A_t)^2 - 1\right]}} \tag{18.7}$$

Hot film anemometry is used as the basis of mass flow measurement in automotive engine applications. A thin film of nickel is maintained at a constant temperature on a cylinder or alternative mounting located in the fluid stream, Figure 18.5. A constant temperature is achieved by an electronic circuit, which rapidly varies the current supplied, depending on the flow conditions. The convection heat transfer characteristics for a cylinder in cross flow are well understood, or alternatively a calibration can be undertaken for the specific geometry concerned to relate the current flow to the local mass flow. Such sensors can be manufactured fit-for-purpose and with care the total cost minimized, as there is only a single measurement than need be made.

18.2.3 Temperature Measurement

Temperature cannot be measured directly. Instead, its measurement requires the use of a transducer to convert a measurable quantity to temperature. Examples of temperature sensors

Table 18.1: Examples of transducer properties and the physical property measured for some temperature sensors.

Sensor	Transducer property	Physical quantity measured
Liquid-in-glass thermometer	Thermal expansion	Length
Constant volume gas thermometer	Thermal expansion	Pressure
Constant pressure gas thermometer	Thermal expansion	Volume
Bimetallic strip	Thermal expansion	Length
Thermocouple	Seebeck effect	Voltage
Platinum resistance thermometer	Electrical resistance	Resistance
Thermistor	Electrical resistance	Resistance
Transistor	Electrical resistance	Voltage
Capacitance thermometers	Electric permittivity	Capacitance
Noise thermometers	Johnson noise	Power
Thermochromic liquid crystals	Reflection	Color
Thermographic phosphors	Fluorescence	Intensity
Heat sensitive crayons and paints	Chemical	Color
Pyrometric cones	Chemical	Shape
Infrared thermometer	Thermal radiation	Radiation intensity
Thermal imager	Thermal radiation	Radiation intensity
Absorption spectroscopy	Absorption	Wavelength
Emission spectroscopy	Emission	Wavelength
Rayleigh scattering	Elastic scattering	Wavelength
Raman scattering	Inelastic scattering	Wavelength
CARS (coherent anti-Stokes Raman scattering)	Scattering	Wavelength
Degenerative four-wave mixing	Scattering	Wavelength
Laser-induced fluorescence	Emission	Time
Acoustic thermography	Sound velocity	Time

Source: Reproduced with modifications from Childs (2002).

and the associated transducer property used and the measured quantity are listed in Table 18.1. As can be seen from this table, a wide variety of sensors are available; within each type listed, a further selection is usually available.

The selection of a specific sensor for a temperature measurement can become a complex activity requiring consideration of a number of aspects:

- uncertainty,
- temperature range,
- thermal disturbance,
- level of contact,
- size of the sensor,
- transient response,
- sensor protection,
- availability, and
- cost.

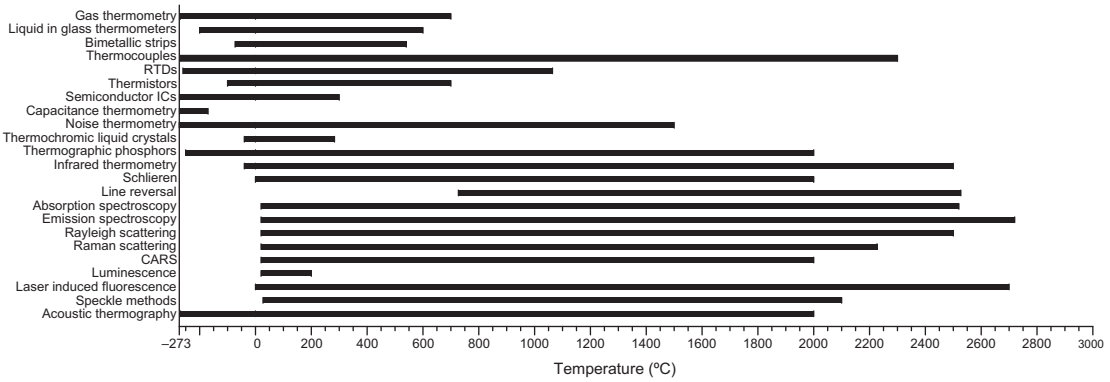


Figure 18.6
Approximate temperature ranges for various temperature measurement techniques.

The range of application, defined as the span from the minimum temperature of operation to the maximum temperature for a particular measurement technique or for a specific instrument, is a critical parameter in selection. Some types of measurement techniques such as thermocouples are suitable for use over a wide temperature range, in this case from approximately −273 to 3000 °C, but this can only be achieved using several different types of thermocouples as no single thermocouple is suitable for the entire range. Figure 18.6 illustrates the approximate temperature range capability for a variety of methods.

Before selecting a temperature measuring system, a number of aspects have to be considered. Some of the issues that might have to be addressed are given below (see ESDU Item 02006).

1. What size and shape of sensor will be required? If there is no special requirement, most types of invasive sensor are available as cylindrical probes, which usually incorporate a protective cover. These may range from about 0.25 to 15.0 mm in diameter.
2. Will an invasive or noninvasive method be the most suitable? In most circumstances, an invasive sensor can be used, and this may provide the most economic solution. However, with some more difficult measuring situations, noninvasive methods are essential; any disadvantages such as complexity, cost, and increased uncertainty have to be accepted.
3. What is the temperature range? Some techniques provide a wide temperature range with a single sensor, some a wide overall temperature range but individual sensors are limited, and with others the temperature range possible with the technique itself may be quite limited.
4. What uncertainty can be accepted? Is the temperature needed with low uncertainty in absolute terms, or are temperature differences or changes in temperature really required? With many types of sensor, the gradient of the output versus temperature characteristic may remain more stable with time than the apparent origin. If low uncertainty is required, the sensors will need to be calibrated against known standards before use. If this uncertainty has to be maintained, regular substitution by new calibrated sensors may be needed.

5. Are steady state or transient phenomena to be measured? Some types of invasive sensor are better suited to fast response, e.g. fine thermocouple or resistance wires.
6. Will the sensor need protection? Sensors may be fragile and also susceptible to contamination. Many sensors can be fitted in tubes, sleeves, or pockets, but by so doing their uncertainty and response characteristics may be impaired.
7. Will the sensor take up the required temperature?
8. Will the presence of the sensor modify the local temperature significantly? If this is likely to be a problem, it may be better to consider a noninvasive method.
9. Can a standard, catalogued system be employed? Although special sensors and systems can be designed and manufactured, they are inevitably likely to be more expensive.
10. Is it a multichannel scheme? If so, thermocouple or resistance temperature detectors are generally more suitable.

For further details on temperature measurement, see Childs (2001) and Childs (2013).

18.3 Hydraulic Pumps

Hydraulic systems rely on a pressurized liquid to energize the various components. Pressurization of the hydraulic fluid, normally an oil, is typically achieved using a hydraulic pump. Hydraulic pumps are usually driven by a three-phase electrical motor running at constant speed rotating at 1500 or 1000 rpm on a 50 Hz supply or 1800 or 1200 rpm on a 60 Hz supply. The three principal types of pumps used for hydraulic systems are gear pumps, vane pumps, and piston pumps. These can be obtained from a wide range of suppliers, and most provide the motor and pump as a single unit. A typical hydraulic pump and associated motor circuit components are illustrated in Figure 18.7.

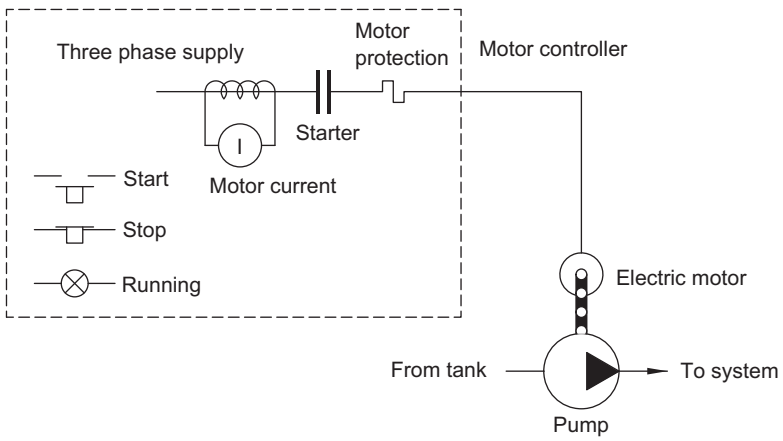


Figure 18.7
Hydraulic pump and associated components.

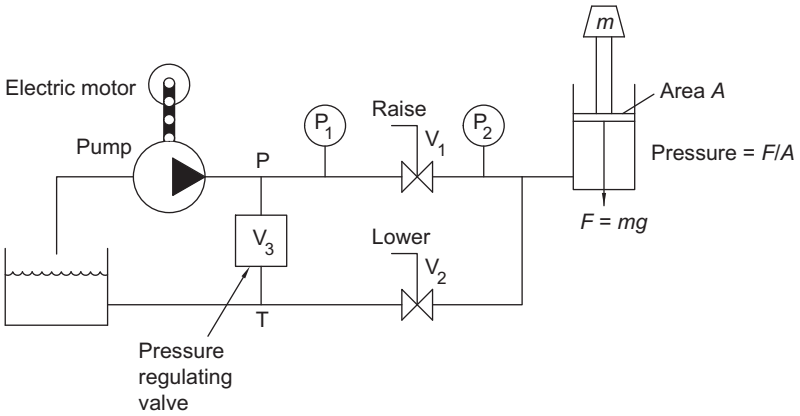


Figure 18.8
Hydraulic circuit with a pressure regulating valve, V3.

A means of maintaining pressure at a safe operating level is necessary in hydraulic circuits; this can be achieved by using a pressure regulating valve or a number of valves. One scheme for maintaining pressure for a hydraulic ram circuit is illustrated in Figure 18.8. The pressure regulating valve shown, V_3, is normally closed when the pressure is below a preset value known as the cracking pressure. When the pressure reaches the cracking pressure, the regulating valve begins to open. The valve becomes fully open when the pressure reaches a value known as the full flow pressure. When valve V_1 is closed, all of the fluid from the pump will return to the tank via the pressure regulating valve V_3. The operation of a simple pressure relief valve is illustrated in Figure 18.9. Examples of pressure regulating valves are illustrated in Figure 18.10.

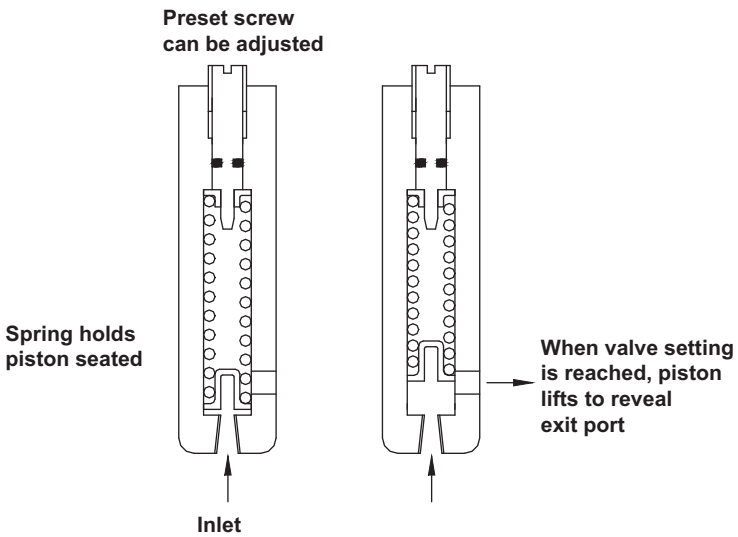


Figure 18.9
Pressure relief valve.

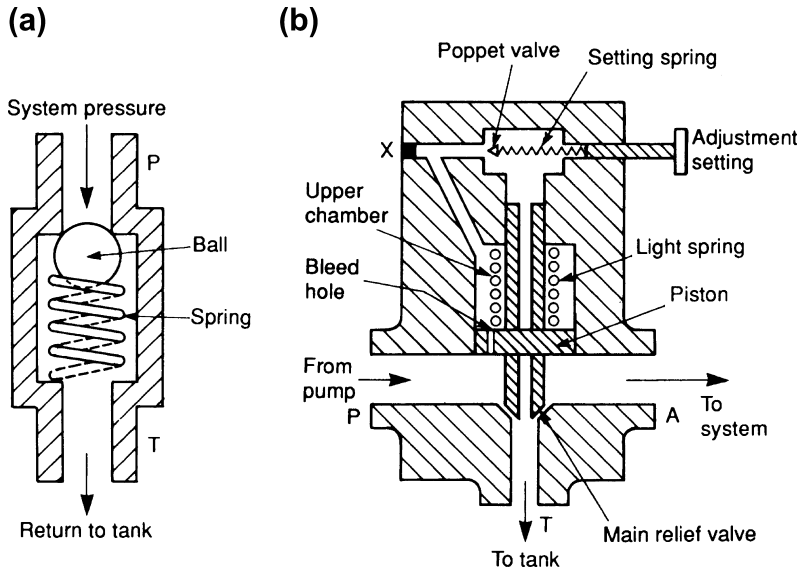


Figure 18.10
(a) Simple pressure regulator, (b) balance piston relief valve. *Figures courtesy of Parr (2011).*

18.4 Air Compressors and Receivers

Most pneumatic systems use air as the working fluid. The vast majority of pneumatic systems are open, with the air sourced from ambient atmosphere and vented back to the atmosphere.

The ideal gas law is given by

$$pV = nRT \tag{18.8}$$

where

p = pressure (N/m^2),
V = volume (m^3),
n = number of moles of the gas ($n = m/M$ (m = mass, M = molar mass)),
R = the universal gas constant (=8.314510 J/mol K (Cohen and Taylor, 1999)), and
T = temperature (K).

From Eqn (18.8), the general gas equation for a gas subjected to changes in pressure, volume, and temperature between two states 1 and 2 can be obtained.

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \tag{18.9}$$

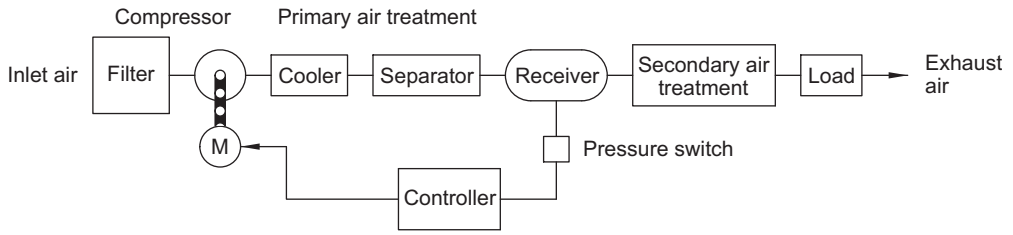


Figure 18.11
Typical pneumatic system.

where

p = absolute pressure (N/m^2),
T = temperature (K), and
V = volume (m^3).

The two principal classes of air compressors are as follows:

- Positive displacement compressors where a fixed volume of air is delivered with each rotation of the compressor shaft; and
- Centrifugal and axial compressors.

The compressor must be selected such that it provides the pressure required as well as the volume of gas at the working pressure. A typical system will be designed to operate with the pressure in the receiver at a slightly higher pressure than that in the remainder of the circuit with a pressure regulator being used. A typical pneumatic system is illustrated in Figure 18.11.

The use of an air receiver tends to enable a smaller air compressor to be specified. The air receiver needs to have a capacity that is sufficiently large to enable to provide the peak flow

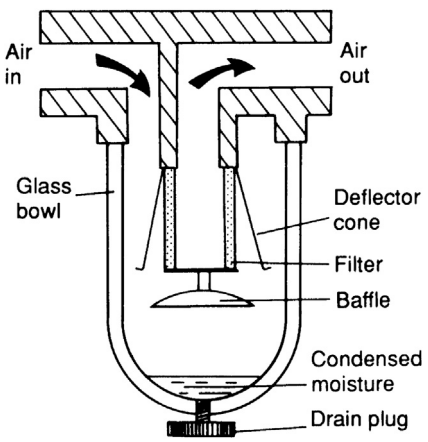


Figure 18.12
Air filter and water trap. *Figure courtesy of Parr (2011).*

requirements for sufficient periods compatible with the application. The volume of the receiver tends to reduce fluctuations in pressure in comparison to a system directly connected to a compressor. Air exiting from a compressor will be hotter than the inlet air, and the receiver surface volume, sometimes finned, is used to dissipate the thermal energy by natural convection. Water will tend to condense within the receiver; regular drainage of this is

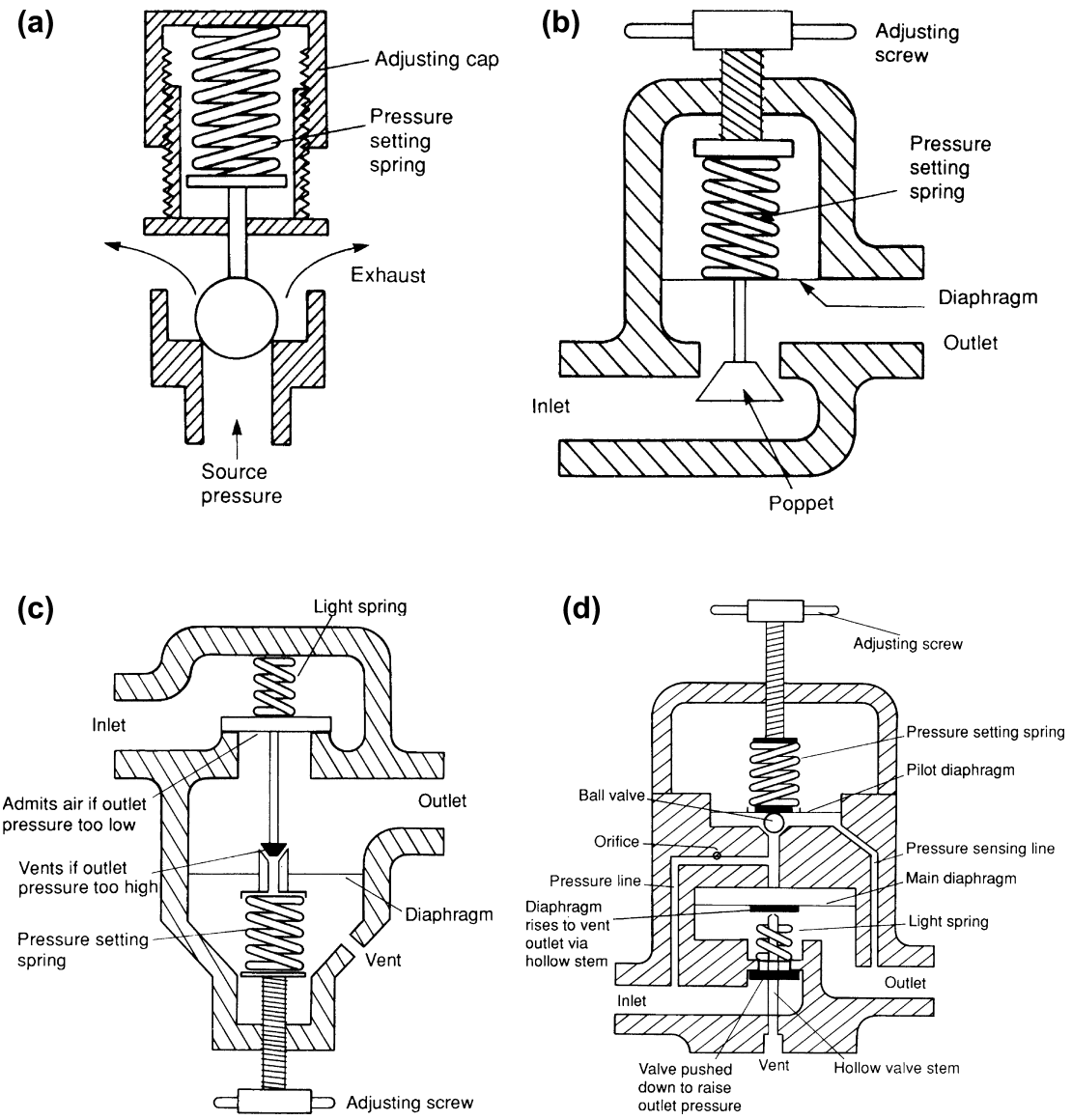


Figure 18.13

(a) Relief valve, (b) nonrelieving pressure regulator, (c) relieving pressure regulator, and (d) pilot operated regulator. *Figures courtesy of Parr (2011).*

necessary. Some applications require drying of air, in which case a simple water trap, Figure 18.12, refrigerator dryer, deliquescent dryer, or adsorption dryer may be necessary.

Typical regulating valves are illustrated in Figure 18.13.

18.5 Filters

Dirt in hydraulic and pneumatic systems can cause sticking of valves, failure of seals, and wear. Filters can be installed to limit or prevent dirt from contaminating vulnerable components. Simple course mesh strainers are usually fitted inside tanks to trap large particles. In addition, separate filters are necessary to remove finer particles with typical filter positions, as shown in Figure 18.14.

18.6 Control Valves

The building blocks for a hydraulic or pneumatic circuit are the pump or compressor, the valves, the hydraulic actuators, the sensors, and the control circuit. Valves are used to control the flow direction, pressure, and flow rate. There are two principal types of valve:

- Infinite position valves that can take up any position between open and closed; and
- Finite position valves that are used to block or allow flow.

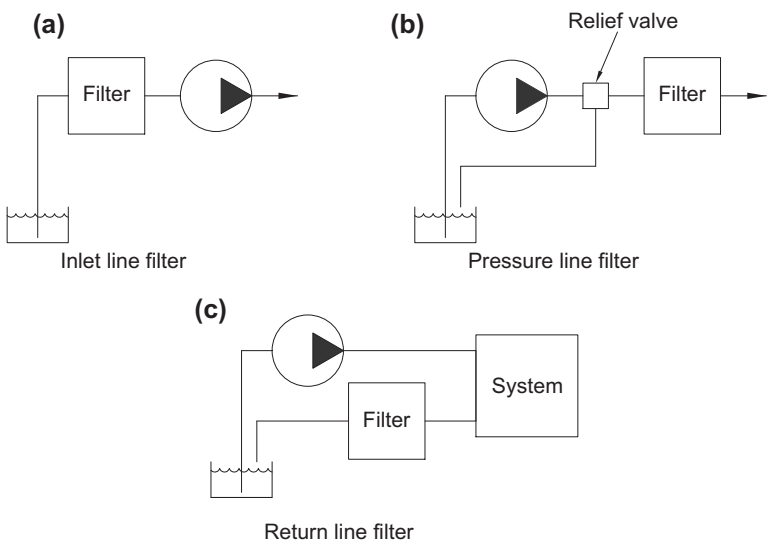


Figure 18.14
Filter positions.

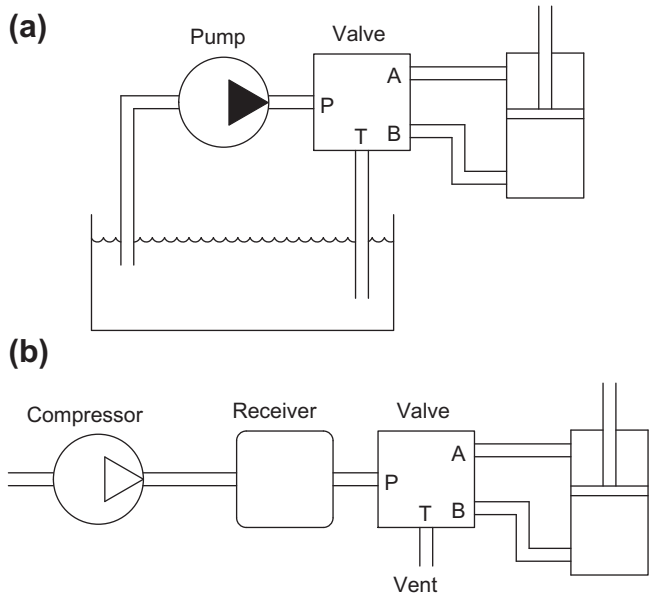


Figure 18.15
Typical four-port control valves: (a) hydraulic, (b) pneumatic.

Connections to a valve are called ports. A simple on or off valve will have two ports. Most valves, however, have four ports, as illustrated in Figure 18.15. In the hydraulic valve application illustrated, hydraulic fluid is returned to the tank from port T. In the pneumatic valve application, air is vented to atmosphere from port T. Finite position valves are commonly described using a number of ports/number of positions notation. As an example, a 2/2 valve would be a two-port, two-position valve, and a 4/3 valve would be a four-port, three-position valve. Figure 18.16 shows the possible valve action for a 4/3 valve.

The scope of valves available is extensive. A set of graphic symbols has evolved to indicate the function of the valve with the need to show the physical construction of the valve and have

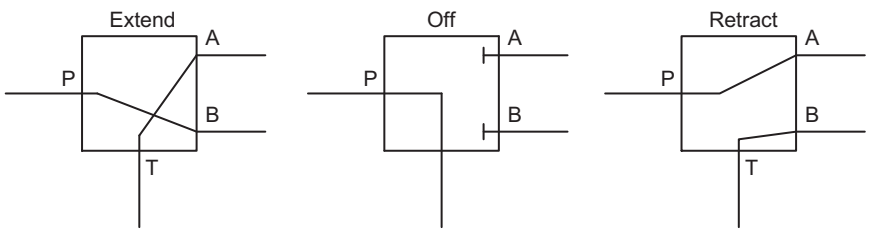


Figure 18.16
Possible valve action for a 4/3 valve.

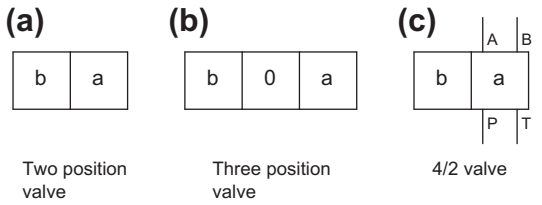


Figure 18.17
Selected valve symbols.

been standardized in, for example, DIN24300, BS2917, ISO1219, ISO5599, CETOP (P.3). A valve is represented with a square for each of its switching positions. So if a valve has three different possible switching positions, it will be represented by three squares. An example is given in Figure 18.17 for a three-position valve. The valve positions can be indicated with letters, with 0 being used for a central neutral position.

The general designations used are as follows:

- Ports are shown on the outside of boxes;
- Arrows are used to represent the direction of flow;
- A T is used to show shut-off positions; and
- A series of specific symbols are used to designate whether the valve is actuated by a button, spring, pressure line, level, solenoid, detent, etc. Some examples are given in Figure 18.18.

There are three principal types of control valve:

- Poppet valves: discs, cones, or balls are used in combination with simple valve seats to control the flow;

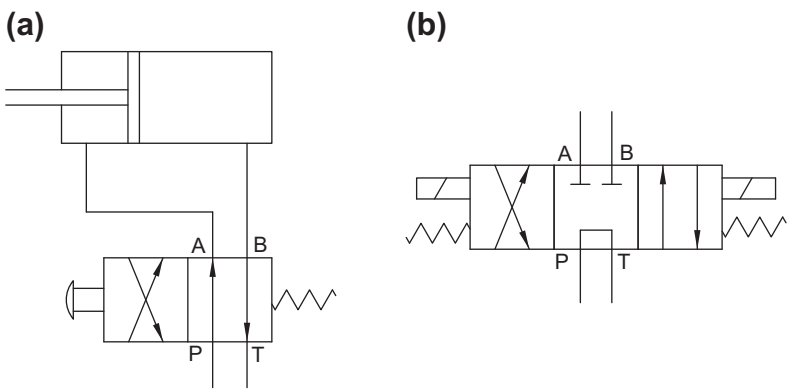


Figure 18.18
Example symbols. (a) Push button extend, spring retract when push button is released; (b) 4/3 valve, solenoid operated, spring return to center. Pressure line uploads to tank and load locked in center position.

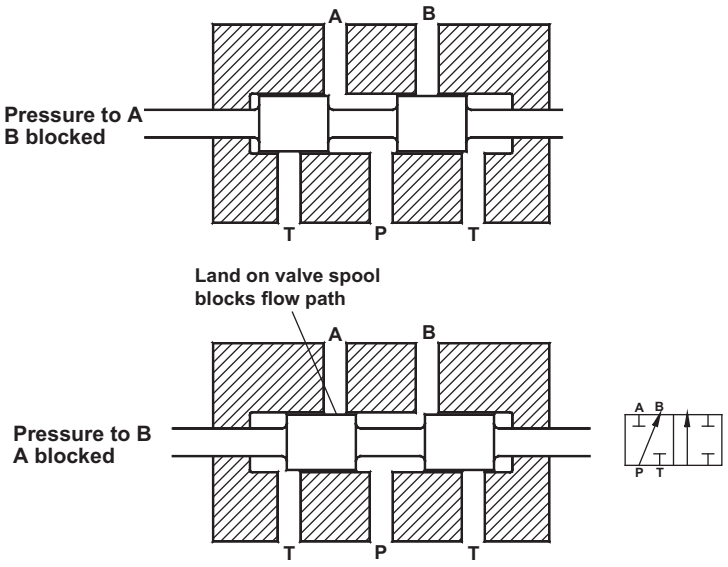


Figure 18.19
Spool positions inside a two-way valve.

- Spool valves: a cylindrical spool with raised lands moves horizontally within the valve housing to block or open ports; and
- Rotary valves: a rotating spool aligns with holes in the casing.

Figure 18.19 illustrates a two-way valve along with the corresponding symbolic representation. In two-way valves the flow path can be controlled by the position of a spool. Two-way valves are useful for directing flow to either of two different parts of a flow circuit depending on the spool position. The diagrams in Figure 18.19 illustrate the flow path options. In the first position, flow can pass from P to A. In the second position, flow can pass from P to B.

A very common method of actuating a spool valve is by using a solenoid. A solenoid is a specially designed type of electromagnet comprising a coil and plunger connected to an application of interest. They are used very widely in industry, with applications including the actuation of valves and locks. When a current flows through a wire, an electromagnetic field is set up. If a wire is formed into a coil, the magnetic field becomes many times stronger. Although a magnetic field can be formed in air it flows much more easily through iron or steel. This is the principle in adding an iron path or C stack around the coil, which concentrates the field in order to give high power densities and therefore more powerful and compact devices. If an iron path, known as a T plunger, is also added at the center of the coil, this will concentrate the magnetic field even further. Because iron provides a more attractive path for the magnetic field than air, a moveable T plunger will be drawn in further to the

coil so that the maximum possible magnetic flux can pass through the plunger. In order to limit stray eddy currents in an electromagnet, the C stack and plunger can be made of laminated sheets of alternate conducting and insulating material. The magnetic field can still flow through its path but the eddy currents are limited to a two-dimensional plane. Unfortunately for electromagnets, although increasing the current will increase the power of the magnetic field, the heat generated in a solenoid increases approximately with the square of the current. Doubling the current will increase the heat generated by a factor of four. Each time an electromagnet is cycled its temperature rises and its force decreases. If cycled too fast it will overheat and burn out. Solenoid operated pneumatic and hydraulic control valves are available as stock items from a wide range of suppliers and represent a well-developed technology.

18.7 Pneumatic and Hydraulic Actuators

An actuator is a device that puts a parameter into action or physical motion. In the case of machine elements actuators are as follows:

- pneumatics,
- hydraulics,
- magnets,
- electromagnets and solenoids,
- electric motors, and
- piezoelectric devices.

Fluid power actuators extract energy from a fluid and covert it to a mechanical output to perform a useful task. Fluid actuators can be classified as hydraulic or pneumatic depending on whether a liquid or gas is used as the working fluid. Fluid actuators include devices for

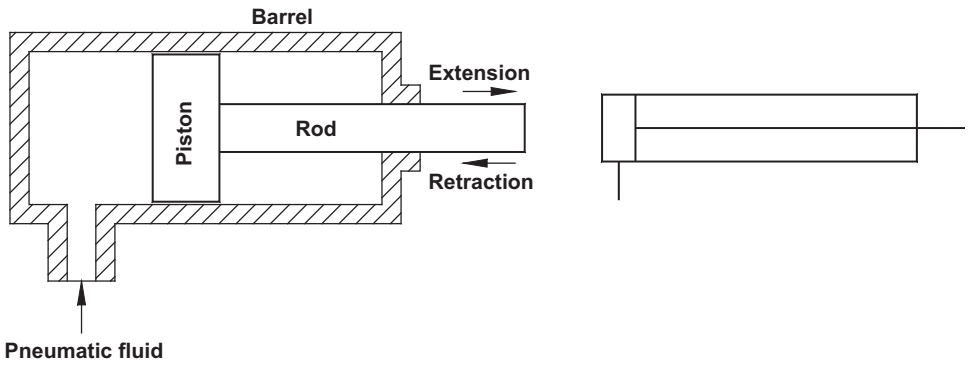


Figure 18.20
Single-acting hydraulic cylinder and corresponding symbolic representation.

producing either linear motion such as hydraulic cylinders, which extend and retract, or rotary motion such as hydraulic motors.

The simplest type of linear actuator is the single-acting hydraulic or pneumatic cylinder as illustrated in Figure 18.20. This consists of a piston inside a cylindrical housing called a barrel. Attached to one end of the piston is a rod that extends through a hole and seal in the cylinder. At the other end is a port for the admission and exit of hydraulic fluid such as oil. Single acting cylinders can only provide actuation in the extending direction. Retraction is achieved by means of gravity or by including a compression spring in the rod end.

The double-acting hydraulic cylinder illustrated in Figure 18.21 can be extended and retracted under hydraulic power. Pneumatic and hydraulic cylinders are available in a wide variety of sizes and end configurations and the rod end is usually threaded so that it can be connected directly to a load, a clevis, yoke, or some other mating device.

The force in the extension stroke from a double-acting cylinder is given by

$$F = pA \tag{18.10}$$

where

$F =$ force (N),
$p =$ pressure (N/m^2), and
$A =$ piston area (m^2).

The velocity is given by

$$V = \frac{Q}{A} \tag{18.11}$$

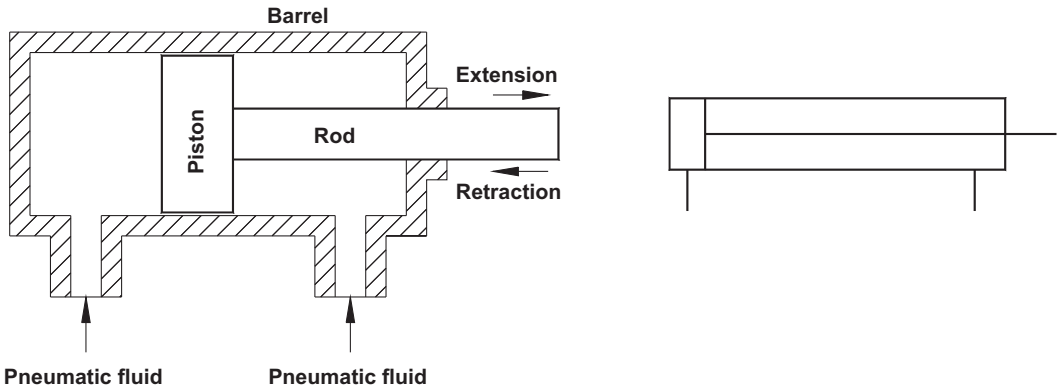


Figure 18.21
Double-acting hydraulic cylinder and corresponding symbolic representation.

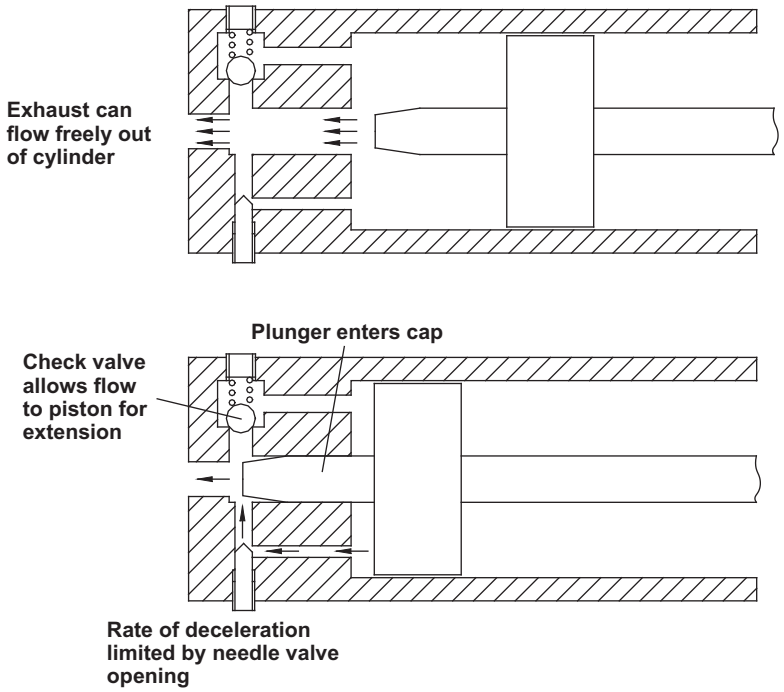


Figure 18.22
Cylinder cushioning.

where

$Q =$ volumetric input flow (m^3/s).

In the retraction stroke, the force and speed are moderated by the effects of the rod area:

$$F = p(A - A_{rod}) \tag{18.12}$$

where

$A_{rod} =$ cross-sectional area of the rod (m^2)

$$V = \frac{Q}{A - A_{rod}} \tag{18.13}$$

The power delivered by a hydraulic cylinder can be determined from

$$\text{Power} = FV = pQ \tag{18.14}$$

Some double-acting cylinders include cylinder cushions to decelerate the piston near the end of the stroke. These typically comprise channels and associated pressure relief valves, as illustrated in Figure 18.22.

18.8 Conclusions

Pneumatic and hydraulic systems can be used in a wide range of applications to apply a force to a component or material. Due to the compressible nature of air, pneumatic systems tend to enable softer action in comparison with hydraulic systems, which can generally be operated at much higher pressures, producing higher forces. For both pneumatic and hydraulic systems, the pressurization of the fluid can be remote from the actuators, thereby providing a compact solution in comparison, say, to the use of an electric motor.

References

Childs, P.R.N., 2001. Practical Temperature Measurement. Butterworth Heinemann.
Childs, P.R.N., 2013. Temperature measurement. In: Kutz, M. (Ed.). Handbook of Measurement in Science and Engineering, vol. 1. Wiley.
Childs, P.R.N., 2002. Advances in temperature measurement. In: Hartnett, J.P. (Ed.). Advances in Heat Transfer, vol. 36, pp. 111–181.
Cohen, E.R., Taylor, B.N., 1999. The fundamental physical constants. Physics Today, BG5–BG9.
Forster, I., 2002. Electronics and engine control, Visteon Advanced Powertrain Systems. Institution of Mechanical Engineers Automotive Division Southern Centre presentation. Private communication.
Parr, A., 2011. Hydraulics and Pneumatics, third ed. Butterworth Heinemann.

Further Reading

Barber, A. (Ed.), 1997. Pneumatic Handbook, eighth ed. Elsevier Science Ltd.
Engineering Sciences Data Unit, 2002. Temperature measurement: techniques. ESDU Item 02006.
ESDU 12003, 2012. Pressure and flow measurement. ESDU.

Standards

BS 2917:1977, ISO 1219-1976. Specification for graphical symbols used on diagrams for fluid power systems and components.
BS ISO 5599-1:2001. Pneumatic fluid power. Five-port directional control valves. Mounting interface surfaces without electrical connector.
BS ISO 5599-2:2001. Pneumatic fluid power. Five-port directional control valves. Mounting interface surfaces with optional electrical connector.
CETOP, 2004. RE-2004.04/P. Pneumatics & control. (P.3) Programme. Comité Européen des Transmissions Oléohydrauliques et Pneumatiques.
DIN24300, Schaltsymbole für Ölhydraulik und Pneumatik.

Web Sites

At the time of going to press the world-wide-web contained useful information relating to this chapter at the following sites.

www.airlines-pneumatics.co.uk
www.airtec-usa.com
www.atlascopco.co.uk
www.boschrexroth.co.uk
www.cpi-pneumatics.co.uk/

www.crane-bel.com/
www.drakepneumatics.co.uk/
www.eaton.com
www.festo.com
www.globalhydraulics.co.uk/
www.harrisonpneumatics.co.uk/
www.hydrastore.co.uk/
www.hydrauliccomponents.co.uk/
www.hydraulicpartsdirect.co.uk/
www.hydrovaneproducts.com/
www.johnguest.com
www.metalwork.co.uk/
www.mhpneumatics.co.uk/
www.mhs.co.uk/
www.norgren.com
www.parker.com
www.phoenixhydraulics.co.uk
www.pneumaticsnorthern.co.uk/
www.pneumatic-source.com/
www.sauer-danfoss.com/
www.smc.com
www.spiraxsarco.com
www.thoritedirect.co.uk

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

A	area (m^2)
A_{rod}	cross-sectional area of the rod (m^2)
AC	alternating current
C	a proportionality constant, coefficient of discharge
d	diameter of orifice (d)
D	upstream internal pipe diameter (d)
DC	direct current
F	force (N)
l	length (m)
LVDT	linear variable differential transformers
$\dot{m}$	mass flow (kg/s)
n_s	synchronous speed (rpm)
N	number of teeth in the rotating toothed wheel
NPT	national pipe thread
p_o	stagnation pressure (N/m^2)

p	pressure (N/m^2)
q_m	mass flow (kg/s)
$\dot{Q}$	volumetric flow (m^3/s)
R	characteristic gas constant (J/kg K)
TPS	throttle position sensor
T	time (s)
u	speed (m/s)
$\overline{u}$	average flow speed (m/s)
V	voltage (V), velocity (m/s)
β	diameter ratio
ε_1	expansibility factor
ρ	density (kg/m^3)
ρ_1	density of the fluid (kg/m^3)
ω	angular velocity (rad/s)
Δp	differential pressure (Pa).

CHAPTER 19

Engineering Tolerancing

Chapter Outline

19.1 Introduction 761
19.2 Component Tolerances 762
19.2.1 Standard Fits for Holes and Shafts 762
19.2.2 Interference Fits 767
19.2.3 Machine Capability 769
19.2.4 Geometric Tolerancing 771
19.3 Statistical Tolerancing 774
19.3.1 Sure-Fit or Extreme Variability 778
19.3.2 Linear Functions or Tolerance Chains 779
19.3.3 Several Independent, Uncorrelated Random Variables 782
19.3.4 Statistical Design Techniques and Quality Assurance 792
19.4 Conclusions 794
References 794
Further Reading 795
Nomenclature 795

Abstract

The aims of this chapter are to introduce the concepts of component and process variability and the allocation of component tolerances. The selection of suitable tolerances for the assembly of components is a key requirement in the development of a machine design for given functionality, and this requires an understanding of process capability as well as careful attention to the specification of the allowable variability component dimensions. The principles of engineering tolerancing can also be applied to model the variability of a process or function.

19.1 Introduction

A solid is defined by its surface boundaries. Designers typically specify a component's nominal dimensions such that it fulfills its requirements. In reality, components cannot be made repeatedly to nominal dimensions due to surface irregularities and intrinsic surface roughness. Some variability in dimensions must be allowed to ensure that manufacture is possible. However, the variability permitted must not be so great that the performance of the assembled parts is impaired. The allowed variability on the individual component dimensions is called the tolerance.

Mechanical Design Engineering Handbook. http://dx.doi.org/10.1016/B978-0-08-097759-1.00019-8
Copyright © 2014 Elsevier Ltd. All rights reserved.

The term "tolerance" applies not only to the acceptable range of component dimensions produced by manufacturing techniques but also to the output of machines or processes. For example, the power produced by a given type of internal combustion engine varies from one engine to another. In practice, the variability is usually found to be modeled by a frequency distribution curve, for example the normal distribution (also called the Gaussian distribution). The modeling and analysis of this kind of variable output are outlined in Section 19.3. One of the tasks of the designer is to specify a dimension on a component and the allowable variability on this value that will yield an acceptable performance. A general introduction to the specification of component tolerances is given in Section 19.2.

19.2 Component Tolerances

Control of dimensions is necessary in order to ensure the assembly and interchangeability of components. Tolerances are specified on critical dimensions that affect clearances and interference fits. One method of specifying tolerances is to state the nominal dimension followed by the permissible variation. So a dimension could be stated as 40.000 ± 0.003 mm. This means that the dimension should be machined so that it is between 39.997 and 40.003 mm. The tolerance is called bilateral when the variation can vary on either side of the nominal dimension. For unilateral tolerance, one tolerance is zero, e.g. $40.000\ ^{+0.006}_{0.000}$ mm.

Most organizations have general tolerances that apply to dimensions when an explicit dimension is not specified on a drawing. For machined dimensions, a general tolerance may be ± 0.5 mm. So a dimension specified as 15.0 mm may range between 14.5 and 15.5 mm. Other general tolerances can be applied to features such as angles, drilled and punched holes, castings, forgings, weld beads, and fillets.

When specifying a tolerance for a component, reference can be made to previous drawings or general engineering practice. Tolerances are typically specified in bands as defined in British or ISO (International Organisation for Standardisation) standards. Table 19.1 gives a guide for the general applications of tolerances. For a given tolerance, e.g. H7/s6, a set of numerical values is available from a corresponding chart for the size of the component under consideration. The following section gives specific examples of this for a shaft or cylindrical spigot fitting into a hole.

19.2.1 Standard Fits for Holes and Shafts

A standard engineering task is to determine tolerances for a cylindrical component, e.g. a shaft, fitting or rotating inside a corresponding cylindrical component or hole. The tightness of fit will depend on the application. For example, a gear located onto a shaft would require a "tight" interference fit where the diameter of the shaft is actually slightly greater than the inside diameter of the gear hub in order to be able to transmit the desired torque.

Table 19.1: Examples of tolerance bands and typical applications.

Class	Description	Characteristic	ISO code	Assembly	Application
Clearance	Loose running fit	For wide commercial tolerances	H11/c11	Noticeable clearance	IC engine exhaust valve in guide
	Free running fit	Good for large temperature variations, high running speeds or heavy journal pressures	H9/d9	Noticeable clearance	Multiple bearing shafts, hydraulic piston in cylinder, removable levers, bearings for rollers
	Close running fit	For running on accurate machines and accurate location at moderate speeds and journal pressures	H8/f7	Clearance	Machine tool main bearings, crankshaft and connecting rod bearings, shaft sleeves, clutch sleeves, guide blocks
	Sliding fit	When parts are not intended to run freely, but must move and turn and locate accurately	H7/g6	Push fit without noticeable clearance	Push on gear wheels and clutches, connecting rod bearings, indicator pistons
	Location clearance fit	Provides snug fit for location of stationary parts, but can be freely assembled	H7/h6	Hand pressure with lubrication	Gears, tailstock sleeves, adjusting rings, loose bushes for piston bolts and pipelines
Transition	Location transition fit	For accurate location (compromise between clearance and interference fit)	H7/k6	Easily tapped with hammer	Pulleys, clutches, gears, flywheels, fixed handwheels, and permanent levers
	Location transition fit	For more accurate location	H7/n6	Needs pressure	Motor shaft armatures, toothed collars on wheels
Interference	Locational interference fit	For parts requiring rigidity and alignment with accuracy of location	H7/p6	Needs pressure	Split journal bearings
	Medium drive fit	For ordinary steel parts or shrink fits on light sections	H7/s6	Needs pressure or temperature difference	Clutch hubs, bearing bushes in blocks, wheels, connecting rods. Bronze collars on gray cast iron hubs

Alternatively, the diameter of a journal bearing must be greater than the diameter of the shaft to allow rotation. Given that it is not economically possible to manufacture components to exact dimensions, some variability in sizes of both the shaft and hole dimension must be specified. However, the range of variability should not be so large that the operation of the assembly is impaired. Rather than having an infinite variety of tolerance dimensions that

could be specified, national and international standards have been produced defining bands of tolerances, examples of which are listed in Table 19.1, e.g. H11/c11. To turn this information into actual dimensions, corresponding tables exist defining the tolerance levels for the size of dimension under consideration (see Tables 19.2 and 19.3). In order to use this information the following list and Figure 19.1 give definitions used in conventional tolerancing. Usually the hole-based system is used (data sheet BS4500A, Table 19.2) as this results in a reduction in the variety of drill, reamer, broach, and gauge tooling required within a company.

Size	A number expressing in a particular unit the numerical value of a dimension.
Actual size	The size of a part as obtained by measurement.
Limits of size	The maximum and minimum sizes permitted for a feature.
Maximum limit of size	The greater of the two limits of size.
Minimum limit of size	The lesser of the two limits of size.
Basic size	The size by reference to which the limits of size are fixed.
Deviation	The algebraic difference between a size and the corresponding basic size.
Actual deviation	The algebraic difference between the actual size and the corresponding basic size.
Upper deviation	The algebraic difference between the maximum limit of size and the corresponding basic size.
Lower deviation	The algebraic difference between the minimum limit of size and the corresponding basic size.
Tolerance	The difference between the maximum limit of size and the minimum limit of size.
Shaft	The term used by convention to designate all external features of a part (including parts that are not cylindrical).
Hole	The term used by convention to designate all internal features of a part.

BS4500 data sheets 4500A and 4500B list selected ISO clearance, transitional fits, and interference fits for internal and external components. When using the Standard, upper case letters refer to the hole and lower case letters to the shaft.

Example 19.1

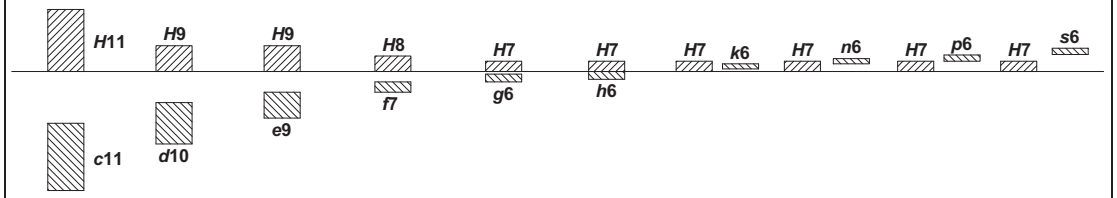
Find the shaft and hole dimensions for a loose running fit with a 35-mm-diameter basic size.

Solution

From Table 19.1, the ISO symbol is 35H11/c11 (the 35 designating a 35 mm nominal tolerance).

From data sheet BS4500A (Table 19.2), the hole tolerance, for a 35 mm hole, is $\begin{matrix}+0.16\\0.00\end{matrix}$ mm and the shaft tolerance, for a 35 mm shaft, is $\begin{matrix}-0.12\\-0.28\end{matrix}$ mm.

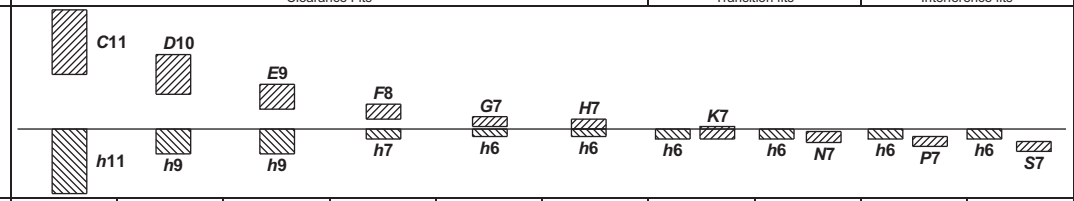
Table 19.2: Selected ISO fits. Holes basis.



Nominal sizes		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Nominal sizes	
Over	To	H11	c11	H9	d10	H9	e9	H8	f7	H7	g6	H7	h6	H7	k6	H7	n6	H7	p6	H7	s6	Over	To
mm	mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	mm	mm
—	3	+60 0	−60 −120	+25 0	−20 −60	+25 0	−14 −39	+14 0	−6 −16	+10 0	−2 −8	+10 0	−6 0	+10 0	+6 0	+10 0	+10 +4	+10 0	+12 +6	+10 0	+20 +14	—	3
3	6	+75 0	−70 −145	+30 0	−30 −78	+30 0	−20 −50	+18 0	−10 −22	+12 0	−4 −12	+12 0	−8 0	+12 0	+9 +1	+12 0	+16 +8	+12 0	+ 20 +12	+12 0	+27 +19	3	6
6	10	+90 0	−80 −170	+36 0	−40 −98	+36 0	−25 −61	+22 0	−13 −28	+15 0	−5 −14	+15 0	−9 0	+15 0	+10 +1	+15 0	+19 +10	+15 0	+24 +15	+15 0	+32 +23	6	10
10	18	+110 0	−95 −205	+43 0	−50 −120	+43 0	−32 −75	+27 0	−16 −34	+18 0	−6 −17	+18 0	−11 0	+18 0	+12 +1	+18 0	+23 +12	+18 0	+29 + 18	+18 0	+39 +28	10	18
18	30	+130 0	−110 −240	+52 0	−65 −149	+52 0	−40 −92	+33 0	−20 −41	+21 0	−7 −20	+21 0	−13 0	+21 0	+15 +2	+21 0	+28 +15	+21 0	+35 +22	+21 0	+48 +35	18	30
30	40	+160 0	−120 −280	+62	−80	+62	−50	+39	−25	+25	−9	+25	−16	+25	+18	+25	+33	+25	+42	+25	+59	30	40
40	50	+160 0	−130 −290	0	−180	0	−112	0	−50	0	−25	0	0	0	+2	0	+17	0	+26	0	+43	40	50
50	65	+190 0	−140 −330	+74	−100	+74	−60	+46	−30	+30	−10	+30	−19	+30	+21	+30	+39	+30	+51	+30 0	+72 +53	50	65
65	80	+190 0	−150 −340	0	−220	0	−134	0	−60	0	−29	0	0	0	+2	0	+20	0	+32	+30 0	+78 +59	65	80
80	100	+220 0	−170 −390	+87	−120	+87	−72	+54	−36	+35	−12	+35	−22	+35	+25	+35	+45	+35	+59	+35 0	+93 +71	80	100
100	120	+220 0	−180 −400	0	−260	0	−159	0	−71	0	−34	0	0	0	+3	0	+23	0	+37	+35 0	+101 +79	100	120
120	140	+250 0	−200 −450																	+40 0	+117 +92	120	140
140	160	+250 0	−210 −460	+100 0	−145 −305	+100 0	−84 −185	+63 0	−43 −83	+40 0	− 14 − 39	+40 0	−25 0	+40 0	+28 +3	+40 0	+52 +27	+40 0	+68 +43	+40 0	+125 +100	140	160
160	180	+250 0	−230 −480																	+40 0	+133 +108	160	180
180	200	+290 0	−240 −530																	+46 0	+151 +122	180	200
200	225	+290 0	−260 −550	+115 0	−170 −355	+115 0	−100 −215	+72 0	−50 −96	+46 0	−15 −44	+46 0	−29 0	+46 0	+33 +4	+46 0	+60 +31	+46 0	+79 +50	+46 0	+159 +130	200	225
225	250	+290 0	−280 −570																	+46 0	+169 +140	225	250
250	280	+320 0	−300 −620	+130	−190	+130	−110	+81	−56	+52	−17	+52	−32	+52	+36	+52	+66	+52	+88	+52 0	+190 +158	250	280
280	315	+320 0	−330 −650	0	−400	0	−240	0	−108	0	−49	0	0	0	+4	0	+34	0	+56	+52 0	+202 +170	280	315
315	355	+360 0	−360 −720	+140	−210	+140	−125	+89	−62	+57	−18	+57	−36	+57	+40	+57	+73	+57	+98	+57 0	+226 +190	315	355
355	400	+360 0	−400 −760	0	−440	0	−265	0	−119	0	−54	0	0	0	+4	0	+37	0	+62	+57 0	+244 +208	355	400
400	450	+400 0	−440 −840	+155	−230	+155	−135	+97	−68	+63	−20	+63	−40	+63	+45	+63	+80	+63	+108	+63 0	+272 +232	400	450
450	500	+400 0	−480 −880	0	−480	0	−290	0	−131	0	−60	0	0	0	+5	0	+40	0	+68	+63 0	+292 +252	450	500

Source: After BS4500 data sheet 4500A.

Table 19.3: Selected ISO fits. Shafts basis.

	Clearance Fits												Transition fits				Interference fits				
																					Holes Shafts
Nominal sizes		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Nominal sizes	
Over	To	h11	C11	h9	D10	h9	E9	h7	F8	h6	G7	h6	H7	h6	K7	h6	N7	h6	P7	h6	S7	Over	To
mm	mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	mm	mm
—	3	0 –60	+120 +60	0 –25	+60 +20	0 –25	+39 +14	0 –10	+20 +6	0 –6	+12 +2	0 –6	+10 0	0 –6	0 –10	0 –6	–4 –14	0 –6	–6 –16	0 –6	–14 –24	—	3
3	6	0 –75	+145 +70	0 –30	+78 +30	0 –30	+50 +20	0 –12	+28 +10	0 –8	+16 +4	0 –8	+12 0	0 –8	+3 –9	0 –8	–4 –16	0 –8	–8 –20	0 –8	–15 –27	3	6
6	10	0 –90	+170 +80	0 –36	+98 +40	0 –36	+61 +25	0 –15	+35 +13	0 –9	+20 +5	0 –9	+15 0	0 –9	+5 –10	0 –9	–4 –19	0 –9	–9 –24	0 –9	–17 –32	6	10
10	18	0 –110	+205 +95	0 –43	+120 +50	0 –43	+75 +32	0 –18	+43 +16	0 –11	+24 +6	0 –11	+18 0	0 –11	+6 –12	0 –11	–5 –23	0 –11	–11 –29	0 –11	–21 –39	10	18
18	30	0 –130	+240 +110	0 –52	+149 +65	0 –52	+92 +40	0 –21	+53 +20	0 –13	+28 +7	0 –13	+21 0	0 –13	+ 6 –15	0 –13	–7 –28	0 –13	–14 –35	0 –13	–27 –48	18	30
30	40	0 –160	+280 +120	0	+180	0	+112	0	+64	0	+34	0	+25	0	+7	0	–8	0	–17	0	–34	30	40
40	50	0 –160	+290 +130	–62	+80	–62	+50	–25	+25	–16	+9	–16	0	–16	–18	–16	–33	–16	–42	–16	–59	40	50
50	65	0 –190	+330 +140	0	+220	0	+134	0	+76	0	+40	0	+30	0	+9	0	–9	0	–21	0 –19	–42 –72	50	65
65	80	0 –190	+340 +150	–74	+100	–74	+60	–30	+30	–19	+10	–19	0	–19	–21	–19	–39	–19	–51	0 –19	–48 –78	65	80
80	100	0 –220	+390 +170	0	+260	0	+159	0	+90	0	+47	0	+35	0	+10	0	–10	0	–24	0 –22	–58 –93	80	100
100	120	0 –220	+400 +180	–87	+120	–87	+72	–35	+36	–22	+12	–22	0	–22	–25	–22	–45	–22	–59	0 – 22	–66 –101	100	120
120	140	0 –250	+450 +200																	0 – 25	–77 –117	120	140
140	160	0 –250	+460 +210	0 –100	+305 +145	0 –100	+185 +85	0 –40	+106 +43	0 –25	+54 +14	0 –25	+40 0	0 –25	+12 –28	0 –25	–12 –52	0 –25	–28 –68	0 – 25	–85 –125	140	160
160	180	0 –250	+480 +230																	0 – 25	–93 –133	160	180
180	200	0 –290	+530 +240																	0 –29	–105 –151	180	200
200	225	0 –290	+550 +260	0 –115	+355 +170	0 –115	+215 +100	0 –46	+122 +50	0 –29	+61 +15	0 –29	+46 0	0 –29	+13 –33	0 –29	–14 –60	0 –29	–33 –79	0 –29	–113 – 159	200	225
225	250	0 –290	+570 +280																	0 –29	–123 –169	225	250
250	280	0 –320	+620 +300	0	+400	0	+240	0	+137	0	+62	0	+52	0	+16	0	–14	0	–36	0 –32	–138 –190	250	280
280	315	0 –320	+650 +330	–130	+190	–130	+110	–52	+56	–32	+17	–32	0	–32	–36	–32	–66	–32	–88	0 –32	–150 –202	280	315
315	355	0 –360	+720 +360	0	–440	0	+265	0	+151	0	+75	0	+57	0	+17	0	–16	0	–41	0 –36	–169 –226	315	355
355	400	0 –360	+760 +400	–140	+210	–140	+125	–57	+62	–36	+18	–36	0	–36	–40	–36	–73	–36	–98	0 –36	–187 – 244	355	400
400	450	0 –400	+840 +440	0	–480	0	+290	0	+165	0	+83	0	+63	0	+18	0	–17	0	–45	0 –40	–209 –272	400	450
450	500	0 –400	+880 +480	–155	+230	–155	+135	–63	+68	–40	+20	–40	0	–40	–45	–40	–80	–40	–108	0 –40	–229 –292	450	500

Source: After BS4500 data sheet 4500B.

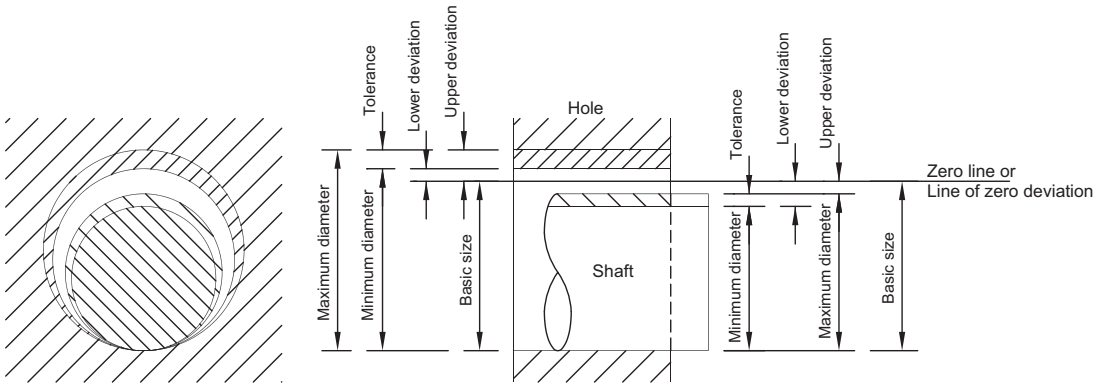


Figure 19.1
Definitions of terms used in conventional tolerancing. *After BS4500.*

So the hole should be given dimensions $35.00\,^{+0.16}_{0.00}$ mm and the shaft $35.00\,^{-0.12}_{-0.28}$ mm.

Example 19.2

Find the shaft and hole dimensions for a medium drive fit using a basic hole size of 60 mm.

Solution

From Table 19.1, the ISO symbol is 60H7/s6.

From data sheet BS4500A (Table 19.2), the hole tolerance is $^{+0.030}_{0.000}$ mm and the shaft tolerance is $^{+0.072}_{+0.053}$ mm.

So the hole should be given dimensions $60.00\,^{+0.030}_{0.000}$ mm and the shaft $60.00\,^{+0.072}_{+0.053}$ mm.

19.2.2 Interference Fits

Interference fits are those for which, prior to assembly, the inside component is larger than the outside component. There is some deformation of the components after assembly, and a pressure exists at the mating surfaces. This pressure is given by Eqn (19.1) if both components are of the same material or by Eqn (19.2) if the two components are of differing materials (see Figure 19.2 for a definition of radii).

$$p = \frac{E\delta}{2b}\frac{\left(c^2 - b^2\right)\left(b^2 - a^2\right)}{2b^2\left(c^2 - a^2\right)} \tag{19.1}$$

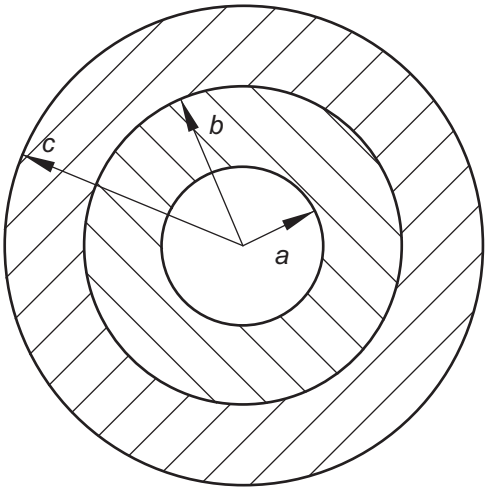


Figure 19.2
Terminology for cylindrical interference fit.

$$p = \frac{\delta}{2b\left[\frac{1}{E_o}\left(\frac{c^2+b^2}{c^2-b^2}+\mu_o\right)+\frac{1}{E_i}\left(\frac{b^2+a^2}{b^2-a^2}-\mu_i\right)\right]} \tag{19.2}$$

where p is the pressure at the mating surface (N/m^2), δ is the total diametral interference (m), a is the internal radius of the inner cylinder (m), b is the outer radius of the inner cylinder (m), c is the outer radius of the outer cylinder (m), E is Young's modulus (N/m^2), and μ is Poisson's ratio.

The subscripts i and o refer to the inner and outer components, respectively.

The tensile stress in the outer component, σ_o, can be calculated from Eqn (19.3) and the compressive stress in the inner component, σ_i, from Eqn (19.4).

$$\sigma_o = p\left(\frac{c^2+b^2}{c^2-b^2}\right) \tag{19.3}$$

$$\sigma_i = -p\left(\frac{b^2+a^2}{b^2-a^2}\right) \tag{19.4}$$

The increase in diameter of the outer component, δ_o, due to the tensile stress can be calculated from Eqn (19.5) and the decrease in diameter of the inner component due to the compressive stress, δ_i, from Eqn (19.6).

$$\delta_o = \frac{2bp}{E_o}\left(\frac{c^2+b^2}{c^2-b^2}+\mu_o\right) \tag{19.5}$$

$$\delta_i = -\frac{2bp}{E_i}\left(\frac{b^2 + a^2}{b^2 - a^2} - \mu_i\right) \tag{19.6}$$

The torque, which can be transmitted between cylindrical components by an interference fit, can be estimated by

$$T = 2fp\pi b^2 L \tag{19.7}$$

where T is the torque (N m), f is the coefficient of friction, p is the interference pressure (N/m^2), b is the interface radius (m), and L is the length of interference fit (m).

Example 19.3

What interference fit is required in order to transmit 18 kN m torque between a 150-mm-diameter steel shaft and a 300-mm-outside-diameter cast iron hub that is 250 mm long? (Take Young's modulus for the steel and cast iron to be 200 and 100 GPa respectively, and assume the coefficient of friction is 0.12 and that Poisson's ratio is 0.3 for both materials).

Solution

The torque is given by $T = 2fp\pi b^2 L$ and this equation can be rearranged to evaluate the pressure required to transmit the torque:

$$p = \frac{T}{2f\pi b^2 L} = \frac{18 \times 10^3}{2 \times 0.12\pi(0.075)^2 \times 0.25} = 16.97 \times 10^6 \text{ N/m}^2.$$

The interference required to generate this pressure is given by

$$\begin{aligned}\delta &= 2bp\left[\frac{1}{E_o}\left(\frac{c^2 + b^2}{c^2 - b^2} + \mu_o\right) + \frac{1}{E_i}(1 - \mu_i)\right] \\ &= 2 \times 0.075 \times 16.97 \times 10^6\left[\frac{1}{100 \times 10^9}\left(\frac{0.15^2 + 0.075^2}{0.15^2 - 0.075^2} + 0.3\right) + \frac{1}{200 \times 10^9}(1 - 0.3)\right] \\ &= 5.89 \times 10^{-5}\text{m}\end{aligned}$$

So the required interference fit is 0.059 mm. This could be specified using the H7/s6 tolerance band with the hole diameter dimension as $150.000\,^{+0.04}_{+0}$ mm and the shaft diameter as $150.000\,^{+0.125}_{+0.100}$ mm.

19.2.3 Machine Capability

A given manufacturing technique can only produce a component within a range of accuracy around the specified nominal dimension. For instance, center lathes are capable of machining

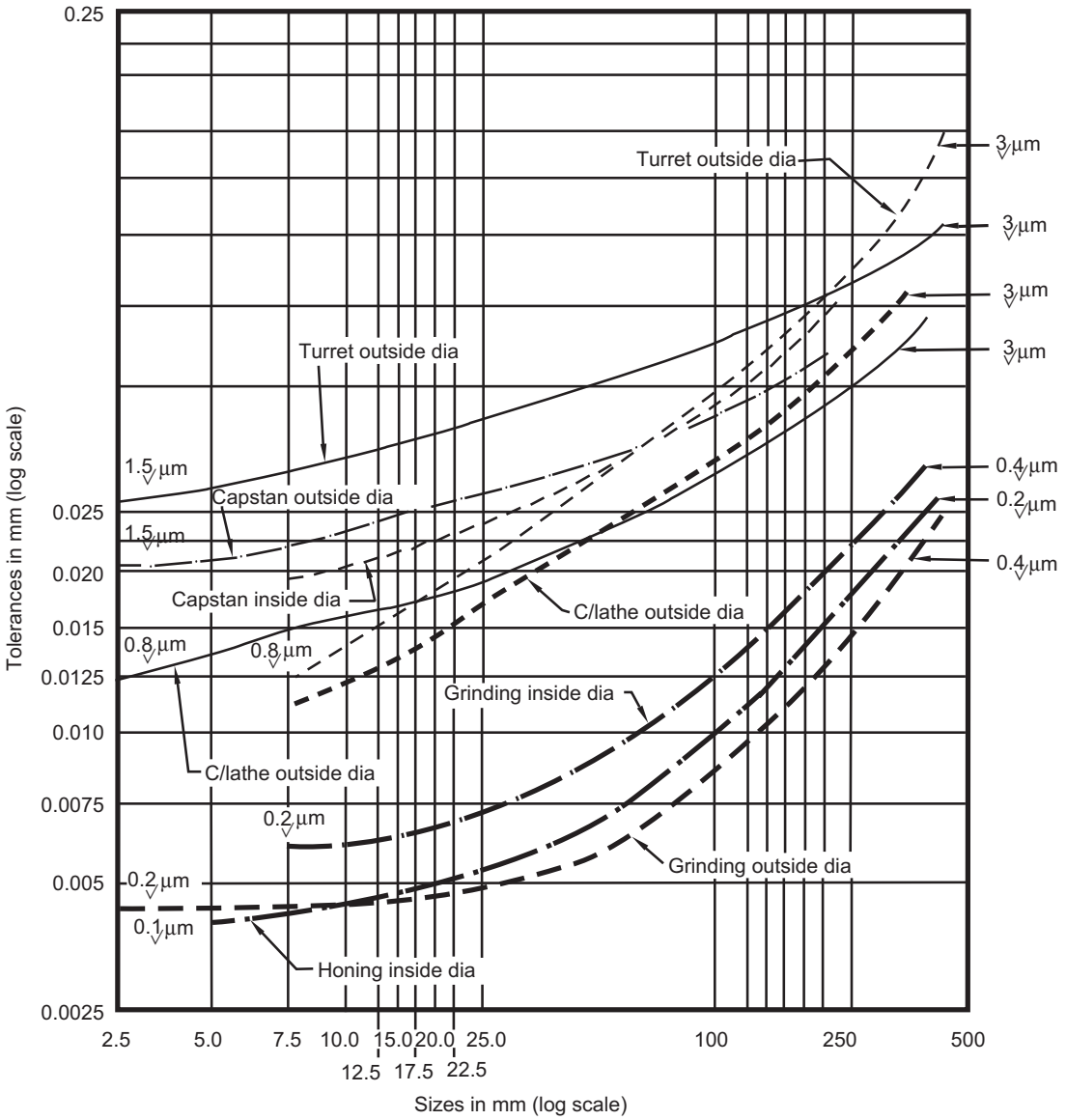


Figure 19.3
Accuracy to be expected from machining operators using various machines. *BSI (1991)*.

10-mm-diameter components to tolerances of 0.01 mm, as illustrated in Figure 19.3. In addition, different manufacturing techniques produce differing levels of surface roughness, as shown in Table 19.4. Given that drilling cannot be normally used to give a surface roughness of better than 0.8 μm, tolerances for drilled components should not be specified smaller than 0.0008 mm. When defining a tolerance, the designer should have in mind the "total design" of the product. This encompasses efficient operation of the product, the ability of the

Table 19.4: Surface roughness values produced by common production processes.

Process	Roughness value, R_a (μm)											
	50	25	12.5	6.3	3.2	1.6	0.8	0.4	0.2	0.1	0.05	0.025	0.0125
Flame cutting													
Sawing													
Planing, Shaping													
Drilling													
Electric discharge machining													
Milling													
Broaching													
Reaming													
Boring, Turning													
Roller burnishing													
Grinding													
Honing													
Polishing													
Lapping													
Superfinishing													
Sand casting													
Hot rolling													
Forging													
Permanent mould casting													
Investment casting													
Extruding													
Cold rolling, Drawing													
Die casting													

Average application
Less frequent application

Source: After BSI (1991).

company to manufacture the component, and total costs (design, manufacture, servicing, performance, etc.). Different manufacturing techniques cost different amounts, as illustrated in Figure 19.4, and this should be considered.

19.2.4 Geometric Tolerancing

The location for a drilled hole has been specified in Figure 19.5 by means of a dimension with bilateral tolerances. The dimensions given imply a square rectangular tolerance zone as identified in Figure 19.6a, anywhere within which it would be permissible to set the hole

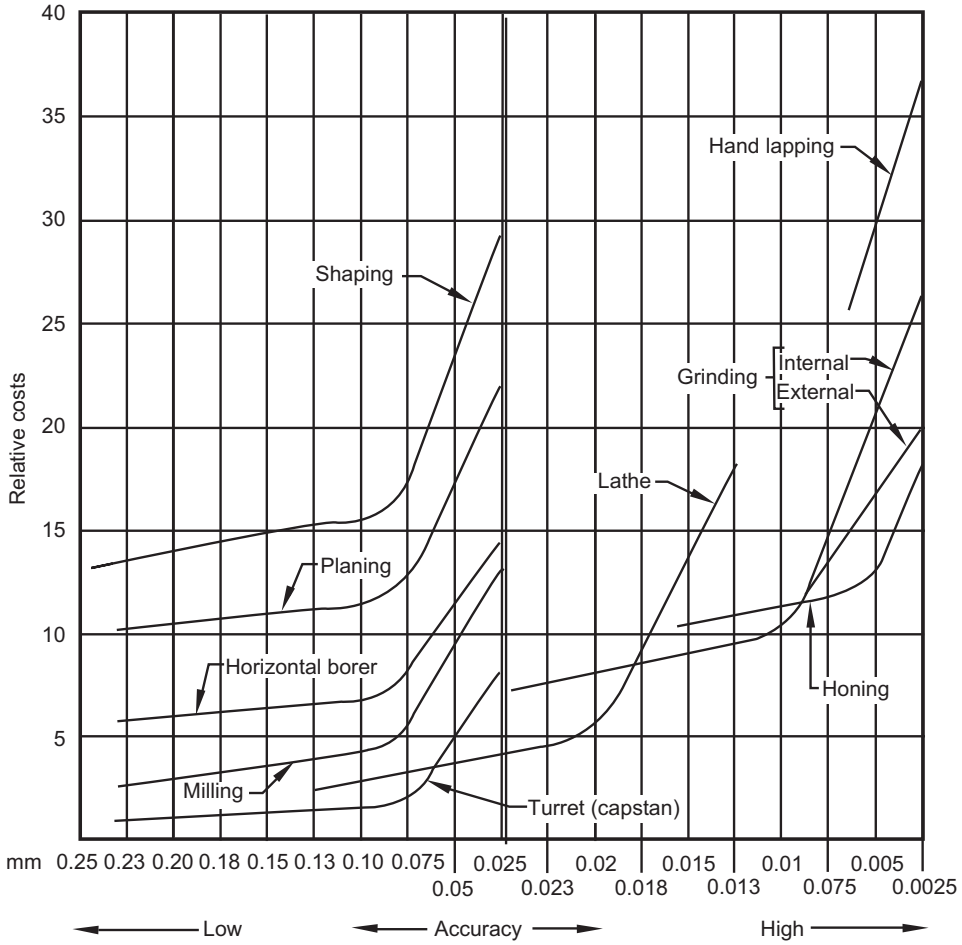


Figure 19.4
Relative cost of various machine and hand processes for achieving set tolerances. *BSI (1991)*.

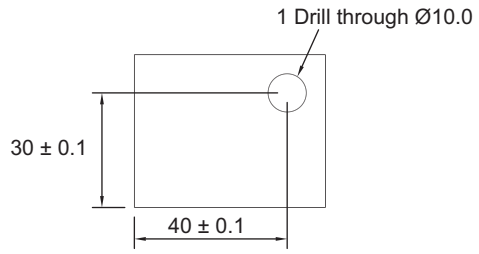


Figure 19.5
Drilled hole location.

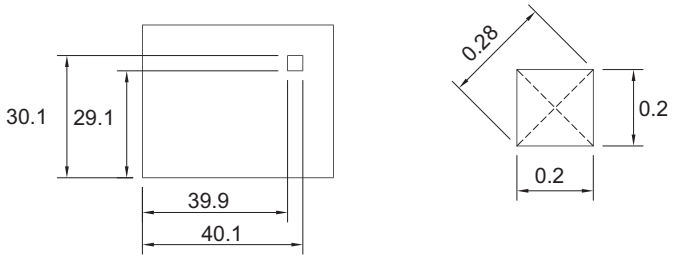


Figure 19.6
Implied tolerance zone.

center. Examination of the tolerance zone shows that the maximum deviation in the location of the hole center actually occurs along the diagonals, as shown in Figure 19.6b. Thus although the drawing has superficially specified the maximum tolerance deviation as ± 0.1 or 0.2 mm, the maximum actual deviation within the specification of the drawing is 0.28 mm. If the deviation of 0.28 mm provides correct functioning of the device in the diagonal directions, it is likely to be acceptable in all directions. If this is the case, then a circular tolerance could be specified with a diameter of 0.28 mm, as shown in Figure 19.7. The area of this circular zone is 57% larger than the square tolerance zone and could have substantial implications for cost reduction in the manufacturing process.

The specification of a circular tolerance zone can be achieved by specifying the exact desired location of the hole and a permissible circular tolerance zone centered at the exact location, as illustrated in Figure 19.8. Here the exact location of the hole is indicated by the boxed dimensions and the tolerance zone is indicated by the ⌖ symbol within the rectangular box. The diameter of the tolerance zone is specified to the right of the ⌖ symbol within the rectangular box.

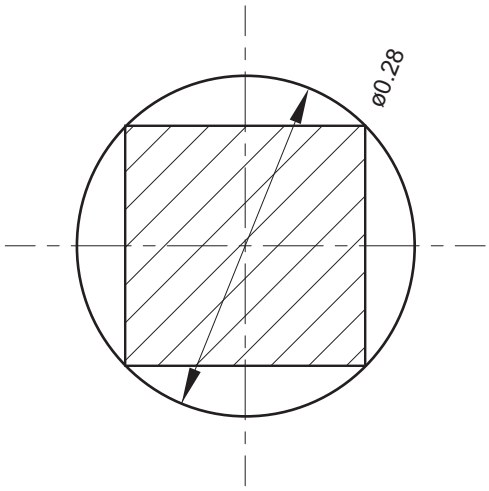


Figure 19.7
Circular tolerance zone.

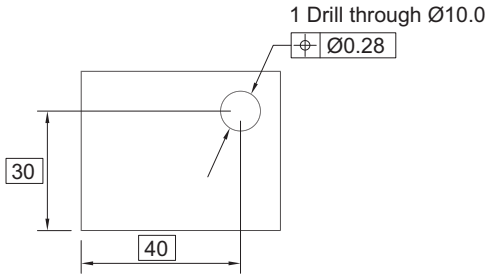


Figure 19.8
Circular tolerance zone specification.

The symbol used for the positional tolerance is one of a series used in engineering drawing, as illustrated in Table 19.5, and their use is referred to as geometric tolerancing. Geometric tolerancing allows the engineer to specify control over feature locations and deviation of form, such as flatness, parallelism, and concentricity. The various symbols used and a brief description are given in Table 19.5. Figure 19.9 illustrates their use for defining the locating radius and shoulder for a centrifugal compressor impeller. In order to avoid significant out-of-balance forces and correct clearance between the compressor blade tips and the stationary shroud, it is critical that the impeller is located concentrically on the shaft and the back face is perpendicular to the shaft. Control over the shaft radial dimensional variation has been specified by means of the runout symbol, which defines the limits of variability for the cylindrical surface, and the squareness symbol for the shoulder.

BS 8888:2011 provides information on the British Standard approach to specifying and graphically representing products using engineering drawing, computer-aided design (CAD), and product specification methods and symbols, which are also reviewed by Simmons et al. (2012). The texts by Spotts (1983), Drake (1999), and Krulikowski (2012) provide very useful information on tolerancing of components.

19.3 Statistical Tolerancing

Many processes produce outputs, such as component dimensions or power, that follow a frequency distribution such as the "normal" or Gaussian distribution illustrated in Figure 19.10. The average value of the output, say a component dimension, is called the mean. The spread of the output values around the mean represents the variability of the process and is represented by a quantity called the standard deviation. The normal distribution is defined such that 68.26% of all the output is within one standard deviation of the mean, 95.44% of all the output is within two standard deviations of the mean, and 99.73% of all the output is within three standard deviations of the mean.

Table 19.5: Geometric tolerancing symbols.

Symbol	Definition	Description
	Straightness	This defines the straightness tolerance for an edge, axis of revolution, or line on a surface. The tolerance zone is either the area between two parallel straight lines in the plane containing the toleranced edge or surface line or a cylindrical region for an axis of revolution.
	Flatness	This is the condition of a surface having all elements in one plane.
	Roundness	This is the condition on a surface of revolution (cylinder or cone or sphere) where all the points of the surface intersected by any plane (i) perpendicular to a common axis (cylinder/cone) or (ii) passing through a common center (sphere) are equidistant from the center.
	Cylindricity	A cylindricity tolerance zone is the annular space between two coaxial cylinders. The tolerance zone is the radial separation of the two cylinders.
	Profile of a line	The profile of a line is the condition permitting a uniform amount of profile variation, either unilaterally or bilaterally, along a line element of a feature.
	Profile of a surface	This defines the tolerance zone of variability for a surface relative to the perfect or exact form.
	Parallelism	This is the condition of a surface, line, or axis that is equidistant at all points from a datum plane or axis.
	Squareness	The squareness tolerance defines the angular tolerance for perpendicularity (90°).
	Angularity	This defines the allowable variability for angular location. The angular tolerance zone is defined by either two parallel lines or surfaces.
	Position	This defines a zone within which the axis or center plane of a feature is permitted to vary from the true, theoretically correct, position.
	Concentricity	A concentricity tolerance limits the deviation of the center or axis of a feature from its true position.
	Symmetry	This typically defines the area between two parallel lines or the space between two parallel surfaces, which are symmetrically located about a datum feature.
	Runout	This is the deviation in position of a surface of revolution as a part is revolved about a datum axis.

The normal distribution is defined by the equation:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \quad -\infty < x < \infty \tag{19.8}$$

where $f(x)$ is the probability density function of the continuous random variable x, μ is the mean, and σ is the standard deviation.

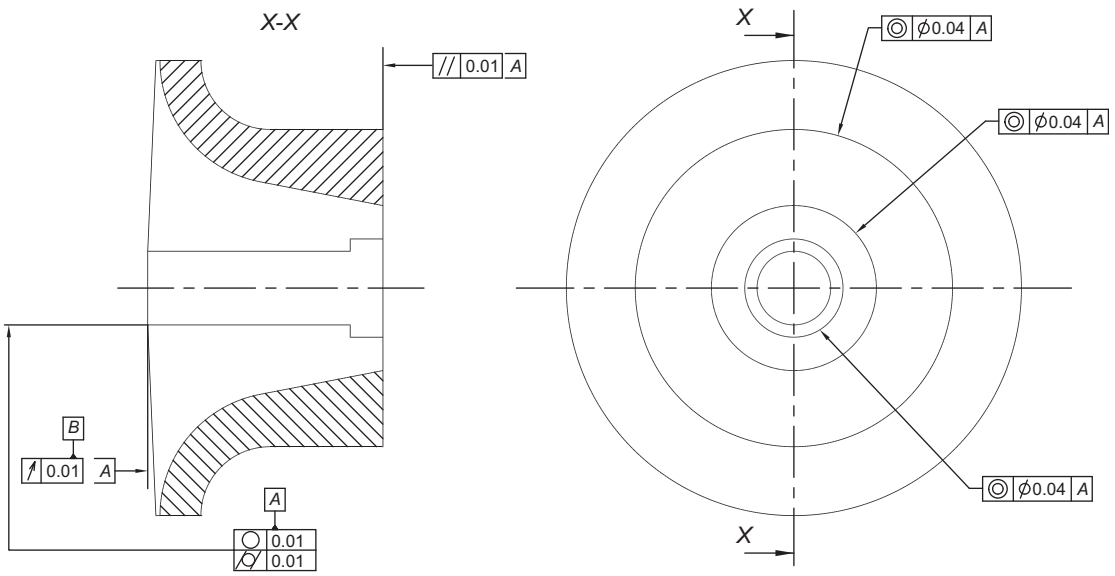


Figure 19.9
Example of the use of geometric tolerancing for specifying control over a centrifugal compressor's shaft dimensions.

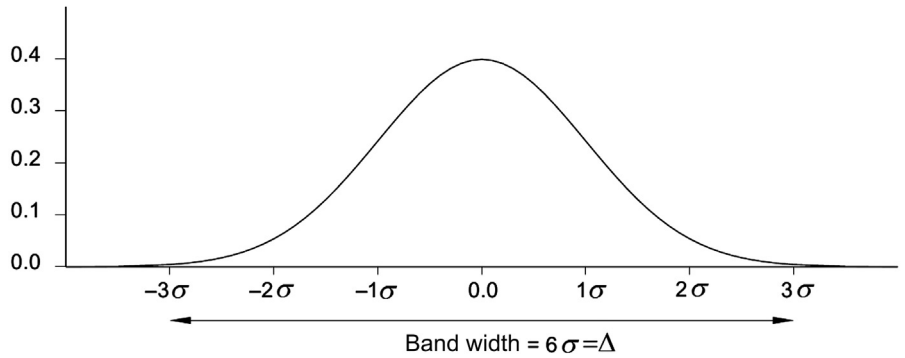


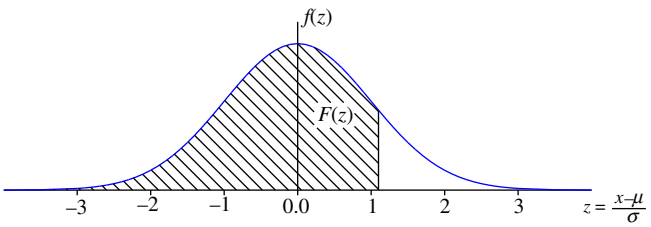
Figure 19.10
The normal distribution.

Equation (19.8) can be solved numerically or with use of a table of solutions (as given in Table 19.6). As an illustration of the use of the normal table with $z = 3$, we obtain $F(z) = 0.998650$. So $2 \times (1 - 0.998650) = 0.0027 = 0.27\%$ of all items can be expected to fall outside of the tolerance limits. In other words, 99.73% of all items produced will have dimensions within the tolerance limits.

In practice, 80% of all processes, e.g. many machined dimensions and engine power outputs, are well modeled by the normal distribution, BSI (1983). Given a set of data, a statistical test can be undertaken to determine whether the data are normally distributed.

Table 19.6: The normal distribution (Chatfield (1983)).

z	*f*(*z*)	*F*(*z*)
0.0	0.398942	0.5
0.1	0.396952	0.539827
0.2	0.391043	0.559260
0.3	0.381388	0.617911
0.4	0.368270	0.655422
0.5	0.352065	0.691465
0.6	0.333225	0.725747
0.7	0.312254	0.758036
0.8	0.289692	0.788145
0.9	0.266085	0.815940
1.0	0.241971	0.841345
1.1	0.217852	0.864334
1.2	0.194186	0.884930
1.3	0.171369	0.903195
1.4	0.149727	0.919243
1.5	0.129518	0.933193
1.6	0.110921	0.945201
1.7	0.094049	0.955435
1.8	0.078950	0.964069
1.9	0.065616	0.971284
2.0	0.053991	0.977250
2.1	0.043984	0.982136
2.2	0.035475	0.986097
2.3	0.028327	0.989276
2.4	0.022395	0.991803
2.5	0.017528	0.993791
2.6	0.013583	0.995339
2.7	0.010421	0.996533
2.8	0.007915	0.997495
2.9	0.005952	0.998134
3.0	0.004432	0.998650
3.5	0.000873	0.999768
4.0	0.000134	0.999968
4.5	0.000016	0.999996
5.0	0.0000015	0.9999997



If measurements of a dimension for a particular component are found to be normally distributed with mean μ and standard deviation σ, then the tolerance limits are often taken as $\mu \pm 3\sigma$. Tolerance limits defined as $\pm$ three standard deviations from the mean are called "natural tolerance limits." So if the mean and standard deviation of a component are 8.00 and 0.01, respectively, the dimension and bilateral tolerances would be defined as $8.00^{+0.03}_{-0.03}$ (i.e. the natural tolerance $= 6\sigma = 6 \times 0.01 = 0.06$, so the bilateral tolerance is $0.06/2 = 0.03$).

19.3.1 Sure-Fit or Extreme Variability

The sure-fit or extreme variability of a function of several uncorrelated random variables could be found by substituting values for the variables to find the maximum and the minimum value of the function, e.g. for the function

$$y = \frac{x_1 + 2x_2}{x_3^3}$$

The maximum value of $y_{\max}$ is given by

$$y_{\max} = \frac{x_{1,\max} + 2x_{2,\max}}{x_{3,\min}^3}$$

Similarly, for the minimum value,

$$y_{\min} = \frac{x_{1,\min} + 2x_{2,\min}}{x_{3,\max}^3}$$

The sure-fit or extreme variability can then be determined as

$$\Delta y = y_{\max} - y_{\min}$$

If $x_1 = 12 \pm 0.1$, $x_2 = 15 \pm 0.2$, and $x_3 = 2 \pm 0.15$, then $x_{1,\max} = 12.1$, $x_{1,\min} = 11.9$, $x_{2,\max} = 15.2$, $x_{2,\min} = 14.8$, $x_{3,\max} = 2.15$, and $x_{3,\min} = 1.85$. $\overline{y} = 5.25$.

$$y_{\max} = \frac{x_{1,\max} + 2x_{2,\max}}{x_{3,\min}^3} = \frac{12.1 + (2 \times 15.2)}{1.85^3} = 6.712$$

$$y_{\min} = \frac{x_{1,\min} + 2x_{2,\min}}{x_{3,\max}^3} = \frac{11.9 + (2 \times 14.8)}{2.15^3} = 4.176$$

$\Delta y = y_{\max} - y_{\min} = 6.712 - 4.176 = 2.536$, so the variable $y = \overline{y} \pm (\Delta y/2) = 5.25 \pm 1.268$.

Alternatively, the expression given in Eqn (19.13), considered later in Section 19.3.3, can be used to approximate this extreme variability.

$$\Delta y \cong \left|\frac{\partial y}{\partial x_1}\right|\Delta x_1 + \left|\frac{\partial y}{\partial x_2}\right|\Delta x_2 + \left|\frac{\partial y}{\partial x_3}\right|\Delta x_3$$

where $\Delta x_1 = x_{1,\max} - x_{1,\min}$, $\Delta x_2 = x_{2,\max} - x_{2,\min}$, and $\Delta x_3 = x_{3,\max} - x_{3,\min}$.

For the numerical example,

$\partial y/\partial x_1 = 1/x_3^3 = 1/2^3 = 0.125,$
$\partial y/\partial x_2 = 2/x_3^3 = 2/2^3 = 0.25,$
$\partial y/\partial x_3 = -3(x_1 + 2x_2)/x_3^4 = 7.875,$
$\Delta x_1 = 0.2,\ \Delta x_2 = 0.4,\ \Delta x_3 = 0.3,$
$\Delta y_{\text{approx}} = 0.125 \times 0.2 + 0.25 \times 0.4 + 7.875 \times 0.3 = 2.4875.$

So $y = 5.25 \pm 1.244$ (a value within a few percent of the value calculated in the above exact method).

19.3.2 Linear Functions or Tolerance Chains

Often we are required to know the overall tolerance in a dimension chain, which dimensions can be slackened without a deleterious effect on performance, or which tolerances need to be tightened to improve the overall tolerance.

Taking a simple linear chain of components of overall dimension z:

$$z = x_1 + x_2 + x_3 + \ldots + x_n \tag{19.9}$$

The expected value of z is given by

$$E(z) = \mu_1 + \mu_2 + \ldots + \mu_n = \mu_z \tag{19.10}$$

and the variance by

$$\text{variance}(z) = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots + \sigma_n^2 = \sigma_z^2 \tag{19.11}$$

If the response variable is a linear function of several measured variables, each of which is normally distributed, then the response variable will also be normally distributed.

Example 19.4

A product is made by aligning four components (see Figure 19.11) whose lengths are x_1, x_2, x_3, and x_4. The overall length is denoted by z. The tolerance limits of the lengths are known to

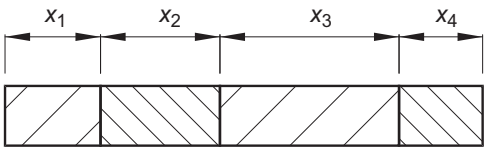


Figure 19.11
Dimension chain.

be 8.000 ± 0.030, 10.000 ± 0.040, 15.000 ± 0.050, and 7.000 ± 0.030 mm, respectively. If the lengths of the four components are independently normally distributed, estimate the tolerance limits for the length of the overall assembly.

Solution

Assuming $\pm 3\sigma$ natural tolerance limits:

$$\sigma_1 = 0.03/3 = 0.01 \text{ mm}$$

$$\sigma_2 = 0.04/3 = 0.0133333 \text{ mm}$$

$$\sigma_3 = 0.05/3 = 0.0166667 \text{ mm}$$

$$\sigma_4 = 0.03/3 = 0.01 \text{ mm}$$

$$\sigma_z^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 = 0.01^2 + 0.013333^2 + 0.0166667^2 + 0.01^2 = 6.555 \times 10^{-4}$$

$$\sigma_z = 0.0256$$

Overall variability $= 6\sigma_z$.

Tolerance limits $= \pm(6\sigma_z/2) = \pm 3\sigma_z = \pm 0.077$.

The mean length of the overall assembly is given by

$$\mu_z = \mu_{x_1} + \mu_{x_2} + \mu_{x_3} + \mu_{x_4} = 8 + 10 + 15 + 7 = 40 \text{ mm}$$

$$z = 40.000 \pm 0.077 \text{ mm.}$$

So 99.73% of all items would have an overall dimension within these tolerance limits.

(By comparison, the sure fit or extreme variability tolerance limits would be $z = 40.000 \pm 0.150$ mm, i.e. 100% of all items would be within this tolerance limit.)

The basic normal model indicated a better, or more attractive (certainly for marketing purposes), tolerance band. If the sure-fit model was used and an assembly tolerance of ± 0.077 mm was desired, the tolerances on the individual components would need to

be 8.000 ± 0.019, 10.000 ± 0.019, 15.000 ± 0.019, and 7.000 ± 0.019 mm. This is a very fine or tight tolerance and would be costly to achieve. As the statistical model takes account of 99.73% of the assemblies, use of the sure-fit approach only gives 0.27% of the assemblies with better quality and would require considerable extra effort and expense.

Example 19.5

Suppose the statistical tolerance bandwidth for the overall length of the previous example (± 0.077 mm) is too large for the particular application. An overall statistical tolerance of ± 0.065 mm is required. How can this be achieved?

Solution

This can be achieved in several different ways. One way would be to set the tolerance (and hence the standard deviation) equal on all the components:

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma$$

Set $\sigma_z^2 = (0.065/3)^2 = 4\sigma^2$.

Hence $\sigma = 0.0108$ mm.

Hence the tolerance limits for each individual component would be $\pm 3\sigma = \pm 0.033$ mm.

Alternatively, the tolerance limits on, say, components 1 and 4 could be kept the same and the tolerance limits on components 2 and 3 could be tightened.

Suppose we decide that the tolerance limits on items 2 and 3 should be equal ($\sigma_2 = \sigma_3$).

$$\sigma_z^2 = \sigma_1^2 + \sigma_4^2 + 2\sigma_2^2$$

$$2\sigma_2^2 = 2.69444 \times 10^{-4}$$

$$\sigma_2 = 0.0116 \text{ mm}$$

So the tolerance limits on components 2 and 3 would need to be ± 0.035 mm in order for the overall tolerance of the assembly to be ± 0.065 mm.

Example 19.6

Which of the following two methods of manufacturing a product 150 mm in length would give the best overall tolerance?

1. An assembly of 10 components, each of length 15.000 mm with tolerance limits of ± 0.050 mm.
2. A single component of 150 mm also with tolerance limits of ± 0.050 mm.

Solution

1. $$\sigma_1 = 0.05/3$$
$$\sigma_z^2 = 10\sigma_1^2$$
$$\sigma_z = 0.0527$$

Tolerance limits for overall length $\pm 3\sigma = \pm 0.158$ mm.

$$\mu_z = 10\mu_1 = 150.000 \text{ mm}$$
$$z = 150.000 \pm 0.158 \text{ mm}$$

2. $$z = 150.000 \pm 0.050 \text{ mm}.$$

As this method gives smaller tolerance limits, this is the most suitable way of achieving the tighter tolerance.

19.3.3 Several Independent, Uncorrelated Random Variables

Statistical analysis of tolerances can be applied to calculate the variation in physical phenomena such as stress, fluid flow rates, compression ratio, etc., as well as component dimension chains. BS5760, Part 2, states that the normal distribution model can be considered to be an appropriate distribution for modeling 80% or more cases of distributional analysis in nonlife circumstances. As such, this analysis has wide-ranging capability.

The theoretical preliminaries that enable analysis of functions of several independent, uncorrelated random variables are given below (after Furman, 1981).

Let y be a function of several independent, uncorrelated random variables.

$$y = f\left(x_1, x_2, \ldots, x_j, \ldots, x_n\right) \tag{19.12}$$

The sure-fit or extreme variability of the variable y is given by

$$\delta y \cong \sum_{j=1}^{n} \frac{\partial f}{\partial x_j} \delta x_j \tag{19.13}$$

The statistical mean and standard deviation of a function of several independent, uncorrelated random variables are given by

$$\mu_y \cong f(\mu_1, \mu_2, \ldots, \mu_n) + \frac{1}{2} \sum_{j=1}^{n} \frac{\partial^2 f}{\partial x_j^2} \sigma_{x_j}^2 \tag{19.14}$$

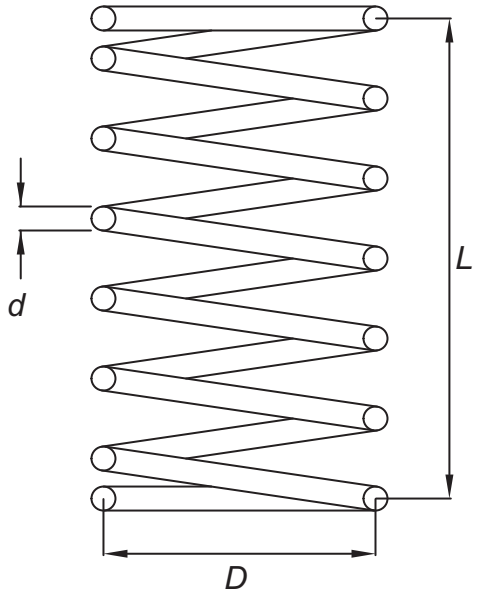


Figure 19.12
Helical compression spring.

$$\sigma_y^2 \cong \sum_{j=1}^{n} \left(\frac{\partial f}{\partial x_j}\right)^2 \sigma_{x_j}^2 + \frac{1}{2}\sum_{j=1}^{n} \left(\frac{\partial^2 f}{\partial x_j^2}\right)^2 \sigma_{x_j}^4 \approx \sum_{j=1}^{n} \left(\frac{\partial f}{\partial x_j}\right)^2 \sigma_{x_j}^2 \tag{19.15}$$

These equations may appear complex, but their application is procedural and various examples are given to show their use.

Example 19.7

The stiffness of a helical compression spring (see Figure 19.12) can be calculated from

$$k = \frac{d^4 G}{8D^3 n}$$

where d is the wire diameter, G is the modulus of rigidity, D is the coil diameter, n is the number of coils, and k is the spring stiffness.

The mean and standard deviation for each variable are known:

$\mu_d = 2.34$ mm, $\mu_D = 16.71$ mm, $\mu_G = 79.29 \times 10^3$ N/mm^2, $\mu_n = 14$ coils,
$\sigma_d = 0.010$ mm, $\sigma_D = 0.097$ mm, $\sigma_G = 1.585 \times 10^3$ N/mm^2, $\sigma_n = 0.0833$ coils.

Calculate the sure-fit extreme tolerance limits and the statistical basic normal tolerance limits for the spring stiffness. Assume $\pm 3\sigma$ natural tolerance limits: i.e. 99.73% of measurements lie within ± 3 standard deviations.

Solution

Substituting values into the equation for k to give the average value of the spring constant:

$$\overline{k} = \frac{2.34^4 \times 79.29 \times 10^3}{8 \times 16.71^3 \times 14} = 4.549 \text{ N/mm}.$$

$\sigma_d = 0.01$ mm,
$\Delta d = 6 \times \sigma_d = 0.06$ mm,
$\sigma_G = 1.585 \times 10^3$,
$\Delta G = 6 \times \sigma_G = 9510$ N/mm^2,
$\sigma_D = 0.097$,
$\Delta D = 6 \times \sigma_D = 0.582$ mm,
$\sigma_n = 0.0833$ coils,
$\Delta n = 6 \times \sigma_n = 0.5$.

$$\frac{\partial k}{\partial d} = \frac{4d^3G}{8D^3n} = \frac{2.34^3 \times 79.29 \times 10^3}{2 \times 16.71^3 \times 14} = 7.776.$$

$$\frac{\partial k}{\partial G} = \frac{d^4}{8D^3n} = 5.7374 \times 10^{-5}.$$

$$\frac{\partial k}{\partial D} = -\frac{3d^4G}{8D^4n} = -8.1673 \times 10^{-1}.$$

$$\frac{\partial k}{\partial n} = -\frac{d^4G}{8D^3n^2} = -3.2494 \times 10^{-1}.$$

$$\begin{aligned}\Delta k_{\text{sure-fit}} &= \left|\frac{\partial k}{\partial d}\right|\Delta d + \left|\frac{\partial k}{\partial G}\right|\Delta G + \left|\frac{\partial k}{\partial D}\right|\Delta D + \left|\frac{\partial k}{\partial n}\right|\Delta n \\ &= 7.776 \times 0.06 + 5.7374 \times 10^{-5} \times 9510 + 0.81673 \times 0.582 + 0.32494 \times 0.5 \\ &= 0.46656 + 0.5456 + 0.4753 + 0.16247 = 1.65 \text{ N/mm}.\end{aligned}$$

This figure ($\Delta k_{\text{sure-fit}}$) gives the overall worst-case variability of the spring stiffness (i.e. the total tolerance). The bilateral tolerance can be obtained by dividing Δk by 2 (i.e. $1.65/2 = 0.825$).

So the spring stiffness can be stated as $k = 4.549 \pm 0.825$ N/mm.

Calculation of the basic normal variability is

$$\begin{aligned}\Delta k^2_{\text{basic normal}} &= \left|\frac{\partial k}{\partial d}\right|^2 \Delta d^2 + \left|\frac{\partial k}{\partial G}\right|^2 \Delta G^2 + \left|\frac{\partial k}{\partial D}\right|^2 \Delta D^2 + \left|\frac{\partial k}{\partial n}\right|^2 \Delta n^2 \\ &= 0.21768 + 0.297708 + 0.22594 + 0.026397 = 0.7677.\end{aligned}$$

$$\Delta k_{\text{basic normal}} = 0.8762 \text{ N/mm}.$$

The overall variability in the spring stiffness is calculated as 0.8762 N/mm using the basic normal method. The bilateral tolerance can be calculated by dividing this value in 2, i.e. the spring stiffness $k = 4.549 \pm 0.4381$ N/mm.

The basic normal variation model implies a narrower bandwidth of variation $k = 4.549 \pm 0.438$ N/mm versus $k = 4.549 \pm 0.825$ N/mm for a sure-fit. This is more attractive from a sales perspective and accounts for 99.73% (i.e. nearly all) of the items.

If we wanted to decrease the variation in k, we would need to tighten the individual tolerances on d, G, D, and n with particular attention to d and D, as these are controllable and influential on the overall variability.

(The standard deviation of the spring stiffness can be calculated, if required, by $\sigma_k = \Delta k/6 = 0.146$).

Example 19.8

In the design work for a model single cylinder reciprocating engine, the work done during the expansion stroke is to be represented as:

$$W = P\pi D^2 C\left(1 - Q^{n-1}\right)/(4(n-1))$$

where

$$Q = C/(C + 2R)$$

and

$$C = M - R - L - H$$

The quantities are defined as follows:

W is the work done during the expansion stroke,
P is the cylinder pressure at start of stroke,
D is the piston diameter,
C is the length of the clearance volume,
n is the expansion stroke polytropic exponent,
R is the crank radius,
M is the distance from the crank center to the underside of the cylinder head,
L is the connecting rod length,
H is the distance from gudgeon pin to piston crown,
Q is the ratio of clearance to clearance-plus-swept volumes.

It is recognized that the independent quantities are subject to variability, and it is desired to determine the consequent extreme and probable variability in the work done.

If

$$P = 20 \times 10^5 \pm 0.5 \times 10^5\,\mathrm{N/m^2},$$

$$n = 1.3 \pm 0.05,$$

$$D = 4 \times 10^{-2} \pm 25 \times 10^{-6}\,\mathrm{m},$$

$$R = 2 \times 10^{-2} \pm 50 \times 10^{-6}\,\mathrm{m},$$

$$M = 10.5 \times 10^{-2} \pm 50 \times 10^{-6}\,\mathrm{m},$$

$$H = 2 \times 10^{-2} \pm 25 \times 10^{-6}\,\mathrm{m},$$

$$L = 6 \times 10^{-2} \pm 50 \times 10^{-6}\,\mathrm{m},$$

determine the extreme and probable (normal model) limits of the work (Ellis, 1990).

To help with your solution, certain partial derivatives have already been calculated.

$$\frac{\partial W}{\partial R} = -2599.46.$$

$$\frac{\partial W}{\partial M} = 2888.39.$$

$$\frac{\partial W}{\partial H} = -2888.39.$$

$$\frac{\partial W}{\partial L} = -2888.39.$$

Note: It may be useful to recall that if $y = a^x$, then $\mathrm{d}y/\mathrm{d}x = x \log_e(a)$.

Solution

The sure-fit "extreme" limit of the work done is given by

$$\Delta W_{\text{sure-fit}} = \left|\frac{\partial W}{\partial P}\right|\Delta P + \left|\frac{\partial W}{\partial D}\right|\Delta D + \left|\frac{\partial W}{\partial M}\right|\Delta M + \left|\frac{\partial W}{\partial R}\right|\Delta R + \left|\frac{\partial W}{\partial L}\right|\Delta L + \left|\frac{\partial W}{\partial H}\right|\Delta H + \left|\frac{\partial W}{\partial n}\right|\Delta n$$

The average values of C, Q, and W are $\overline{C} = 0.005$, $\overline{Q} = 0.1111$, and $\overline{W} = 20.22$.

$$\frac{\partial W}{\partial P} = \frac{\pi D^2 C\left(1 - Q^{n-1}\right)}{4(n-1)} = \pi 0.04^2 0.005 \frac{1 - 0.1111^{0.3}}{4 \times 0.3} = 1.011035 \times 10^{-5}.$$

$$\Delta P = 2 \times 0.5 \times 10^5 = 1 \times 10^5.$$

$$\left|\frac{\partial W}{\partial P}\right|\Delta P = 1.011035.$$

$$\frac{\partial W}{\partial D} = 2P\pi DC\frac{1-Q^{n-1}}{4(n-1)} = 2\pi 20\times 10^5\times 0.04\times 0.005\frac{1-0.1111^{0.3}}{4\times 0.3} = 1011.035.$$

$$\Delta D = 50\times 10^{-6}.$$

$$\left|\frac{\partial W}{\partial D}\right|\Delta D = 0.0505517.$$

$$\frac{\partial W}{\partial M} = 2888.39, \quad \Delta M = 100\times 10^{-6}.$$

$$\left|\frac{\partial W}{\partial M}\right|\Delta M = 0.288839.$$

$$\frac{\partial W}{\partial R} = -2599.46, \quad \Delta R = 100\times 10^{-6}.$$

$$\left|\frac{\partial W}{\partial R}\right|\Delta R = 0.259946.$$

$$\frac{\partial W}{\partial L} = -2888.39, \quad \Delta L = 100\times 10^{-6}.$$

$$\left|\frac{\partial W}{\partial L}\right|\Delta L = 0.288839.$$

$$\frac{\partial W}{\partial H} = -2888.39, \quad \Delta H = 50\times 10^{-6}.$$

$$\left|\frac{\partial W}{\partial H}\right|\Delta H = 0.1444195.$$

Let

$$W = K_3\left(\frac{1-Q^{n-1}}{n-1}\right), \quad K_3 = \frac{P\pi D^2 C}{4}.$$

$$\left|\frac{\partial W}{\partial n}\right| = K_3\left(\frac{-(n-1)\frac{\partial}{\partial n}Q^{n-1} - \left(1-Q^{n-1}\right)1}{(n-1)^2}\right) = K_3\left(\frac{-(n-1)Q^{n-1}\ln Q - \left(1-Q^{n-1}\right)}{(n-1)^2}\right),$$

which on substituting for K_3 gives

$$= \frac{W(n-1)}{1-Q^{n-1}}\left(\frac{-(n-1)Q^{n-1}\ln Q - \left(1-Q^{n-1}\right)}{(n-1)^2}\right)$$

$$\frac{\partial W}{\partial n} = -\frac{W}{n-1}\left(\frac{(n-1)Q^{n-1}\ln Q}{1-Q^{n-1}}+1\right) = \frac{20.22}{0.3}\left(\frac{0.3\times 0.1111^{0.3}\ln 0.1111}{1-0.1111^{0.3}}+1\right)$$
$$= 19.7909.$$

$$\Delta n = 2\times 0.05 = 0.1.$$

$$\left|\frac{\partial W}{\partial n}\right|\Delta n = 1.979.$$

$$\Delta W_{\text{sure-fit}} = 1.011,035 + 0.0,505,505517 + 0.288,839 + 0.259,946 + 0.288,839$$
$$+ 0.1,444,444195 + 1.979 = 4.022.$$

$$W = \overline{W} \pm (\Delta W/2) = 20.22 \pm 2.011 \text{ J}.$$

Statistical tolerance limits are

$$\Delta W^2_{\text{normal}} = \left(\frac{\partial W}{\partial P}\right)^2\Delta P^2 + \left(\frac{\partial W}{\partial D}\right)^2\Delta D^2 + \left(\frac{\partial W}{\partial M}\right)^2\Delta M^2 + \left(\frac{\partial W}{\partial R}\right)^2\Delta R^2 + \left(\frac{\partial W}{\partial L}\right)^2\Delta L^2$$
$$+ \left(\frac{\partial W}{\partial H}\right)^2\Delta H^2 + \left(\frac{\partial W}{\partial n}\right)^2\Delta n^2 = 5.196.$$

$$\Delta W = 2.2796.$$

So $W = \overline{W} \pm (\Delta W/2) = 20.22 \pm 1.1398$ J.

If required, $\sigma_W = 1.1398/3 = 0.38$.

Example 19.9

The torque capacity, T, for a multiple disc clutch can be calculated using the relationship

$$T = \frac{2}{3}\mu F_a N\left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right)$$

where μ is the coefficient of friction, F_a is the axial clamping force, N is the number of disc faces, r_o is the outer radius of clutch ring, and r_i is the inner radius of clutch ring.

Determine the sure-fit and basic normal probable limits for the torque capacity if

$\mu = 0.3 \pm 0.03$,
$F_a = 4000 \pm 200$ N,
$N = 12$,
$r_o = 60 \pm 0.5$ mm,
$r_i = 30 \pm 0.5$ mm.

Which design parameter should be reduced to create a maximum reduction in the variability of the torque capacity?

Solution

$$T_{av} = \frac{2}{3} \times 0.3 \times 4000 \times 12 \times \left(\frac{0.06^3 - 0.03^3}{0.06^2 - 0.03^2} \right) = 672 \text{ N m}$$

For sure-fit variability,

$$\delta y \approx \sum_{j=1}^{n} \frac{\partial f}{\partial x_j} \delta x_j$$

$$\Delta T_{\text{sure-fit}} \approx \left| \frac{\partial T}{\partial F_a} \right| \Delta F_a + \left| \frac{\partial T}{\partial \mu} \right| \Delta \mu + \left| \frac{\partial T}{\partial r_o} \right| \Delta r_o + \left| \frac{\partial T}{\partial r_i} \right| \Delta r_i$$

$$\frac{\partial T}{\partial F_a} = \frac{2}{3} \mu N \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} = \frac{2}{3} \times 0.3 \times 12 \times \frac{0.06^3 - 0.03^3}{0.06^2 - 0.03^2} = 0.168$$

$$\frac{\partial T}{\partial \mu} = \frac{2}{3} F_a N \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} = \frac{2}{3} \times 4000 \times 12 \times \frac{0.06^3 - 0.03^3}{0.06^2 - 0.03^2} = 2240$$

Quotient rule, $\left(\frac{u}{v} \right)' = \frac{vu' - uv'}{v^2}$.

$$\begin{aligned} \frac{\partial T}{\partial r_o} &= \frac{2}{3} \mu F_a N \frac{\left(r_o^2 - r_i^2\right) 3 r_o^2 - \left(r_o^3 - r_i^3\right) 2 r_o}{\left(r_o^2 - r_i^2\right)^2} \\ &= \frac{2}{3} \times 0.3 \times 4000 \times 12 \times \frac{2.7 \times 10^{-3} \times 3 \times 0.06^2 - 1.89 \times 10^{-4} \times 2 \times 0.06}{\left(0.06^2 - 0.03^2\right)^2} \\ &= 0.8889 \times 9600 = 8533 \end{aligned}$$

$$\frac{\partial T}{\partial r_i} = \frac{2}{3}\mu F_a N \frac{-\left(r_o^2 - r_i^2\right)3r_i^2 + \left(r_o^3 - r_i^3\right)2r_i}{\left(r_o^2 - r_i^2\right)^2}$$

$$= \frac{2}{3} \times 0.3 \times 4000 \times 12 \times \frac{-2.7 \times 10^{-3} \times 3 \times 0.03^2 - 1.89 \times 10^{-4} \times 2 \times 0.03}{\left(0.06^2 - 0.03^2\right)^2}$$

$$= 0.5556 \times 9600 = 5333$$

$$\Delta F_a = 2 \times 200 = 400 \text{ N}$$

$$\Delta \mu = 2 \times 0.3 = 0.6 \text{ N}$$

$$\Delta r_o = 2 \times 0.0005 = 0.001 \text{ m}$$

$$\Delta r_i = 2 \times 0.0005 = 0.001 \text{ m}$$

$$\Delta T_{\text{sure-fit}} = (2240 \times 0.06) + (0.168 \times 400) + (0.8889 \times 0.001 \times 9600) + (0.5556 \times 0.001 \times 9600)$$

$$= 134.4 + 67.2 + 8.533 + 5.333 = 215.5 \text{ N m}$$

$$T_{\text{sure-fit}} = T_{av} \pm \left(\frac{\Delta T_{\text{sure-fit}}}{2}\right) = 672 \pm 108 \text{ N m}$$

Basic normal probable limits are $\sigma_y^2 \approx \sum_{j=1}^{n} \left(\frac{\partial f}{\partial x_j}\right)^2 \sigma_{x_j}^2$.

Assuming natural tolerance limits,

$$\Delta T_{\text{basic normal}}^2 \approx \left|\frac{\partial T}{\partial F_a}\right|^2 \Delta F_a^2 + \left|\frac{\partial T}{\partial \mu}\right|^2 \Delta \mu^2 + \left|\frac{\partial T}{\partial r_o}\right|^2 \Delta r_o^2 + \left|\frac{\partial T}{\partial r_i}\right|^2 \Delta r_i^2$$

$$= 134.4^2 + 67.2^2 + 8.533^2 + 5.333^2 = 22{,}680$$

$$\Delta T_{\text{basic normal}} = 150.6 \text{ N m}$$

$$T_{\text{basic normal}} = T_{av} \pm \left(\frac{\Delta T_{\text{basic normal}}}{2}\right) = 672 \pm 75 \text{ N m}$$

The coefficient of friction contributes the most to the variability. This would require more stringent manufacture control.

Example 19.10

For a standard series "A" Belleville spring, the spring rate or stiffness can be determined by the equation

$$k = \frac{4E}{(1-\mu^2)} \times \frac{t^3}{K_1 D_e^2}$$

where E is Young's modulus of the washer material, t is the washer thickness, μ is Poisson's ration, K_1 is a dimensionless constant, and D_e is the external diameter.

Determine the sure-fit and probable limits for the spring stiffness stating any assumptions made if

$$t = 2.22 \pm 0.03 \text{ mm}, \; D_e = 40.00 \pm 0.08 \text{ mm}, \; \mu = 0.30 \pm 0.003,$$

$$E = 207 \times 10^9 \pm 2 \times 10^9 \text{ N/m}^2, \; K_1 = 0.69.$$

Comment on which tolerance could be altered to best reduce the overall variability.

Solution

$$k = \frac{4E}{(1-\mu^2)} \times \frac{t^3}{K_1 D_e^2}.$$

$$K_1 = \text{constant}.$$

Assuming quantities are subject to variability and are uncorrelated and random,

$$\Delta k_{\text{sure-fit}} = \left|\frac{\partial k}{\partial E}\right| \Delta E + \left|\frac{\partial k}{\partial \mu}\right| \Delta \mu + \left|\frac{\partial k}{\partial t}\right| \Delta t + \left|\frac{\partial k}{\partial D_e}\right| \Delta D_e.$$

$$\Delta k^2_{\text{basic normal}} = \left|\frac{\partial k}{\partial E}\right|^2 \Delta E^2 + \left|\frac{\partial k}{\partial \mu}\right|^2 \Delta \mu^2 + \left|\frac{\partial k}{\partial t}\right|^2 \Delta t^2 + \left|\frac{\partial k}{\partial D_e}\right|^2 \Delta D_e^2.$$

$$\frac{\partial k}{\partial E} = \frac{4t^3}{(1-\mu^2) K_1 D_e^2} = \frac{4 \times \left(2.22 \times 10^{-3}\right)^3}{(1-0.3^2) \times 0.69 \times 0.04^2} = 4.356 \times 10^{-5}.$$

$$\frac{\partial k}{\partial \mu} = -\frac{2\mu}{(1-\mu^2)^2} \times \frac{4Et^3}{K_1 D_e^2} = -\frac{2 \times 0.3}{(1-0.09)^2} \times \frac{4 \times 207 \times 10^9 \times \left(2.22 \times 10^{-3}\right)^3}{0.69 \times 0.04^2}$$

$$= -5.946 \times 10^6.$$

Quotient rule, $\left(\frac{u}{v}\right)' = \frac{uv' - vu'}{v^2}$.

$$\frac{\partial k}{\partial t} = \frac{12Et^2}{(1-\mu^2)K_1 D_e^2} = \frac{12 \times 207 \times 10^9 \times \left(2.22 \times 10^{-3}\right)^2}{(1-0.09) \times 0.69 \times 0.04^2} = 1.219 \times 10^{10}.$$

$$\frac{\partial k}{\partial D_e} = -\frac{8Et^3}{(1-\mu^2)K_1 D_e^3} = -\frac{8 \times 207 \times 10^9 \times (0.00222)^3}{(1-0.09) \times 0.69 \times (0.04)^3} = -4.509 \times 10^8.$$

Assume natural tolerance limits $\pm 3\sigma$,

$$\Delta E = 2 \times 2 \times 10^9 = 4 \times 10^9.$$

$$\Delta \mu = 2 \times 0.003 = 0.006.$$

$$\Delta t = 2 \times 0.03 \times 10^{-3} = 6 \times 10^{-5}.$$

$$\Delta D_e = 2 \times 0.08 \times 10^{-3} = 1.6 \times 10^{-4}.$$

$$\begin{aligned}\Delta k_{\text{sure-fit}} &= (4.356 \times 10 - 5 - 5 \times 4 \times 109) + (5.946 \times 106 \times 0.006) \\ &\quad + \left(1.219 \times 10^{10} \times 6 \times 10^{-5}\right) + \left(4.509 \times 10^8 \times 1.6 \times 10^{-4}\right) \\ &= 174{,}240 + 35{,}676 + 731{,}400 + 72{,}144 = 1{,}013{,}460.\end{aligned}$$

$$\overline{k} = \frac{4 \times 207 \times 10^9}{1 - 0.3^2} \times \frac{\left(2.22 \times 10^{-3}\right)^3}{0.69 \times 0.04^2} = 9{,}017{,}347.3 \text{ N/m}.$$

$$k_{\text{sure-fit}} = 9{,}017{,}000 \pm 506{,}700 \text{ N/m}.$$

$$\Delta k^2{}_{\text{basic normal}} = 174{,}240^2 + 35{,}676^2 + 731{,}400^2 + 72{,}144^2 = 5.718 \times 10^{11}.$$

$$\Delta k_{\text{basic normal}} = 756{,}200.$$

$$k_{\text{basic normal}} = 9{,}017{,}000 \pm 378{,}100 \text{ N/m}.$$

To reduce the variability, tighten the tolerances on t as this has the highest sensitivity coefficient.

19.3.4 Statistical Design Techniques and Quality Assurance

The traditional approach to handling quality problems is based upon defining quality as a conformance to specifications. For example, if the dimension and tolerance for a component were defined as 40.0 ± 0.1 mm, it would not matter whether the component dimension was 40.1, 40.5, 40.0, or 39.9 mm, the specification would have been satisfied. A Japanese engineer named Genichi Taguchi has defined a new approach for solving quality problems based on product uniformity around the target value. In the example above, the target value is 40.0 and the approach proposed by Taguchi is to endeavor to ensure as many components as

possible are near to this value. Essentially, Taguchi's methods are based on the premise that quality is the avoidance of financial loss to society after the article is shipped. Within this definition, quality is related to quantifiable monetary loss rather than emotional gut reaction. Taguchi techniques of ensuring good quality require a three-step approach to product or process design consisting of system, parameter, and tolerance design. These are defined below:

System design	First, all the possible systems that can perform the required functions must be considered, including new ones that have not yet been developed.
Parameter design	During this stage, the appropriate system parameters should be specified to improve quality and reduce costs.
Tolerance design	Decisions concerning the tolerance specifications for all the components and appropriate grades of materials must be made. The objective of tolerance design is to decide on tradeoffs between quality levels and cost in designing new systems.

As a qualitative illustration of this technique, a case study is outlined of the production of the same design of an automotive transmission by a multinational corporation but at two different sites, one in Japan and one in the United States. Both factories manufactured the transmission within a tolerance band as specified by the designer, but in market surveys it was found that American customers preferred the Japanese manufactured transmissions. Various factors were investigated, including the dimensional tolerance chain, which is shown for the two factories in Figure 19.13. Too tight a tolerance chain results in high wear or binding of the transmission, and too loose a tolerance chain results in high noise and component impact levels. The Japanese manufactured transmission was found to be approximately normally distributed about the target value, with 0.27% of items falling outside the tolerance bandwidth. The output from the U.S. factory was rectangularly distributed with all the items within the tolerance band. So the product from the U.S. factory was always within the tolerance specifications, unlike the output from the Japanese factory, which was occasionally outside the tolerance specification. However, the market surveys showed that customers had preferred the product that had originated from the Japanese factory. This initially

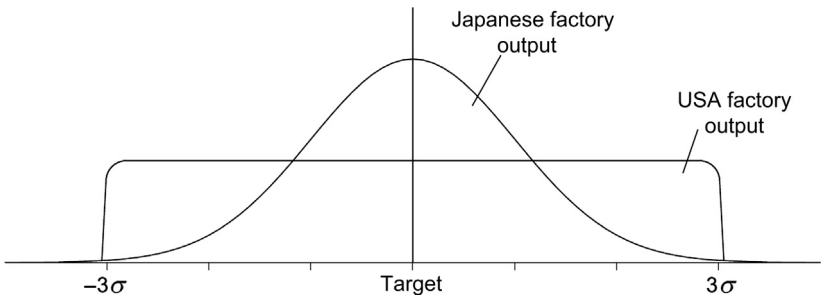


Figure 19.13
Distribution of automotive transmission tolerance.

surprising result, considering the on-target nature of the U.S. product, can be identified as being due to proportionally more customers getting a higher-quality, or nearer the target, product if supplied from the Japanese factory. The customers' perception of quality responded to the Japanese factory paying more attention to meeting the target, as opposed to U.S. factory's approach of meeting the tolerances.

Application of Taguchi's methods requires an understanding of statistical information along with budgetary information for each stage of the design, production, and marketing process. Further information on these methods can be found in the original texts by Taguchi (1989, 1993), Phadke (1989), Krishnamoorthi and Ram Krishnamoorthi (2011), and Su (2013).

Customers require products that have characteristics that satisfy their needs and expectations. Because of the competitive nature of business as well as the desire to improve service, organizations are continually driven to improve their products and processes. The ISO 9000 and 9001 series of standards describe the fundamentals of quality management systems and the requirements that an organization must demonstrate its ability to provide products that fulfill customer and regulatory requirements. An increasing proportion of business has embraced such approaches, and the reader is referred to the standards themselves for full details, or to Hoyle (1998).

19.4 Conclusions

Tolerances have to be applied to components in order to define acceptable limits for manufacturing purposes. Generally, the larger the tolerance, the easier it is to ensure that components will conform. The subject of tolerances is, however, not limited to components and assemblies of components; it also applies to processes. This chapter has introduced the subject of tolerances and methods to model the overall tolerance for an assembly or process based on statistical methods. In addition, the subject of quality assurance has been briefly introduced.

References

Books and Papers

Drake, P.J., 1999. Dimensioning and Tolerancing Handbook. McGraw Hill.
Ellis, J., 1990. Mechanical Design, BEng Final Examination. University of Sussex.
Furman, T.T., 1981. Approximate Methods in Engineering Design. Academic Press.
Hoyle, D., 1998. ISO 9000 Pocket Guide. Butterworth Heinemann.
Krishnamoorthi, K.S., Ram Krishnamoorthi, V.R., 2011. A First Course in Quality Engineering: Integrating Statistical and Management Methods of Quality, second ed. CRC Press.
Krulikowski, A., 2012. Fundamentals of Geometric Dimensioning and Tolerancing, third ed. Delmar Cengage Learning.
Phadke, M.S., 1989. Quality Engineering Using Robust Design. Prentice Hall.

Simmons, C.H., Maguire, D.E., Phelps, N., 2012. Manual of Engineering Drawing: Technical Product Specification and Documentation to British and International Standards, fourth ed. Butterworth-Heinemann.
Spotts, M.F., 1983. Dimensioning and Tolerancing for Quantity Production. Prentice Hall.
Su, Chao-Ton, 2013. Quality Engineering: Off-line Methods and Applications. CRC Press.
Taguchi, G., Elsayed, E.A., Hsiang, T., 1989. Quality Engineering in Production Systems. McGraw Hill.
Taguchi, G., 1993. Taguchi on Robust Technology Development. ASME Press.

Further Reading

Chatfield, C., 1983. Statistics for Technology. Chapman and Hall.
Haugen, E.B., 1968. Probabilistic Approaches to Design. Wiley.
Taguchi, G., 1983. Taguchi on Robust Technology Development. ASME Press.

Standards

BIP 3092 ISO 9000 Collection. The ISO 9000 Standards Collection. Quality Management Systems
British Standards Institution, 1953. BS 1916: Part 1: 1953. Specification for Limits and Fits for Engineering. Limits and Tolerances.
British Standards Institution. BS 4500A: 1970. Specification for ISO Limits and Fits. Data Sheet: Selected ISO Fits—Holes Basis.
British Standards Institution. BS 4500B: 1970. Specification for ISO Limits and Fits. Data Sheet: Selected ISO Fits—Shafts Basis.
British Standards Institution. BS 8888:2011. Technical Product Documentation and Specification.
British Standards Institution. BS5760: Part 2: Guide to the Assessment of Reliability.
British Standards Institution. BSI Handbook 22, Quality Assurance, 1983.
BS 5760−18:2010. Reliability of Systems, Equipment and Components. Guide to the Demonstration of Dependability Requirements. The Dependability Case.
BS EN ISO 9000:2005. Quality Management Systems. Fundamentals and Vocabulary.
BS EN ISO 9001:2008. Quality Management Systems. Requirements.
BS EN ISO 9004:2009. Managing for the Sustained Success of an Organization. A Quality Management Approach.
BS ISO 10006:2003. Quality Management Systems. Guidelines for Quality Management in Projects.
BS ISO 10007:2003. Quality Management Systems. Guidelines for Configuration Management.
International Organization for Standardization. ISO 9000−1:1994. Quality Management and Quality Assurance Standards—Part 1: Guidelines for Selection and Use.

Nomenclature

The following symbols have been used in this chapter. Generally, preferred SI units have been stated:

a	internal radius of inner cylinder (m)
b	outer radius of inner cylinder (m), interface radius (m)
c	outer radius of outer cylinder (m)
C	length of the clearance volume (m)
d	wire diameter (m)
D	coil diameter (m), piston diameter (m)

E Young's modulus (N/m^2)
f coefficient of friction
$f(x)$ probability density function of the continuous random variable x
G modulus of rigidity (N/m^2)
H distance from gudgeon pin to the piston crown (m)
k spring stiffness (N/m)
L length of interference fit (m), connecting rod length (m)
M distance from crank center to underside of cylinder head (m)
n number of coils, expansion stroke polytropic exponent
p pressure (N/m^2)
P cylinder pressure at start of stroke (N/m^2)
Q ratio of clearance to clearance-plus-swept volumes
R crank radius (m)
T torque (N m)
W work done during the expansion stroke (N m)
δ total diametral interference (m)
μ Poisson's ratio, mean
σ standard deviation

Index

Note: Page numbers followed by "f" denote figures; "t" tables.

A

ABS. *See* Antilock braking systems (ABS)
Acme thread, 694f, 695t, 697–698
Acrylics
  applications of, 709t
  characteristics of, 709t
Actuators
  hydraulics, 754–756, 754f, 755f
  pneumatics, 754–756
Addendum, 327, 331, 445
Adequacy assessment, 9
Adhesives, 104, 133, 708–714
  applications of, 709t
  characteristics of, 709t
  limitations of, 708
  loading of, 710f
Advantages, Limitations and Unique features (ALoU), 73
AGMA. *See* American Gear Manufacturers Association (AGMA)
Air compressors
  hydraulics, 747–750, 748f, 749f
  pneumatics, 747–750, 748f, 749f
Alignment, 256f
Allowable bending stress
  bevel gears, 410, 421–424, 423t, 426, 430, 435
  helical gear, 372–373, 383, 385, 385t, 388–390, 394–396, 399
  spur gear, 372–373, 383, 385, 385t, 388–390, 394–396, 399, *see also* Bending stress
Allowable contact stress
  bevel gears, 409, 421, 424–425, 424t, 429, 434
  helical gear, 372, 385t, 386, 389–390, 394, 396, 399
  spur gear, 373, 385t, 386, 389–390, 394, 396, 399, *see also* Contact stress
Allowable transmitted power
  bevel gears, 409–410
Allowance, 410–411, 588–589
Alloys, 288, 290, 619, 627–628
ALoU. *See also* Advantages, Limitations and Unique features (ALoU)
Alphabet brainstorming, 64, 65t
Altschuller, Genrich
  TRIZ, 53, 55t, 56, 75, 87–88, 87f, 92t
American Gear Manufacturers Association (AGMA), 346, 367–368, 405
  bending stress, equations for, 370–397, 374t, 376f, 376t, 377f, 378t, 379t, 380t, 381t, 382t, 384f, 385t
  contact stress, equations for, 370–397, 374t, 376f, 376t, 377f, 378t, 379t, 380t, 381t, 382t, 384f, 385t

American Gear Manufacturers Association (AGMA) (*Continued*)
equations, for worm gears, 449–452
American National Standard
rivets, 705t, 706t
American Society for Testing and Materials (ASTM), 627–628
American Society of Mechanical Engineers (ASME)
Boiler Code, 700, 702
design code for transmission shafting, 282–301
Anaerobics
applications of, 709t
characteristics of, 709t
Analysis of design, 5t, 9
Angular contact ball bearing, 202f
single row, 238f
Angular deflection, 659–660
Angular velocity ratio, 459
ANSI/AGMA standard, 407, 409–410, 445–446, 449–451
ANSYS, 266
Antilock braking systems (ABS), 560
control loop for, 561f
Aperture
seal, 570–574, 574f
strengthening, 712f
Arbitration process, 26
Arc welding
manual metal, 714
submerged, 714
ASME. *See* American Society of Mechanical Engineers (ASME)
ASTM. *See* American Society for Testing and Materials (ASTM)
Asymmetry, 93
Attribute Value Chain, 75
Attribution
direct, 58
equal, 58
Automotive clutch, 532f, 533f
Automotive disc brake, 537f
Axial seals, 612–616, 615f

B

Backlash, 328
Ball bearings, 201–203
Band brakes, 557–562, 558f, 559f
Barometers, 738
Barrel springs, 633f
Barth equation, 346
Basic dynamic load rating, 204
Basic static load rating, 206
Beams
cantilever, 661–662, 662f
curved, 637, 661–662
reflection, Macaulay's method for, 268–278, 269f, 271f, 273f, 275f, 275t, 277f
Bearings, 126–128, 126f, 127f
angular contact ball, 202f
boundary-lubricated, 143–144, 143f, 144f, 146–149
characteristic number. *See* Sommerfield number
classification of, 140f
cylindrical roller, 202f
deep groove ball, 140f
full-film hydrodynamic. *See* Full-film hydrodynamic bearings
journal, 139–200
installation, 245–248
mixed-film lubricated, 143–144, 143f, 144f
parameter, 144
partial surface, 151f
performance for continuous rotation, comparison of, 142t
rolling element, 201–253
selection of, 141f
self-aligning ball, 202f
sliding, 142–146, 143f, 144f, 145f
spherical roller, 202f
taper roller, 202f
thrust ball, 202f
Belleville springs, 531
characteristics of, 626t
stacking combinations for, 667f
washers, 664–671, 664f, 665f, 666f, 667f, 668t
Bellows, 738

Belt drives, 128–130, 129f, 460f, 461–495
comparison with chain drives, 497t
comparison with gear drives, 497t
configuration of, 462, 462f
cross sections of, 461, 462f
flat belt, 462–463, 492–495, 492f, 494t
merits of, 463t
selection of, 463–464
flat belts, 492–495, 492f, 494t
synchronous belt, 473, 473t–490t, 475t, 476t, 491f
wedge belt, 464, 464f, 466t, 467f, 468t, 469t, 470t, 471t
synchronous belt, 463, 473, 473f–490f, 475t, 476t, 491f
V belt, 463–464, 464f, 466t, 467f, 468t, 469t, 470t, 471t
Bending moment, of shafts, 270, 279, 294f, 297f
horizontal, 293–294, 294f, 297f
vertical, 293, 293f, 297f
Bending stress
AGMA equations for, 370–397, 374t, 376f, 376t, 377f, 378t, 379t, 380t, 381t, 382t, 384f, 385t
allowable. *See* Allowable bending stress
of bevel gears, 405, 410, 421–424, 423t
of gear materials, 356t
of gears, factor analysis of, 346–348, 347t
of helical extension springs, 649–650
of wire rope, 728–729
Bevel gears, 322f, 325, 403–438
calculation procedure summary, 425–435, 428t
capacity of, 408–409
design procedure for, 405
failure of, 408
force analysis of, 405–407
nomenclature, 406f
spiral, 404–405, 404f
straight, 403, 404f
stress analysis of, 407–424, 411t, 412f, 413f, 414f, 416f, 417f, 418f, 419f, 420f, 421f, 422f, 423t, 424t
zerol, 405
Bilateral tolerances, 762, 771–773, 776–778, 784–785
Boeing 747, 677
Bolted joints, 685–686, 685f, 686f, 687f
forces, 687f
load and deflection in, 687–688, 687f
modeling, 686f
Boundary conditions
helical compression springs and, 638
journal bearings and, 160, 161f, 165
shaft deflection and, 266, 270, 272–274, 278
Boundary shifting, 113
Boundary-lubricated bearings, 143–144
design of, 146–149
performance of, 144, 144f
surface roughness for, 143f
Bourdon gauges, 738
Brainstorming, 12, 53, 56–71, 75
alphabet, 64, 65t
brainwriting, 59–60, 64–66
evaluation, 69–71, 70t, 71t
facilitation, 68–69
flip chart technique, 60–62, 68
grid, 55t, 66–68, 66f, 116–117
group dynamics, 67–68
intellectual property rights and, 58–59
methods, 59–60
Post-it, 55t, 58, 62–64, 63f, 68, 116–117
principal rules of, 58
scenarios, 57–58, *see also* Creative process; Ideation
Brainwriting, 59–60, 64–66
Brake(s), 130, 531–562
band, 557–562, 558f, 559f
classification of, 535f
defined, 513–514
disc, 535–540, 537f, 538t

Brake(s) (*Continued*)
drum, 540
long-shoe external, 543–554, 543f, 544f, 547f, 549f, 550f, 551f
long-shoe internal, 554–557, 555f, 556f
short-shoe external, 540–543, 541f, 542f
friction type, 515–516, 515f
performance, comparison of, 536t
Brief, 25
British Standard (BS), 9, 346, 567, 627–628, 639, 751–752
for hot forged rivets, 703t
for cold forged rivets, 704t
British Standards Institution (BSI), 367–368
Brinell hardness, 290, 386–387, 391–392, 418
BS. *See* British Standard (BS)
BSI. *See* British Standards Institution (BSI)
Buckling, 636, 637f
Bush seals, 612–616, 615f
Butt joint, 700
cylindrical, 678f
double riveted, 701f
edge, 678f
performance of, 710f
tee, 678f
Buttress thread, 694f

C

CAD. *See* Computer-aided design (CAD)
Caliper disc brake, 537f
Cantilever beams, 661–662, 662f
Carry over correction factor, 597
Casing, 310f, 312, 554, 566, 611–612
Castigliano's theorem, for shaft deflection, 279–282, 279f, 281f
CDIO. *See also* Conceive, design, implement, operate (CDIO)
Centrifugal clutch, 520, 520f
applications of, 521t
characteristics of, 521t
Ceramics
applications of, 709t
characteristics of, 709t
CETOP RP3, 751–752
Chain conveyor drive shaft, 296f
Chain drives, 128–130, 129f, 299f, 460f, 495–509, 496f, 497f
comparison with belt drives, 497t
comparison with gear drives, 497t
conveyor, 499, 499f
duplex, 496f
failure of, 500
layouts of, 501f
leaf, 499, 499f
roller, 496f, 498f
selection of, 501–509, 502t, 503f, 506t
silent, 499–500, 500f
triplex, 496f
types of, 498f
Chatham house rules, 58
Checklists, for creative process, 55t, 56
Circlips, 263f
Circular pad disk brakes, 538, 538t
Circular pitch, 327
Clearance circle, 327f
Clutch(es), 130, 516–531
automotive, 532f
centrifugal, 520, 520f
classification of, 516f
cone, 521t
defined, 513–514
disc
design of, 521–531, 523f, 524f, 526f, 526t, 529f
multiple, 521t, 525f
single, 521t
friction type, 515–517, 515f, 521
magnetic, 520
multiple serration, 518f
over-running, 518, 519f
roller, 518–520, 519f
selection criteria for, 520–521, 521t
service factors for, 522t
single face, 529f
sprag, 518–520, 519f
spring wound, 518–520, 520f
square jaw, 518f
Coarse series threads, 679–680

Coefficient of friction, 144, 152, 168, 170f, 448, 455, 521, 524, 528, 540–541, 696
Color change, 103
Combining principle, 93–94
Commercivity, 114
Component location
  keys for, 262f
  pins for, 262f
Component stress concentration factor, 285–287
Component tolerances, 762–774
  geometric tolerancing, 771–774, 772f, 773f, 774f, 775t, 776f
  holes, standard fits for, 762–767, 765t
  interference fits, 767–769, 768f
  machine capability, 769–771, 770f, 771t, 772f
  shafts, standard fits for, 762–767, 766t
Composite materials use, 106
Compressors
  air, 747–750, 748f, 749f
  transmission
    for engine-driven compressor, 123–124, 123f
    product design specification for, 33–34
Computer-aided design (CAD), 329–331
Conceive, design, implement, operate (CDIO), 4, 17
Concentricity, 774
Concept evaluation, 56
Conceptual design, 15
  phase, of total design, 12
Concurrent engineering, 12
Cone clutch
  applications of, 521t
  characteristics of, 521t
Cone drives, 319f
Conjugate curves, 329
Contact stress, 368
  AGMA equations for, 370–397, 374t
  allowable. *See* Allowable contact stress
  bevel gears, 405, 409, 424, 424t
  Hertzian, 368, 725, 732
  permissible, 409, 429, 434
Continuity of a useful action, 99
Contradiction matrix, 106–107
Control valves
  hydraulics, 750–754, 751f, 752f, 753f
  pneumatics, 750–754, 751f, 752f, 753f
Convert harm into benefit principle, 99
Conveyor chain, 499, 499f
Copier rollers, 149f
Copying, 101
Corner rounding, 602–603, 603f
Cosine load integral, 189, 190t
Cost optimization, 9–10, 18
Counterweight principle, 94
Coupling(s)
  flexible, 264f
  fluid, 520
  rigid, 264f
  shaft–shaft connection, 263, 264f
CPS. *See* Creative problem solving (CPS)
Create Process, 75, 77t
Create Project, 75, 116–117
Creative problem solving (CPS), 71–74, 72f
Creative process, 53–56
  phases of, 54
  tools, 55t, *see also* Ideation
Creativity, 124, 126–128, 130
Creep, 639
Critical speed, of shaft deflection, 265–282, 266f, 267f
  Castigliano's theorem for, 279–282, 279f, 281f
  Macaulay's method for, 268–278, 269f, 271f, 273f, 275f, 275t, 277f

Critical to process (CTP), 18t
Critical to quality (CTQ), 18t
Crossed axis helical gears, 323f
Crowning factor, 408
  bevel gears, 414
CTP. *See* Critical to process (CTP)
CTQ. *See* Critical to quality (CTQ)
Cushion in advance principle, 95
Cyanoacrylates
  applications of, 709t
  characteristics of, 709t
Cylinder cushioning, 756f
Cylindrical butt joint, 678f
Cylindrical flange joint, 678f
Cylindrical lap joint, 678f
Cylindrical roller bearing, 202f
  single row, 210f

D

Dedendum, 327, 445
Deep groove ball bearing, 140f
  single row, 210t–223t
"Deferment of judgment" principle, 59–60
Deflection
  of bolted joints, 688–689, 689f
  of snap fasteners, 716, 716f
Deployment, quality function, 25–26, 36–47, 37t, 38f, 39f, 40f, 41f, 42f, 45f, 46f
Design, 1–24
  brief, 27, 28f
  conceptual, 15
  defined, 1–4
  detail, 15
  double diamond, 17, 17f
  embodiment, 15
  engineering, 3–4
  intent, 27
  inventor's approach to, 4f, 5
  optimization, 18–19
  process, 4–10, 4f, 5t, 6f, 7f
    case study, 8–10
  stage-gate process, 19–20, 20t
  systematic, 15–16, 16f
  technology base, 20–22
  total. *See* total design
Design, measure, analyze, design, verify (DMADV), 17
Design for assembly (DFA), 12
Design for manufacture (DFM), 12
Design for six sigma (DFSS), 17, 18t
Design of Experiments (DoE), 18t
Detailed design
  of systematic design, 15
  of total design, 12
Deutsche Industrie Normen (DIN), 627–628, 751–752
Deviation, 764t, 771–773
DFA. *See* Design for assembly (DFA)
DFM. *See* Design for manufacture (DFM)
DFSS. *See* Design for six sigma (DFSS)
Diametral pitch, 328
Diaphragm springs, 531, 534f
Dimension chain, 779–782
Dimensions, 762–764, 769–771
DIN. *See* Deutsche Industrie Normen (DIN)
Disc brakes, 535–540
  automotive, 537f
  caliper, 537f
  circular pad, 538, 538t
Disc clutch(es)
  design of, 521–531
  multiple, 521t
  single, 521t
Discard and renewal principle, 103–104
Disk drives, 319f
DMADV. *See also* Design, measure, analyze, design, verify (DMADV)
DoE. *See* Design of Experiments (DoE)
Double acting hydraulic cylinder, 755, 755f
Double diamond, 17, 17f
  design process, 54, 54f
Double helical gears, 322f, 324–325
Double lap joint, 678f, 701f, 710f

Double strap joints, 710f
tapered, 710f
Drawbar springs, characteristics of, 626t
Drives
belt. *See* Belt drives
chain. *See* Chain drives
cone, 319f
disk, 319f
roller, 319f
Drum brakes, 540
long-shoe external, 543–554, 543f, 544f, 547f, 549f, 550f, 551f
long-shoe internal, 554–557, 555f, 556f
short-shoe external, 540–543, 541f, 542f
Dunkerley equation, 267
Duplex chain, 496f
Duty cycle factor, 285
Dynamic factor
bevel gears, 411, 412f
Dynamic seals, 566, 577–580
for rotating machinery, 577–580, 579f, 580f
Dynamics, 96–97
group, 67–68

E

Eberle, Bob
SCAMPER, 12, 56, 74–75
Eccentricity, 168
Eccentricity ratio, 168
Effective radius, 537–538
Elastic coefficient, 368–370, 369t
Elastomeric seal rings (O rings), 567–574, 570f, 571t, 572f, 573t
geometry of, 570f
operating principle of, 570f
Embodiment design, 15
Enclosures, 134–135
End treatments, 629–631, 634f
Endurance limit, 284
Endurance stress, 282
Engine-driven compressor, transmission for, 123–124, 123f
Engineering, 56
concurrent, 12
design, 3–4
tolerancing. *See* Engineering tolerancing
value, 56
Engineering tolerancing, 761–796, 767f
bands of, 763t
component tolerances, 762–774
geometric tolerancing, 771–774, 772f, 773f, 774f, 775t, 776f
holes, standard fits for, 762–767, 765t
interference fits, 767–769, 768f
machine capability, 769–771, 770f, 771t, 772f
shafts, standard fits for, 762–767, 766t
statistical tolerancing, 774–794
independent, uncorrelated random variables, 782–792
linear functions, 779–782
statistical design techniques, quality assurance for, 792–794
sure-fit or extreme variability, 778–779
tolerance chains, 779–782, 780f
Epicyclic gear trains, 325, 325f, 341–344, 342f, 360f
Epoxies
applications of, 709t
characteristics of, 709t
Equipotentiality, 95
Equivalent load, 206–207
Euler number, 159
Evaluation matrices, 73, 75, 77t
Evaluation
brainstorming, 69–71, 70t, 71t
of design, 5t, 10
Extraction, 93
Extreme variability, 778–779, 782
Eytlewein's formula, 493

F

Face joint, 678f
Face seals, 579–580, 580f
Face width, 327f, 348, 352
Facilitation
brainstorming, 68–69
Factor of safety, 283–284

Failure
of bevel gear teeth, 408
of chain drives, 500
of helical compression spring, 634
of wire rope, 725
Failure mode and effects analysis (FMEA), 18t
Fasten(ing/ers), 133, 677–720
adhesives, 708–714
design considerations for, 677–678
nonpermanent, 677–678
permanent, 677–678
power screws, 693–699
rivets, 699–708, 700f, 701f, 703t, 704t, 705t, 706t
selection of, 684–685
snap, 714–716, 716f
threaded, 679–693
welding, 714, 715f
Fatigue, 204
bending, 725
Fretting, 725
of helical compression spring, 634, 637
limit, 282
strength parameter, 729
stress concentration factor, 285–287, 286f, 287f, 287t
surface, 725
tensile, 725
of wire rope, 725, 726f, 727f
Feedback, 100
Fillet radius, 327f
Film pressure, 150f, 152, 174f, 175f, 178
Film termination angle, 168, 176f
Film thickness, 150f, 152, 168, 169f, 173f, 178–179, 191
Filter(s)
hydraulics, 750
pneumatics, 750
positions, 750f
Finite element analysis, 367–368
Fin height, 585, 590t
Fin-tip rounding. *See* Corner rounding
Fishbone diagram, 56
Fixed bearing, 245
Flange joint, 678f, 689–690, 689f
combination lap and, 678f
cylindrical, 678f
Flat belts, 462–463
geometry definition of, 492f
maximum permissible stress, guideline values for, 494t
selection of, 492–495
Flexible coupling, 264f
Flexible membranes, 102
Floating bearing, 245–246
Flip chart technique, 60–62, 68
Fluid couplings, 520
Fluid flow measurement
hydraulics, 739–742
pneumatics, 739–742
FMEA. *See* Failure mode and effects analysis (FMEA)
Foodstuffs containers, seals for, 575–577, 578f
Force analysis, 345–348, 345f, 359f
bending stresses, 346–348, 347t
of bevel gears, 405–407
gear stresses, 346
for worm gears, 446–449, 447f, 447t, 449t
Fourier's equation, 532–533
Free-body diagram, 634, 636f
Free length, 635f
Frequency equation, 525
Friction clutches, 515–517
Friction coefficients, 526t
Froude number, 159
Full-film hydrodynamic bearings, 143–144, 177f, 179f, 182f
design of, 150–196, 150f, 151f, 152f, 157f, 158f, 161f
alternative method for, 189–196, 190t, 192f
charts, 168–189, 169f, 170f, 171f, 172f, 173f, 174f, 175f, 176f
Reynolds equation derivation, 153–167
performance of, 144, 144f
surface roughness for, 143f

G
Gallery method, 56
Garter springs, characteristics of, 626t
Gas welding, 714
  metal inert, 714
  Tungsten inert, 714
Gaskets, 574–575, 575f, 576t, 577f
Gauge(s), 738
  Bourdon, 738
  pressure, 737
  thin film, 740
Gaussian distribution. *See* Normal distribution
Gear(s), 128–130, 128f, 317–366, 403–438
  bevel, 322f, 325, 403–438
  classification of, 321–324
  crossed axis helical, 323f
  double helical, 322f
  drives
    comparison with belt drives, 497t
    comparison with chain drives, 497t
  force analysis, 345–348, 345f, 359f
    bending stresses, 346–348, 347t
    gear stresses, 346
  helical, 321f, 324–325
  material matches for, 324, 324t
  primitive teeth, 320, 320f
  ratios, range of, 326t
  selection procedure for, 348–349, 397–399, 398f
  shaft, arrangement proposal for, 305f
  spur, 321f, 324, 327f, 348
  stressing
    helical, 367–402
    spur, 367–402
  sun, 325, 341
  tooth profiles, construction of, 329–331, 329f, 330f, 332f, 344t
  tooth systems, 344–345, 344t
  trains, 325, 331–344, 333f, 334f, 335f, 336f
    epicyclic gear trains, 341–344, 342f, 360f
    manually shifted automotive transmissions, 337–341, 338f, 339f
    nonreverted compound, 333f
    reverted compound, 333f
    simple, 333f
  worm, 323f, 325, 439–458
Geometric stress concentration factor, 285–287
Geometric tolerancing, 771–774, 772f, 773f, 774f, 776f
  symbols, 775t
Geometry factor
  bevel gears, 415–416, 416f, 417f, 418f, 419f, 420f, 421f
Grid brainstorming, 55t, 66–68, 66f, 116–117. *See also* Brainstorming
Grease, 145
Grinding, of bevel gear teeth, 408
Group dynamics, 67–68
Grub screw, 261t

H
Hard-finish cutting, of bevel gear teeth, 408
Hardness ratio factor
  bevel gears, 417–419, 422f
Helical compression springs, 629–648, 633f, 634f, 635f, 636f, 783f
  barrel, 633f
  characteristics of, 626t
  conical, 633f
  constant pitch, 633f
  creep under load, 639
  dimensional parameters for, 635f
  free-body diagram for, 634, 636f
  hourglass, 633f
  prestressing of, 639
  styles of end treatments for, 634f
  in use, lengths associated with, 635f
  variable pitch, 633f
Helical extension springs, 648–659, 649f, 650f
  characteristics of, 626t
Helical gears, 321f, 324–325
  double, 322f
  stressing, 367–402
    wear failure, 368–370

Helical torsion springs, 659–661, 660f
Hertzian stress, 368, 725, 732
Hits, 73
Hodkinson's carryover factor, 597
Holes, standard fits for, 762–767, 765t
Homeogeneity, 103
Honeycomb cell, 601, 602f
Hot film air mass flow sensor, 742f
Hot film anemometry, 742
Hot melts
  applications of, 709t
  characteristics of, 709t
Hotspots, 73
Hub, 260–263, 261f
Hunting tooth, 326
Hydraulics, 134, 735–760, 736f
  actuators, 754–756, 754f, 755f
  air compressors, 747–750
  construction of, 102
  control valves, 750–754, 751f, 752f, 753f
  filters, 750, 750f
  fluid flow measurement, 739–742
  pressure measurement, 738–739
  pumps, 745–746, 745f, 746f, 747f
  receivers, 747–750
  temperature measurement, 742–745
Hydrodynamic bearings, 150–196
Hydrostatic pressure, 737

I

IC engine, torque speed characteristics of, 317, 318f
Idea engine, 116f
Idea generation, 52, 56, 59, 62, 67–69, 75, 77t
Idea-spurring questions, 74–75
Ideation, 51–120
  brainstorming. *See* Brainstorming
  Create Process, 75, 77t
  creative problem solving, 71–74, 72f
  creative process, 53–56, 55t
  and innovation engine, 114–117, 116f
  morphological analysis, 55t, 56, 75, 77–85, 78t, 80t, 82t, 83t, 84t
  SCAMPER, 74–75
  standard solutions to, 85f, 86–113, 86f
    contradiction matrix, 106–107
    contradiction of, 88–91
    principles, 91–106, 92t
    tradeoff, 88–91
Identify, design, optimize, verify (IDOV), 17
IDEO, 52, 58, 117
Idler, 334–336
IDOV. *See also* Identify, design, optimize, verify (IDOV)
Independent, uncorrelated random variables, 782–792
Inert environment, 105–106
Inexpensive short-life, 101
Influence coefficients, 267–268
Initial clamping load, 692
Innovation engine, creativity and, 114–117, 116f
Installed force, 640
Installed length, 632–634, 635f, 640
Intellectual property rights (IPRs)
  and brainstorming, 58–59
Interference fits, 767–769, 768f
International Standards Organization (ISO), 9, 346, 367–368, 751–752
  threads, 679–680, 681f, 681t, 690
Interstitial seal, 580
Inversion, 95–96
Inverted tooth chain. *See* Silent chain
Involute, 320, 328–329, 329f, 331
IPRs. *See* Intellectual property rights (IPRs)
ISO. *See* International Standards Organization (ISO)

J
Joint(s)
adhesives, 708–714
bolted, 685–686, 685f, 686f, 687f
butt, 678f, 700, 701f
face, 678f
flange, 678f, 689–690, 689f
lap, 678f, 700, 701f, 702, 710f
for sheet materials, 711f
strap, 710f
Journal bearings, 139–200, 140f
boundary-lubricated bearings, 143–144, 143f, 144f, 146–149
full-film hydrodynamic. *See* Full-film hydrodynamic bearings
mixed-film lubricated bearings, 143–144, 143f, 144f
partial surface, 151f
sliding bearings, 142–146, 143f, 144f, 145f

K
Kanban principle, 95
Kelley, Tom, 58
Keys, 256, 260, 261t
Keyseat, 261–263, 262f
Keyway, 261–263, 285–287, 287t
Knife-to-knife (KTK) model, 593
KTK. *See* Knife-to-knife (KTK) model

L
Labyrinth seals, 581–612, 581f
with abradable rub-in surface, 587–588, 588f
applications of, 584–585, 584f
characteristic flow through, 581–582, 581f
forms of, 582
geometry of, 582f, 586t
Martin's equation, 590–595
open-celled honeycomb mounted with, 588, 588f
orifice flow approach, 595–596
principal fluid dynamic parameters for, 585t
radial, 612, 615t
selection of, 582, 583f
specification guidelines for, 590t
staggered, 611–612, 611f
stepped, 589, 608–611, 608f, 609f, 609t
straight-through, 587–590
Zimmermann and Wolff's model, 596–608
Lap joint, 678f, 700
combination flange and, 678f
cylindrical, 678f
double, 678f, 701f, 710f
performance of, 710f
simple, 710f
single, 701f, 702
stepped, 710f
tapered, 710f
Laser Doppler anemometry, 740
Lateral frequency of vibration, 265
Lateral thinking, 53, 55t, 56
Lead, 442f, 443–445, 444f, 453
Lead angle, 442f, 443–445, 444f, 453
Lead screws, 693, 695f
Leading shoe, 554, 555f
Leaf chain, 499, 499f
Leaf springs, 661–663, 661f, 662f, 663f
characteristics of, 626t
types of, 661f
Leakage, 566–567, 579–580, 584, 587–589, 593, 596, 601, 611–616
Left handed thread, 679, 679f
Lengthwise curvature factor
bevel gears, 414
Leverage, 124
Lewis equation, 367
Lewis form factor, 346, 347t, 348
Lewis formula, for bending stress, 346–348, 347t
Life, 383, 405, 416, 425
Life equation, 203–248
Life factor, 383, 384f, 421
Limiting PV value, 146–148, 147t
Line of action, 328
Linear functions, 779–782
Linkages, 124, 125f

Lining material, 515–516, 520–521, 526, 528, 543
Lip seals, 577–578, 579f
Load(s/ing)
  of adhesive joints, 710f
  of bolted joints, 688–689, 689f
  equivalent, 206–207
  of helical compression spring, 634
  shaft, 257f, 291f
    fluctuating, 259f
    horizontal, 293f, 300f
    principal, 277f, 296f
    radial, 292
    reversed axial, 285f
    stepped shaft with single load, 279f, 281f
    tangential, 292
    two concentrated, 267f
    vertical, 292f, 300f
  transmitted, 345
Load capacity, 141, 144, 146–148, 147t, 167, 177
Load distribution factor
  bevel gears, 413–414, 414f
Local quality, 93
Locking ring, 245
Long-shoe external drum brakes, 543–554, 543f, 544f, 547f, 549f, 550f, 551f
Long-shoe internal drum brakes, 554–557, 555f, 556f
Long-term memory, 59
Lower deviation, 764t, 767f
Lubricants, for sliding bearings, 145–146, 145f
Lubrication
  methods, for chain drives, 505f
  recirculation, 246–247
  rolling element bearings, 246–247

M

Macaulay's method, for beam reflection, 268–278, 269f, 271f, 273f, 275f, 275t, 277f
Mach number, 595–596
Machine capability, 769–771, 770f, 771t, 772f
Machine elements, 21t
Magnetic clutch, 520
  applications of, 521t
  characteristics of, 521t
Manometers, 738
Manual metal arc welding (MMA), 714
Manually shifted automotive transmissions, 337–341, 338f, 339f
Manufacturing phase, of total design, 12
Mapping
  external, 75, 77t
  internal, 75, 77t
Market
  brief, 8, 25
  phase, of total design, 10, 13
Martin's equation, 590–595
Maximum allowable tension, 493
Maximum film pressure ratio, 174f, 178
Maximum permissible pressure, 526, 528, 540–541
Maximum permissible stress, 494t
McLeod gauges, 738
MDO. *See* Multidisciplinary design optimization (MDO)
Mean, 774, 776–778, 782–783
Mechanical engineering, *xiii*
Mechanical face seals, 579–580, 580f
Mechanical substitution, 101–102
Mechanical vibration, 98
Mediator, 100
Memory
  defined, 56
  long-term, 59
  short-term, 56, 59, *see also* Brainstorming; Creative process; Ideation
Metal inert gas welding (MIG), 714
METUS methodology, 19
MIG. *See* Metal inert gas welding (MIG)
Mind mapping, 55t
Mineral oils, 145–146

Minimum film thickness, 150f, 152, 168, 169f, 173f, 178–179
Misalignment, 256f
Mixed-film lubricated bearings, 143–144
  performance of, 144, 144f
  surface roughness for, 143f
MMA. *See* Manual metal arc welding (MMA)
Module, 327
Morphological analysis, 12, 55t, 56, 75, 77–85
  applications of, 82–85
  generic chart, 78t
  for image-recording device, 83t
  for pallet-moving device, 79–80, 80t, 82t
  for vehicle design, 84t
Multidisciplinary design optimization (MDO), 4, 19
Multigrade oil, 145–146
Multi-leaf spring, 662
Multiple disc clutch
  applications of, 521t
  characteristics of, 521t
Multiple serration clutch, 518f

N

NASA heavy lift winch, 721, 722f
Natural frequency, 258, 265–267
Natural tolerance limits, 776–778
Navier–Stokes equation, 152–155, 159, 593
Nesting, 94
Newton's second law of motion, 122, 737
Nominal dimensions, 761–762, 769–771
Nodal analysis, 266
Nonparallel co-planar gears, 321, 321f
Nonparallel noncoplanar gears, 323–324, 323f
Nonpermanent fastening, 677–678
Nonreverted compound gear trains, 333f
Normal distribution, 774–778, 776f, 777t
No-slip condition, 160
Notch sensitivity, 285f
Number of active coils, 637–638, 641, 645, 661
Number of equivalent coils, 661

O

O rings. *See* Elastomeric seal rings
Observation, 52–53
Oil, 145–146
Oil ring, 578
Oil spot lubrication, 246–248
Operating force, 640–641, 645
Operating length, 632–634, 635f, 640, 645
Optimization, 5t, 9–10
  cost, 9–10
  design, 18–19
  multidisciplinary design, 4, 19
  statistical, 18t
Orifice flow approach, 595–596
Orifice plates, 740–741
Orthogonal distance, 544–545
Overload factor
  bevel gears, 410–411, 411t
Over-running clutches, 518, 519f

P

Packing seals, 616–618, 617f
Pads, 535–538, 537f, 560
Paired comparison analysis (PCA), 73
Parallel axis gears, 321, 321f, 322f
Parameter design, 793t
Partial or excessive action, 97
Partial surface journal bearings, 151f
PCA. *See* Paired comparison analysis (PCA)
PDS. *See* Product design specification (PDS)
Periodic action, 98
Permanent fastening, 677–678
Permissible bending stress, 410
Permissible contact stress
  bevel gears, 409, 429, 434, *see also* contact stress
Phase change, 104–105

Phenolics
applications of, 709t
characteristics of, 709t
Physical or chemical state of an object, transforming, 104
Piezoelectric pressure transducers, 738
Pin, 260–263, 261f, 261t, 262f
Pinion gears, 326–328
material matches for, 324, 324t
Pinion shaft
arrangement proposal for, 305f
general arrangement for, 309f
Piston rings, 618–619, 618f, 619t
sections of, 618f
Pitch
circle, 326–327
cone angle, 406–407
diametral, 328
helical compression spring, 633f
line velocity, 345, 411–412
thread, 680t
worm gear, 445
Pitot tubes, 740, 740f
Pitting, 367
Plain bearings, 141f, 142, 142t
Plain surface bearings, 139, 142
Planetary gear trains. *See* Epicyclic gear trains
Plastic gears, 325
Plastisols
applications of, 709t
characteristics of, 709t
Pneumatics, 134, 735–760, 736f
actuators, 754–756
air compressors, 747–750, 748f, 749f
construction of, 102
control valves, 750–754, 751f, 752f, 753f
filters, 750, 750f
fluid flow measurement, 739–742
pressure measurement, 738–739
receivers, 747–750
temperature measurement, 742–745
Polyurethanes
applications of, 709t
characteristics of, 709t
Polyvinyl acetates
applications of, 709t
characteristics of, 709t
Porous materials, 102–103
Positional tolerance, 774
Positive contact clutches, 516–517
Post-it brainstorming, 55t, 58, 62–64, 63f, 68, 116–117. *See also* Brainstorming
Power screws, 693–699
Predisposition, 75, 77t
Preloading
threaded fasteners, 690–691
Presetting, of springs. *See also* Prestressing, of springs
Press fit, 261–263, 261t
Pressure
defined, 737–745
gauge, 737
hydrostatic, 737
measurement
hydraulics, 738–739
pneumatics, 738–739
relief valve, 746f
Pressure angle, 328, 328f, 330–331, 344–345, 348
Pressure line, 328, 328f, 330–331, 332f
Prestressing, of springs, 639
Primitive teeth gears, 320, 320f
Prior action principle, 95
Prior counteraction principle, 94–95
Probability density function, 775
Problem definition, 5t, 8–9
Problem exploration, 56
Product design specification (PDS), 10–11, 25–30, 27t, 28f, 29t
for compressor transmission, 33–34
documentation format for, 27t, 29t
evaluation criteria for, 36
information content of, 28f

for snow sports activity vehicle, 30t–31t, *see also* Specification
Product planning, 15
PV factor, 146–148

Q

QFD. *See* Quality function deployment (QFD)
Quadring. *See* X ring
Quality control number, 373–374
Quality function deployment (QFD), 18t, 25–26, 36–47, 56
benefis of, 47
defined, 37
disadvantages of, 47
phases of, 36f, 37
characteristic activities associated with, 37t
matrices, 38f, 39–47, 39f, 40f, 41f, 45f, 46f
timeline of, 38f
versus traditional industrial approach, 42f
"Quantity breeds quality" principle, 59–60
Quasiobjective technique, 69
Question asking, 56

R

Rabinow, Jacob, 54–55
Radial bush seal, 612–616, 615f
Radial labyrinths, 612, 615t
Radial lip seal, 578, 579f
Radial location, of rolling element bearings, 248, 248t, 249t–249t, 250t–250t
Raimondi and Boyd charts, 168
Random stimulation, 53
Random variables, 782–792
Rated life, 204
Ratio of side flow to total flow, 168, 172f, 192
Rayleigh–Ritz equation, 266–267, 271–272, 276, 307
Reason and Narang, 189
Receivers, 189–196
hydraulics, 747–750, 748f, 749f
pneumatics, 747–750, 748f, 749f
Reciprocating components, seals for, 616–619
Reciprocity, 268
Recirculation lubrication system, 246–247
Recognition of need, 5t, 8
Rectangular seal, 570, 574f
Reliability factor, 284, 285t
bevel gears, 420, 423t
Resistance welding, 714
Reverted compound gear trains, 333f
Reynolds equation, 152–167
Reynolds number, 155–156, 598–599, 605–608, 610
Rigid coupling, 264f
Rivets, 699–708, 700f, 701f
American National Standard for, 705t, 706t
applications of, 700
blind, 700
button head, 705t
British Standard for, 703t, 704t
Cold forged, 704t
cone head, 705t
countersunk and raised countersunk head, 706t
double riveted butt-head, 701f
double riveted lap-joint, 701f
efficiency of, 702
flat countersunk head, 706t
flat head, 704t
high button, 705t
hot forged, 703t
oval countersunk head, 706t
pan head, 705t
single riveted lap-joint, 701f
snap head, 703t
tubular, 700
universal head, 703t
Roller chain, 496f
components of, 498f
lubrication methods of, 505f
selection of, 501–509, 502t, 503f, 506t
Roller clutch, 518–520, 519f
applications of, 521t
characteristics of, 521t

Roller drives, 319f
Rolling element bearings, 201–253
  installation, 245–246
  life and selection of, 203
    modified life equation, 207f–209f
  merits of, 203t
  radial location, 248, 248t, 249t–249t, 250t–250t
  types of, 202f
Rolls-Royce Trent series of engines, 206, 206f, 207f–207f
Rotating machinery, seals for, 577–580, 579f, 580f
Rubber adhesives
  applications of, 709t
  characteristics of, 709t
Rubbing materials, characteristics of, 147t
Rule-breaking, and brainstorming, 59
Runout, 686, 691t
Rushing through principle, 99

S

SAE, 146
Safety factor
  bevel gears, 411
Sales phase, of total design, 13
SAW. *See* Submerged arc welding (SAW)
SCAMPER, 12, 56, 74–75
Screws
  power, 693–699
  styles of, 684f
Screw jack, 693–694
Seals, 132, 132f, 565–624
  aperture, 570–574, 574f
  axial, 612–616, 615f
  bush, 612–616, 615f
  classification of, 568f
  dynamic, 566, 577–580
    for rotating machinery, 577–580, 579f, 580f
  geometry of, 566f
  interstitial, 580
  labyrinth. *See* Labyrinth seals
  packing, 616–618, 617f
  radial lip, 578, 579f
  for reciprocating components, 616–619
  rectangular, 570, 574f
  selection procedure for, 566–567, 569f
  static, 566–577
    elastomeric seal rings (O rings), 567–574, 570f, 571t, 572f, 573t
    for foodstuffs containers, 575–577, 578f
    gaskets, 574–575, 575f, 576t, 577f
  X ring, 570, 574f
Second moment of area, 270
Segmentation, 91–93
Self de-energizing, 542, 545–547, 557
Self-aligning ball bearing, 202f
Self energizing, 535, 540, 542, 544–547, 554, 557–558, 559f
Self-locking
  in band brakes, 559
  in long-shoe external drum brakes, 545
  in power screws, 694, 698
  in short-shoe external drum brakes, 542
  in worm gears, 441, 444–445, 454–455
Self-service, 100
Sequencing, 73
Series springs, 629, 632f, 667f
Series wound direct current motor, torque speed characteristics of, 318f
Set screw, 261f
Service factors, for clutches, 522t
Shaft(s), 255–316
  arrangements, 256f, 257f
  ASME design code for transmission shafting, 282–301
  case study, 302–311, 302f, 303f, 304t, 305f, 306f, 309f, 310f
  deflection, 257f
  deflection, critical speed of, 265–282, 266f, 267f
    Castigliano's theorem for, 279–282, 279f, 281f
    Macaulay's method for, 268–278, 269f, 271f, 273f, 275f, 275t, 277f
  design, 255–260

with fluctuating loading, design procedure flow chart for, 259f
loading, 257f, 291f
fluctuating, 259f
horizontal, 293f, 300f
principal, 277f, 296f
radial, 292
reversed axial, 285f
stepped shaft with single load, 279f, 281f
tangential, 292
two concentrated, 267f
vertical, 292f, 300f
standard fits for, 762–767, 766t
strength and rigidity, design procedure flow chart for, 258f
Shaft-hub connection, 260–263, 261f, 262f
merits of, 261t
Shaft–shaft connection couplings, 263, 264f
Shear stress
adhesive joints, 712, 713f
of compression coil springs, 640t
Sheet(s)
materials, joints for, 711f
stiffening of, 712f
Short-shoe external drum brakes, 540–543, 541f, 542f
Short-shoe internal drum brakes, 540
Short-term memory, 56
Shoulder, 256, 257f, 261–263
Shrink fit, 260, 261t, 763t
Shut height, 632–634, 635f
Silent chain, 499–500, 500f
Silicones
applications of, 709t
characteristics of, 709t
Simple chain, 496f, 502
Sine load integral, 189, 190t
Single acting hydraulic cylinder, 754f
Single disc clutch
applications of, 521t
characteristics of, 521t
Single face clutch, 529f
Six hats, 55t
Six sigma, design for. *See* Design for six sigma (DFSS)
Size factor
bevel gears, 412–413, 413f
Skiving, 408
Sled runner keyseat, 262f
Sliding bearings, 142–146, 143f, 144f
defined, 142
lubricants, 145–146, 145f
lubrication for, 142–144
performance of, 144, 144f
surface roughness for, 143f
Snap fasteners, 714–716, 716f
Snap rings, 263f
Society for Automotive Engineers, 146
Solenoid, 753–754
Sommerfield number, 168, 169f, 170f, 171f, 172f, 173f, 174f, 175f, 176f, 178, 189–191, 190t, 193
Specification, 25–50
phase, of total design, 10–11
product design, 25–30, 27t, 28f, 29t
quality function deployment, 25–26, 36–47, 37t, 38f, 39f, 40f, 41f, 42f, 45f, 46f
Speed ratio, 331–333, 459
Spherical roller bearing, 202f
Spheroidality, 96
Spiral bevel gears, 404–405, 404f
force analysis of, 407
Splines, 263f
Sprag clutch, 518–520, 519f
applications of, 521t
characteristics of, 521t
Spring wound clutch, 518–520, 520f
Springs, 133, 625–676
Angular deflection, 659–660
Belleville, 531, 626t, 664–671, 664f, 665f, 666f, 667f, 668t
characteristics of, 626t
classification of, 626t
constant, 629
creep under load, 639
deflection, 625, 628, 636, 638, 649, 651, 661–667, 665f
diaphragm, 531, 534f
drawbar, 626t
Garter, 626t

Springs (*Continued*)
helical compression, 626t, 629–648, 633f, 634f, 635f, 636f, 783f
helical extension, 626t, 648–659, 649f, 650f
helical torsion, 659–661, 660f
index, 636–637, 642, 649, 653
leaf, 626t, 661–663, 661f, 662f, 663f
materials, properties of, 629t
of minimum volume, materials for, 627f
of minimum weight, materials for, 628f
multi-leaf, 662, 663f
in parallel, 629, 631f
prestressing of, 639
rate, 628–629, 638–641, 649, 660, 663–665
scale, 629
in series, 629, 632f
surge, 638–639
torsion, 626t
volute, 626t
wire diameters, 630t–631t
Sprockets, 459–461, 460f, 495–502, 496f, 497f, 504–505, 506t
Spur gears, 321f, 324, 327f, 348
stressing, 367–402
bending stress, AGMA equations for, 370–397, 374t, 376f, 376t, 377f, 378t, 379t, 380t, 381t, 382t, 384f, 385t
contact stress, AGMA equations for, 370–397, 374t, 376f, 376t, 377f, 378t, 379t, 380t, 381t, 382t, 384f, 385t
wear failure, 368–370
Square jaw clutch, 518f
Square thread, 694f, 698
Squared ends, 634f, 637–638
Squeeze number, 156
Stage-gate process, 19–20, 20t
Staggered labyrinths, 611–612, 611f
Standard solutions to ideation process, 85f, 86–113, 86f
contradiction matrix, 106–107
contradiction of, 88–91
principles, 91–106, 92t
tradeoff, 88–91
Static seals, 566–577
elastomeric seal rings (O rings), 567–574, 570f, 571t, 572f, 573t
for foodstuffs containers, 575–577, 578f
gaskets, 574–575, 575f, 576t, 577f
Statistical design techniques, quality assurance for, 792–794
Statistical optimization, 18t
Statistical tolerancing, 774–794
independent, uncorrelated random variables, 782–792
linear functions, 779–782
statistical design techniques, quality assurance for, 792–794
sure-fit or extreme variability, 778–779
tolerance chains, 779–782, 780f
Steel gears
allowable bending stress for, 421–424, 423t
allowable contact stress for, 424, 424t
Steel shafts
failures, avoiding, 287–288
fatigue stress concentration factors for, 285–287, 286f, 287f, 287t
physical properties of, 289t
strength cycle diagram for, 283f
ultimate tensile strength of, 289t
yield strength of, 289t
Stepped labyrinth seals, 589, 608–611, 608f, 609f, 609t
“Sticky plaster” approach, 42–44
Stiffness, of bolted joints, 688
Stock gears, 130, 348, 350
Straddle mounted pinion, 375

Straight bevel gears, 403, 404f
Straight-through labyrinth seal, 587–590, 600f
Strain energy, 279
Strap joints, 710f
  double, 710f
  tapered double, 710f
Stress, 368, 725
  analysis, for bevel gears, 407–424, 411t, 412f, 413f, 414f, 416f, 417f, 418f, 419f, 420f, 421f, 422f, 423t, 424t
  bending, 346–348, 347t, 370–397, 374t, 376f, 376t, 377f, 378t, 379t, 380t, 381t, 382t, 384f, 385t
  contact, 368, 370–397, 374t, 376f, 376t, 377f, 378t, 379t, 380t, 381t, 382t, 384f, 385t
  endurance, 282
  gear, 346
  Hertzian, 368, 725, 732
Stress concentration factor, 284–287, 286f, 287f
Stress cycle factor
  bevel gears, 416–417, 422f
Strong oxidants, 105
Submerged arc welding (SAW), 714
Sun gear, 325, 341
Sure-fit or extreme variability, 778–779
Surface condition factor, 372, 376
Surface finish factor, 284, 284t
Surface strength geometry factor, 376
Surface roughness, 143–144, 143f, 151–152, 179, 761, 769–771, 771t
SWOT analysis, 56, 75
Sychromesh, 337–341, 339f
Synchronous belts, 463
  Fenner torque drive plus, 476t
  length correction factors for, 476t
  pitch selection guide, 491f
  power ratings for, 475t
  selection of, 473
  service factors for, 476t
  width factors for, 476t
Synectics, 53, 55t
Synthesis, 5t, 9
Synthetic oils, 145
System design, 793t
Systematic design, 15–16, 16f

T

Tabular evaluation matrix, 71
Taguchi methods, 56
Taper roller bearing, 202f
Tapered double strap joints, 710f
Task clarification, 15
Taylor number, 157–158, 180, 184, 186–188, 193, 195
Technology base, 20–22
Temperature factor, 285
  bevel gears, 419
Temperature measurement
  hydraulics, 742–745
  pneumatics, 742–745
  techniques, temperature ranges for, 744, 744f
Temperature sensors, 743t
Tensile strength, of spring wire, 640f
Tensile stress
  adhesive joints, 712, 713f
Thermal expansion, 105
Thin films, 102
  auges, 740
Thread(s)
  ACME, 694f, 695t, 697–698
  buttress, 694f
  pitch, 680t
  square, 694f, 698
Threaded fasteners, 679–693
  left-hand, 679f
  right-hand, 679f
  terminology, 680f, 680t
Three-phase induction alternating current motor, torque speed characteristics of, 318f
Thrust ball bearing, 202f
TIG. *See* Tungsten inert gas welding (TIG)
Timing belts. *See* Synchronous belts
Tolerance, 761–762, 769–794
Tolerance chains, 779–782, 780f
Tolerance design, 793t

Tolerance limits, 762–774, 776
Tolerance zone, 771–773
Tooth systems, 344–345, 344t
Torque speed characteristics
  of IC engine, 317, 318f, 337
  of series wound direct current motor, 318f
  of three-phase induction alternating current motor, 318f
  transformation of, using reduction gear ratio, 318, 318f
Torque, defined, 515
Torque transmission
  keys for, 262f
  pins for, 262f
Torsion springs, characteristics of, 626t
Torsional deflection, 256
Torsional stress
  helical extension springs, 650
Total design, 10–15, 11f, 13f, 14f
  conceptual design, 12
  detailed design, 12
  information flows and activities, 13–15
  manufacturing, 12
  market, 10
  marketing/sales, 13
  specification, 10–11
Trailing shoe, 554, 555f
Trains, gear, 325, 331–344, 333f, 334f, 335f, 336f
  epicyclic gear trains, 341–344, 342f, 360f
  manually shifted automotive transmissions, 337–341, 338f, 339f
  nonreverted compound, 333f
  reverted compound, 333f
  simple, 333f
Train value, 341
Transition to a new dimension principle, 97–98
Transmission(s)
  conceptual arrangements for, 131f
  drive shaft, 291f
  for engine-driven compressor, 123–124, 123f
  manually shifted automotive, 337–341, 338f, 339f
  shafting
    ASME design code for, 282–301
    conceptual arrangements for, 303f
    general arrangement for, 310f
Transmitted load, 304, 345–346, 348, 373–374, 391, 397, 405–406, 411
Tribology, 124–125
Triplex chain, 496f
Triplex sprocket, 497f
TRIZ (*Teoria Resheneya Isobretatelskih Zadach*), 53, 55t, 56, 75, 87–88, 87f, 106
  contradiction matrix, 107
  parameters of, 89t–91t
  principles, 91–106, 92t
Tungsten inert gas welding (TIG), 714
Turbocharger journal, 151f

U

Ultimate tensile strength, 151f
  of rivers, 702
  of springs, 639, 651
  of steel shafts, 284, 289t, 294, 305–306
  of wire rope, 730
UNC, 682, 682t
Uncorrelated random variables, 778, 782–792
UNF. *See* Unified fine pitch (UNF)
Unified coarse pitch thread series, 682
Unified fine pitch (UNF) thread, 682, 683t
Unified National Standard (UNS) threads, 679–680, 682, 682t, 690
Uniform wear, 525, 528, 538
Unilateral tolerance, 762
Universal joints, 265f
Universality, 94
UNS. *See* Unified National Standard (UNS)

Upper deviation, 764t, 767f
UV-curable adhesives
  applications of, 709t
  characteristics of, 709t

V
V belts, 463
  center distances for, 469t
  power increment, 471t
  power ratings for, 470t
  selection of, 464–473, 468t
  service factors for, 466t
V packings, 616–618
Value engineering, 56
Valve(s)
  control, 750–754, 751f, 752f, 753f
  three-position, 752f
Variance, 779
Velocity factor, 368–369
Velocity ratio, 331–333, 459
Venturi meters, 740–742, 741f
Viscosity, 145–146, 145f, 151, 159–160, 168–177
VOC. *See* Voice of the customer (VOC)
Voice of the customer (VOC). *See* Quality function deployment
Volute springs, characteristics of, 626t
Voting systems, 69

W
Wacky ideas, and brainstorming, 59
Wahl factor, 637, 642, 653
Washers, 683, 685f
Wear failure, 368–370
Wear rate, 525
Wedge belts. *See* V belts
Welding, 714, 715f
  types of, 714
Wire rope, 134, 721–734, 722f
  configuration of, 723t–724t
  construction of, 722f
  failure of, 725
  lay, 721, 724f
  selection of, 725–731
Wire diameters, 639–641, 644, 651
Woodruff key, 257f, 261f, 262f
Worm gears, 323f, 325, 439–458, 440f
  AGMA equations for, 449–452
  design procedure for, 453–455
  double enveloping, 442f
  force analysis for, 446–449, 447f, 447t, 449t
  gear ratio for, 441
  geometric features and dimensions for, 443, 443f
  lines of contact for, 443, 443f
  rotation and hand relations for, 440, 441f
  single enveloping, 442f
  winch configuration, 439–440, 440f

X
X ring, 570, 574f

Y
Yield strength, of shafts, 283, 289t, 305–307
Young's modulus
  bevel gears and, 420–421
  shafts and, 269–270, 272, 275, 277
  springs and, 629t, 659–660, 662
  threaded fasteners and, 686

Z
Zerol bevel gears, 405
Zimmermann and Wolff's model, 596–608
Zimmermann's carryover factor, 597, 598f